Observational Bounds on Cosmic Doomsday

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Recently it was found, in a broad class of models, that the dark energy density may change its sign during the evolution of the universe. This may lead to a global collapse of the universe within the time $t_c \sim 10^{10} - 10^{11}$ years. Our goal is to find what bounds on the future lifetime of the universe can be placed by the next generation of cosmological observations. As an example, we investigate the simplest model of dark energy with a linear potential $V(\phi) = V_0(1 + \alpha\phi)$. This model can describe the present stage of acceleration of the universe if α is small enough. However, eventually the field ϕ rolls down, $V(\phi)$ becomes negative, and the universe collapses. The existing observational data indicate that the universe described by this model will collapse not earlier than $t_c \gtrsim 10$ billion years from the present moment. We show that the data from SNAP and Planck satellites may extend the bound on the "doomsday" time to $t_c \gtrsim 40$ billion years at the 95% confidence level.

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I. INTRODUCTION

Recent discovery of the acceleration of the universe [1–3] is often interpreted as a proof that the universe is going to expand forever. This is indeed the case if the acceleration occurs due to the existence of a positive vacuum energy density Λ (cosmological constant), constituting approximately 70% of the energy density of the universe $\rho_0 \sim 10^{-120} M_p^4 \sim 10^{-29}$ g/cm^3 today [4]. However, the only existing theoretical model based on string theory and describing an accelerating universe in a state with a positive cosmological constant predicts that this state is metastable, and therefore acceleration of the universe cannot persist for an indefinitely long time [5].

The situation becomes even more complicated in the models where the present acceleration of the universe is related not to the cosmological constant, but to a slowly changing energy density of a scalar field, called dark energy. One of the first models of dark energy (and by far the simplest one) was proposed back in 1986 in Ref. [6], where it was suggested to replace the cosmological constant by the energy density of a slowly changing scalar field ϕ with the linear effective potential

$$V(\phi) = V_0(1 + \alpha\phi) \ . \tag{1}$$

Here we use the units $M_p = (8\pi G)^{-1/2} = 1$. If the slope of the potential is sufficiently small, $\alpha V_0 \lesssim 10^{-120}$, the field ϕ practically does not change during the last 10^{10} years, its kinetic energy is very small, so at least until the present stage of the evolution of the universe its total potential energy $V(\phi)$ acts nearly like a cosmological constant. The anomalous flatness of the effective potential in this scenario is a standard feature of most of the models of dark energy [7, 8].

The main reason to introduce this model in [6] was to address the cosmological constant problem. The main idea can be explained as follows. Even though the energy density of the field ϕ in this model practically does not change at the present time, it changed substantially during inflation. Since ϕ is a massless field, it experienced quantum jumps with the amplitude $H/2\pi$ during each time H^{-1}. These jumps move the field ϕ in all possible directions. In the context of the eternal inflation scenario this implies that the field becomes randomized by quantum fluctuations: The universe becomes divided into an infinitely large number of exponentially large parts containing all possible values of the field ϕ. In other words, the universe becomes divided into an infinitely large number of 'universes' with all possible values of the effective cosmological constant $\Lambda(\phi) = V(\phi)$. This quantity may range from $-M_p^4$ to $+M_p^4$ in different parts of the universe, but we can live only in the 'universes' with $|\Lambda| \lesssim O(10)\rho_0 \sim 10^{-28}$ g/cm^3.

Indeed, if $\Lambda \lesssim -10^{-28}$ g/cm^3, the universe collapses within the time much smaller than the present age of the universe $\sim 10^{10}$ years [6, 9, 10]. On the other hand, if $\Lambda \gg 10^{-28}$ g/cm^3, the universe at present would expand exponentially fast, energy density of matter would be exponentially small, and life as we know it would be impossible [6, 9]. This means that we can live only in those parts of the universe where the cosmological constant does not differ too much from its presently observed value $|\Lambda| \sim \rho_0$. This approach proposed in [6] constituted the basis for many subsequent attempts to solve the cosmological constant problem using the anthropic principle in inflationary cosmology [11–18].

However, the simplest dark energy model with the linear potential (1) has a disturbing consequence: even though the field ϕ moves down very slowly, eventually the potential energy density $V(\phi)$ becomes negative, and the universe collapses, just as the universe with a negative cosmological constant. A detailed description of this process can be found in [19]. Nevertheless, since the collapse will occur only in a distant future, the linear model (1) with a sufficiently small α is quite satisfactory from the point of view of all existing observational data.

The existence of the vacuum instability leading to a global collapse of the universe is a property of a large class of the models of dark energy, [6, 7, 20–24]. However, it takes some time for the universe to switch from acceleration to collapse. Recent observational data allow us to rule out many of the models with steep potentials predicting rapid collapse of the universe. Most of these models compatible with the existing observational data predict the global collapse of the universe within the time exceeding 10-30 billion years [16, 20–25]. This is the time comparable with the present age of the universe $t_0 \sim 13.7$ billion years. In this respect it becomes very interesting to check whether one could find any indication of the vacuum instability and the future "doomsday," or, *vice versa*, whether it is possible to increase the "life expectancy" of the universe, using the most advanced data to be obtained, for example, by the Supernova/Acceleration Probe (SNAP: [26]) distance-redshift measurements, the Planck Surveyor cosmic microwave background satellite [27], and weak gravitational lensing from the SNAP wide field survey [28] or the LSST survey [29].

