Simple Stringy Dynamical SUSY Breaking

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We present simple string models which dynamically break supersymmetry without non-Abelian gauge dynamics. The Fayet model, the Polonyi model, and the O’Raifeartaigh model each arise from D-branes at a specific type of singularity. D-brane instanton effects generate the requisite exponentially small scale of supersymmetry breaking.
1. Introduction

Dynamical supersymmetry breaking (DSB) is a promising candidate solution to the hierarchy problem \[1\]. Many field theories which dynamically break supersymmetry have been discovered (see \[2,3,4,5\] for reviews). In each of these examples, non-Abelian gauge dynamics plays a crucial role. In general, the constructions are rather complicated, though they have become simpler over the years \[3\].

One way to dynamically break supersymmetry (SUSY) in string theory is to embed a non-Abelian gauge theory which dynamically breaks supersymmetry into the low-energy spectrum. Here, we propose an alternative. We find simple D-brane theories which dynamically break supersymmetry after including D-brane instanton effects. The low-energy theories are Fayet, Polonyi or O’Raifeartaigh models. The terms in the superpotential which are responsible for supersymmetry breaking arise due to stringy D-instanton generated perturbations, which have recently been investigated in \[3,7,8,9\] and many subsequent papers. Non-Abelian gauge dynamics plays no role, and the SUSY breaking “hidden sectors” are extremely modest in size, including a single Abelian gauge field with two charged chiral multiplets or even more minimal field content. One can view our results as indicating that stringy instantons make retrofitting of simple supersymmetry-breaking models \[13\] a natural feature of D-brane constructions. Because of the importance of the stringy instanton effect, in these models the brane construction plays a more fundamental role than just serving as a way to embed a known low-energy field theory mechanism into string theory.

In §2, we describe the simplest models we have found. All of these models can arise from D-branes at a specific singularity, which can be chosen to be an orientifold of an orbifold of the conifold. In §3, we briefly discuss the prospects for making fully realistic models using our SUSY breaking hidden sectors. The construction of complete models utilizing our SUSY breaking models as hidden sectors is left for future work.

2. Some Simple Models

In this section we present simple D-brane theories where stringy instanton effects yield vacua with exponentially small SUSY breaking scale. The low-energy theories are Fayet, Polonyi and O’Raifeartaigh models.

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1 Early work on similar instanton effects appears in \[10,11,12\].
2 Some other papers which study stringy mechanisms to break supersymmetry using systems of branes, anti-branes and fluxes are \[13-22\].
2.1. The Fayet Model

The Fayet model consists of a $U(1)$ gauge field coupled with strength $e$ to charged chiral multiplets $\Phi_\pm$ with equal and opposite charges and canonical Kähler potential. The superpotential is

$$W = m\Phi_+ \Phi_-,$$

so the F-term equations for supersymmetric vacua require the scalar components to satisfy $\phi_\pm = 0$. The D-term constraints require supersymmetric vacua to satisfy

$$|\phi_+|^2 - |\phi_-|^2 = r,$$

where $r$ is the Fayet-Iliopoulos (FI) D-term for the Abelian gauge symmetry.

For generic values of the FI term $r \neq 0$, the F-term equations and (2.2) cannot be simultaneously satisfied. The energy grows without bound at infinity in field space, so this model has a stable ground state which spontaneously breaks supersymmetry. Specifically, for $r \gg m^2/(2e^2)$, the minimum of the scalar potential is at $|\phi_+|^2 = r - m^2/(2e^2) \simeq r$, $\phi_- = 0$, and the breaking of supersymmetry is dominated by the $F$-term

$$F_{\Phi_-} \simeq m\sqrt{r}.$$ 

The limit $m \to 0$ in (2.1) restores the non-anomalous axial symmetry $\Phi_\pm \to e^{i\lambda}\Phi_\pm$. Therefore, any model where $m$ is generated by exponentially small effects is natural in the sense of ’t Hooft and Wilson. We will now exhibit a simple brane realization of this model, with an exponentially small supersymmetry breaking scale obtained by generating $m$ from a stringy instanton effect.

