# Transverse Beam Dynamics in Plasma-based Linacs

Ralph Assmann*

*Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309*

Kaoru Yokoya

*High Energy Accelerator Research Organization (KEK), Tsukuba, Ibaraki 305, Japan*

The transverse beam dynamics in plasma channels of possible future plasma-based linacs is discussed. We represent the transverse focusing of both a beam-driven and a laser-driven plasma wakefield accelerator by a uniform focusing channel. The transverse beam sizes and a basic offset tolerance are calculated, finding that sub-micron beams must be transported with even smaller offset tolerances. The results emphasize the need to pursue further ideas for plasma structures with high acceleration gradients but reduced transverse wakefields.

## I. INTRODUCTION

Experiments on laser- and beam-driven wakefield accelerators have demonstrated impressive achievements over the last years. Accelerating gradients of up to 100 GeV/m have been measured, though the acceleration lengths generally were in the mm range. Those experiments have addressed beam acceleration from longitudinal wakefields. Another set of experiments has demonstrated the focusing effects of the transverse wakefields that are excited in plasma structures ("plasma lens"), again over lengths of several mm.

In order to produce usable beams with a high *absolute* energy, the lengths of plasma structures must increase significantly. Second generation experiments will test plasma wakefield acceleration on lengths of up to 1 m. On this length the transverse beam dynamics in long plasma chambers becomes important, as the beam performs several betatron oscillations within a single plasma module. In order to preserve the beam quality (emittance) the strong transverse fields must be handled appropriately, putting stringent requirements on beam sizes and alignment tolerances. This will be discussed in the following.

*Work supported by Department of Energy contract DE-AC03-76SF00515.

## II. TRANSVERSE BEAM DYNAMICS IN A PLASMA CHANNEL

We briefly summarize the transverse beam dynamics in a plasma-based linear accelerator. The beam dynamics is different from standard RF-based linacs because the accelerating plasma structures produce large transverse fields. As agreed during the workshop, two cases are considered for our discussion:

1. The beam-driven plasma-wakefield accelerator in the underdense case or "blowout regime" (PWFA). The plasma density ($2 \times 10^{14}$ cm$^{-3}$) is lower than the beam density and all plasma electrons are blown out by the drive beam. The accelerated beam experiences a constant focusing field. The acceleration gradient is about 1 GeV/m.
2. The laser-driven plasma-wakefield accelerator (LWFA). The plasma density is high ($10^{17}$ cm$^{-3}$) and the accelerated bunch sees a focusing field that varies along its length. The acceleration gradient is 30 GeV/m.

We limit the discussion to a homogeneous plasma channel and to a part of the beam that experiences a constant focusing field. For the beam-driven case we further assume that the accelerated bunch with radial size $\sigma_r$ is located at a point where the plasma electrons are rushing back in to the beam axis but are still outside of $\sigma_r$. The focusing force experienced by the accelerated bunch is then constant as a function of $r$ and proportional to the ion density. In this case the following treatment of the beam dynamics is fully correct. If not in the blowout regime, the transverse focusing force is a function of both the longitudinal and transverse positions for the laser-driven plasma-wakefield accelerator. For this case our treatment should be considered as a simplified estimate of the transverse focusing involved.

It is important to note that the choice of plasma density $n_0$ determines the wavelength $\lambda_p$ of the accelerating plasma wakefield [1]:

$$\lambda_p \approx 1\text{mm} \cdot \sqrt{\frac{10^{15}\text{cm}^{-3}}{n_0}}. \tag{1}$$

For the two cases we find wavelengths of 2 mm (PWFA) and 100 $\mu$m (LWFA). This determines the maximum lengths of the accelerated bunches that are compatible with those schemes. As the bunch length is preserved, it is desirable to keep $\lambda_p$ and therefore the plasma density $n_0$ constant in the linac. For the PWFA case the transverse focusing field $g$ can be written as:

$$g = 960\,\pi \cdot \left(\frac{n_0}{10^{14}\ \mathrm{cm}^{-3}}\right)\ \mathrm{T/m}\,. \tag{2}$$

The focusing field is constant in time and only a function of the plasma density $n_0$. As $n_0$ is constant, the focusing field will also be constant along the linac. The same is true for the LWFA case in the blowout regime. The general case is more complicated and must include the accelerated bunch self-fields (addressed by the plasma response) as well as the phase dependent plasma focusing field. The above formula applies to an homogeneous plasma channel.