This is the main goal of our paper. We will study this issue in the context of the simplest dark energy model (1). The reason to do it is that this model (up to the field redefinition) has only one free parameter α, so one can relatively easily study this model in all possible regimes. Also, this model is quite typical: in most of the models discussed in [16, 20–25] the potential looks linear with respect to ϕ near the point where $V(\phi) = 0$, where the acceleration gives way to a collapse.[1]

II. DARK ENERGY WITH A LINEAR POTENTIAL

We assume that the scalar field ϕ, with the linear potential (1), represents the dark energy density of the universe. There is also the usual matter energy density $\rho_M = \frac{C}{a^3}$, so that the equations of motion are given by

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{\partial V}{\partial \phi} = 0 \ , \qquad (2)$$

$$\frac{\ddot{a}}{a} = \frac{V - \dot{\phi}^2 - \frac{1}{2}\rho_M}{3}. \qquad (3)$$

Here $a(t)$ is the scale factor of the flat FRW metric, $ds^2 = dt^2 - a(t)^2 d\vec{x}^2$. The Hubble parameter is given by

$$H^2(t) \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho_M(t) + \rho_D(t)}{3} \equiv \frac{\rho_T(t)}{3} \ . \qquad (4)$$

We will solve these equations numerically, more details on this can be found in [23]. The dark energy density ρ_D and the pressure p_D are given by $\rho_D = \dot{\phi}^2/2 + V$ and $p_D = \dot{\phi}^2/2 - V$, and the total energy density includes also the energy density of matter, $\rho_T = \rho_M + \rho_D$.

A dimensionless dark energy (matter) density is given by a ratio of the dark energy (matter) to the total energy.

$$\Omega_D = \frac{\rho_D}{\rho_T} \ , \qquad \Omega_M = \frac{\rho_M}{\rho_T} \ . \qquad (5)$$

Observations suggest that now $\Omega_M \approx 0.28$, $\Omega_D \approx 0.72$, and $\Omega_T = \Omega_M + \Omega_D \approx 1$. The present time will be specified in our numerical solutions by the moment when $\Omega_D = 0.72$. We will study separately the effect of changing the current value of Ω_D between 0.7 and 0.73. Another important characteristic of the dark energy is its pressure-to-energy ratio defining the dark energy equation of state:

$$w_D = \frac{p_D}{\rho_D} = \frac{\dot{\phi}^2/2 - V}{\dot{\phi}^2/2 + V} \ . \qquad (6)$$

In what follows we will drop the index D in w_D and use w for dark energy w_D.

Without any loss of generality one can assume that the initial value of the field ϕ in the linear potential is zero, since any change of ϕ_0 can be absorbed into a redefinition of V_0. The value of the constant part of the potential V_0 for any choice of the slope is not independent: it is chosen in a way that today at $z = 0$ the value of Ω_D equals 0.72. Thus there is only one independent parameter in the linear potential model, α, or, equivalently, the slope of the potential, αV_0. We will assume that $\alpha > 0$, but all results for the lifetime of the universe depend only on $|\alpha|$.

We solved the equation of motion of the theory numerically and we present below a set of solutions of the linear potential model with various slopes. The scale factor $a(t)$ and the equations of state $w(z)$ are plotted in Figs. 1 and 2, respectively. These models are designed for the analysis of the current and future observational data. Complete information on each model is given in the Table I. This includes the color of the curve, the values of α and V_0, the slope αV_0, the value of w at present

[1] A future curvature singularity may also appear in the models where the null energy condition is violated, $\rho + p < 0$, so that $w < -1$. We will not discuss such "phantom" models [30] here because their physical interpretation is rather obscure and they are expected to lead to a very rapid development of instability at the quantum level [31].

TABLE I: Parameters for the Models on Figs. 1 and 2

Parameter	Cosmological Constant	SNAP[SN] + Planck + SNAP[WL] (95% cl)	SNAP[SN] +Planck (95% cl)	SNAP[SN] $+\sigma_\Omega$ (95% cl)	Minimum Lifetime Model
curve color	red	orange	purple	blue	black
α	0	0.71	0.76	0.86	1.13
V_0	0.72 ρ_0	0.83 ρ_0	0.85 ρ_0	0.91 ρ_0	1.77 ρ_0
αV_0	0	$0.72 \times 10^{-120} M_p^3$	$0.79 \times 10^{-120} M_p^3$	$0.96 \times 10^{-120} M_p^3$	$2.46 \times 10^{-120} M_p^3$
$w(0)$	-1	-0.89	-0.87	-0.82	-0.0001
t_c	∞	39.5 Gyr	35.5 Gyr	28.7 Gyr	11.3 Gyr

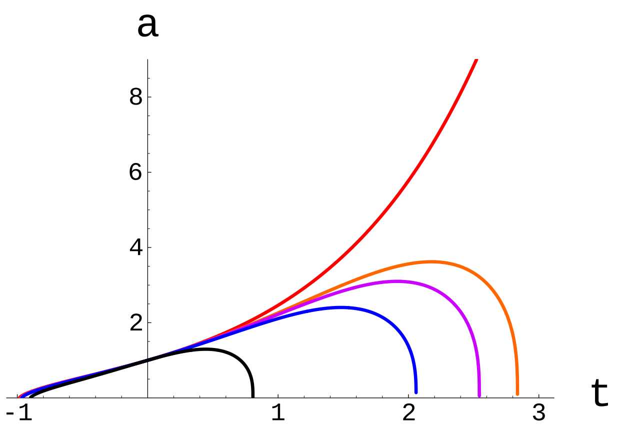

FIG. 1: Scale factor $a(t)$ in five models, the present moment is $t = 0$. The upper (red) curve corresponds to the cosmological constant model with the vanishing slope α; classically it has an infinite future lifetime. The curves below (orange, purple, blue, and black) correspond to a steepening slope. The time remaining from today to the future collapse in these models is shown in the table. Time is given in units of $H_0^{-1} \approx 13.7/0.983$ billions of years. In these units the current age of the universe $t \approx 13.7$ Gyr is given by 0.983.

