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A Calculation of Cosmological Scale from Quantum Coherence* †

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Abstract

We use general arguments to examine the energy scales for which a quantum coherent description of gravitating quantum energy units is necessary. The cosmological dark energy density is expected to decouple from the Friedman-Lemaitre energy density when the Friedman-Robertson-Walker scale expansion becomes sub-luminal at $\dot{R} = c$, at which time the usual microscopic interactions of relativistic quantum mechanics (QED, QCD, etc) open new degrees of freedom. We assume that these microscopic interactions cannot signal with superluminal exchanges, only superluminal quantum correlations. The expected gravitational vacuum energy density at that scale would be expected to freeze out due to the loss of gravitational coherence. We define the vacuum energy which generates this cosmological constant to be that of a zero temperature Bose condensate at this gravitational de-coherence scale. We presume a universality throughout the universe in the available degrees of freedom determined by fundamental constants during its evolution. Examining the reverse evolution of the universe from the present, long before reaching Planck scale dynamics one expects major modifications from the de-coherent thermal equations of state, suggesting that

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the pre-coherent phase has global coherence properties. Since the arguments presented involve primarily counting of degrees of freedom, we expect the statistical equilibrium states of causally disconnected regions of space to be independently identical. Thus, there is no horizon problem associated with the lack of causal influences between spatially separated regions in this approach. The scale of the amplitude of fluctuations produced during de-coherence of cosmological vacuum energy are found to evolve to values consistent with those observed in cosmic microwave background radiation and galactic clustering.

1 Introduction

There is general (although not universal) agreement among physical cosmologists that the current expansion phase in the evolution of our universe can be extrapolated back toward an initial state of compression so extreme that we can neither have direct laboratory nor indirect (astronomical) observational evidence for the laws of physics needed to continue that extrapolation. Under these circumstances, lacking a consensus "theory of everything", and in particular a theory of "quantum gravity", we believe that the prudent course is to rely as much as possible on general principles rather than specific models. This approach is adopted in this paper.

We believe that the experimental evidence for currently accepted theories of particle physics is relevant up to about 5 TeV—the maximum energy or temperature we need consider in this paper. We further assume that our current understanding of general relativity as a gravitational theory is adequate over the same range, and consequently that the cosmological Friedman-Lemaitre (FL) (Hubble) dynamical equations are reliable guides once we have reached the observational regime where the homogeneity and isotropy assumptions on which those equations are based become consistent with astronomical data to requisite accuracy. Although the elementary particle theories usually employed in relativistic quantum field theories have well defined transformation properties in the flat Minkowski space of special relativity, we hold that their fundamental principles still apply on coordinate backgrounds with

cosmological curvature. In fact there is direct experimental evidence that quantum mechanics does apply in the background space provided by the Schwarzschild metric of the earth thanks to the beautiful experiments by Overhauser and collaborators[1, 2]. These experiments show that the interference of a single neutron with itself changes as expected when the plane of the two interfering paths is rotated from being parallel to being perpendicular to the "gravitational field" of the earth.

Since quantum objects have been shown to gravitate, we expect that during some period in the past, quantum coherence of gravitating systems will qualitatively alter the thermodynamics of the cosmology. Often, the onset of the importance of quantum effects in gravitation is taken to be at the Planck scale. However, as is the case with Fermi degenerate stars, this need not be true of the cosmology as a whole. By quantum coherence, we refer to the entangled nature of quantum states for space-like separations. This is made evident by superluminal correlations (without the exchange of signals) in the observable behavior of such quantum states. Note that the exhibition of quantum coherent behavior for gravitating systems does not require the quantization of the gravitation field.

The horizon problem for present day cosmology examines the reason for the large scale homogeneity and isotropy of the observed universe. The present age of the universe can be estimated from the Hubble scale to be $H_o t_o \cong 0.96 \Rightarrow t_o \cong 13.2 \times 10^9$ years$= 4.16 \times 10^{17}$ seconds. If the size of the observable universe today is taken to be of the order of the Hubble scale $\frac{c}{H_o} \approx 10^{28} cm$, then if the universe expanded from the Planck scale, its size at that scale would have been of the order $\sim 10^{-4} cm$ at that time. Since the Planck length is of the order $L_P \sim 10^{-33} cm$, then there would be expected to be $(10^{29})^3 \sim 10^{87}$ causally disconnected regions in the sky. Further, examining the ratio of the present conformal time η_o with that during recombination $\frac{\eta_o}{\eta_*} \sim 100$, the subsequent expansion is expected to imply that light from the cosmic microwave background would come from $100^3 = 10^6$ disconnected regions. Yet, angular correlations of the fluctuations have been accurately measured by several experiments[3].

Our approach is to start from well understood macrophysics and end at the onset of microcosmology. We refer to this period as gravitational decoherence. The FRW

scale factor is used to compare cosmological scales with those microscopic quantum scales we are familiar with, which define the lengths of rulers, ticks of clocks, mass of particles, and temperatures of thermodynamic systems. We will insist that our calculations not depend on the present horizon scale, which is an accident of history. It will be argued that the equilibration of microscopic interactions can only occur post-decoherence. Global quantum coherence prior to this period solves the horizon problem, since quantum correlations are in this sense supraluminal.

Present data examining the luminosities of distant Type Ia supernovae, which have a somewhat well understood time and frequency dependency, indicate clearly that the rate of expansion of the universe has been accelerating for several billion years[4]. This conclusion is independently confirmed by analysis of the Cosmic Microwave Background radiation[5]. Both results are in quantitative agreement with a (positive) cosmological constant fit to the data. Our interpretation of this cosmological "dark energy" will be due to the vacuum energy of a quantum coherent cosmology.

One physical system in which vacuum energy density directly manifests is the Casimir effect[6]. Casimir considered the change in the vacuum energy due to the placement of two parallel plates separated by a distance a. He calculated an energy per unit area of the form

$$\frac{\frac{1}{2}\left(\sum_{modes}\hbar c k_{plates} - \sum_{modes}\hbar c k_{vacuum}\right)}{A} = -\frac{\pi^2}{720}\frac{\hbar c}{a^3} \tag{1.1}$$

resulting in an attractive force of given by

$$\frac{F}{A} \cong \frac{-0.013 dynes}{(a/micron)^4} cm^{-2}, \tag{1.2}$$

independent of the charges of the sources. Lifshitz and his collaborators[7] demonstrated that the Casimir force can be thought of as the superposition of the van der Waals attractions between individual molecules that make up the attracting media. This allows the Casimir effect to be interpreted in terms of the zero-point motions of the sources as an alternative to vacuum energy. Boyer[8] and others subsequently demonstrated a repulsive force for a spherical geometry of the form

$$\frac{1}{2}\left(\sum_{modes}\hbar c k_{sphere} - \sum_{modes}\hbar c k_{vacuum}\right) = \frac{0.92353\hbar c}{a}. \tag{1.3}$$

This means that the change in electromagnetic vacuum energy *is* dependent upon the geometry of the boundary conditions.

The introduction of energy density ρ into Einstein's equation introduces a preferred rest frame with respect to normal energy density. However, as can be seen in the Casimir effect, the vacuum need not exhibit velocity dependent effects which would break Lorentz invariance. Although a single moving mirror does experience dissipative effects from the vacuum due to its motion, these effects can be seen to be of 4th order in time derivatives[9].

Another system which manifests physically measurable effects due to zero-point energy is liquid ^{4}He. One sees that this is the case by noting that atomic radii are related to atomic volume V_a (which can be measured) by $R_a \sim V_a^{1/3}$. The uncertainty relation gives momenta of the order $\Delta p \sim \hbar/V_a^{1/3}$. Since the system is non-relativistic, we can estimate the zero-point kinetic energy to be of the order $E_o \sim \frac{(\Delta p)^2}{2m_{He}} \sim \frac{\hbar^2}{2m_{He}V_a^{2/3}}$. The minimum in the potential energy is located around R_a, and because of the low mass of ^{4}He, the value of the small attractive potential is comparable to the zero-point kinetic energy. Therefore, this bosonic system forms a low density liquid. The lattice spacing for solid helium would be expected to be even smaller than the average spacing for the liquid. This means that a large external pressure is necessary to overcome the zero-point energy in order to form solid helium.

Applying this reasoning to relativistic gravitating mass units with quantum coherence within the volume generated by a Compton wavelength λ_m^3, the zero point momentum is expected to be of order $p \sim \frac{\hbar}{V^{1/3}} \sim \frac{\hbar}{\lambda_m}$. This gives a zero point energy of order $E_0 \approx \sqrt{2}mc^2$. If we estimate a mean field potential from the Newtonian form $V \sim \frac{G_N m^2}{\lambda_m} = \frac{m^2}{M_P^2}mc^2 << E_0$, it is evident that the zero point energy will dominate the energy of such a system.

For the reverse time extrapolation from the present, we adopt the currently accepted values[5][10] for the cosmological parameters involving dark energy and matter:

$$h_0 \cong 0.73; \quad \Omega_\Lambda \cong 0.73; \quad \Omega_M \cong 0.27 \tag{1.4}$$

Here h_0 is the normalized Hubble parameter. Note that this value implies that the universe currently has the critical energy density $\rho_c = 5.6 \times 10^{-4} Gev\ cm^{-3}$. We can

make the backward time extrapolation with confidence using known physics in the customary way back to the electro-weak unification scale $\sim 100\ Gev$, with somewhat less confidence into the quark-gluon plasma then encountered and beyond the top quark regime, and expect that — unless unexpected new particles and/or new physics are encountered —we can continue up to an order of magnitude higher energies with at most modest additions to the particle spectrum. In this radiation-dominated universe this backward extrapolation (which taken literally *must* terminate when the Friedman-Robertson-Walker scale factor $R(t)$ goes to zero and its time rate of change $\dot{R}(t)$ goes to infinity) is guaranteed to reach the velocity of light $\dot{R}(t_c) = c$ at some finite time t_c when the scale factor $R(t_c)$) still has a small, but finite, value.

As we discuss more carefully below, using our extrapolation beyond the limit just established (i.e. $\dot{R}(t_c) = c$) would seem to conflict with our basic methodological assumption that we invoke no unknown physics. It is true that *as a metric theory of space-time* the curved space-times of general relativity used in the homogeneous and isotropic cosmological models we employ are not restricted in this way. However, if we wish drive these models by mass-energy tensors derived from either particulate or thermodynamic models relying on some equation of state, and hence the hydrodynamics of some form of matter, we must not use them in such a way as to allow causal signalling at speeds greater than c by non-gravitational interactions. The exception to this stricture which *is* allowed by known physics is that *coherent quantum systems* have supraluminal *correlations* which cannot be used for supraluminal *signaling*. Consequently, we are allowed — as we assert in this paper — to start our examination of the universe at the $\dot{R} = c$ boundary if it is a fully coherent quantum system. Note that the beautiful experiments by Overhauser and collaborators already cited[1, 2] justify our invocation of such systems when they are primarily (or even exclusively) dependent on gravitational interactions. Thus we claim that it is consistent to start our cosmology with the cosmological *decoherence* of a quantum system at the $\dot{R} = c$ boundary. This quantum decoherence process is discussed in detail in Section 3:*Dark Energy De-coherence*.