From the focusing field $g$ in T/m and the beam energy $E$ in GeV the focusing strength $k_\beta$ is easily obtained:

$$k_\beta^2 = 0.2998\,\frac{g}{E}\,, \tag{3}$$

and the corresponding $\beta$-function is calculated from:

$$\beta = \frac{1}{k_\beta}\,. \tag{4}$$

Once the beam emittance $\epsilon$ is decided, the matched beam size $\sigma_0$ is calculated from

$$\sigma_0 = \sqrt{\beta\epsilon}\,. \tag{5}$$

It is an important observation that the choice of the plasma density $n_0$ determines the $\beta$ function and the matched beam size in the plasma-based linac. This is different from standard linacs, where the transverse focusing is independent of the choice of the accelerating structure. We will discuss the implications in the next section. If the beam is injected with a mismatched beam size at a sharp plasma boundary, emittance growth will occur such that the matching condition is fulfilled. It has been shown that adiabatic matching can occur if there is a transition section of length of order $\beta$, in which the accelerating and focusing amplitude increases [3].

We now have to include the scalings for the beam acceleration. The scalings for $k_\beta$ and the $\beta$ function follow immediately from Eq. 3:

$$k_\beta \propto \frac{1}{\sqrt{E}} \qquad \text{and} \qquad \beta \propto \sqrt{E}\,. \tag{6}$$

It is well known that the geometric emittance $\epsilon$ shrinks proportionally to the beam energy (adiabatic emittance damping):

$$\epsilon \propto \frac{1}{E}\,. \tag{7}$$

With that we obtain for the scaling of the matched beam size:

$$\sigma_0 \propto E^{-1/4}\,. \tag{8}$$

The beam size shrinks with the fourth root of the beam energy. This result has been obtained before [4]. Now we consider betatron oscillations in the plasma channel. An offset $x_0$ at the longitudinal location $s = 0$ with an initial beam energy $E_0$ produces a betatron oscillation $x(s)$:

$$x(s) = x_0 \cdot \cos(\psi(s)) \cdot \left(\frac{E_0}{E}\right)^{1/4}\,. \tag{9}$$

Note, that the phase advance of the oscillation is given by

$$\psi(s) = \int k_\beta s\, ds \propto \sqrt{E}\,. \tag{10}$$

The dispersion $\eta(s)$ that is produced from this oscillation is:

$$\eta(s) = \frac{x_0 k_\beta}{2}\left(\frac{\cos(k_\beta s)}{k_\beta} - s\sin(k_\beta s)\right)\,. \tag{11}$$

Plasma channels have large $k_\beta$'s and are therefore inherently sensitive to dispersive growth $\Delta\epsilon$ in the projected transverse emittance:

$$\frac{\Delta\epsilon}{\epsilon_0} = \left(\frac{\eta}{\sigma_0}\delta\right)^2\,, \tag{12}$$

with $\delta$ being the rms relative energy spread in the accelerated beam. The above result is only valid for small phase mixing (filamentation) of the beam particles:

$$\psi(s) \cdot \delta \ll \pi/2\,. \tag{13}$$

If $\psi(s)\delta$ becomes close to $\pi/2$ then complete phase mixing occurs. The betatron oscillation with an initial rms of $\sigma_x$ has completely "filamented" away and caused an asymptotic emittance growth:

$$\frac{\Delta\epsilon}{\epsilon_0} = \left(\frac{\sigma_x}{\sigma_0}\right)^2\,. \tag{14}$$

The ratio of beam offset to beam size is the important parameter. Correspondingly the dispersion reaches a maximum of $\eta_{\mathrm{max}} = \sigma_x/\delta$ and comes back to zero after filamentation.

