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Tetraquark interpretation of $e^+e^- \to \Upsilon\pi^+\pi^-$ Belle data and $e^+e^- \to b\bar{b}$ BaBar data

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We summarize the main features of the spectroscopy, production and decays of the $J^{PC} = 1^{--}$ tetraquarks in the $b\bar{b}$ sector, concentrating on the lowest state called $Y_b(10890)$. The tetraquark framework is used to analyze the BaBar data on the $e^+e^- \to b\bar{b}$ cross section (R_b energy scan) between $\sqrt{s} = 10.54$ and 11.20 GeV and the Belle data on the processes $e^+e^- \to \Upsilon(1S)\pi^+\pi^-, \Upsilon(2S)\pi^+\pi^-$ near the peak of the $\Upsilon(5S)$ resonance. The BaBar R_b energy scan is consistent with an additional state at a mass of 10.90 GeV and a width of about 28 MeV, in broad agreement with the state $Y_b(10890)$ GeV seen by Belle in the exclusive final states. We argue that the decay widths and the dipion invariant mass distributions measured by Belle are naturally explained by the tetraquark interpretation of $Y_b(10890)$.

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1. Introduction

Experiments at the B factories and Tevatron have in the past several years revived the interest in the spectroscopy of the Quarkonium-like exotic states. Labeled tentatively as X, Y and Z, due to a lack of consensus on their interpretation, they have masses above the open charm ($D\bar{D}$) threshold, with the $X(3872)$ being the lightest and $Y(4660)$ the heaviest state observed so far [1]. There is also evidence for an exotic $s\bar{s}$ bound state $Y_s(2175)$ having the quantum numbers $J^{PC} = 1^{--}$, first observed in the initial state radiation (ISR) process $e^+e^- \to \gamma_{\rm ISR}\, f_0(980)\,\phi(1020)$, where $f_0(980)$ is the 0^{++} scalar state. In the $b\bar{b}$ sector, Belle [2] has observed enhanced production for the processes $e^+e^- \to \Upsilon(1S)\pi^+\pi^-, \Upsilon(2S)\pi^+\pi^-, \Upsilon(3S)\pi^+\pi^-$ in the e^+e^- center-of-mass energy between 10.83 GeV and 11.02 GeV, which does not agree with the conventional $\Upsilon(5S)$ line shape [3]. The enigmatic features of the Belle data are the anomalously large decay widths for the mentioned final states and the dipion invariant mass distributions, which are strikingly different from the conventional QCD expectations for such dipionic transitions. A fit of the Belle data, using a Breit-Wigner resonance, yields a mass of $10888^{+2.7}_{-2.6}(\text{stat}) \pm 1.2(\text{syst})$ MeV and a width of $30.7^{+8.3}_{-7.0}(\text{stat}) \pm 3.1(\text{syst})$ MeV [2]. This particle is given the tentative name $Y_b(10890)$. In [4, 5], $Y_b(10890)$ is interpreted as a $b\bar{b}$ tetraquark state, which is a linear superposition of the $J^{\rm PC} = 1^{--}$ flavour eigenstates $Y_{[bd]} \equiv [bd][\bar{b}\bar{d}]$ and $Y_{[bu]} \equiv [bu][\bar{b}\bar{u}]$. The mass eigenstates $Y_{[b,l]}$ (for the lighter) and $Y_{[b,h]}$ (for the heavier) of the two are almost degenerate, with their small mass difference arising from isospin-breaking [6]. A dynamical model for the decay mechanisms of $Y_b(10890)$ and the final state distributions measured by Belle was developed in [4] and refined in [5], yielding good fits of the Belle data. One anticipates that $Y_b(10890)$ is also visible in the energy scan of the $e^+e^- \to b\bar{b}$ cross section, which was undertaken by the BaBar collaboration between $\sqrt{s} = 10.54$ GeV and 11.20 GeV [7]. A fit of the BaBar data on R_b-scan is consistent with a structure around $Y_b(10890)$ and yields a better χ^2/d.o.f. than the fits without the tetraquark states. More data are required to resolve this and related structures in the R_b line shape. This contribution summarizes the work done in [4, 5, 6] interpreting the Belle [2] and BaBar [7] data in terms of the $b\bar{b}$ tetraquark states.