In Section 2:*Motivation* we examine an earlier paper[11] which gave cosmological reasons why $\sim 5Tev$ might be the threshold for new physics. This paper was based

on much earlier work by E.D.Jones[12, 13, 14] and a more recent collaboration with L.H.Kaufmann and W.R.Lamb[15]. We find that, in contrast to Jones' *Microcosmology* which starts with an expansion from the Planck scale, we can identify the transition from speculative physics to a regime which can be reached with some confidence by the backward extrapolation from the present already assumed above. This section shows that the identification of the critical transition with the decoherence of a cosmological gravitationally quantum coherent system allows us to recover the semi-quantitative results of the Jones theory *without* having to introduce speculative physics.

In Section 3:*Dark Energy De-coherence* we will motivate our explanation of the dark energy driving the observed acceleration of the cosmology as the gravitational vacuum energy density of a zero temperature Bose condensate. Prior to a sub-luminal rate of expansion of the FRW scale factor, we assert that only gravitational and quantum coherence properties are relavant to the dynamics of the expanding cosmology. We will develop a single parameter model in terms of the cosmological constant, and use this to predict a mass and temperature scale for decoherence. We give an argument to support our assumption that the condensate remains at zero temperature during the pre-coherent phase of the cosmology. We will end by examining the expected amplitudes of density fluctuations if such fluctuations are the result of dark energy de-coherence.

Finally, in Section 4:*Discussion and Conclusions*, we will discuss the nature of cosmological dark energy, especially with regards to the distant future. It is especially interesting to question the constancy of vacuum energy density after a future gravitational re-coherence event $\dot{R} \geq c$. Some thoughts on our present and future efforts will be given.

2 Motivation

2.1 Jones' Microcosmology

Our present work originated in the re-examination of a paper by one of us[11]

emphasizing the likelihood of some threshold for new physics at $\sim 5\ Tev$. This in turn was based on a discussion with E.D. Jones in 2002[14]; our understanding of this discussion and his earlier ideas[12, 13] has been published by us in collaboration with L.H.Kaufmann and W.R.Lamb[15]. Briefly, Jones envisages an extremely rapid ("inflationary") expansion from the Planck scale (i.e. from the Planck length $L_P = \frac{\hbar}{M_P c} \cong 1.6 \times 10^{-33} cm$, where $M_{Pk} = [\frac{\hbar c}{G_N}]^{\frac{1}{2}} \cong 2.1 \times 10^{-8} kg \cong 1.221 \times 10^{19} GeV/c^2$, and $G_N = \frac{\hbar c}{M_P^2}$ is Newton's gravitational constant) to a length scale $R_\epsilon \sim \frac{1}{\epsilon}$. For the reader's convenience, will will also display the Planck time $T_P = \frac{L_P}{c} \cong 5.4 \times 10^{-44} sec$ and the Planck temperature $\theta_P = \frac{M_P c^2}{k_B} \cong 1.4 \times 10^{32}\, {}^o K$. Unless necessary for clarity, we will generally choose units such that $\hbar = 1, c = 1, k_B = 1$. This expansion, whose details are not examined, is characterized by the dimensionless ratio

$$\mathcal{Z}_\epsilon \sim \frac{R_\epsilon}{L_P} \sim \frac{M_P}{\epsilon} \tag{2.5}$$

When this expansion has occurred, the virtual energy which drives it makes a thermodynamic equilibrium transition to normal matter (i.e. dark, baryonic and leptonic, electromagnetic,...) at a mass-energy scale characterized by the mass parameter m_θ and length scale $\frac{1}{m_\theta}$. Jones uses the energy parameter ϵ as a unit of energy which he calls one *Planckton* defined as one Planck mass's worth of energy distributed over the volume $V_\epsilon \sim \frac{1}{\epsilon^3}$ measured by the scale parameter $R_\epsilon \sim \frac{1}{\epsilon}$. The virtual energy is assumed to consist of N_{Pk} Plancktons of energy ϵ, corresponding to an energy density $\sim \frac{N_{Pk}\epsilon}{V_\epsilon}$. This virtual energy makes an (energy-density) equilibrium transition to normal matter, so that

$$N_{Pk}\epsilon^4 \sim m_\theta^4 \tag{2.6}$$

It is assumed that one Planckton's worth of energy is "left behind" and hence that $\sim \epsilon^4$ can be interpreted as the cosmological constant density ρ_Λ at this and succeeding scale factors. It then can be approximately evaluated at the present day using $\Omega_\Lambda \sim 0.73$ as we show in the Sec. 2.4. In this sense both Jones' theory and ours can be thought of as *phenomenological* theories which depend only on a *single* parameter (outside of the constants from conventional physics and astronomy). We will review in the concluding section how many additional *observed or potentially observable* numbers can then be *predicted*.

2.2 Dyson-Noyes-Jones Anomaly

We note that our sketch of the Jones theory as expressed by Equation 2.6 introduces two new parameters (N_{Pk},m_θ) which must be expressed in terms of ϵ if we are to justify our claim that this is a single parameter theory. These parameters refer to a very dense state of the universe. According to our methodology, we must be able extrapolate back to this state using only current knowledge and known physics. Jones assumes that this dense state can be specified by using an extension to gravitation[17] of an argument first made by Dyson[19, 20]. Dyson pointed out that if one goes to more that 137 terms in e^2 in the perturbative expansion of renormalized QED, and assumes that this series also applies to a theory in which e^2 is replaced by $-e^2$ (corresponding to a theory in which like charges attract rather than repel), clusters of like charges will be unstable against collapse to negatively infinite energies. Schweber[20] notes that this argument convinced Dyson that renormalized QED can never be a *fundamental theory*. Noyes[17] noted that *any* particulate gravitational system consisting of masses m must be subject to a similar instability and could be expected to collapse to a black hole.

We identify $Z_{e^2} = 1/\alpha_{e^2} \cong 137$ as the number of electromagnetic interactions which occur within the Compton wave-length of an electron-positron pair ($r_{2m_e} = \hbar/2m_e c$) when the Dyson bound is reached. If we apply the same reasoning to gravitating particles of mass m (and if we are able to use a classical gravitational form), the parameter α_{e^2} is replaced by $\alpha_m = G_N m^2 = \frac{m^2}{M_P^2}$; the parameter fixing the Dyson-Noyes (DN) bound becomes the number of *gravitational* interactions within $\hbar/mc$ which will produce another particle of mass m and is given by

$$\mathcal{Z}_m \sim \frac{M_P^2}{m^2} \tag{2.7}$$

If this dense state with Compton wavelength $\lambda_m \sim 1/m$, contains $\mathcal{Z}_m$ interactions within λ_m, then the Dyson-Noyes-Jones (DNJ) bound is due to the expected transition transition $\mathcal{Z}_m m \rightarrow (\mathcal{Z}_m + 1)m$, indicating instability against gravitational collapse due to relativistic particle creation.

It is particularly interesting to examine the production channel for the masses m due to $\mathcal{Z}_m$ interactions within λ_m. This is diagrammatically represented in Figure 1.

If there are $\mathcal{Z}_m$ scalar gravitating particles of mass m within the Compton wavelength

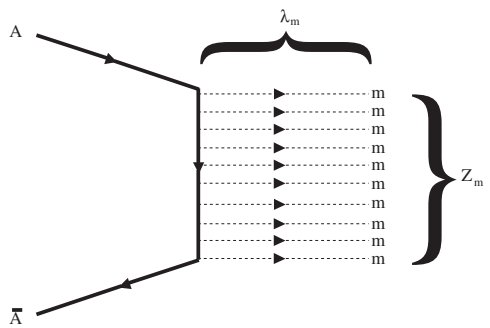

Figure 1: Noyes-Jones collapse of gravitating quanta

of that mass, a particle falling into that system from an appreciable distance will gain energy equal to mc^2, which could produce yet another gravitating mass m. Clearly, the interaction becomes anomalous.

Generally, when the perturbative form of a weak interaction becomes divergent, it is a sign of a phase transition, or a non-perturbative state of the system (eg bound states). One expects large quantum correlations between systems of mass m interacting on scales smaller than or comparable to the Compton wavelength of those masses. We will assume that if the (intensive) number of gravitational interactions (with no more than a Planck mass worth of interaction energy per Planckton, as defined in Section 2.1) of mass units m which can occur within a region of quantum coherence is greater than the DNJ limit, a phase transition into systems with quantum coherence scales of the Compton wavelength of those mass units will occur. We then expect de-coherence of subsystems of vastly differing quantum coherence scales when the Jones transition occurs.

We could be concerned that such a concentration of mass might form a black hole. Although we will be primarily assuming an FRW global geometry, if the Schwarzschild radius of a concentration of mass is considerably less than the proper radial size of the cosmology, one can sensibly discuss smaller regions which approximate a Schwarzschild geometry. For a system of a large number $\mathcal{Z}_m$ of gravitating particles (given by the onset of the DNJ anomaly), the Schwarzschild radius of these

masses is given by

$$R_S \;=\; \frac{2G_N(\mathcal{Z}_m m)}{c^2} \;=\; 2\mathcal{Z}_m \frac{m^2}{M_P^2}\frac{\hbar}{mc} \;=\; 2\lambda_m \tag{2.8}$$

Therefore, such a gravitating system of masses would be expected to be unstable under gravitational collapse. We determine the maximum number $\mathcal{Z}_m$ of coherent interactions energy units m beyond which the system will become unstable under gravitational collapse as

$$R_S \;=\; \lambda_m \Rightarrow \mathcal{Z}_m \;=\; \frac{M_P^2}{2m^2} \tag{2.9}$$

This argument does not assume that Newtonian gravitation.

A general comparison of the dependence of the Compton wavelength and Schwarzschild radius on the mass of the system as shown in Figure 2 gives some insight into regions of quantum coherence. To the left of the point of intersection of the two curves, the

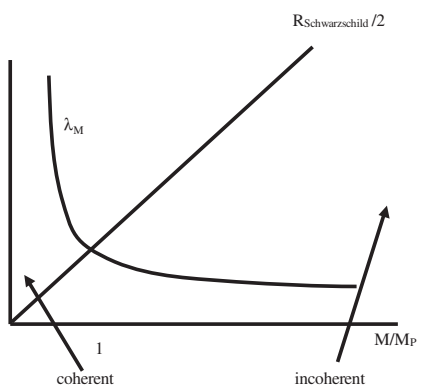

Figure 2: Functional dependence of Compton wavelength and Schwarzschild radius on system mass

Compton wavelength is larger than the Schwarzschild radius, and for such localized mass distributions the quantum coherence properties are important in any gravitational considerations. On the other hand, for masses much larger than the Planck mass, the gravitational distance scales are well outside of the quantum coherence scales for isolated masses. For the present discussion, it is the transition region that is of interest. An elementary particle is not expected to have a mass greater than a Planck mass. If the mass were greater than M_P, then the Compton wavelength would

be less than the Schwarzschild radius of the particle, thereby dis-allowing global coherence (for local experiments) of the particle due to Hawking radiation, as will be discussed.