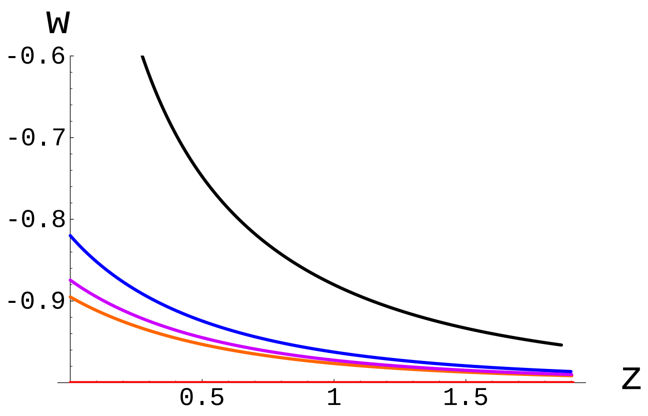

FIG. 2: Evolution of dark energy equation of state $w(z)$ in five models; the present moment is at $z = 0$. The value $w(0)$ is also given in the table.

at $z = 0$, and, finally, the time t_c of the collapse of the universe (from the present moment).

Model 1, the curve with ever expanding universe in Fig. 1, is a fiducial cosmological constant model with vanishing slope α; it has an infinite future lifetime. Models 2, 3, 4 with increasing slope will later be associated with some limits on lifetime based on specific observations. Model 5 has the largest slope $\alpha V_0 = 2.46 \times 10^{-120} M_p^3$, for which the value of $\Omega_D = 0.72$ is barely reached. Any further increase of the slope will make the model ruled out by the data (assuming that $\Omega_D \approx 0.72$ at present) since Ω_D in these models will never reach 0.72. The scale factor of the minimal model is plotted in Fig. 1, where one can see that the universe will collapse in a time of the order $t_c = 11$ Gyr from now.

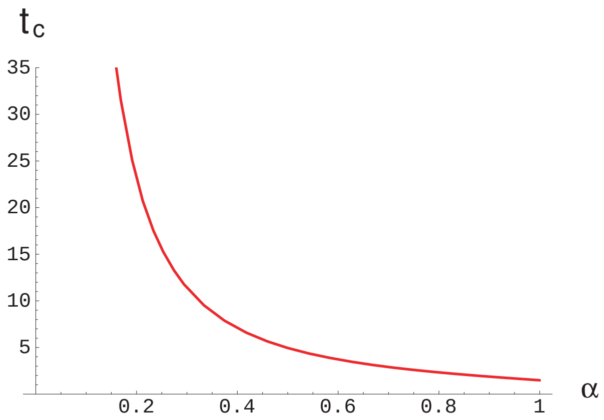

FIG. 3: Dependence of the lifetime of the universe (starting from the present moment), in units of H_0^{-1}) on α in the linear model.

Fig. 3 shows the dependence of the time t_c of the collapse of the universe (from the present moment) on α. As we see, the lifetime sharply increases for $\alpha \ll 1$.

By looking at Figs. 1, 2 one can easily conclude that the models with the lifetime smaller than 10 billion years are at odds with the existing observational data. First of all, in these models one would have $\Omega_D < 0.72$. Secondly, while current data cannot see time variation in $w(z)$, the average value of w, defined as $\bar{w} = \frac{\int da\ \Omega_D(a)\ w(a)}{\int da\ \Omega_D(a)}$, would be rather large, $\bar{w} > -0.6$. However, for less extreme models, the average value of w is generically quite close to -1, so data on this quantity is not a reliable guide. One requires an experiment that is capable of seeing the time variation in $w(z)$. This requires a new generation of surveys, which we discuss in the next section.

Instead of trying to find out the best constraint on the lifetime of the universe on the basis of the present observational data, we will try to understand to what extent the future data can allow us to predict the fate of the universe. Our results will be limited to the simplest linear model, but they will be quite indicative of the general situation.

III. CONSTRAINTS ON THE LIFETIME OF THE UNIVERSE FROM NEXT GENERATION OBSERVATIONS

To constrain the dark energy models and their impact on the fate of the universe we use observational data from next generation cosmological probes. The centerpiece is the Supernova/Acceleration Probe (SNAP) satellite, proposed as a dedicated dark energy mission designed to measure precise luminosity distances to some 2000 Type Ia supernovae covering the redshift range $z = 0.1 - 1.7$. This data is supplemented by another part of the primary mission, involving wide area measurements of weak gravitational lensing. Valuable complementarity is provided by data from the Planck Surveyor cosmic microwave background satellite that will use the temperature fluctuation power spectrum to make precision determinations of several combinations of cosmological parameters. For our purposes the key quantity will be measurement of the angular diameter distance to the CMB last scattering surface at $z = 1089$.

Of course, we do not know the results of these future experiments. They may indicate that the universe already began decelerating, which could be a precursor for the future collapse of the universe. Here we would like to study the most optimistic possibility. Let us assume that the new observational data will favor the simplest of all possible options: the universe is dominated by the positive cosmological constant with the equation of state $w = -1$. In terms of our simplest model of dark energy with a linear potential this would mean that the slope of the potential cannot be distinguished from zero with the accuracy of the combined set of experiments mentioned above. What kind of predictions for the future evolution of the universe we would be able to make? In particular, we wonder whether we will be able to say, as is often asserted, that the acceleration of the observable part of the universe will continue forever, and in about 150 billion years our galaxy and its closest neighbors will remain the only inhabitants of the otherwise empty observable part of the universe.

In order to study this question, we examine the types of cosmological information we will obtain. The luminosity and angular diameter distances are both related to the comoving distance by factors of $1+z$. The comoving distance is given in terms of quantities from Section II by

$$d(z) = H_0 \int_0^z \frac{dz'}{H(z')}. \tag{7}$$

The Friedmann equations of general relativity define the expansion history of the universe, the evolution of the scale factor $a(t)$, in terms of the components of the energy density by

$$(\dot{a}/a)^2 \equiv H^2 = H_0^2[\Omega_M^0(1+z)^3 + \Omega_D(z)] \tag{8}$$

The dark energy density evolves with redshift $z = a^{-1} - 1$ as

$$\begin{aligned} \Omega_D(z) &= \Omega_D^0 e^{3\int_0^z [dz'/(1+z')][1+w(z')]} && (9) \\ &\to \Omega_D^0 (1+z)^{3(1+w_0+w_a)} e^{-3w_a z/(1+z)} && (10) \end{aligned}$$

The second line gives the result for a commonly used parametrization $w(z) = w_0 + w_a\,(1-a)$ giving a good approximation for slowly rolling fields [32]. Here H_0, Ω_M^0, and Ω_D^0 are the values of $H(z)$, $\Omega_M(z)$, and $\Omega_D(z)$ today at $z = 0$, respectively.