2. Spectrum of bottom diquark-antidiquark states

The mass spectrum of tetraquarks $[bq][\overline{bq'}]$ with $q = u, d, s$ and c can be described in terms of the constituent diquark masses, $m_Q = m_{[bq]}$, spin-spin interactions inside the single diquark, spin-spin interaction between quark and antiquark belonging to two diquarks, spin-orbit, and purely orbital term [8], i.e., with a Hamiltonian

$$H = 2m_Q + H_{SS}^{(QQ)} + H_{SS}^{(Q\bar{Q})} + H_{SL} + H_{LL}, \tag{2.1}$$

where:

$$\begin{aligned} H_{SS}^{(QQ)} &= 2(\mathcal{K}_{bq})_{\bar{3}}[(\mathbf{S}_b \cdot \mathbf{S}_q) + (\mathbf{S}_{\bar{b}} \cdot \mathbf{S}_{\bar{q}})], \\ H_{SS}^{(Q\bar{Q})} &= 2(\mathcal{K}_{b\bar{q}})(\mathbf{S}_b \cdot \mathbf{S}_{\bar{q}} + \mathbf{S}_{\bar{b}} \cdot \mathbf{S}_q) + 2\mathcal{K}_{b\bar{b}}(\mathbf{S}_b \cdot \mathbf{S}_{\bar{b}}) + 2\mathcal{K}_{q\bar{q}}(\mathbf{S}_q \cdot \mathbf{S}_{\bar{q}}), \\ H_{SL} &= 2A_Q(\mathbf{S}_Q \cdot \mathbf{L} + \mathbf{S}_{\bar{Q}} \cdot \mathbf{L}), \qquad H_{LL} = B_Q \frac{L_{Q\bar{Q}}(L_{Q\bar{Q}}+1)}{2}. \end{aligned} \tag{2.2}$$

Here $(\mathcal{K}_{bq})_{\bar{3}}$ is the coupling of the spin-spin interaction between the quarks inside the diquarks, $\mathcal{K}_{b\bar{q}}$ are the spin-spin couplings ranging outside the diquark shells, A_Q is the spin-orbit coupling of diquark and B_Q characterizes the contribution of the total angular momentum of the diquark-antidiquark system to its mass.

The parameters involved in the above Hamiltonian (2.2) can be obtained from the known meson and baryon masses by resorting to the constituent quark model [9]: $H = \sum_i m_i + \sum_{i<j} 2\mathscr{K}_{ij}(\mathbf{S}_i \cdot \mathbf{S}_j)$, where the sum runs over the hadron constituents. The coefficient $\mathscr{K}_{ij}$ depends on the flavour of the constituents i, j and on the particular colour state of the pair. Using the entries in the PDG [3] for hadron masses along with the assumption that the spin-spin interactions are independent of whether the quarks belong to a meson or a diquark, the results for the masses corresponding to the tetraquarks $[bq][\bar{b}\bar{q}]$ $(q = u,d,s,c)$ were calculated in [6]. The lowest eight 1^{--} tetraquark states $[bq][\bar{b}\bar{q}]$ $(q = u,d)$, which are all orbital excitations with $L_{Q\bar{Q}} = 1$, have the following spin and orbital angular momentum eigenvalues: $Y^{(1)}_{[bq]}\left(S_Q = 0,\ S_{\bar{Q}} = 0,\ S_{Q\bar{Q}} = 0,\ L_{Q\bar{Q}} = 1\right)$, $Y^{(2)}_{[bq]}\left(S_Q = 1,\ S_{\bar{Q}} = 0,\ S_{Q\bar{Q}} = 1,\ L_{Q\bar{Q}} = 1\right)$, $Y^{(3)}_{[bq]}\left(S_Q = 1,\ S_{\bar{Q}} = 1,\ S_{Q\bar{Q}} = 0,\ L_{Q\bar{Q}} = 1\right)$, and $Y^{(4)}_{[bq]}\left(S_Q = 1,\ S_{\bar{Q}} = 1,\ S_{Q\bar{Q}} = 2,\ L_{Q\bar{Q}} = 1\right)$. Identifying the lowest lying $J^{PC} = 1^{--}$ state $Y^{(1)}_{[bq]}$ with the $Y_b(10890)$ measured by Belle, and using the estimates for the other parameters entering in Eq. (2.2), fixes the diquark mass $m_Q = m_{[bq]} = 5.251$ GeV. The uncertainties on the masses of the other six states $M_{Y^{(n)}_{[bq]}}$ $(n = 2,3,4)$ are higher, as they depend in addition on the mass-splittings between the *good* and *bad* diquarks, $\Delta = m_Q(S_Q = 1) - m_Q(S_Q = 0)$, estimated as $\Delta \simeq 200$ MeV [10, 11]. The central values of their masses are: $M_{Y^{(2)}_{[bq]}} = 11133$ MeV, $M_{Y^{(3)}_{[bq]}} = 11257$ MeV, and $M_{Y^{(4)}_{[bq]}} = 11227$ MeV. Assuming isospin symmetry, the states $Y^{(n)}_{[bu]}$ and $Y^{(n)}_{[bd]}$ are degenerate for each n. Including isospin-symmetry breaking lifts this degeneracy with the mass difference between the lighter and the heavier of the two states estimated as $M(Y^{(n)}_{[b,l]}) - M(Y^{(n)}_{[b,h]}) = (7 \pm 3)\cos(2\theta)$ MeV, where θ is a mixing angle and the mass eigenstates are defined as: $Y^{(n)}_{[b,l]} = \cos\theta Y^{(n)}_{[bd]} + \sin\theta Y^{(n)}_{[bu]}$ and $Y^{(n)}_{[b,h]} = -\sin\theta Y^{(n)}_{[bd]} + \cos\theta Y^{(n)}_{[bu]}$ [6]. The resulting mass differences are small. However, depending on θ, the electromagnetic couplings of the $Y^{(n)}_{[b,l]}$ and $Y^{(n)}_{[b,h]}$ may turn out to be significantly different from each other, and hence also their contributions to R_b.