In our previous discussions[15], we considered N_m particles of mass m within $\sim \lambda_m$ and hence $\mathcal{Z}_m = \frac{N_m(N_m-1)}{2}$ interacting pairs. The Schwarzschild radius in this case was considerably smaller than λ_m

$$R_S \cong 2G_N\sqrt{2\mathcal{Z}_m}m \sim \frac{\lambda_m}{\sqrt{\mathcal{Z}_m}} << \lambda_m \tag{2.10}$$

However, in the present discussion, $\mathcal{Z}_m$ counts the number of interactions carried by a quantum of mass m. Such interactions are expected to have a coherence length of the order found in the propagator of a Yukawa-like particle, $\lambda_m = \frac{\hbar}{mc}$ The DN argument applied to gravitation does allow us to partition 1 Planck mass worth of energy into $\mathcal{Z}_m$ interactions within the Compton wavelength λ_m. One way of examining Dyson's argument is to note that if one has $\mathcal{Z}_{e^2} \cong 137$ photons of approriate energy incident on an electron, all within its Compton wavelength λ_{m_e}, we expect a high likelihood of pair creation. By analogy, if there are $\mathcal{Z}_m$ coherent masses m within λ_m, there is high likelihood of the production of a scalar mass m.

2.3 Coherent Gravitating Matter

As noted in the Introduction, our approach is to examine the physical principles that we feel most comfortable using, and then extrapolate those principles back to the earliest period in the evolution of the universe for which this comfort level persists. Those conclusions that can be deduced from these principles will in this sense be model independent. We will refrain from engaging in constructing micro-cosmological models during earlier stages.

Following Jones we have associated a Planckton (cf. Sec.2.1) with a region that has quantum coherent energy of one Planck mass M_P. There are expected to be many Planck units of energy within a scale radius of the universe. Planckta will be considered to be internally coherent units that become incoherent with each other during the period of de-coherence.

In general, we define ϵ as a gravitational energy scale associated with the scale factor R of the FRW metric. On a per Planckton basis, the average number $\mathcal{Z}_\epsilon$ of (virtual quantum) energy units $\epsilon < M_P$ that are localizable within a region of quantum coherence $R_\epsilon \sim 1/\epsilon$ is given by

$$\mathcal{Z}_\epsilon \epsilon \; = \; M_P \Rightarrow \epsilon \sim \frac{R_\epsilon}{L_P} = R_\epsilon M_P \tag{2.11}$$

Note that for us this understanding of the meaning of $\mathcal{Z}_\epsilon$ *replaces* the Jones "inflationary" definition motivated by Microcosmology. This allows us to *start* our "cosmological clock" at a *finite* time calculated by backward extrapolation to the transition. In this paper we need not consider "earlier" times or specify a specific $t = 0$ achievable by backward extrapolation.

The localization of interactions has to be of the order $\hbar/mc$ in order to be able to use the DNJ argument. This allows us to use the argument just presented to obtain the number of gravitational interactions that can occur within a Compton wavelength of the mass m which provide sufficient energy to create a new mass or cause gravitational collapse, namely $\mathcal{Z}_m \sim \frac{M_P^2}{m^2}$. The process of de-coherence occurs when there are a sufficient number of available degrees of freedom such that gravitational interactions of quantum coherent states of Friedman-Lemaitre (FL) matter-energy could have a DNJ anomaly. We assume that a Planck mass M_P represents the largest meaningful scale for energy transfer at this boundary. A single Planckton of coherent energy in a scale of R_ϵ will have $\mathcal{Z}_\epsilon$ partitions of an available Planck mass of energy that can constitute interactions of this type. Since there is global gravitational de-coherence at later times, the quantity $\mathcal{Z}_\epsilon$ can only be calculated prior to and during de-coherence.

When the number of partitions of a given Planckton energy unit equals the DNJ limit, in principle there could occur an energetic breakdown of the DNJ type involving interaction energies equal to a Planck mass (or less than a Planck mass at later times), thus allowing us to conclude that de-coherence gives a mass scale from the relationship

$$\mathcal{Z}_\epsilon = \mathcal{Z}_m \equiv \mathcal{Z} \tag{2.12}$$

Expressed in terms of the energy scales, this gives the fundamental equation connecting (via the current value of Ω_Λ) the observable parameter ϵ to the mass scale at

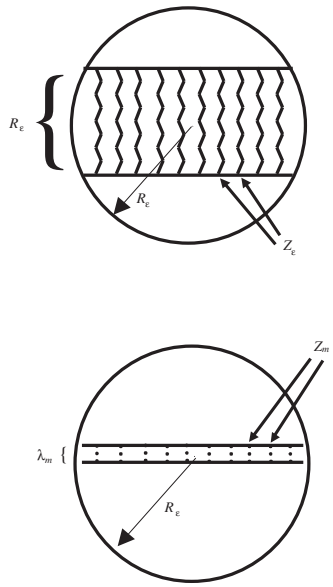

Figure 3: Counting of gravitating quanta during de-coherence

decoherence m

$$m^2 \approx \epsilon M_P \tag{2.13}$$

This connection is a succinct summary of Jones' theory; henceforth we will refer to it as the *Jones equation*. Note that, viewed in this way, we no longer need the (Jones) thermodynamic Eqn. 2.6 to derive Eqn. 2.13. Therefore the temperature scale we called m_θ need no longer be *directly* identified with the particulate mass m which is associated with our *quantum decoherence* transition.

2.4 Correspondence with the Measured Cosmological Constant

We use the present day measurement of the cosmological density ρ_Λ to determine the energy scale m of cosmological de-coherence. The present cosmological constant energy density ρ_Λ is usually given in terms of the critical density ρ_c and the reduced Hubble parameter h

$$\rho_c \equiv \frac{3H_o^2}{8\pi G_N} \approx 1.0537 \times 10^{-5}\, h^2\, GeV/cm^3. \tag{2.14}$$

As already noted, we will take the value of the reduced Hubble parameter to be given by $h = 0.73$; current estimates of the reduced cosmological constant energy density $\Omega_\Lambda \equiv \frac{\rho_\Lambda}{\rho_c} \approx 0.73$, which means that the vacuum energy sets the de-coherence scale as

$$\rho_\Lambda \;\approx\; 4.10 \times 10^{-6} \frac{GeV}{cm^3} \approx 3.13 \times 10^{-47} GeV^4 \sim \epsilon^4 \tag{2.15}$$

Using the value $\hbar c \cong 1.97 \times 10^{-14} GeV\, cm$ we can immediately calculate the de-coherence scale energy and FRW scale radius

$$\epsilon \;\sim\; 10^{-12} GeV \qquad , \qquad R_\epsilon \;\sim\; 10^{-2} cm. \tag{2.16}$$

The Planck energy scale and DNJ limit at this scale is given by

$$\mathcal{Z}_\epsilon \;\equiv\; \frac{M_P}{\epsilon} \;\sim\; 10^{30} \;\sim\; \mathcal{Z}_m \tag{2.17}$$

The equality of the interaction factors $\mathcal{Z}$ gives the Jones equation $m^2 = \epsilon M_P$ from which we calculate a value for the mass scale for quantum de-coherence

$$m \;\sim\; 5\, TeV/c^2 \tag{2.18}$$

The number of Planck energy units per scale volume during de-coherence is given by

$$N_{Pk} \;\sim Z^2 \;\sim\; 10^{60}. \tag{2.19}$$

At this point we will examine the Hubble rate equation during this transition. If we substitute the expected energy density into the Freedman-Lamaitre equation, we obtain a rate of expansion given by

$$\dot{R}_\epsilon \;\sim\; R_\epsilon \sqrt{\frac{8\pi G_N}{3}(\rho_m + \rho_\Lambda)} \;\sim\; \frac{1}{\epsilon}\sqrt{\frac{1}{M_P^2}\mathcal{Z}_\epsilon^2 \epsilon^4} \;\sim\; 1. \tag{2.20}$$

This is a very interesting result, which implies that the transition occurs near the time that the expansion rate is the same as the speed of light. In Section 3 on dark energy de-coherence, we will develop this argument as the primary characteristic of this transition.

2.5 Gravitating massive scalar particle

If the mass m represents a universally gravitating scalar particle, we expect the coherence length of interactions involving the particle to be of the order of its Compton wavelength with regards to Yukawa-like couplings with other particulate degrees of freedom present. If the density is greater than

$$\rho_m \sim \frac{m}{\lambda_m^3} \sim m^4 \tag{2.21}$$

we expect that regions of quantum coherence of interaction energies of the order of m and scale λ_m will overlap sufficiently over the scale R_ϵ such that we will have a macroscopic quantum system on a universal scale. As long as the region of gravi-