The basic idea is as follows. We can realize the theory described above as a quiver gauge theory, arising at low energies on D-branes probing a non-compact singular Calabi-Yau space in type IIB string theory (or, equivalently, from D-branes stretched between NS-branes in type IIA string theory). The relevant quiver for us is quite simple and could potentially arise from many singularities; it appears below in Figure 1. It has two $U(r)$ nodes of rank $r = 1$ and one $USp(r)$ node of rank $r = 0$. In the geometrical language, the space locally contains two 2-cycles on which space-filling 5-branes (often called “fractional branes”) are wrapped, and another 2-cycle $C$ which is not wrapped by a 5-brane. There are two chiral multiplets arising from open strings between the 5-branes, with charges $(\pm 1, \mp 1)$ under the $U(1) \times U(1)$ gauge group. The superpotential is zero perturbatively.
A Euclidean D1-brane wrapped on $C$ contributes an instanton effect with precisely the right zero-mode structure to generate the superpotential (2.1); this cannot be interpreted as an ordinary field-theoretic instanton, since there is no field theory associated with this cycle, and no non-Abelian gauge dynamics is required for the effect. $m$ and $r$ are fixed parameters at the level of the non-compact system since they arise from non-normalizable modes.

\[ \begin{array}{ccc}
1 & \overset{\alpha}{\rightarrow} & 2 \\
\beta & \rightarrow & X_{23} \\
0 & \rightarrow & 1 \\
& & X_{32} \\
& & 1 \\
\end{array} \]

**Figure 1:** The quiver diagram that leads to the Fayet model. The first, square, node corresponds to a $USp(r_1)$ group, while the circular nodes correspond to $U(r_i)$ groups. For our application we need to have $r_2 = r_3 = 1$, and $r_1 = 0$ (this is the node wrapped by the D-instanton); the bifundamentals connecting node 1 and node 2 are then Ganor strings.

Concretely, we can obtain the simple subquiver in Figure 1, as well as a generalization relevant for gauge mediation to be discussed in §3, starting from the singular geometries

\[(xy)^n = zw.
\]

These are $\mathbb{Z}_n$ orbifolds of the conifold, studied in [23]. The quivers describing the effective gauge theories living on D3 and D5-branes at these singularities have $2n$ $U(r_i)$ nodes with bifundamentals $X_{i,i+1}, X_{i+1,i}$ going each way between adjacent nodes, as in the left-hand side of Figure 2, and with a superpotential

\[ W = h \sum_{i=1}^{2n} (-1)^i X_{i,i+1}X_{i+1,i+2}X_{i+2,i+1}X_{i+1,i} .
\]

Specific orientifolds of this theory which lead to interesting stringy instanton effects were described in [24,25]. In the case where the quiver nodes are occupied by space-filling wrapped branes, these modify the field content such that nodes 1 and $n + 1$ correspond

\[ \text{In a compact model with finite four dimensional Planck scale, these modes become dynamical. Then, as with all proposals for dynamical supersymmetry breaking in string theory, one must stabilize the closed string moduli which control the scales of the gauge theory.}
\]

\[ \text{The quivers we use can probably be obtained from many other singularities as well.} \]
to symplectic gauge groups instead of unitary groups, while the remaining $U(r_i)$ nodes are pairwise identified by the obvious reflection symmetry. The identification of node 1 with itself by the orientifold is important because it reduces the number of fermion zero modes on the Euclidean D1-brane wrapping the corresponding cycle $C$ to the two that are required for a contribution to the space-time superpotential. The T-dual type IIA string description of the branes at this orientifolded orbifolded conifold is shown in Figure 3.

![Figure 2: The quiver gauge theories of the orbifolded conifold and of its orientifold for $n = 3$. The circular nodes have $U(r_i)$ gauge groups, and the square nodes have $USp(r_i)$ groups. More generally there are $2n$ nodes before orientifolding and $n + 1$ nodes after orientifolding.](image)

The model we are interested in arises when we have $n \geq 3$, and we have single (space-filling) branes on nodes 2 and 3 ($r_2 = r_3 = 1$), and vanishing occupation numbers elsewhere. The tree-level superpotential (2.5) vanishes in this case. The D-instanton wrapping node 1 has bifundamental fermionic “Ganor strings” $\alpha$ and $\beta$ stretching to node 2 [12,24] (see Figure 1). These modes have a coupling analogous to (2.5) to the fields $X_{23}, X_{32}$; performing the path integral over $\alpha$ and $\beta$ then generates a superpotential [24,26]

$$W = \Lambda_1 X_{23} X_{32},$$

(2.6)

where $\Lambda_1$ is the instanton action controlled by the size of node 1 in the geometry, and it can naturally be exponentially small.