Since the equation of state enters via two integrals, small deviations between the model behavior and the $w_0 - w_a$ fit will be unimportant. This allows us to plot results in the $w_0 - w_a$ space, easing comparison with other dark energy models. Note that a measure of the time variation of the dark energy equation of state is often given by $w' \equiv dw/d\ln(1+z)|_{z=1} = w_a/2$.

Specifically, we take the SNAP baseline mission presented in [33], including statistical and systematic errors amounting to 1% in distance at the depth of the survey, $z = 1.7$. We marginalize over the absolute magnitude parameter $\mathcal{M}$ (including the Hubble constant H_0) and take a fiducial cosmological constant model with $\Omega_D = 0.72$, see Appendix for details. For those cases where we only consider supernova data, we also impose a gaussian prior on Ω_D of 0.03; when we include CMB or weak lensing (WL) data, the natural determination of Ω_D in complementarity with the supernovae is better than this. When we incorporate WL data we use only information from the linear part of the mass power spectrum, in the manner of [34]. Note that WL is roughly equivalent to the CMB in complementary power with the supernova data. But if the fiducial model has time varying equation of state $w(z)$ then there is a further gain in precision with all three data sets.

Constraints on the dark energy model from the data are analysed within the Fisher matrix method [35] which is also explained in the Appendix. The equation of state function $w(z)$ for each model is represented by 2 parameters, w_0 and w_a. The fit suggested in [32],

$$w(z) = w_0 + w_a\,(1-a) = w_0 + w_a \frac{z}{1+z}\ , \tag{11}$$

TABLE II: Parameters for the ellipses and models in Fig. 4

	Cosmological Constant	SNAP[SN] + Planck + SNAP[WL] (68% cl)	SNAP[SN] + Planck + SNAP[WL] (95% cl)	SNAP[SN] + Planck (68% cl)	SNAP[SN] + Planck (95% cl)	SNAP[SN] + σ_Ω (68% cl)	SNAP[SN] + σ_Ω (95% cl)
curve color	red	orange dashed	orange	purple dashed	purple	blue dashed	blue
α	0	0.576	0.71	0.63	0.76	0.72	0.86
V_0	0.72 ρ_0	0.79 ρ_0	0.83 ρ_0	0.80 ρ_0	0.85 ρ_0	0.84 ρ_0	0.91 ρ_0
$w(0)$	-1	-0.94	-0.89	-0.92	-0.87	-0.89	-0.82
t_c	∞	55.3 Gyr	39.5 Gyr	47.9 Gyr	35.5 Gyr	38.6 Gyr	28.7 Gyr

works well for our models, especially for the models with larger lifetime. Plots in the $w_0 - w_a$ plane marginalize over the value of Ω_D.

Each case that we studied is shown in a $w_0 - w_a$ plane as a (black) triangle in Fig. 4. Each case (apart from the cosmological constant) is chosen so that the point in the $w_0 - w_a$ plane is at the boundary of one of the six ellipses, corresponding to future data. Complete information on each case is given in Tables I and II. The color code for the confidence ellipses, orange, purple and blue, is the same as in previous figures, for SNAP+Planck+WL, SNAP+Planck, and SNAP, respectively with 95% confidence. Dashed lines in the same colors are used for 68% confidence data. We give the values of α, V_0 and w_0, and the lifetime before the collapse for all these cases.

The triangles at the boundary of each ellipse give us information about the constraint on the lifetime before collapse in each case. Any model whose point in the $w_0 - w_a$ plane lies outside the corresponding ellipse will be ruled out if the actual observation will favor the cosmological constant as the most likely point in the parameter space. This will rule out models with lifetimes before the collapse smaller than that of each model at the boundary. When drawing the ellipses around the (future, expected) measurement point, we made the common assumption that the probability distribution has the same shape around the measurement value as around the true value. This assumption is justified, as, indeed, we confirmed that ellipses drawn around the triangles are very similar in shape and size.

The ellipses in $w_0 - w_a$ parameter space for SNAP, for SNAP and Planck, as well as for SNAP and Planck and WL given in Fig. 4 are generic, model-independent, and can be used in connection with any model—not just the linear potential studied here—that has its $w(z)$ reasonably approximated by (11). The only information that enters are the properties of the measurements and the parametrization of the fit for $w(z)$ used. Therefore our results can be easily used for investigation of other models of dark energy.

The lifetimes, on the other hand, that become associated with each point in the $w_0 - w_a$ parameter space depend on the individual model studied. The Fisher ellipses for SNAP, SNAP+Planck, and SNAP+Planck+WL are shown in Fig. 4. Thus, the lifetimes that we attribute to the triangles there are particular for the linear model.

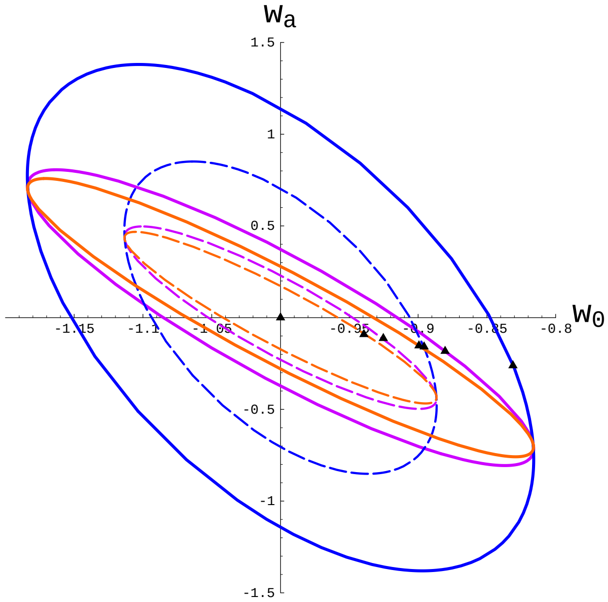

FIG. 4: Confidence contours for different combinations of data sets are plotted assuming a fiducial cosmological constant ($w_0 = -1$, $w_a = 0$) model. The innermost, orange (dashed) ellipse represents SNAP[supernovae] + Planck + SNAP[WL] at 95% (68%) confidence level. The slightly wider purple ellipses use only SNAP[supernovae] + Planck, and the rounder, blue ellipses use only SNAP[supernovae].