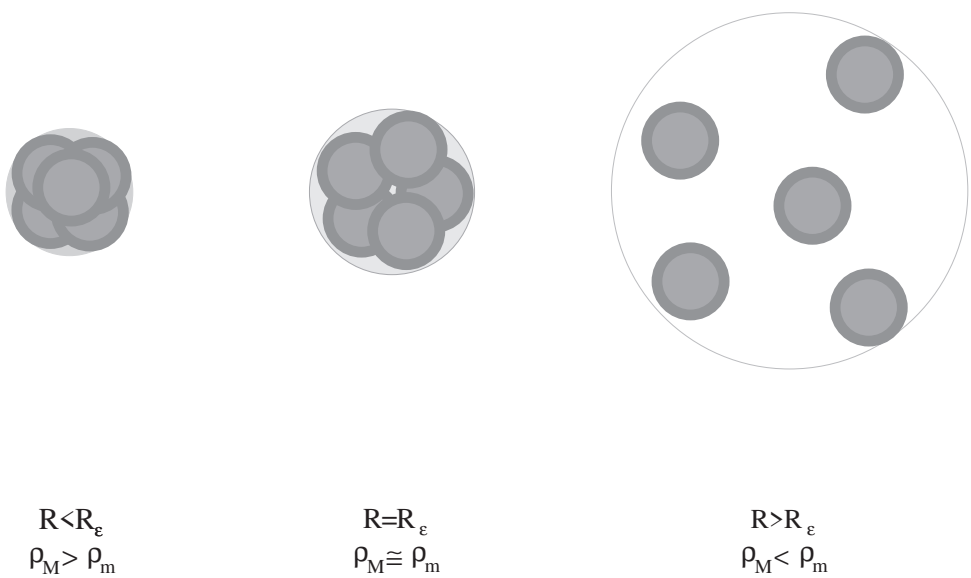

Figure 4: Overlapping regions of coherence during expansion

tational coherence is of FRW scale $R < R_\epsilon$, cosmological (dark) vacuum energy is determined by this scale. However, when the density of FL energy becomes less than ρ_m, we expect that since the coherence length of the mass m given by its Compton wavelength is insufficient to cover the cosmological scale, the FL energy density will break into domains of cluster decomposed (AKLN de-coherent[21]) regions of local quantum coherence. This phase transition will decouple quantum coherence of gravitational interactions on the cosmological scale R_ϵ. At this stage (de-coherence), the

cosmological (dark) vacuum energy density ρ_Λ is frozen at the scale determined by R_ϵ. The cosmological dark energy contribution to the expansion rate is so small, and its coupling to de-coherent FL energy so insignificant, that its value is frozen at the value just prior to de-coherence given by

$$\rho_\Lambda \sim \frac{\epsilon}{R_\epsilon^3} = \epsilon^4 \tag{2.22}$$

We should note that using the DNJ argument, if the mass m were engaged in active cosmological energy exchanges involving non-gravitational microscopic interactions prior to decoherence, then, since those interactions have considerably larger coupling constants, it can be concluded that $\mathcal{Z}_g = \frac{1}{g^2} < \mathcal{Z}_m$. This would mean that this interaction would have broken coherence prior to our expected gravitational de-coherence event. We expect the mass m to be dark during this period (as must be all other particles).

During de-coherence, we assume that the FL energy contained in R_ϵ is given by N_{Pk} Planck mass units appropriately red-shifted to the de-coherence epoch. This gives an intensive FL energy density (for a spatially flat universe) of the form

$$\rho_{FL} \sim N_{Pk}\epsilon^4 \tag{2.23}$$

Since de-coherence is expected to occur when this density scale is given by the quantum coherence density scale for the mass by ρ_m, we obtain the following relationship between the de-coherence energy scale ϵ and the scalar mass m:

$$N_{Pk}\epsilon^4 \cong m^4 \tag{2.24}$$

This allows us to consistently relate the number of Plancktonic energy units in the region of coherence R_ϵ to the DN counting parameters:

$$N_{Pk} \cong \frac{m^4}{\epsilon^4} = \frac{m^4}{M_P^4}\frac{M_P^4}{\epsilon^4} = \frac{\mathcal{Z}_\epsilon^4}{\mathcal{Z}_m^2} = \mathcal{Z}^2 \tag{2.25}$$

This insures that all quantities relevant to our theory can be reduced to a single parameter in the Jones equation $m^2 \cong \epsilon M_P$.

It has already been suggested[11] that if we identify the Jones mass parameter m with a massive, scalar gravitating particle, this could be a candidate for particulate

dark matter. Unfortunately if we assume that the mass m interacts *only* gravitationally, such a particle would be difficult to discover in accelerator experiments due to the extremely small coupling of gravitational scale forces.

We can estimate the expected dark matter to photon number ratio from available phenomenological data if the mass is known. The FL equations satisfy energy conservation $T^{\mu\nu}_{;\nu} = 0$, which implies $\dot{\rho} = -3H(\rho + P)$. The first law of thermodynamics relates the pressure to the entropy density $\dot{P} = \frac{S}{V}\dot{T}$, which then implies an adiabaticity condition on the expansion given by

$$\frac{d}{dt}\left(\frac{S}{V}R^3\right) \;=\; 0. \tag{2.26}$$

Assuming adiabatic expansion, we expect $g(T)\,(RT)^3$ to be constant far from particle thesholds. Here, $g(T)$ counts the number of low mass particles contributing to the cosmological entropy density at temperature T. This gives a red shift in terms of the photon temperature during a given epoch

$$\frac{R_o}{R} \;\equiv\; 1 + z \;=\; (1 + z_{dust})\left(\frac{g(T)}{g_{dust}}\right)^{1/3}\frac{T}{T_{dust}}. \tag{2.27}$$

In terms of photon temperature, we can count the average number of photons using

$$\frac{N_\gamma}{N_{\gamma\,o}} \;=\; \frac{(R\,T)^3}{(R_o\,T_o)^3} \;\cong\; \frac{g_o}{g(T)}. \tag{2.28}$$

This allows us to write a formula for the dark matter - photon ratio at the temperature of dark matter number conservation, in terms of its mass and the measured baryon-photon ratio:

$$\frac{N_{dm}}{N_\gamma} \;\cong\; \frac{\Omega_{dm}}{\Omega_{baryon}}\frac{N_{b\,o}}{N_{\gamma\,o}}\frac{m_N}{m}\frac{g(T)}{g_o}, \tag{2.29}$$

which gives $\frac{N_{dm}}{N_\gamma} \;\cong\; 2.9 \times 10^{-9}\,\frac{g(T_{freeze})}{g_o}\,\frac{m_N}{m}$.

If there is DNJ collapse, one can estimate the lifetime of the resulting black holes. These collapsed objects would be expected to emit essentially thermal low mass quanta at a rate determined by the barrier height near the horizon $\sim MG_N$ and the wavelength of the quanta $\sim (MG_N)^{-1}$ giving a luminosity of order $dM/dt \sim -1/M^2G_N^2$. This can be integrated to give a lifetime of the order

$$t_{evaporation} \;\sim\; M^3G_N^2. \tag{2.30}$$

This means that a collapsed DNJ object has an approximate lifetime of

$$\tau_{BH} \sim \frac{(\mathcal{Z}_m m)^3}{M_P^4} \sim \mathcal{Z}_m \frac{\hbar}{mc^2} \tag{2.31}$$

Substituting the expected mass, the lifetime is expected to be $\tau_{BH} \sim 400 sec$, which is long compared to the inverse Hubble rate $H_\epsilon^{-1} \sim 10^{-13} sec$ during decoherence.

We can also estimate the number of low mass quanta that would result from the evaporation. We will examine the quantum mechanics of massive scalar particles $g_{\mu\nu}p^\mu p^\nu = -m^2$ in a Schwarzschild metric.

$$ds^2 = -(1 - \frac{2G_N M}{r})dt^2 + \frac{dr^2}{1 - \frac{2G_N M}{r}} + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2. \tag{2.32}$$

Using tortoise coordinates, $r^* \equiv r + 2G_N M log\left(\frac{r}{2G_N M} - 1\right)$, the action is taken to be

$$W = \frac{1}{2}\int dt dr^* d\theta d\phi\, r^2 \left(1 - \frac{2G_N M}{r}\right) sin\theta \left[\frac{-\chi_t^2 + \chi_{r^*}^2}{1 - \frac{2G_N M}{r}} + m^2\chi^2 + \frac{\chi_\theta^2}{r^2} + \frac{\chi_\phi^2}{r^2 sin^2\theta}\right]. \tag{2.33}$$

The equation of motion generated for the reduced function $\psi \equiv r\chi$ for stationary states is given by

$$\psi_{r^*r^*} - \left(1 - \frac{2G_N M}{r}\right)\left(m^2 + \frac{\ell(\ell+1)}{r^2} + \frac{2G_N M}{r^3}\right)\psi = \psi_{tt} = (m^2 + k_\infty^2)\psi. \tag{2.34}$$

The effective potential barrier height is seen to be of the order of the inverse Schwarzschild radius. The asymptotic solution takes satisfies $\psi_{rr}(r \to \infty) \cong -k_\infty^2\psi$, whereas the solution near the Schwarzschild radius (for s-waves) is given by $\psi_{r^*r^*}(r \to 2G_N M) \cong -(m^2 + k_\infty^2)\psi$. We expect that when the temperature is above mass threshold, the particle can be radiated, and that for temperatures above the barrier height $V_{max} \sim \frac{0.3}{2G_N M}$ the transmission rate of particles is of order $\frac{k_\infty}{\sqrt{m^2 + k_\infty^2}}$.

Writing the luminosity and number rate as

$$\begin{aligned} \frac{dM}{dt} &= \frac{\eta(M)}{M^2 G_N^2} \\ \Rightarrow\ t &= \frac{1}{3}\frac{M^3 G_N^2}{\bar{\eta}} \\ \frac{dN}{dt} &= \frac{\eta(M)}{M G_N} \cong \frac{\bar{\eta}^{2/3}}{(3G_N t)^{1/3}} \end{aligned} \tag{2.35}$$

Here $\eta(M)$ is expected to be a slowly varying function of the temperature that counts the number of low mass thermal states at the temperature of the black hole. This

factor is expected to be essentially constant between particle thresholds. The solution then takes the form

$$N(M) \cong \frac{1}{2}\frac{M^2}{M_P^2}. \tag{2.36}$$

More generally, the total number of low mass quanta resulting from evaporation from mass M to mass M' is expected to satisfy

$$N(M \to M') \cong \frac{M^2 - M'^2}{2M_P^2}. \tag{2.37}$$

If the black hole has formed due to DNJ collapse, substituting $M = \mathcal{Z}_m m$ gives

$$N(\mathcal{Z}_m m) \cong \frac{1}{2}\mathcal{Z}_m^2 \frac{m^2}{M_P^2} \cong \mathcal{Z}_m. \tag{2.38}$$

Therefore, the intermediate quanta in the collapse are expected to produce an essentially equal number of low mass quanta during evaporation.

We can estimate the relative number of quanta of mass m evaporated by a black hole formed by NJ collapse. If the mass cannot be radiated prior to temperature T_m, this ratio is given by

$$\frac{N_m}{N_{Total}} \cong \frac{1}{g(T_m)}\left(\frac{M(T > T_m)}{M}\right)^2, \tag{2.39}$$

where $g(T_m)$ is the number of low mass states available for radiation at temperature T_m. Substituting $M = Z_m m$, $T_m = \frac{1}{8\pi G_N M(T_m)} \cong m$ and $g(T_m) = \frac{427}{4}$) gives an estimate of $\frac{N_m}{N_{Total}} \approx 10^{-5}$ from each black hole thermalization.

To summarize, our re-examination of the Jones theory has led us to the conclude that the $\mathcal{Z}_\epsilon = \mathcal{Z}_m$ relation is best interpreted in that context as the equality of the (intensive) number of gravitational quanta of mass m exchanged between all gravitating systems between the cosmological scale R_ϵ and the particulate scale λ_m, when the DNJ bound $\mathcal{Z}m \to (\mathcal{Z}+1)m$ is reached. One way of examining Dyson's argument is to note that if one has $\mathcal{Z}_{e^2} \cong 137$ photons of approriate energy incident on an electron, all within its Compton wavelength λ_{m_e}, we expect a high likelihood of pair creation. By analogy, if there are $\mathcal{Z}_m$ coherent masses m within λ_m, there is high likelihood of the production of a scalar mass m. This interpretation *requires* us to be talking about *quantum coherent systems* when the Jones transition from

microcosmology to a universe where we can use conventional physics and cosmology takes place. This line of reasoning suggested to us that this transition itself *must* in some sense correspond to *quantum decoherence* and to the title of this paper. The consequences of pursuing this line of thought constitute the rest of this paper.