The sum of the $U(1)$’s associated to nodes 2 and 3 acts trivially on all fields and decouples. The low-energy theory consists of a single $U(1)$ gauge field (the difference of the $U(1)$’s at the two nodes), with $X_{23}$ and $X_{32}$ carrying equal and opposite charges. This $U(1)$ does not decouple at low energies, because its renormalization group running stops below the scale of the mass of the charged fields. Thus, we obtain precisely the

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5 For $n = 3$ there would be a similar contribution arising also from node 4.
Figure 3: The T-dual type IIA brane configuration for our Fayet model when it is embedded in the $n = 3$ orientifold. NS branes stretch in the 012345 directions, NS' branes in the 012389 directions, and D4 branes stretch in the 01236 directions. The O6 planes extend along the 01237 directions, and lie at a 45 degree angle with respect to the 45 and 89 planes. The $x^6$ direction is compact and becomes an interval after the orientifolding.

Fayet model, with $\Phi_+ = X_{23}$, $\Phi_- = X_{32}$, and with the parameter $m$ of (2.4) having been dynamically generated by a D-instanton. For generic choices of the FI term $r$ (which is a non-normalizable mode in the non-compact geometry), this model breaks supersymmetry at an exponentially low scale $F \sim \Lambda_1 \sqrt{r} \ (2.3)$. This can be considered a retrofitted Fayet model, in the spirit of [13]. However, no non-Abelian gauge dynamics is invoked in the retrofitting; it is automatically implemented by string theory. In the type IIA language of Figure 3, the FI term corresponds to the $x^7$ position of the NS’ between the two D4-branes.

The effective action of the model described above (and of the models we will discuss below) will in general be corrected by higher-dimension operators. These generically shift the location of the vacuum slightly, and can also introduce a supersymmetric vacuum elsewhere in field space, rendering the SUSY breaking vacuum metastable.

2.2. The Polonyi model

An even simpler model of SUSY breaking is the Polonyi model. This is the theory of a single chiral superfield with superpotential

$$W = \mu^2 X . \quad (2.7)$$

$F_X = \mu^2$ provides the order parameter of SUSY breaking. At tree level, this model has a flat direction. The existence of a stable non-SUSY vacuum at $X = 0$ depends on the sign of the leading quartic correction to the Kähler potential

$$K = X^\dagger X + \frac{c}{M_*^2} (X^\dagger X)^2 + \ldots . \quad (2.8)$$
\(M_*\) denotes the scale of high-energy physics which has been integrated out and corrects \(K\). For one sign of \(c\) there is a stable vacuum, and for the other the theory runs away to large values of \(X\). In any given completion of the Polonyi model by a larger field theory or string theory, there will be some corresponding value of \(c\).

In fact, a particularly simple completion manifesting a stable vacuum is provided by the Fayet model discussed above, which reduces to the Polonyi model in a limit. At the level of the field theory model, as \(r\) grows large, with \(m\sqrt{r} \equiv \mu^2\) fixed, the \(U(1)\) under which \(\Phi_{\pm}\) are charged becomes very massive along with \(\Phi_{\pm}\). The remaining \(U(1)\) is free and decouples as before. The low-energy theory therefore reduces to a free \(U(1)\) theory with a singlet \(X = \Phi_\pm\) that has mass squared \(2m^2 = 2\mu^2/\sqrt{r}\) (which goes to zero in our limit) and a linear superpotential \(W = \mu^2X\) as in (2.7). In the string construction realizing this model, we must keep \(r\) smaller than the string scale to avoid introducing new degrees of freedom; this still leaves a regime where the low energy effective theory is the Polonyi model with a locally stable minimum.

In the brane construction of Figure 3, turning on a large FI term corresponds to moving an NS' brane far away in the \(x^7\) direction. One could also obtain the Polonyi model directly, with a dynamically generated small scale \(\mu^2\), by considering the brane configuration without this NS' brane, such that we have a single D4-brane stretched between two parallel NS5-branes (with orientifolds as in Figure 3). This corresponds to the quiver shown in Figure 4, with \(r_2 = 1\) and \(r_1\) vanishing. In the brane language, the field \(X\) arises as the translation mode of the D4 along the \(x^4\) and \(x^5\) directions, and the stringy instanton is in this language the Euclidean D0 brane wrapping the interval between the NS 5-brane and the O6-plane. The Ganor strings now have an action of the form \(S = \alpha/\beta X\), so that this instanton gives precisely a superpotential of the form (2.7).