3. Decay Widths of $Y_b(10890)$ and other $J^{PC} = 1^{--}$ tetraquarks

As the masses of all the eight $J^{PC} = 1^{--}$ $[bq][\bar{b}\bar{q}]$ tetraquark states lie above the thresholds for the decays $Y^{(n)}_{[bq]} \to B^{(*)}_q \bar{B}^{(*)}_q$, they decay readily into these final states. For the $n = 3$ state (having a mass of 11257 MeV), also the decay $Y^{(3)}_{[bq]} \to \Lambda_b \overline{\Lambda_b}$ is energetically allowed. In [6], the decay widths $\Gamma(Y^{(n)}_{[bq]} \to B^{(*)}_q \bar{B}^{(*)}_q)$ have been estimated (up to a tetraquark hadronic size parameter κ) in terms of the corresponding partial decay widths $\Gamma(\Upsilon(5S) \to B^{(*)}_q \bar{B}^{(*)}_q)$, which can be calculated with the help of the entries in the PDG [3]. Specifically, the following relations are assumed

$$\kappa^2 \langle B^+ B^- |\hat{H}| Y^{(n)}_{[bu]}\rangle = \kappa^2 \langle B^0 \bar{B}^0 |\hat{H}| Y^{(n)}_{[bd]}\rangle = \langle B^+ B^- |\hat{H}| \Upsilon(5S)\rangle = \langle B^0 \bar{B}^0 |\hat{H}| \Upsilon(5S)\rangle \,, \tag{3.1}$$

and likewise for the $B\bar{B}^*$ and $B^*\bar{B}^*$ decays. Noting that the decays $\langle B^+ B^- |\hat{H}| Y^{(n)}_{[bd]}\rangle$, $\langle B^0 \bar{B}^0 ||\hat{H}| Y^{(n)}_{[bu]}\rangle$ as well as the decays $\Gamma(Y^{(n)}_{[bq]} \to B^{(*)}_s \bar{B}^{(*)}_s)$ are Zweig-forbidden, one expects, concentrating on the lowest mass state, $\Gamma(Y^{(1)}_{[bq]}) \simeq 0.4\Gamma(\Upsilon(5S)$. Using the PDG value [3] $\Gamma(\Upsilon(5S) = 110$ MeV, we get $\Gamma(Y^{(1)}_{[bd]}) = \Gamma(Y^{(1)}_{[bu]} = (44 \pm 8)\kappa^2$ MeV for the total decay widths. Equating this decay width to the measured value of the total decay width $\Gamma[Y_b(10890)] = 30.7^{+8.3}_{-7.0}(\text{stat}) \pm 3.1(\text{syst})$ MeV by Belle [2], one gets $\kappa = \sqrt{\frac{28\pm2}{44\pm8}} = 0.8 \pm 0.1$. This suggests that the tetraquarks $Y_{[bq]}^{(n)}$ have a hadronic size of the same order as that of the $\Upsilon(5S)$. The hadronic widths of the other $J^{PC} = 1^{--}$ tetraquarks are estimated as [6]: $\Gamma(Y_{[bq]}^{(2)}) = 80 \pm 16$ MeV, $\Gamma(Y_{[bq]}^{(3)}) = 114 \pm 22$ MeV and $\Gamma(Y_{[bq]}^{(4)}) = 102 \pm 20$ MeV.