3 Dark Energy De-coherence

We will now make quantitative arguments to develop the general ideas motivated by the previous sections. In most of what follows we will assume flat spatial curvature $k = 0$. Prior to the scale condition $\dot{R}_\epsilon = c$, which we will henceforth refer to as the time of dark energy decoherence, gravitational influences are propagating (at least) at the rate of the gravitational scale expansion, and microscopic interactions (which can propagate no faster than c) are incapable of contributing to cosmological scale equilibration. Since the definition of a temperature requires an equilibration of interacting "microstates", there must be some mechanism for the redistribution of those microstates on time scales more rapid than the cosmological expansion rate, which can only be gravitational.

3.1 Dark Energy

As we have discussed in the motivation section, we expect that dark energy de-coherence occurs when the FRW scale is $R_\epsilon \sim 1/\epsilon$. The gravitational dark energy scale associated with de-coherence is given by ϵ, independent of the actual number of energy units N_ϵ in the scale region. Since the dark energy density must be represented by an intensive parameter which should be the same for the universe as a whole, we will express this density as the coherent vacuum state energy density of this macroscopic quantum system. In the usual vacuum state, the equal time correlation function

$$< vacuum|\Psi(x, y, z, t)\, \Psi(x', y', z', t')|vacuum >$$

does not vanish for space-like separations. (For example, for massless scalar fields, this correlation function falls off with the inverse square of the distance between the

points). Since we assume no physical distinction between spatially separated points, our correlation functions would be expected to be continuous, with periodic boundary conditions defined by the cosmological scale factor. Given a cosmological scale factor R_ϵ, periodic boundary conditions on long range (massless or low mass) quanta define momentum quantization in terms of this maximum wavelength. The energy levels associated with these quanta would satisfy the usual condition

$$E_{N_\epsilon} \;=\; \left(N_\epsilon + \frac{1}{2}\right)\hbar\omega \;=\; \left(N_\epsilon + \frac{1}{2}\right)\frac{hc}{\lambda_\epsilon} \;=\; (2N_\epsilon + 1)\,\epsilon. \tag{3.40}$$

This is associated with quanta of wavelength of the order of the cosmological scale factor with vacuum energy density, given by

$$\rho_\Lambda \;\equiv\; \frac{\epsilon}{(2R_\epsilon)^3} \;=\; \left(\frac{k_\epsilon}{2\pi}\right)^3 \frac{\sqrt{m^2_{condensate} + k^2_\epsilon}}{2} \tag{3.41}$$

for a translationally invariant universe with periodicity scale $2R_\epsilon$. In effect, this provides the infrared cutoff for cosmological quantum coherent processes,

$$k_\epsilon \;=\; \frac{2\pi}{\lambda_\epsilon} \;=\; \frac{\pi}{R_\epsilon}. \tag{3.42}$$

If, for instance, one chooses the wave vector scale for observed power spectrum fluctuations of the order $k_* \approx 10^{-2}/Mpc$, and assumes a temperature scale associated with the condensate mass m, equation 3.41 predicts a mass scale for the condensate of the order $m \sim 10^9 GeV$.

The vacuum energy scale associated with a condensate of gravitationally coherent massless quanta is given by

$$\epsilon \;=\; \frac{1}{2}\hbar\omega_\epsilon \;=\; \frac{1}{2}\hbar k_\epsilon c \;=\; \frac{\pi}{2}\frac{\hbar c}{R_\epsilon}, \tag{3.43}$$

and the vacuum energy density for such massless quanta

$$\rho_\Lambda \;=\; \frac{\epsilon^4}{(\pi)^3}. \tag{3.44}$$

We might inquire into the nature of the dark energy, in the sense as to whether it is geometric or quantum mechanical in origin. From the form of Einstein's equation

$$\mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R} \;=\; \frac{8\pi G_N}{c^4}T_{\mu\nu} + \Lambda g_{\mu\nu} \qquad , \qquad (\Lambda g^{\mu\nu})_{;\nu} = 0 \tag{3.45}$$

if the term involving the cosmological constant should most naturally appear on the left hand side of the equation, we would consider it to be geometric in origin. If the cosmological term is geometric in origin, we would expect it to be a fundamental constant of the cosmology which scales with the FRW/FL cosmology consistently with the vanishing divergence of the Einstein tensor. However, if the previous arguments are interpreted literally, the dark energy density freezes out to a constant determined by the period of last quantum coherence with the FL energy density and the onset of the equilibration of states involving microscopic non-gravitational interactions, supporting its interpretation as a gravitational quantum vacuum energy density. This means that it is fixed by a physical condition being met, and thus would not be a purely geometric constant.

3.2 Rate of Expansion during De-Coherence–massless condensate

We will next examine the rate of the expansion during the period of de-coherence. We examine the Friedmann-LeMaitre (FL)/Hubble equations, which relates the expansion rate and acceleration to the densities.

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G_N}{3}\left(\rho + \rho_\Lambda\right) - \frac{k}{R^2} \tag{3.46}$$

$$\frac{\ddot{R}}{R} = -\frac{4\pi G_N}{3}(\rho + 3P - 2\rho_\Lambda) \tag{3.47}$$

where we have written

$$\rho_\Lambda = \frac{\Lambda}{8\pi G_N}, \tag{3.48}$$

and ρ represents the FL energy density. The only scale dependent term in this equation involves the spatial curvature k. If k is non-vanishing, we have no reason to assume that any scale other than R_ϵ de-coherence determines the cosmological scale. In our discussion, the dark energy density will have negligible contribution to the FL expansion during de-coherence, but will become significant as the FL energy density decreases due to the expansion of the universe.

It is unclear whether one can speak of causal horizons and causal communications in the usual ways prior to the period of de-coherence, since the scale expansion rate is larger than c. Assuming that local inertial physics satisfies the principle of equivalence with a limiting velocity of c, it would be difficult to extrapolate the type of physics we do presently into a domain with local expansion rates greater than c. Only the FRW gravitation interacts with rates which can equilibrate states defining a thermal system in this domain, since the other interactions cannot have superluminal exchanges, only superluminal quantum correlations. If the expansion rate is superluminal $\dot{R} > c$, scattering states cannot form decomposed (de-coherent) clusters of the type described in reference [21]. We see from the above discussion that, assuming the validity of an FL universe back to the stage of de-coherence, our usual ideas of microscopic causality become obscure beyond this period.

Since we find the expansion rate equation $\dot{R}_\epsilon = c$ a compelling argument for the quantitative description of gravitational de-coherence, it is this relationship that we will use to determine the form for the energy density during dark energy de-coherence ρ_{FL}, which counts the number of gravitating quanta above vacuum energy in the condensed state. The Hubble equation takes the form

$$H_\epsilon^2 = \left(\frac{c}{R_\epsilon}\right)^2 = \frac{8\pi G_N}{3}\left(\rho_{FL} + \rho_\Lambda\right) - \frac{kc^2}{R_\epsilon^2} = \frac{8\pi G_N}{3}\left(2N_\epsilon + 1\right)\rho_\Lambda - \frac{kc^2}{R_\epsilon^2}. \tag{3.49}$$

We see that $2N_\epsilon$ counts the number of Jones-Planck energy units per scale factor in the pre-coherent universe (referred to by Jones as $N_{Planckton}$).

If this condition is to describe the onset of dark energy de-coherence, we can see that a so called "open" universe ($k = -1$) is excluded from undergoing this transition. In this case, the cosmological constant term in equation 3.46 already excludes a solution with $\dot{R}_\epsilon \leq c$.

Likewise, for a "closed" universe that is initially radiation dominated, we can compare the scale factors corresponding to $\dot{R}_\epsilon = c$ and $\dot{R}_{max} = 0$. From the Hubble equation

$$\frac{1}{R_{max}^2} = \frac{8\pi G_N}{3}(\rho + \rho_\Lambda) \cong \frac{8\pi G_N}{3}\rho_\epsilon \frac{R_\epsilon^4}{R_{max}^4} \Rightarrow R_{max}^2 = 2R_\epsilon^2. \tag{3.50}$$

Clearly, this closed system never expands much beyond the transition scale. Henceforth, we will only consider flat spaces.

We will assert that de-coherence cannot occur prior to $\dot{R} = c$ since incoherent decomposed clusters [22] cannot be cosmologically formulated. Using the equation

$$\left(\frac{c}{R_\epsilon}\right)^2 = \frac{8\pi G_N}{3}\left(2N_\epsilon + 1\right)\rho_\Lambda \tag{3.51}$$

and the form of ρ_Λ from equation 3.41 we can directly determine number of quanta in the condensed state

$$N_\epsilon = \frac{1}{2}\left(\frac{3}{2}\frac{M_P^2}{\epsilon^2} - 1\right) \cong \frac{3}{4}\mathcal{Z}_\epsilon^2 \tag{3.52}$$

where, as before $\mathcal{Z}_\epsilon \equiv \frac{M_P}{\epsilon}$. The energy density during dark energy decoherence is therefore given by

$$\rho_{FL} = 2N_\epsilon\rho_\Lambda \cong \frac{3}{2\pi^3}M_P^2\epsilon^2. \tag{3.53}$$

3.3 Phenomenological correspondence–massless condensate

To make correspondence with observed cosmological values, we will utilize parameters obtained from the Particle Data Group[5]. We will use the value for the critical density given by

$$\rho_c \equiv \frac{3H^2}{8\pi G_N} \cong 5.615 \times 10^{-6}\, GeV/cm^3 \cong 4.293 \times 10^{-47} GeV^4. \tag{3.54}$$

The dark energy cosmological density parameter $\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c}$ is taken to have the value $\Omega_\Lambda \cong 0.73$. We will therefore use the value

$$\rho_\Lambda \cong 4.099 \times 10^{-6} GeV/cm^3 \cong 3.134 \times 10^{-47} GeV^4. \tag{3.55}$$

This gives a dark energy scale and FRW scale given by

$$\epsilon \cong 5.58 \times 10^{-12}\, GeV \tag{3.56}$$

$$R_\epsilon \cong 5.54 \times 10^{-3}\, cm. \tag{3.57}$$

The Friedman-Lemaitre energy density from equation 3.53 is then given by

$$\rho_{FL} \cong 2.93 \times 10^{55}\, GeV/cm^3 \cong 2.24 \times 10^{14}\, GeV^4, \tag{3.58}$$

with the Planck energy partition $\mathcal{Z}_\epsilon$ and number of "gravons" N_ϵ at dark energy de-coherence given by

$$\mathcal{Z}_\epsilon \cong 2.19 \times 10^{30} \tag{3.59}$$

$$N_\epsilon \cong 3.59 \times 10^{60}. \tag{3.60}$$

The coherent mass density scale $\frac{m}{\lambda_m^3} = m^4$ corresponding to the FL density ρ_{FL} is given by $m \sim 3800\, GeV/c^2$. However, we will obtain a precise determination of the coherent mass scale in terms of the UV cutoff scale of the gravitational dynamics when we discuss the thermal ground state shortly.