Instead of moving away an NS' brane along \(x^7\) as described above, one can also obtain this brane configuration from the one in Figure 3 by moving the NS' in the \(x^6\) direction so that it trades places with an NS brane (annihilating the D4-branes ending on it in the process). Such a position-switching process actually happens during the renormalization group cascade [27] which arises for branes with large occupation numbers at the singularities described in the previous subsection. As described in [28], the cascade steps lead to adjoints as in Figure 4, with trilinear couplings of the adjoints to the adjacent bifundamentals replacing (some of) the quartic couplings of (2.5). These trilinear couplings imply that the Ganor strings have the action \(S = \alpha/\beta X\) as above, which upon performing their path integral leads to the superpotential (2.7). Thus, the quiver of Figure 4 can arise from
D-branes at the same singularity described in the previous subsection. This raises the possibility of UV completing the SUSY breaking configuration with a cascading non-Abelian gauge theory. Then, the Polonyi model would arise as the effective low-energy description of the SUSY breaking in much the same way that an O’Raifeartaigh model captures the SUSY breaking vacua of SUSY QCD with slightly massive quark flavors \[28\]. Of course in the spirit of simplicity and minimality, we are free to consider the final brane configuration of interest (UV completed by string theory) without invoking the RG cascade and the consequent increased complexity of our hidden sector.

**Figure 4:** The two-node quiver which gives rise directly to a Polonyi model, after considering the instanton wrapping symplectic node 1. The arrow from node 2 to itself is a chiral superfield in the adjoint representation of \(U(r_2)\).

An advantage of obtaining the Polonyi model from a limit of the Fayet model as above, is that (for suitable \(r\)) one is certain of the existence of the stable SUSY breaking minimum; this is not clear when we obtain the Polonyi model directly from a brane configuration. It would be interesting to compute the constant \(c\) in the latter case, to see if it leads to a stable SUSY breaking vacuum.

2.3. An O’Raifeartaigh model

We obtained the Polonyi model by removing the NS’ brane between the two NS 5-branes in our type IIA brane construction of the Fayet model. Now, we can make an O’Raifeartaigh model (retrofitted by a stringy instanton) by inserting another NS-5 brane where the NS’ brane originally was. There are then adjoint fields both for node 2 and for node 3, as in Figure 5.\footnote{Again, one can also obtain this configuration by performing several steps in the RG cascade of the theories described in §2.1 \[26\]. Thus, it corresponds to branes on the same geometrical singularity of §2.1 (with different blow-up parameters).}
We now have a $U(1) \times U(1)$ gauge group. Let us call the two “adjoints” of $U(1) \times U(1)$ arising at nodes 3 and 2, respectively, $X$ and $\tilde{X}$. In addition, there are bifundamentals $\Phi, \tilde{\Phi}$. The tree-level superpotential is

$$W_{\text{tree}} = \tilde{\Phi} \tilde{X} \Phi + \tilde{\Phi} X \Phi.$$  \hspace{1cm} (2.9)

A stringy instanton at node 1 generates a perturbation

$$\delta W = \mu^2 \tilde{X},$$  \hspace{1cm} (2.10)

as in §2.2. The resulting full superpotential is

$$W_{\text{tot}} = X \tilde{\Phi} \Phi + \tilde{X} \left( \tilde{\Phi} \Phi + \mu^2 \right).$$  \hspace{1cm} (2.11)

The $X$ and $\tilde{X}$ F-terms conspire to break supersymmetry. In absence of (2.10), one could solve the D-term constraint $|\phi|^2 - |\tilde{\phi}|^2 = r$ by setting one of $\phi, \tilde{\phi}$ to $\sqrt{r}$ and the other to zero. This would yield a supersymmetric vacuum. The presence of the stringy instanton effect (2.10) instead leads to supersymmetry breaking, with an exponentially small scale set (in the natural regime $r \gg \mu^2$) by $\mu$.