To calculate the production cross sections, we have derived the corresponding Van Royen-Weisskopf formula for the leptonic decay widths of the tetraquark states made up of point-like diquarks [5]:

$$\Gamma(Y_{[bu/bd]} \to e^+e^-) = \frac{24\alpha^2|Q_{[bu/bd]}|^2}{m_{Y_b}^4}\,\kappa^2 \left|R_{11}^{(1)}(0)\right|^2, \tag{3.2}$$

where α is the fine-structure constant, $Q_{[bu]} = +1/3$, $Q_{[bd]} = -2/3$ are the diquark charges in units of the proton electric charge, and $|R_{11}^{(1)}(0)|^2 = 2.067$ GeV^5 [12] is the square of the derivative of the radial wave function for $\chi_b(1P)$ taken at the origin. Hence, the leptonic widths of the tetraquark states are estimated as [5]

$$\Gamma(Y_{[bd]} \to e^+e^-) = 4\Gamma(Y_{[bu]} \to e^+e^-) \approx 83\,\kappa^2 \text{ eV}, \tag{3.3}$$

which are substantially smaller than the leptonic width of the $\Upsilon(5S)$ [3]. The electronic widths of the mass eigenstates $Y_{[b,l]}$ and $Y_{[b,h]}$ depend, in addition, on the mixing angle θ.

4. Analysis of the BaBar data on R_b-scan

BaBar has reported the $e^+e^- \to b\bar{b}$ cross section measured in a dedicated energy scan in the range 10.54 GeV and 11.20 GeV taken in steps of 5 MeV [7]. Their measurements are shown in Fig. 1 (left-hand frame) together with the result of the BaBar fit which contains the following ingredients: A flat component representing the $b\bar{b}$-continuum states not interfering with resonant decays, called A_{nr}, added incoherently to a second flat component, called A_r, interfering with two relativistic Breit-Wigner resonances, having the amplitudes A_{10860}, A_{11020} and strong phases, ϕ_{10860} and ϕ_{11020}, respectively. Thus,

$$\begin{aligned}\sigma(e^+e^- \to b\bar{b}) \;=\; & |A_{nr}|^2 + |A_r + A_{10860}e^{i\phi_{10860}}BW(M_{10860},\Gamma_{10860}) \\ & + A_{11020}e^{i\phi_{11020}}BW(M_{11020},\Gamma_{11020})|^2\,,\end{aligned} \tag{4.1}$$

with $BW(M,\Gamma) = 1/[(s-M^2)+iM\Gamma]$. The results summarized in their Table II for the masses and widths of the $\Upsilon(5S)$ and $\Upsilon(6S)$ differ substantially from the corresponding PDG values [3], in particular, for the widths, which are found to be 43 ± 4 MeV for the $\Upsilon(10860)$, as against the PDG value of 110 ± 13 MeV, and 37 ± 2 MeV for the $\Upsilon(11020)$, as compared to 79 ± 16 MeV in PDG. As the systematic errors from the various thresholds are not taken into account, this mismatch needs further study. The fit shown in Fig. 1 (left-hand frame) is not particularly impressive having a χ^2/d.o.f. of approximately 2.