We will next estimate the time of dark energy de-coherence assuming a radiation dominated expansion prior to this period. Although we are dubious about using a standard radiation dominated equation of state prior to dark energy de-coherence, we can get some feeling for the time scale of this transition. The period after de-coherence is radiation dominated until the dust driven epoch, with the scale factor satisfying

$$R(t) = R_\epsilon \left(\frac{t}{t_\epsilon}\right)^{1/2}. \tag{3.61}$$

This means that $\frac{\dot{R}(t)}{R(t)} = \frac{1}{2t}$, resulting in an estimate for the time

$$t_\epsilon \cong \frac{R_\epsilon}{2c} \cong 9.30 \times 10^{-14}\, sec. \tag{3.62}$$

This gives a Hubble rate

$$H_\epsilon = \frac{c}{R_\epsilon} \cong 5.38 \times 10^{12}/sec \cong (1.86 \times 10^{-13} sec)^{-1}. \tag{3.63}$$

We can use this Hubble rate to estimate the minimal lifetime of any gravitating mass scale m that can equilibrate during de-coherence. If the mass is to have meaningful coherence during the period of de-coherence, it lifetime in the thermal bath must be of an order greater than the inverse Hubble rate. This means that

$$\tau_m > \frac{1}{H_\epsilon} \sim 10^{-13} sec \tag{3.64}$$

If the mass scale is associated with the Higgs scalar of the symmetry breaking, this mass could ONLY couple to electro-weak bosons to generate mass, since the Yukawa

coupling to masses comparable to the top quark mass would give a width well in excess of this scale.

The estimates are only slightly modified for vector and tensor gravons. Substituting spin degeneracy corresponding to the particle type, the scale factor of de-coherence becomes $R_V \cong 7.3 \times 10^{-3} cm$ for vector gravons, and $R_T \cong 8.3 \times 10^{-3} cm$ for tensor gravons, with the other calculated quantities varying accordingly. We will assume scalar quanta for our further calculations.

3.4 Thermal Ground State

For a hot, thermal system, the ground state is not chosen to be that state which satisfies $\hat{N}|0>= 0$ for all modes (zero occupation), but instead is constructed of a thermal product of occupation number states, weighted by a density matrix. Unlike the zero occupation number state, this ground state need not generally be time translationally invariant. Examining the low energy modes at high temperatures, the thermally averaged occupation of those modes $< \hat{N}_n >\cong \frac{k_B T}{E_n}$ demonstrates large numbers of low energy massless quanta, giving these modes a large number of degrees of freedom. For our system, there are natural infrared and ultraviolet cutoffs provided by the macroscopic scale ϵ and microscopic scale m. We expect macroscopic gravitational physics involving gravitating masses m to be cutoff for momenta $k_{UV} \sim m$.

It is of interest to calculate the energy of the zero-occupation number state using these cutoffs,

$$\hat{H}|0,0,...,0> = \sum_{\vec{k}=\vec{k}_\epsilon}^{\vec{k}_{UV}} \frac{1}{2}\hbar c\hat{k}\,|0,0,...,0> . \tag{3.65}$$

Inserting the density of states to approximate the sum gives an energy density of the form

$$\frac{E_{|0,...,0>}}{V} \sim \frac{1}{(2\pi)^3}\int_{k_\epsilon}^{m} \frac{1}{2}\hbar ck\, 4\pi k^2 dk \sim m^4 - \epsilon^4 \sim \rho_{FL}, \tag{3.66}$$

which is essentially the Jones equilibrium condition equation 2.6. This means that the vacuum energy density corresponding to zero occupancy of the gravitational modes corresponds to the energy density of the normal gravitating matter just after deco-

herence if the ultraviolet cutoff of the long range modes in the superfluid is chosen to be the mass scale m. We will therefore proceed recognizing that the mass scale provides an ultraviolet gravitational cutoff for the decoherent cosmology.

As has been previously discussed, the vacuum energy associated with the condensate is given by $\rho_\Lambda = \epsilon/(2R_\epsilon)^3$, which for a massless condensate gives $\rho_\Lambda = \epsilon^4/\pi^3$. However, once the expansion rate is sub-luminal, global gravitation coherence is expected to be broken due to interactions that propagate at the speed of light. This means that all available modes must thereafter be included in calculations of the vacuum energy. As is the case with superfluids, we will assume that there is an ultraviolet cutoff associated with the (scalar) mass scale m with coherence length $\lambda_m = \hbar/mc$. The vacuum energy density associated with this cosmology transitions during dark energy de-coherence from ρ_Λ (at pre-coherence) to ρ_{vac} (at de-coherence) given by

$$\begin{aligned} \rho_{vac} &= \tfrac{E_{vacuum}}{V_\epsilon} = \int \tfrac{g_m}{2}\hbar c|\vec{k}|\tfrac{d^3k}{(2\pi)^3} \\ \tfrac{\hbar c}{(2\pi)^2}\int_{k_\epsilon}^{k_m} k^3 dk &= \tfrac{1}{4}\tfrac{\hbar c}{(2\pi)^2}(k_m^4 - k_\epsilon^4), \end{aligned} \tag{3.67}$$

where $k_m = \frac{2\pi}{\lambda_m} = \frac{2\pi mc}{\hbar}$, $k_\epsilon = \frac{\pi}{R_\epsilon} = \frac{2\epsilon}{\hbar c}$, and the spin degeneracy g_m will be taken to be unity. Therefore, the vacuum energy at de-coherence is taken to be

$$\rho_{vac} = \pi^2\left(m^4 - \left(\frac{\epsilon}{\pi}\right)^4\right) \cong \pi^2 m^4. \tag{3.68}$$

If we presume minimal parametric input to this model, then this vacuum energy thermalizes as the FL energy density in equation 3.49 for the cosmology $\rho_{FL} = \rho_{vac}$, giving a relationship for the mass scale of dark energy de-coherence

$$m^4 = \frac{3}{2\pi^5}M_P^2\epsilon^2. \tag{3.69}$$

This gives an expected mass scale given by

$$m \cong 2183 GeV/c^2. \tag{3.70}$$

If g_m is the spin degeneracy associated with m, then the left hand side of equation 3.70 is modified by a factor of $g_m^{1/4}$.

The introduction of a gravitational mass scale associated with de-coherence introduces the possibility that the vacuum energies should be calculated in terms of

the vacuum states of this mass rather than in terms of the long range excitations (gravons) treated previously. More generally, the mass scale associated with the condensate need not be the same as that of the ultraviolet cutoff, which introduces another mass scale. In the present context, we will associate these two scales as identical. Thus far, there is nothing in our discussion preventing the use of vacuum energy as

$$\epsilon_m \equiv \frac{1}{2}\sqrt{m^2 + k_\epsilon^2} = \frac{1}{2}\sqrt{m^2 + \left(\frac{\pi}{R_\epsilon}\right)^2}. \tag{3.71}$$

The post-decoherence vacuum energy then is given in general by

$$\rho_{vac} = \frac{E_{vacuum}}{V_\epsilon} = \int_{k_\epsilon}^{k_m} g_m \frac{\sqrt{m^2 + |\vec{k}|^2}}{2} \frac{4\pi|\vec{k}|^2 dk}{(2\pi)^3}, \tag{3.72}$$

which can be used to solve for the mass m by setting $\rho_{vac} = \rho_{FL}$.

3.5 Phenomenological correspondence–massive condensate

We will recalculate the phenomenological parameters for a pre-coherent condensate of massive particles of mass m. The self-consistent mass that satisfies the condition $\rho_{vac} = \rho_{FL}$ is given by $m \cong 19,73 GeV$. The dark energy scale and FRW scale is given by

$$\epsilon \cong 9.86\, GeV \tag{3.73}$$

$$R_\epsilon \cong 67.1\, cm. \tag{3.74}$$

The Friedman-Lemaitre energy density from equation 3.53 is then given by

$$\rho_{FL} \cong 2.00 \times 10^{47}\, GeV/cm^3 \cong 1.53 \times 10^6\, GeV^4, \tag{3.75}$$

with the Planck energy partition $\mathcal{Z}_\epsilon$ and number of "gravons" N_ϵ at dark energy de-coherence given by

$$\mathcal{Z}_\epsilon \cong 1.24 \times 10^{18} \tag{3.76}$$

$$N_\epsilon \cong 2.45 \times 10^{52}. \tag{3.77}$$

The time estimate for a radiation dominated cosmology is given by

$$t_\epsilon \cong \frac{R_\epsilon}{2c} \cong 1.13 \times 10^{-9}\, sec. \tag{3.78}$$

This gives a Hubble rate

$$H_\epsilon \;=\; \frac{c}{R_\epsilon} \;\cong\; 4.44 \times 10^8/sec \;\cong\; (2.25 \times 10^{-9} sec)^{-1}. \tag{3.79}$$

Again, the estimates are only slightly modified for vector and tensor masses. Substituting spin degeneracy corresponding to the particle type, the scale factor corresponding to de-coherence becomes $R_V \cong 62.0cm$ for vector masses $m \cong 15.59$ GeV, and $R_T \cong 59.8cm$ for tensor masses $m \cong 13.97$ GeV, with the other calculated quantities varying accordingly. We will assume scalar masses for our calculations.