This model has a flat direction at this level of analysis. Lifting the flat direction by “UV completing” the model with a slightly larger quiver, in analogy with what we did for the Polonyi model in §2.2, is one way to potentially stabilize the flat direction.

3. Discussion

For realistic model building, there are various options for communicating supersymmetry breaking to the Standard Model sector. If a Standard Model brane system sits far away from our SUSY-breaking system, we may obtain gravity mediation. We can also generalize the models above in a straightforward way to obtain messengers appropriate for
Consider (for example) the extension of the brane system of §2.1 depicted in Figure 6, where we now occupy node 4 with a toy “Standard Model.” This introduces a second set of chiral fields \( \eta, \tilde{\eta} \) charged under the new gauge group, and a superpotential of the form

\[
W = \Lambda_1 \Phi_+ \Phi_- + \frac{1}{M_*} \eta \tilde{\eta} \Phi_+ \Phi_- + M \eta \tilde{\eta},
\]

(3.1)

where the quartic term arises from the superpotential (2.5), and we have included a possible supersymmetric mass term \( M \) for \( \eta, \tilde{\eta} \). In the supersymmetry breaking vacuum with \( \phi_+ \sim \sqrt{r} \) and \( F_{\Phi_-} \sim \Lambda_1 \sqrt{r} \), the operator \( \Phi_+ \Phi_- \) has zero VEV and an \( F \) component of order \( \langle \phi_+ \rangle F_{\Phi_-} \). As a result, the superpotential (3.1) is of the form appropriate for gauge mediation with messengers \( \eta, \tilde{\eta} \) of mass \( M \), and with an effective SUSY-breaking \( F \)-term of order \( \langle \phi_+ \rangle F_{\Phi_-}/M_* \sim r \Lambda_1 / M_* \). The quartic term in (3.1) leads to the existence of additional (supersymmetric) vacua far away in field space, but it does not affect the non-supersymmetric vacuum that we are interested in (which is now metastable).

\[\begin{array}{ccccccc}
& 2 & & 3 & & 4 & \\
0 & \rightarrow & & \rightarrow & & \rightarrow & 0 \text{SM}
\end{array}\]

**Figure 6:** A quiver with a coupling to the “Standard Model” at node 4 and symplectic nodes (with stringy D-instantons) at nodes 1 and 5.

For high-scale gauge mediation, one requires a messenger mass \( M \) well below the string scale but much higher than the TeV scale. One possibility for obtaining such a mass is by turning on closed string moduli (blow-up modes), and this then involves a small tune of parameters. If one prefers a dynamical mechanism to obtain \( M \), which is particularly important for lower-scale gauge mediation, one can (as in Figure 6) make node 5 another (unoccupied) symplectic node. Then, if we put a single brane at node 4, we get a mass term for \( \eta, \tilde{\eta} \) of magnitude \( \Lambda_5 \) from the stringy instanton at node 5. This provides a tunable messenger mass. Since node 4 must be a \( U(1) \) for this to happen, we would need to consider an extension of the Standard Model by this \( U(1) \) symmetry, with appropriate charges to get gauge-mediated masses from this setup.

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7 For a review of gauge mediation, see [29]. For recent attempts to engineer such models using branes, see [30][31][32][33].

8 This has various phenomenological advantages [34][35].
In order to obtain a realistic model, we could investigate the possibility of replacing node 4 above with a full brane realization of the Standard Model (rather than the toy version described above), which is (classically) mutually supersymmetric with our SUSY-breaking sector. In doing so we must require that the new open strings $\eta, \tilde{\eta}$ connecting our SUSY-breaking theory to the Standard Model have the couplings (3.1) (which are the lowest order couplings allowed by the gauge symmetries). Again, one would need to generate messenger masses by an appropriate choice of closed string moduli, or by a dynamical mechanism similar to the one described above. It would be interesting to construct an explicit model of this sort, and to explore to what extent our simple DSB sectors (or obvious analogues) can easily be incorporated in existing semi-realistic brane constructions of the Standard Model (such as [36,37]). It would also be worthwhile to find analogous DSB models in the limits of string theory which more readily admit unification of coupling constants. Another natural generalization may be to apply D-instanton retrofitting to the recently studied O’Raifeartaigh models which spontaneously break R-symmetry [38].

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