The BaBar R_b-data is refitted in [6] by modifying the model in Eq. (4.1) by taking into account two additional resonances, corresponding to the masses and widths of $Y_{[b,l]}$ and $Y_{[b,h]}$. Thus, formula (4.1) is extended by two more terms

$$A_{Y_{[b,l]}}e^{i\phi_{Y_{[b,l]}}}BW(M_{Y_{[b,l]}},\Gamma_{Y_{[b,l]}}) \qquad \text{and} \qquad A_{Y_{[b,h]}}e^{i\phi_{Y_{[b,h]}}}BW(M_{Y_{[b,h]}},\Gamma_{Y_{[b,h]}}), \tag{4.2}$$

which interfere with the resonant amplitude A_r and the two resonant amplitudes for $\Upsilon(5S)$ and $\Upsilon(6S)$ shown in Eq. (4.1). Using the same non-resonant amplitude A_{nr} and A_r as in the BaBar

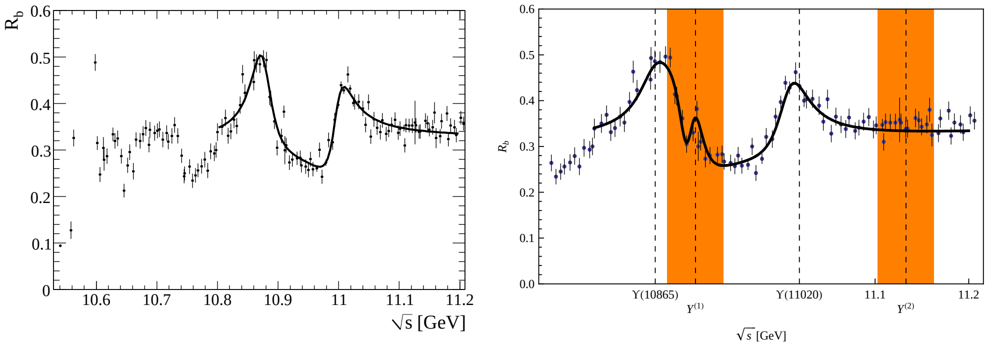

Figure 1: Measured R_b as a function of $\sqrt{s}$ with the result of the fit with 2 Breit-Wigners described in Fig. 1 of B. Aubert *et al.* [BaBar Collaboration] [7] (left-hand frame). The result of the fit of the R_b data with 4 Breit-Wigners [6] (right-hand frame). Location of the $\Upsilon(5S)$, $\Upsilon(6S)$, the tetraquark states $Y^{(1)}_{[b,q]}$ (labelled as $Y^{(1)}$) and $Y^{(2)}_{[b,q]}$ (labelled as $Y^{(2)}$) are indicated. The shaded bands around the mass of $Y^{(1)}$ and $Y^{(2)}$ reflect the theoretical uncertainty in their masses. (From [6]).

analysis [7]. the resulting fit is shown in Fig. 1 (right-hand frame). Values of the best-fit parameters yield the masses of the $\Upsilon(5S)$ and $\Upsilon(6S)$ and their respective full widths which are almost identical to the values obtained by BaBar [7]. However, quite strikingly, a third resonance is seen in the R_b-line-shape at a mass of 10.90 GeV, tantalisingly close to the $Y_b(10890)$-mass in the Belle measurement of the cross section for $e^+e^- \to Y_b(10890) \to \Upsilon(1S,2S)\,\pi^+\pi^-$, and a width of about 28 MeV. In the region around 11.15 GeV, where the $Y^{(2)}_{[bq]}$ states are expected, our fits of the BaBar R_b-scan do not show a resonant structure due to the larger decay widths of the states $Y^{(2)}_{[bq]}$. The resulting $\chi^2/\text{d.o.f.} = 88/67$ with the 4 Breit-Wigners shown in Fig. 1 (right frame) is better than that of the BaBar fit [7].