3.6 Thermalization

We will next examine the thermalization of the coherent gravitating cosmology into the familiar particulate states. De-coherence is presumed to occur adiabatically into a radiation dominated cosmology. For each low mass particle state, the standard black body relationships are satisfied:

$$\frac{U}{V} \;=\; g\frac{\pi^2}{30}\left(\frac{k_B T}{\hbar c}\right)^3 k_B T \;=\; \rho\, c^2 \;=\; 3P \tag{3.80}$$

$$\frac{S}{V} \;=\; g\frac{2\pi^2}{45} k_B \left(\frac{k_B T}{\hbar c}\right)^3 \tag{3.81}$$

$$\frac{N}{V} \;=\; g^* \frac{\zeta(3)}{\pi^2}\left(\frac{k_B T}{\hbar c}\right)^3 \tag{3.82}$$

where the statistical factors are given by

$$g/\#\, spin\, states = \begin{cases} 1 & \text{bosons} \\ \frac{7}{8} & \text{fermions} \end{cases} \quad , \quad g^*/\#\, spin\, states = \begin{cases} 1 & \text{bosons} \\ \frac{3}{4} & \text{fermions} \end{cases} \tag{3.83}$$

The temperature of radiation with density ρ_{FL} is given by

$$\rho_{FL} \;=\; g(T_\epsilon)\frac{\pi^2}{30}\frac{(k_B T_\epsilon)^4}{(\hbar c)^3} \;\Rightarrow\; k_B T_\epsilon \;\cong\; \frac{T_{crit}\, GeV}{g^{1/4}(T_\epsilon)} \tag{3.84}$$

For a massless condensate, $T_{crit} \cong 5111$ GeV, and for temperatures above top quark mass, the degeneracy factor $g(T > m_{top}c^2/k_B) = \frac{429}{4}$ is relatively weakly dependent

upon any new degrees of freedom. To a few percent, the temperature of de-coherence is determined to be

$$k_B T_\epsilon \cong 1592\, GeV. \tag{3.85}$$

For a massive condensate $T_{crit} \cong 1.3 \times 10^{15}$ GeV, and we will assume temperatures above bottom quark degrees of freedom with a degeneracy factor of $g(T > m_{top}c^2/k_B) = \frac{345}{4}$. This gives a temperature of de-coherence of

$$k_B T_\epsilon \cong 34.71\, GeV. \tag{3.86}$$

We will next estimate the present day scale corresponding to the dark energy de-coherence scale R_ϵ. We will assume a relatively sharp transition from a radiation dominated expansion to a matter dominated expansion at the dust transition red-shift, corresponding to equal energy densities of the (present day) relativistic and non-relativistic particles. We will use a value calculated from standard references[5] for $z_{eq} \equiv z_{dust} \cong 3629$, where the red shift satisfies the usual formula

$$\frac{\nu(z)}{\nu_o} \equiv 1 + z = \frac{R_o}{R(z)}. \tag{3.87}$$

After relativistic radiation falls out of equilibrium, its temperature satisfies $T \sim 1/R$. Photon fell out of equilibrium at last scattering $z \sim 1100$, whereas neutrinos fell out of equilibrium at a temperature $T \sim 1MeV$. The present cosmic background photon temperature is $2.725K \cong 2.35 \times 10^{-13} GeV$, and that of neutrinos is about 1.9K. We will calculate red shift from CMB photon temperature to de-coherence temperature.

For a massless condensate, the red shift at decoherence is found to be $z_\epsilon \approx 10^{16}$, whereas for a massive condensate $z_\epsilon \approx 10^{14}$. We can use these redshifts to determine the present scale associated with the de-coherence scale R_ϵ. For a massless condensate $R_o \approx 10^{14}$ cm, which is about the distance of Satern from Earth. For a massive condensate, this scale is given by 10^{18} cm, four orders of magnitude larger.

We next examine the entropy of the system during the de-coherence period. The Fleisher-Susskind[25] entropy limit considers a black hole as the most dense cosmological object, limiting the entropy according to

$$S \leq S_{\substack{black \\ hole}} = \frac{k_B c^3}{\hbar} \frac{A}{4G_N} \tag{3.88}$$

For a radiation dominated cosmology at de-coherence, the entropy is proportional to the number of quanta and related to the energy density $\left(\frac{c}{R_\epsilon}\right)^2 \cong \frac{8\pi G_N}{3}\rho_{FL}$ by

$$\frac{S}{V} = \frac{4}{3}\frac{\rho_{FL}}{T_\epsilon} \Rightarrow S = \frac{4}{\pi G_N}\frac{R_\epsilon}{T_\epsilon}. \tag{3.89}$$

Examining this for the space-like area given by the box $a = 6(2R_\epsilon)^2$ the ratio of the entropy in a thermal environment to the limiting entropy during thermalization is given by

$$\frac{S}{A/4G_N} \cong \frac{2}{3\pi}\left(\frac{1}{R_\epsilon T_\epsilon}\right) \sim 10^{-16}. \tag{3.90}$$

Clearly this result satisfies the FS entropy bound regardless of the mass of the condensate.

subsectionBose condensation

We next calculate the critical temperature for condensation of a non-interacting gas of massless Bose quanta just prior to dark energy decoherence. At temperature T, such a gas has energy density satisfying the relation

$$\rho = \rho_{GS} + \frac{\pi^2}{30}\left(\frac{k_B T}{\hbar c}\right)^3 k_B T. \tag{3.91}$$

Critical temperature is defined when the second term is insufficient to contain all particles. For the pre-coherent state, this is given by

$$\rho_{FL} = \frac{\pi^2}{30}\frac{(k_B T_{crit})^4}{(\hbar c)^3}, \tag{3.92}$$

where $\rho_{FL} = \frac{3}{8\pi}\left(\frac{M_P}{R_\epsilon}\right)^2 - \rho_\Lambda$ as before. The critical temperature therefore satisfies $T_{crit} = (g(T_\epsilon))^{\frac{1}{4}} T_\epsilon$. This corresponds to a temperature of around $T_{crit} \approx 5109$ GeV for a pre-thermalized system consisting of only gravons. We expect the system to remain in a zero temperature state prior to de-coherence, defining a vacuum energy density ρ_Λ just prior to de-coherence. The thermodynamics after the availability of sub-luminal degrees of freedom will define the temperature of thermalization using

$$\rho_{FL} = (g(T_\epsilon) + 1)\frac{\pi^2}{30}\frac{(k_B T_\epsilon)^4}{(\hbar c)^3} + \rho_{GS}, \tag{3.93}$$

where $g(T_\epsilon)$ counts the degrees of freedom available to luminal and sub-luminal interactions. Because of the availability of the new degrees of freedom, one expects a solution without condensate ρ_{GS} to be consistent at these temperatures.

Just as de-coherence begins, we expect the fraction of condensate to thermal gravons to satisfy

$$\frac{\rho_{condensate}}{\rho_{FL}} = 1 - \left(\frac{T_\epsilon}{T_{crit}}\right)^4 \tag{3.94}$$

$$\frac{N_{condensate}}{N_{thermal}} = 1 - \left(\frac{T_\epsilon}{T_{crit}}\right)^3, \tag{3.95}$$

where the total number of thermal gravons satisfies

$$\frac{N_{thermal}}{V_\epsilon} = \frac{\zeta(3)}{\pi^2}\left(\frac{k_B T_{crit}}{\hbar c}\right)^3. \tag{3.96}$$

The de-coherent temperature is considerably lower than this temperature due to the degrees of freedom $g(T_\epsilon)$, giving $\frac{\rho_{condensate}}{\rho_{FL}} \cong 0.99$ and $\frac{N_{condensate}}{N_{thermal}} \cong 0.97$ starting thermalization. Expressing ρ_{FL} in terms of the pre-coherent condensate, we obtain a relationship between the pre-coherent and thermal gravons, most of which initially remain in condensate form:

$$\rho_{FL} = \frac{\pi^4}{30\zeta(3)}\frac{N_{thermal}}{V_\epsilon}k_B T_{crit} = 2N_\epsilon \rho_\Lambda. \tag{3.97}$$

This gives a large ratio of pre-coherent to thermal gravons given by

$$\frac{N_\epsilon}{N_{thermal}} = \frac{\pi^4}{60\zeta(3)}\frac{k_B T_{crit}}{\rho_\Lambda V_\epsilon} \sim 1.24 \times 10^{15}. \tag{3.98}$$

Therefore, pre-coherent gravons must rapidly thermalize a large number of states.

We next examine the properties of a (non-interacting) Bose gas of particle of mass m for a system with temperature $T \lesssim m$. At temperature T, such a fluid has energy density satisfying the relation

$$\rho = \rho_{GS} + \frac{\zeta(5/2)\Gamma(5/2)}{(2\pi)^2\hbar^3}\left(2mk_B T\right)^{3/2} k_B T \tag{3.99}$$

where $\rho_{GS} = \frac{N_{condensate}}{V_\epsilon}\sqrt{m^2 + k_\epsilon^2}$. Critical temperature for the pre-coherent state is again determined when $\rho_{GS} = 0$. This corresponds to a temperature of around $T_{crit} \approx 1.31 \times 10^{15}$ GeV for a pre-thermalized system consisting of only scalar particles $m \cong 19.73$GeV. The thermodynamics after the availability of sub-luminal degrees of freedom will define the temperature of thermalization using

$$\rho_{FL} = g(T_\epsilon)\frac{\pi^2}{30}\frac{(k_B T_\epsilon)^4}{(\hbar c)^3} + \rho_{GS} + \frac{\zeta(5/2)\Gamma(5/2)}{(2\pi)^2\hbar^3}\left(2mk_B T_\epsilon\right)^{3/2} k_B T_\epsilon. \tag{3.100}$$

Again, a solution without condensate ρ_{GS} is consistent after thermalization, at a temperature of de-coherence given by 34.69 GeV.

Just as de-coherence begins, we expect the fraction on condensate to thermal scalar masses to satisfy

$$\frac{\rho_{condensate}}{\rho_{FL}} = 1 - \left(\frac{T_\epsilon}{T_{crit}}\right)^{5/2} \tag{3.101}$$

$$\frac{N_{condensate}}{N_{thermal}} = 1 - \left(\frac{T_\epsilon}{T_{crit}}\right)^{3/2}, \tag{3.102}$$

where the total number of thermal scalars satisfies

$$\frac{N_{thermal}}{V_\epsilon} = \frac{\zeta(3/2)\Gamma(3/2)}{(2\pi)^2\hbar^3}\left(2mk_BT_{crit}\right)^{3/2}. \tag{3.103}$$

As decoherence begins, the condensate density fraction is given by $\frac{\rho_{condensate}}{\rho_{FL}} \cong 1 - 4.3\times10^{-21}$ and $\frac{N_{condensate}}{N_{thermal}} \cong 1-1.1\times10^{-24}$ starting thermalization. After de-coherence, $\frac{\rho_m}{\rho_{FL}} \cong 0.0518$. The relationship between the pre-coherent and thermal scalars is then given by

$$\rho_{FL} = \frac{\zeta(5/2)\Gamma(5/2)}{\zeta(3/2)\Gamma(3/2)}\frac{N_{thermal}}{V_\epsilon}k_BT_{crit} = 2N_\epsilon\rho_\Lambda. \tag{3.104}$$

This gives a large ratio of pre-coherent to thermal gravons given by

$$\frac{N_\epsilon}{N_{thermal}} = \frac{\zeta(5/2)\Gamma(5/2)}{2\zeta(3/2)\Gamma(3/2)}\frac{k_BT_{crit}}{\rho_\Lambda V_\epsilon} \sim 1.03\times10^{14}. \tag{3.105}$$

Again, this would drive a very rapid thermalization of the excess condensed gravons.