## III. TWO EXAMPLES FOR PLASMA-BASED LINACS

A generic plasma-based linac design is discussed for both the PWFA and the LWFA case, as they were defined above. Table 1 summarizes parameters for a 1 TeV linac case study. We point out that the PWFA case is consistent with the one in [2].

The properties of the transverse focusing are best characterized by the $\beta$-function. Using the formalism from

| Parameter | PWFA | LWFA |
|---|---|---|
| Plasma density | $2 \times 10^{14}$ cm$^{-3}$ | $10^{17}$ cm$^{-3}$ |
| Accelerating gradient | 1 GeV/m | 30 GeV/m |
| Acc. wavelength | 2 mm | 100 $\mu$m |
| Focusing field | 6,000 T/m | 600,000 T/m |
| Module length | 6 m | 1 m |
| Injection energy | 1 GeV | 1 GeV |
| Final energy | 1 TeV | 1 TeV |
| Number of modules | 167 | 33 |

TABLE I. Parameters of the two 1 TeV linac designs and the accelerated beam. We only list parameters that are important for the discussion of the transverse beam dynamics. The focusing field for the LWFA case is a simplified estimate.

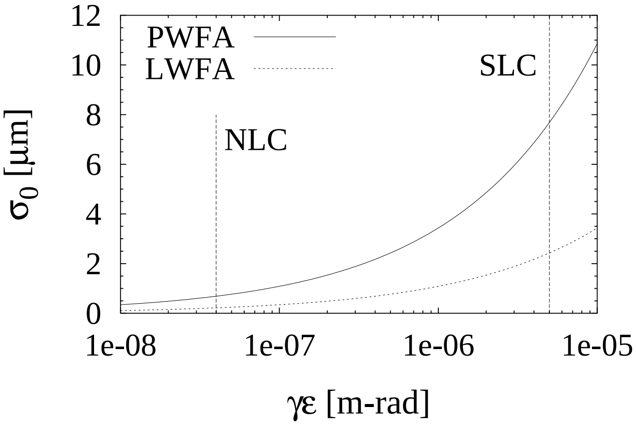


FIG. 1. The matched beam size $\sigma_0$ as a function of normalized emittance $\gamma\epsilon$, calculated at the injection energy of 1 GeV. The typical SLC measurement and the NLC/JLC design values are indicated for the vertical emittance.

the previous section and the parameters from Table 1 we calculate the following $\beta$-functions at the injection energy of 1 GeV:

$$\beta = 2.4 \text{ cm} \quad (\text{PWFA}) \quad , \quad \beta = 0.24 \text{ cm} \quad (\text{LWFA})\,. \quad (15)$$

The $\beta$-functions are extremely small, though they increase with the square root of the beam energy. For comparison, the injection $\beta$-functions for the proposed NLC/JLC linacs are of the order of 10 m. The tight focusing has serious consequences for the beam size. The matched beam sizes at injection energy are shown in Fig. 1 for the two cases as a function of the normalized emittance. With emittances as they are proposed for the next generation of linear colliders the matched beam size is sub-micron for both acceleration schemes. Even for SLC type emittances the beam sizes are well below 10 $\mu$m.

Therefore we conclude that the strong transverse fields in plasma-based linacs require the handling of beams with sub-micron spot sizes. This increases the sensitivity to dispersive emittance growth due to offsets $\Delta x_i$ between the accelerated beam and the axis of the wakefield at the entrance of plasma module $i$. Assuming an NLC/JLC injection emittance of $4 \times 10^{-8}$ m-rad (normalized) [5] the injection beam sizes from Fig. 1 are:

$$\sigma_0 = 0.7\ \mu\text{m} \quad (\text{PWFA}) \quad , \quad \sigma_0 = 0.22 \mu\text{m} \quad (\text{LWFA})\,. \quad (16)$$

Due to beam acceleration those beam sizes are damped proportional to $E^{-1/4}$, meaning a reduction by a factor of 5.6 at 1 TeV. From Eq. 14 we see immediately that any offset that is of the order of the beam size will cause the emittance to double after filamentation. Demanding a total emittance growth of 100% to 200% we see immediately that the offset tolerances for every stage must be below the beam size.