The quantity $\mathscr{R}_{ee}(Y_b) = \Gamma_{ee}(Y_{[b,l]})/\Gamma_{ee}(Y_{[b,h]})$ is given by the ratio of the two amplitudes $A_{Y_{[b,l]}}$ and $A_{Y_{[b,h]}}$, which also fixes the mixing angle θ. From the fit shown in the right-hand frame in Fig. 1, one obtains

$$\mathscr{R}_{ee}(Y_b) = 1.07 \pm 0.05, \tag{4.3}$$

yielding

$$\theta = -19 \pm 1^\circ \quad \text{and} \quad \Delta M = 5.6 \pm 2.8\ \text{MeV}, \tag{4.4}$$

for the mixing angle and the mass difference between the eigenstates, respectively. For the mass eigenstates $Y_{[b,l]}$ and $Y_{[b,h]}$, the electronic widths $\Gamma_{ee}(Y_{[b,l]})$ and $\Gamma_{ee}(Y_{[b,h]})$ are given by [5] $\Gamma_{ee}(\theta) = 0.2\,\kappa^2 Q(\theta)^2$ keV. With the above determination of κ and θ, we get

$$\Gamma_{ee}(Y_{[b,l]}) = 0.033 \pm 0.006\ \text{keV} \quad \text{and} \quad \Gamma_{ee}(Y_{[b,h]}) = 0.031 \pm 0.006\ \text{keV}. \tag{4.5}$$

5. Analysis of the Belle data on $e^+e^- \to Y_b \to (\Upsilon(1S), \Upsilon(2S))\pi^+\pi^-$

With the $J^{PC} = 1^{--}$ for both Y_b and $\Upsilon(nS)$, the dipionic final state is allowed to have the quantum numbers 0^{++} and 2^{++}. There are only three low-lying resonances in the PDG which can contribute as intermediate states, namely, the two 0^{++} states, $f_0(600)$ and $f_0(980)$, which we take as the lowest tetraquark states, and the 2^{++} $q\bar{q}$ meson state $f_2(1270)$. All three states contribute for the

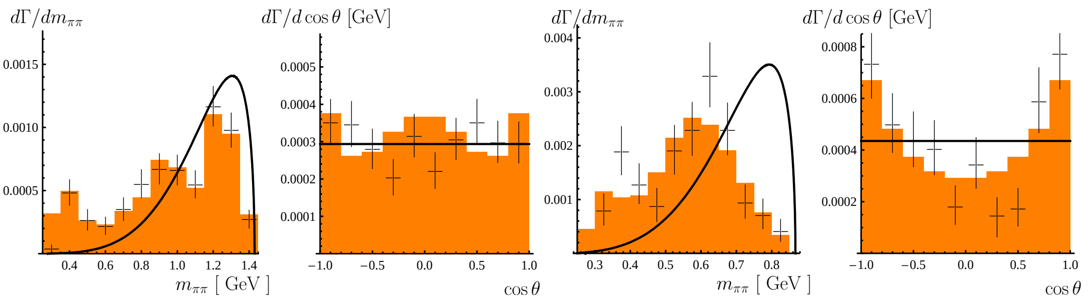

Figure 2: Left-hand frames: Fit results of the $M_{\pi^+\pi^-}$ distribution and the $\cos\theta$ distribution for $e^+e^- \to Y_b \to \Upsilon(1S)\pi^+\pi^-$, normalized by the measured cross section by Belle [2]. Right-hand frames: The same distributions for $e^+e^- \to \Upsilon(2S)\pi^+\pi^-$. In all figures, the histograms represent the fit results based on tetraquarks, while the crosses are the Belle data [2]. The solid curves in the figures show purely continuum contributions. (From [4].)

final state $\Upsilon(1S)\pi^+\pi^-$. However, kinematics allows only the $f_0(600)$ in the final state $\Upsilon(2S)\pi^+\pi^-$. In addition, a non-resonant contribution with a significant D-wave fraction is needed by the data on these final states. This model accounts well the shape of the measured distributions, as shown in Fig. 2 for $e^+e^- \to Y_b \to \Upsilon(1S)\pi^+\pi^-$ (left-hand frames) and for $e^+e^- \to Y_b \to \Upsilon(2S)\pi^+\pi^-$ (right-hand frames). As the decays $Y_b \to (\Upsilon(1S), \Upsilon(2S))\pi^+\pi^-$ are Zweig-allowed, one expects larger decay widths for these transitions, typically of $O(1)$ MeV [5], than the decay widths for the conventional dipionic transitions, such as $\Upsilon(4S) \to \Upsilon(1S)\pi^+\pi^-$, which are of order 1 keV [3]. Further tests of the tetraquark hypothesis involving the processes $e^+e^- \to Y_b \to \Upsilon(1S)(K^+K^-, \eta\pi^0)$ are presented in [5].

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