3.7 Pre-coherence

Although up to this point we have avoided examining the cosmology prior to dark energy de-coherence, it is useful to conjecture on the continuity of the physics of this period. Since the cosmological scale expansion is supraluminal, only gravitational interactions are available for cosmological equilibrations. We will assume that the cosmological scale excitations will have energies that satisfy the usual Planck relation, only with propagation speed determined by the expansion rate:

$$E_\epsilon = h\nu = \frac{h\dot{R}}{\lambda} = \frac{h\dot{R}}{2R} = \pi\hbar H \tag{3.106}$$

For the scalar long range gravitating quanta (collective modes) discussed previously, the density of states is expected to be of the form

$$\Delta^3 n \;=\; \frac{V}{(2\pi)^3} d^3k \;=\; \frac{4\pi}{(\pi\hbar)^3}\frac{E^2 dE}{H^3} \tag{3.107}$$

If there is thermal equilibration, we therefore expect the usual forms for a scalar boson, with the substitution $\hbar c \to \hbar\dot{R}$. In particular, the energy density takes the form

$$\rho \;=\; \frac{\pi^2}{30}\frac{(k_B T)^4}{(\hbar\dot{R})^3} \;=\; \frac{\pi^2}{30}\left(\frac{1}{\hbar R}\right)^3 \frac{(k_B T)^4}{H^3}. \tag{3.108}$$

We assume that the FL equation continues to drive the dynamics, which allows substitution of the Hubble rate in terms of density

$$\rho \;=\; \frac{\pi^2}{30}\left(\frac{1}{\hbar R}\right)^3 \frac{(k_B T)^4}{\left[\frac{8\pi G_N}{3}(\rho+\rho_\Lambda)\right]^{\frac{3}{2}}}. \tag{3.109}$$

Since these gravons are expected to behave like radiation $\frac{\rho}{\rho_{DC}} = \left(\frac{R_\epsilon}{R}\right)^4$ (as any condensate is likewise expected to consistently scale), we determine the scaling of temperature with cosmological scale

$$\left(\frac{R_\epsilon}{R}\right)^7 \;=\; \left(\frac{T}{T_\epsilon}\right)^4 \tag{3.110}$$

Thus, we see that the scaling of temperature with inverse FRW scale factor no longer holds. As suspected, the equation of state could be considerably altered prior to dark energy de-coherence.

If we consistently continue this conjecture to determine the critical temperature for Bose condensation of the gravons, the number of quanta in a scale volume is given by

$$N \;=\; N_{condensate} \;+\; \frac{\zeta(3)}{\pi^2}\left(\frac{T}{\hbar H}\right)^3. \tag{3.111}$$

As usual, the ratio of condensate to "normal" state satisfies

$$\frac{N_{condensate}}{N} \;=\; 1 - \left(\frac{T}{T_c}\right)^3. \tag{3.112}$$

The critical temperature is given by

$$k_B T_c \;=\; \left(\frac{\pi^2}{8\zeta(3)}\right)^{1/3} \hbar\sqrt{\frac{8\pi G_N}{3}\rho}. \tag{3.113}$$

Therefore, since the energy density is expected to scale like R^{-4}, we can conclude that the critical temperature scales as

$$\frac{T_c}{T_{c\epsilon}} = \left(\frac{R_\epsilon}{R}\right)^2 = \left(\frac{T}{T_\epsilon}\right)^{\frac{8}{7}}. \tag{3.114}$$

Since T_c increases more rapidly than T at higher temperatures, such a system would remain condensed at early times. This means that a system obeying this behavior would have a suppressed vacuum energy due to the condensation into the lowest momentum mode until thermalization during de-coherence. For such a system, the coherence of a supraluminal horizon need not be driven by the rapid expansion rates, but rather is a direct consequence of the global quantum coherence of the macroscopic quantum system.

Since the ratio of the temperature to the critical temperature becomes vanishingly small for the earliest times

$$\frac{T}{T_c} \sim 0.31 \left(\frac{T_\epsilon}{T}\right)^{1/7} \Rightarrow 0 \tag{3.115}$$

we feel justified in asserting that the pre-coherent cosmology which starts completely condensed will remain a zero temperature condensate until de-coherence. This condition is required to justify the use of the lowest momentum mode only in the evaluation of cosmological vacuum energy density at de-coherence.

3.8 Fluctuations

Adiabatic perturbations are those that fractionally perturb the number densities of photons and matter equally. For adiabatic perturbations, the matter density fluctuations grow according to[5]

$$\delta = \begin{cases} \delta_\epsilon \left(\frac{R(t)}{R_\epsilon}\right)^2 & radiation - dominated \\ \delta_{dust} \left(\frac{R(t)}{R_{dust}}\right) & matter - dominated \end{cases} \tag{3.116}$$

This allows us to write an accurate estimation for the scale of fluctuations during de-coherence in terms of those at last scattering

$$\delta_\epsilon = \left(\frac{R_{dust}}{R_{LS}}\right)\left(\frac{R_\epsilon}{R_{dust}}\right)^2 \delta_{LS} \cong \frac{z_{dust} z_{LS}}{z_\epsilon^2}\delta_{LS} \tag{3.117}$$

if the fluctuations are textitfixed at dark energy de-coherence. Assuming the values for z_{dust} and z_ϵ calculated previously, along with the red shift at last scattering $z_{LS} \approx 1100$, this requires fluctuations fixed at dark energy de-coherence to have a value

$$\delta_\epsilon \approx 8.46 \times 10^{-27} \delta_{LS} \sim 10^{-31} \quad massless \tag{3.118}$$

$$\delta_\epsilon \approx 1.83 \times 10^{-22} \delta_{LS} \sim 10^{-27} \quad massive. \tag{3.119}$$

If quantum coherence persists such that the fluctuations are fixed at a later scale R_F, this relation gets modified to take the form

$$\delta_F \cong \frac{z_{dust} z_{LS}}{z_F^2} \delta_{LS} \approx \delta_\epsilon \left(\frac{R_F}{R_\epsilon} \right)^2 . \tag{3.120}$$

We expect the energy available for fluctuations to be of the order of the vacuum energy. This means that we should expect the amplitude of the fluctuations to be of the order

$$\delta_{DC} \sim \left(\frac{\rho_\Lambda}{\rho_{FL} + \rho_\Lambda} \right)^{1/2} \cong \left(\frac{1}{2N_\epsilon} \right)^{1/2} . \tag{3.121}$$

This form also appears in the literature on fluctuations[26]. Indeed, we obtain the correct order of magnitude for fluctuations at de-coherence for either massless or massive condensates

$$\delta_{DC} \cong \begin{cases} 3.7 \times 10^{-31} & massless \\ 4.5 \times 10^{-27} & massive. \end{cases} \tag{3.122}$$

At last scattering this gives

$$\delta_{LS} \cong \begin{cases} 3.0 \times 10^{-4} & massless \\ 1.4 \times 10^{-4} & massive, \end{cases} \tag{3.123}$$

whereas for present day, this fluctuation is given by

$$\delta_{LS} \cong \begin{cases} 0.3 & massless \\ 0.2 & massive, \end{cases} \tag{3.124}$$

if the fluctuation grows only linearly (which is not the case for late times). The amplitude of galaxy fluctuations is expected to be $\sigma_8 \cong 0.84$. We see that we are unable to differentiate between the massless and massive condensate in terms of the fluctuation amplitude only.

4 Discussion and Conclusions

We feel that we have given a strong argument for the interpretation of cosmological dark energy as the vacuum energy of a zero temperature condensate of bosons. Prior to de-coherence, the scale of gravitational quantum vacuum energy is given by the FRW scale $R(t)$. During de-coherence, the coherence scale of the FL density changes considerably (most likely to be the Compton wavelength of the mass m associated with the Bose condensate, which is much less than the coherence scale of the dark energy), resulting in a gravitational phase transition, and the onset of new thermal degrees of freedom. This means that thermal interactions between the FL matter will not have sufficient coherence to affect the gravitational dark energy. We have assumed that the quantum vacuum state for gravitation is an intrinsic state, with an energy density scale given by the vacuum energy density of the zero temperature Bose condensate during the period of last quantum coherence (given by $\epsilon = \frac{1}{2}\sqrt{m^2 + k_\epsilon^2}$, which is determined by the current value of the cosmological constant). When the cosmology has global coherence, the vacuum (ground) state is expected to evolve with the contents of the universe. When global coherence is lost, there remains only local coherence within independent clusters, and the prior vacuum state loses scale coherence with the clusters as the new degrees of freedom become available. This dark energy scale will be frozen out as a cosmological constant of positive energy density satisfying $\rho_\Lambda = \frac{\Lambda}{8\pi G_N} = \frac{\epsilon k_\epsilon^3}{(2\pi)^3}$ in terms of the present day cosmological constant.

To determine the ultimate fate of the universe, one needs an understanding of the fundamental nature of the quantum vacuum. The Wheeler-Feynmann interpretation of the propagation of quanta irreducibly binds those quanta to their sources and sinks. A previous paper by these authors directly demonstrates the equivalence of the usual Compton scattering process calculated using photons as the asymptotic states in standard QED with a description that explicitly includes the source and sink of the scattered photons in a relativistic three-particle formalism[23]. According to Lifshitz and others[7], the zero temperature electromagnetic field in the Casimir effect can be derived in terms of the zero-point motions of the sources and sinks upon which the forces act. In the absence of a causal connection between those sources

and sinks, one has a difficult time giving physical meaning to a vacuum energy or Casimir effect. Since the zero-point motions produce classical electromagnetic fields in Landau's treatment, these fields propagate through the"vacuum" at c. This would mean that one expects the Casimir effect to be absent between comoving mirrors in a cosmology with $\dot{R} > c$. If the regions in a future cosmology whose expansion are driven by vacuum energy are indeed causally disjoint, then there could be no driving of that expansion due to the local cosmological constant. Such an expansion requires that gravitational interactions propagate in a manner that causally affects regions requiring superluminal correlations. The expected change in the equation of state for the cosmology as a whole should modify the behavior of the FRW expansion in a manner that would require reinterpretation of the vacuum energy term, as was necessary during the pre-coherence epoch.

This means that we do not view the cosmological constant as the *same* as vacuum energy density. In our interpretation, cosmological vacuum energy changes during pre-coherence and post-coherence. The cosmological constant is frozen at de-coherence due to the availability of luminal degrees of freedom. Since this vacuum energy density is associated with regions of global gravitational coherence, it is not clear whether subsequent expansion in space-time will re-establish coherence on a cosmological scale.

We are in the process of examining the power spectrum of fluctuations expected to be generated by de-coherence as developed. It is our hope that further explorations of the specifics of density and temperature fluctuations will allow us to differentiate between massless vs massive condensate. In addition, we have begun to examine the scale of the symmetry breaking involved in coherence, especially with regards to the mass scales involved.

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