With full filamentation within a single stage the total emittance growth from $n$ stages is just the sum of the emittance growths in the single stages. Betatron oscillations will filament and no trajectory correction is required. In this case we can calculate the tolerances on the offsets $\Delta x$ for a total emittance growth of 200%. We find tolerances on $\sigma_x/\sigma_0$ of 0.08 (PWFA) and 0.18 (LWFA). For both cases we calculate a typical alignment tolerance of about 20 nm along the modules in the 1 TeV linac. We note that full filamentation within a single stage is a good assumption for LWFA, but does overestimate emittance growth for the PWFA case, where betatron oscillations at high energy will leak from one module into the next. The PWFA tolerances will depend on the efficiency of correction algorithms and loosen up less than a factor of $n^{1/2}$ ($\approx 13$), but generally remain tight.

## IV. POSSIBLE SOLUTIONS

The required handling of sub-micron transverse alignment tolerances in order to avoid unacceptable emittance growth, imposes a serious disadvantage for plasma-based acceleration methods. Further research is needed in order to determine how to best decrease transverse wakefields while maintaining strong longitudinal wakefields for beam acceleration. For example the study of inhomogeneous plasmas [6] might lead to important new insights. For an extreme case, the hollow plasma channel, the ion density on the beam axis is zero, minimizing the focusing transverse fields due to the ions, while maintaining strong plasma-wakefield acceleration. Transverse and longitudinal wakefield excitation in inhomogeneous plasmas needs further analysis.

Even in the case of a homogeneous plasma one might be able to place the accelerated bunch at a point with strong longitudinal and weak transverse wakefields. Further studies are needed to examine the transverse and longitudinal wakefields close to the point where the plasma electrons cross the beam axis. The transverse wakefield is now a function of longitudinal and radial positions, changing the relevant beam dynamics significantly. It remains to be decided whether a local optimum exists for a realistic bunch length and transverse beam size.

## V. CONCLUSION

We briefly reviewed the beam dynamics in a plasma-based linac for a homogeneous plasma. The strong transverse wakefields were taken into account as uniform focusing fields. While this is a good representation for the PWFA concept this is a simplified model for the LWFA case, where the transverse wakefield varies significantly along the bunch. However, the treatment allows easy estimates of the transverse beam dynamics for both cases.

We have shown that the strong focusing requires matched beam sizes in the sub-micron range in order to transport and preserve a normalized emittance of $4\times10^{-8}$ (NLC/JLC design). The alignment tolerance between the beam position and the wakefield axis at the entrance of a plasma-module is of the order of 20 nm for both concepts. This tolerance allows for a total emittance growth of 200% in a 1 TeV linac. Though those are only rough estimates it is evident that the alignment tolerances for homogeneous plasmas require advances in beam handling techniques beyond those presently available.

The beam dynamics described in this paper will start to affect 2nd generation plasma wakefield acceleration experiments as soon as the 1 m length scale is approached. The limitations due to strong transverse fields clearly require attention. New approaches to minimize transverse wakefields beyond the homogeneous plasma case, while maintaining high beam acceleration should clearly be pursued. For example, hollow plasma channels promise to be a way to ease the requirements on the transverse beam handling significantly.


[1] T.C. Chiou, T. Katsouleas, C. Decker, W.B. Mori, J.S. Wurtele, G. Shvets, J.J. Su, Phys. Plasmas 2, 310 (1995).

[2] J. Rosenzweig et al, "A Linear Collider Based on Nonlinear Plasma Wakefield Acceleration". Published in Proc. of Snowmass '96.

[3] D. Whittum, "A Continuous Plasma Final Focus". Proc. Topical Conf. on Research Trends in Nonlinear and Relativistic Effects in Plasmas. Ed. V. Stefan. LaJolla 1990.

[4] T. Katsouleas. Private communication.

[5] The NLC design group, "Zeroth Order Design Report for the Next Linear Collider". SLAC Report 474. The JLC design group, "JLC Design Study", KEK 1997.

[6] G. Shvets et al., "Excitation of Accelerating Wakefields in Inhomogeneous Plasmas". IEEE Transactions on Plasma Science, Vol. 24, No. 2, April 1996.