Thus our results can be formulated as follows. If the supernova distance-redshift observations from SNAP will yield the positive cosmological constant, i.e. $w_0 = -1$, $w_a = 0$, as the most likely point in the $w_0 - w_a$ parameter space, our analysis allows us to rule out a future lifetime before collapse that is shorter than 28.7 Gyr at the 95% confidence level (solid blue ellipse), and 38.6 Gyr at the 68% confidence level (dashed blue ellipse).

With the future Planck mission added to the SNAP data, the corresponding lifetime constraints can be raised even further to 35.5 Gyr (solid purple) and 47.9 Gyr (dashed purple) at the 95% and 68% confidence levels, respectively.

Weak lensing similarly tightens the constraints on $w(z)$. Both WL and Planck successfully complement

SNAP (but not each other). The three data sets together lead to 39.5 Gyr (solid orange) and 55.3 Gyr (dashed orange) at the 95% and 68% confidence levels, respectively.

IV. DIFFERENT Ω_D^0

While all our models include Ω_D as a parameter to marginalize over, the fiducial, central value so far has been $\Omega_D^0 = 0.72$ as suggested by a combination of CMB and large scale structure results [3] and new supernova data [36].

It is interesting to estimate what the difference will be in lifetimes if one changes the fiducial Ω_D^0 to 0.7 or 0.73. We have constructed the Fisher ellipses for SNAP and SNAP+Planck for this case, see Fig. 5. We have also evaluated the relevant change in the lifetime bounds between cases $\Omega_D^0 = 0.7$ and $\Omega_D^0 = 0.73$. The bound is changed by few percent, therefore the changes in Ω_D^0 do not lead to significant changes in expected bounds on the lifetimes.

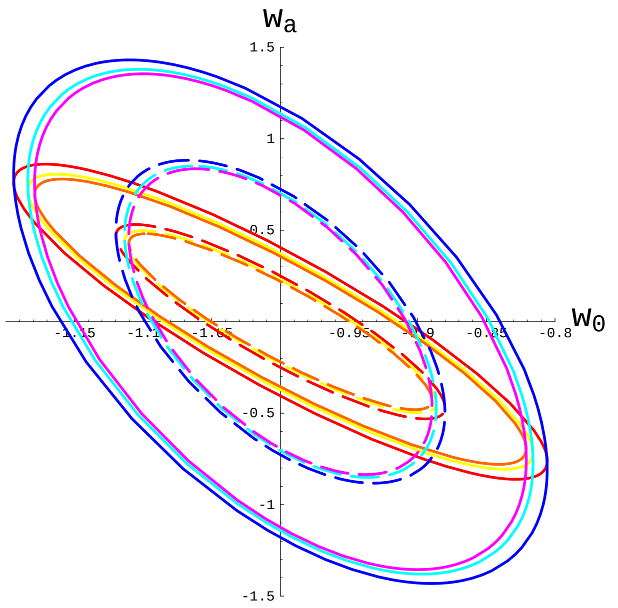

FIG. 5: Saturn plot of Fisher ellipses: Each of the Fisher ellipses shown in Fig. 4 is now given for fiducial Ω_D^0 taking 3 values: 0.70, 0.72, 0.73. The innermost ellipse in each case corresponds to $\Omega_D^0 = 0.73$, the outermost one corresponds to $\Omega_D^0 = 0.70$.

V. POSSIBLE SIGNATURES OF THE FUTURE COLLAPSE

Until now, we considered the possibility that the future observations will produce the simplest result, $w_0 = -1$, $w_a = 0$, which would suggest that the dark energy is nothing but the cosmological constant V_0. In this case, because of the observational uncertainties, we would be unable to claim that the universe is going to accelerate forever, but we will be able to say, that in the context of the simplest dark energy model (1) it will not collapse earlier than in 40 billion years from now, at the 95% confidence level.

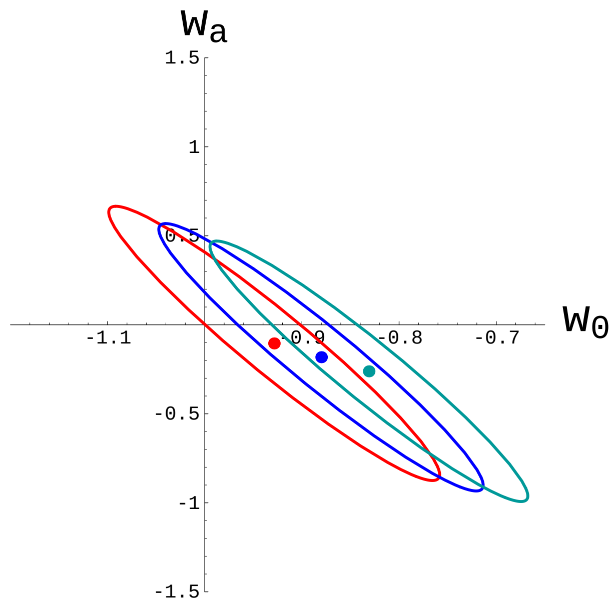

FIG. 6: Fisher ellipses with the centers corresponding to three different possible results of the future observations, including SNAP[supernovae] + Planck + SNAP[WL], see the text. The points outside the ellipses are ruled out at the 95% confidence level. The point $w_0 = -1$, $w_a = 0$, corresponding to the cosmological constant, lies outside of the green and blue ellipses and at the boundary of the red ellipse.

But what if we get a different result? Suppose, for example, that the future observations (including SNAP[supernovae] + Planck + SNAP[WL]) will tell us that $w_0 = -0.83$ and $w_a = -0.26$, which corresponds to our linear model with $\alpha = 0.86$. This would imply that at the 95% confidence level the true values of w_0 and w_a lie inside the green ellipse with the center at $w_0 = -0.83$, $w_a = -0.26$ (see Fig. 6). The blue ellipse corresponds to the possibility that $w_0 = -0.88$, $w_a = -0.18$, and the red one to $w_0 = -0.93$, $w_a = -0.1$.

Note that the point $w_0 = -1$, $w_a = 0$, corresponding to the cosmological constant, lies outside the green and blue ellipses and at the boundary of the red ellipse.

This result has two different implications. First of all, if the future observations find any of the sets of w_0 and w_a discussed above, they will rule out the standard cosmological constant model at the 95% confidence level.

Secondly, within the simplest model of dark energy, which is our linear model (1) [6], finding, e.g., that $w_0 = -0.93$ and $w_a = -0.1$, would imply, at the 95% confidence level, that our universe will not exist forever, but is going to collapse. This would not be a definite proof of the coming collapse, because there may be other, more complicated models of dark energy with similar values of w_0 and w_a, which do not lead to a global collapse. Still, this would be a serious warning sign, which would stimulate further investigation of the fate of the universe.

VI. DISCUSSION

Our investigation leads us to the following set of conclusions.

First of all, now it becomes even more apparent that it is very difficult to predict the future evolution of the universe. The standard textbook illustrations showing that open and flat universes slow down but expand forever, and a closed universe collapses, recently became replaced by the picture of a flat eternally accelerating universe. Now we are coming to a realization that the stage of acceleration may be transient, and even a flat universe may experience global collapse within a time comparable with its present age.

Our investigation shows that even the best experiments to be carried out in the next decade probably will be unable to give us a final answer concerning the destiny of the universe. Even if all experiments will unambiguously support the simplest possibility that dark energy is nothing but a positive cosmological constant with $w(z) = -1$, this will not really mean, as often claimed in the popular press, that in 150 billion years our galaxy and its immediate neighborhood will remain the sole island of matter surrounded by eternally expanding empty space. For all we know now, and for all we are going to learn in the next ten years, we will be unable to rule out the possibility that our part of the universe is going to collapse in the distant future.

But one can look at it from a different perspective. We live at the very beginning of the era of precision cosmology. It is amazing that within the next decade, by combining several different tools such as investigation of supernovae, CMB and weak lensing, we will be able to learn quite a lot about the possible outcome of the universe evolution. For example, if the combination of the experiments discussed in our paper will show that the most probable parameters of the dark energy correspond to the simplest cosmological constant scenario, $w_0 = -1$, $w_a = 0$, we will be able to say that in accordance with the simplest theories of dark energy, such as our linear model, there is no imminent danger of the global collapse at least for the next 40 billion years.

On the other hand, if these observations will favor a different set of parameters, e.g. $w_0 > -0.93$, $w_a = -0.1$, this will rule out, at the 95% confidence level, the simplest cosmological constant scenario. This result would be of great importance for our understanding of the most fundamental issues of physics, such as the structure of the vacuum state. In addition, such a result would mean, at least in the context of the simplest model of dark energy (1), that our universe is going to collapse. This would make further investigation of dark energy even more urgent and interesting.

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Appendix: SNAP, Planck, and Fisher ellipses

This appendix is designed to offer a simple, practical guide on the implementation of the Fisher matrix method for analysis of data constraints on cosmological parameters. For the mathematical basis of the method see [37] and for a general application to cosmology see [35]. Here we give a step by step introduction to allow those unfamiliar with the method to use it immediately (also see the appendix of [33]).

Formally, the Fisher matrix is defined by the expectation value

$$F_{ij} \equiv \left\langle -\frac{\partial^2 \ln L(\mathbf{x},\bar{\mathbf{p}})}{\partial p_i \partial p_j} \right\rangle = \\ = \left\langle \frac{\partial \ln L(\mathbf{x},\bar{\mathbf{p}})}{\partial p_i} \frac{\partial \ln L(\mathbf{x},\bar{\mathbf{p}})}{\partial p_j} \right\rangle, \qquad (12)$$

where $L(\mathbf{x},\mathbf{p}) = \Pi_{k=1}^{K} f(x_k,\mathbf{p})$ is the combined probability distribution, and $f(x_k,\mathbf{p})$ is the probability distribution of the individual measurement x_k that in general also depends on all the model parameters $\mathbf{p} = (p_1,\ldots,p_N)$. $\bar{\mathbf{p}}$ denotes the fiducial or (unknown) true parameter value of $\mathbf{p}$. The second equality in (12) follows from the matrix properties in the maximum likelihood approach, see [37] and references therein for a derivation. In many cases the probability distribution L could be approximated by a Gaussian and its relation to observed quantities, such as supernova magnitudes, becomes very simple [37].

Practically, the Fisher matrix method provides a local approximation to the likelihood surface for model parameters p_i given a set of observations $\mathbf{x} = \{m_k\}$. It involves the sensitivities, or first derivatives, $\partial m_k/\partial p_i$ evaluated at the fiducial model $\{\bar{p}_i\}$ [35]. This means that it only gives good estimates for small fit uncertainties, i.e. where the data constraints are strong enough to limit consideration to models near the input. Biases as well as uncertainties can be treated within the Fisher method.

The sensitivities combine into the symmetric Fisher matrix:

$$F_{ij} = \sum_k \frac{1}{\sigma^2(m_k)} \frac{\partial m_k}{\partial p_i} \frac{\partial m_k}{\partial p_j}. \quad (13)$$

Here we have assumed that the covariance of the data points vanishes, so the covariance matrix of the errors reduces to diagonal entries $1/\sigma^2(m_k)$. This is a reasonable approximation under certain circumstances, such as when the data is grouped into redshift bins wider than the correlation length. Remember that the Fisher matrix is a quick and rough approach; if one wants a more rigorous treatment then one might as well use a full Monte Carlo of the data set (including covariances).

One can see that the larger the derivatives (i.e. more sensitivity), the larger the matrix entries. Similarly, the more (independent) data points or the smaller the data errors, the larger the entries. This leads to the Fisher matrix also being known as the information matrix and this name gives a good guide to the interpretation. The larger a matrix entry, the more information the data has provided on those model parameters: hence it is a more sensitive probe and a better final parameter estimator.

The parameter estimations and their covariances can be read off from the inverse of the Fisher matrix, known as the error or covariance matrix $C = F^{-1}$. So for example $C_{ii} = \sigma_{ii}^2$ gives the one sigma error (68% confidence level) on the parameter p_i, averaged over all the other dimensions of the parameter space. Formally, the Fisher method only gives a lower limit on this error, as given by the Cramér-Rao inequality

$$\Delta p_i \geq \frac{1}{\sqrt{F_{ii}}}, \quad (14)$$

where $\Delta p_i \equiv \sqrt{\langle p_i^2 \rangle - \langle p_i \rangle^2}$ is the variance of the model parameter p_i, given the measurement errors. The Cramér-Rao lower bound theorem (14) is proved readily from the general property

$$\mathrm{Var}(X) \geq \frac{(\mathrm{Cov}(X,Y))^2}{\mathrm{Var}(Y)}, \quad (15)$$

which holds for any two random variables X and Y, by taking $X \equiv p_i(x_1, ..., x_K)$, $Y \equiv \frac{\partial \ln(L(x_1,...,x_K;p_1,...,p_N))}{\partial p_i}$, and using that, for these X and Y, we have $\langle Y \rangle = 0$, $\mathrm{Cov}(X,Y) = 1$ and

$$\mathrm{Var}(Y) = \left\langle -\frac{\partial^2 \ln(L(x_1, \ldots, x_K; p_1, \ldots, p_N))}{\partial p_i^2} \right\rangle. \quad (16)$$

One can deal with the full set of parameters in two basic ways: by fixing those not of immediate interest or by averaging over their probability distribution. The first is accomplished practically by excising the rows and columns corresponding to the fixed parameters from the Fisher matrix before inverting it to find the error matrix. This is a severe step, equivalent to assuming perfect knowledge of that variable. The second approach, known as marginalization, excises the appropriate rows and columns from the covariance matrix. Thus, as stated before, C_{ii} gives the error on a single parameter, averaging over all others.

If one has multiple data sets, then the information simply adds (if the data sets are independent). That is, one simply adds the Fisher matrices, assuming they have a common parameter set (note, the m's in (13) can be completely different quantities for the two Fisher matrices added, what is important is that the p_i's have exactly the same meaning). So if one wants to constrain cosmological parameters using both supernova distance data and CMB power spectrum data, then one just adds the Fisher matrices of each experiment. As a shortcut, sometimes one directly places a parameter constraint on the phase space rather than incorporating the data from the beginning. This is known as a prior, and is also implemented by adding a Fisher matrix; if the prior is on a single parameter, e.g. Ω_D is known to σ_{Ω_D}, then the addition is of a matrix empty except for one entry $F_{ii}^{\mathrm{prior}} = 1/\sigma_{\Omega_D}^2$, where $i = \Omega_D$. Strictly speaking, this should be done only if the prior knowledge truly has no covariance with any other parameters. If one is interested in the effects of different priors on the final parameter estimation, then one can use rules of matrix algebra to quickly determine how this extra bit of information affects the results (see [38]). Biases, where the data has been skewed from the true model behavior by systematic errors, i.e. $m_k \to m_k + \delta m_k$, can also be treated by matrix manipulation (see [39] and the Appendix of [33]).

SNAP

The SNAP data set will comprise high precision and accuracy measurements of the astronomical magnitudes of some 2000 Type Ia supernovae from redshift $z = 0$ out to redshift $z = 1.7$. Magnitudes are logarithmic distance variables and depend on the cosmological variables, say Ω_D, w_0, and w_a, and an unimportant (for cosmology) absolute supernova luminosity variable written as $\mathcal{M}$. We will always marginalize over $\mathcal{M}$, assuming a uniform probability distribution.

For the level of statistical analysis considered here, the supernova magnitude data is placed into 17 redshift bins, each bin having a width of $\delta z = 0.1$. This should be wide enough that observational uncertainties are uncorrelated from bin to bin. The parameter analysis turns out to be not very sensitive to the exact number of supernovae within each bin, as long as the entire redshift range is reasonably represented [40]. The SNAP baseline distribution is tabulated in [33]; generically one also includes data from 300 expected supernovae of the currently running Nearby Supernova Factory [41], as a single point in the lowest bin.

The magnitude error $\sigma(m_k)$ comprises two contributions: a statistical component from observational and intrinsic supernova magnitude dispersion and a systematic component from observational uncertainties. These are added in quadrature:

$$\sigma(m_k) = \sqrt{\frac{\sigma_0^2}{n_k} + \left(m_{sys}\frac{z_k}{1.7}\right)^2}, \tag{17}$$

where $k = 1, \ldots, 17$ labels the 17 bins, $\sigma_0 = 0.15$ mag is the statistical calibrated uncertainty of an individual supernova, n_k is the number of supernovae in bin k, and the redshift z_k is taken at the bin center, i.e. at $z_k = 0.05$ for $k = 1$. The second, systematic term uses the error model discussed in [33], with a linear rise with redshift. This serves as an approximation of observational uncertainties. SNAP instrumentation and observing strategy is specifically designed to limit systematic uncertainties to below $m_{sys} = 0.02$ mag out to the maximum redshift $z = 1.7$.

The data is related to the theoretical parameters by

$$\begin{aligned} m(z) &= 5\log_{10}\Bigg[(1+z)\int_0^z dz'\Big[(1-\Omega_D)(1+z')^3 \\ &+ \Omega_D e^{3\int_0^{\ln(1+z')} d(\ln(1+z''))[1+w(z'')]}\Big]^{-1/2}\Bigg] + \mathcal{M} \,. \end{aligned}$$

While the use of only one constant parameter w to model the function $w(z)$, i.e. assuming a constant equation of state, is physically unrevealing and often a poor approximation, more than two parameters makes it difficult to obtain meaningful constraints. We will write equation for $m(z)$ using the fit (11) containing two parameters, w_0 and w_a:

$$\begin{aligned} m(z) &= 5\log_{10}\Bigg[(1+z)\int_0^z dz'\Big[(1-\Omega_D)(1+z')^3 \\ &+ \Omega_D(1+z')^{3(1+w_0+w_a)}e^{-3w_a\frac{z'}{1+z'}}\Big]^{-1/2}\Bigg] + \mathcal{M} \,. \end{aligned}$$

Note that the offset parameter $\mathcal{M} = M + 25 - 5\log_{10}(H_0/100\mathrm{km/s/Mpc})$ (where M is the supernova absolute magnitude) just adds linearly, so the sensitivity derivative $\frac{\partial m_k}{\partial \mathcal{M}} = 1$ for all bins. Technically, other astrophysical terms enter into $m(z)$ such as a change in supernova magnitude due to dimming by intervening dust, but we assume that we are dealing with fully calibrated data, with only the cosmological dependences remaining.

In Fig. 4 we evaluate the sensitivity derivatives at the fiducial parameter values: $\Omega_D = 0.72$, $w_0 = -1$, $w_a = 0$, corresponding to the cosmological constant model for the dark energy. For the case of only supernova data we also add a gaussian prior of $\sigma_{\Omega_D} = 0.03$, corresponding to a reasonable level of knowledge on this parameter by the time SNAP data is available. Now we have all the elements needed to generate the Fisher matrix. Next we invert it to obtain the covariance matrix and the parameter estimation uncertainties (the diagonal elements). If we want to plot the confidence contours in a two dimensional section of the parameter phase space, e.g. the $w_0 - w_a$ plane, then we marginalize over the other parameters and reinvert the covariance matrix to get a reduced, 2×2 Fisher matrix. This gives the major and minor axes of the elliptical contour (it is always an ellipse within the Fisher approximation) and the orientation, i.e. the direction of the degeneracy between the parameters. This is basically an eigenvector where a particular combination of the parameters is best determined while an orthogonal combination is poorly constrained.

A plotting program takes the eigenvector information and draws the ellipse, with the scale determined by what level of probability one wants to enclose within the contour. For 68% of the probability enclosed (called 1σ joint probability), each axis of the figure is 1.52 ($\sqrt{2.30}$) times larger than the individual 1σ probabilities (called 1σ projected probability, or enclosing 39% joint probability) from the diagonal elements of the covariance matrix. For 95% confidence level, the scaling is 2.45 ($\sqrt{5.99}$; formally the numbers 2.30 and 5.99 are the increments in the χ^2 statistic from the fiducial model to a model on the 68%, resp. 95% contours). As found here, without a biasing systematic magnitude error, the best fit model will always be the fiducial, input model.

Planck

The cosmic microwave background data carries much information on the cosmological model parameters. Here we consider a simple subset of the measurements, involving only the distance to the photon last scattering surface, which is exquisitely determined by the location of the acoustic peaks in the CMB power spectrum. The Planck Surveyor mission will also precisely determine the combination $\Omega_M h^2$, so we employ the reduced distance

$$\tilde{d} = \int_0^{z_{LSS}} dz\, f(z)^{-1/2},$$

where the upper boundary of the integral is taken at the last scattering surface (LSS), $z_{LSS} = 1089$, and where

$$f(z) = \left[(1+z)^3 + \frac{\Omega_D}{1-\Omega_D}(1+z)^{3(1+w_0+w_a)}e^{-3w_a\frac{z}{1+z}}\right]$$

Planck has the ability to determine $\tilde{d}$ up to 0.7%, i.e. $\sigma_{\tilde{d}} = 0.007 \cdot \tilde{d}$; this is translated into a Fisher matrix

$$F_{ij}^{\mathrm{Planck}} = \frac{1}{\sigma_{\tilde{d}}^2}\frac{\partial \tilde{d}}{\partial p_i}\frac{\partial \tilde{d}}{\partial p_j}, \tag{18}$$

where $\mathbf{p} = \{\Omega_D, w_0, w_a\}$. Since we want to add this Fisher matrix to the Fisher matrix from SNAP, we will include also a fourth column and row of zeros (for the supernovae parameter $\mathcal{M}$ that does not enter the CMB data).

The final Fisher matrix for SNAP and Planck combined then is

$$F^{\rm SNAP+Planck} = F^{\rm SNAP} + F^{\rm Planck}. \quad (19)$$

Notice that it does not matter that the Fisher matrices are based on different data quantities from different experiments ($m(z)$ and $\tilde{d}$). The information still adds. When including the Planck data, we can eliminate the step of adding a prior on Ω_D, since the different cosmological quantities measure sufficiently different combinations of Ω_D with the other parameters. This breaks degeneracies well enough to determine Ω_D with superior precision (roughly equivalent to a prior of $\sigma_{\Omega_D} = 0.01$ [40]).

Weak lensing

The procedure for other data sets, such as from measurements of the gravitational distortion of images of background galaxies by foreground mass concentrations, known as WL, is implemented similarly. Here we used only estimates of the future precision of measurements of the linear part of the lensing shear power spectrum (see §3.3 of [34] for details). An even stronger data set employing the full power spectrum, and possibly other lensing methods, should be available from the SNAP wide field survey and other experiments.

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