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Consequences of the BaBar $e^+e^- \to \pi^+\pi^-$ Measurement for the Determination of Model-Dependent ρ-ω Mixing Effects in $\Pi_{\rho\omega}(m_\rho^2)$ and $(g-2)_\mu$

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We update our analysis of ρ-ω mixing effects in the pion form factor to incorporate the recently published BaBar $e^+e^- \to \pi^+\pi^-$ cross-sections. The implications for τ-decay-based Standard Model estimates of the leading order hadronic contribution, $[a_\mu]_{had}^{LO}$, to the anomalous magnetic moment of the muon, and for the extraction of the off-diagonal vector meson self-energy matrix element, $\Pi_{\rho\omega}(m_\rho^2)$, are discussed.

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In the following we update the analysis performed in Ref. [1] of the isospin-breaking (IB) $\rho - \omega$ mixing correction required in order to use τ-decay-based data instead of electroproduction data in the evaluation of $[a_\mu]_{had}^{LO}$, the leading order hadronic vacuum polarization contribution to the anomalous magnetic moment of the muon, $a_\mu \equiv (g - 2)_\mu/2$. This update focusses on the BaBar electroproduction data [2, 3] since it was not released until shortly after publication of the previous analysis.

As is well known, several recent measurements of the $e^+e^- \to \pi^+\pi^-$ cross-section [2, 4–9] together yield estimates of $[a_\mu]_{had}^{LO}$ which are consistent with one another, but lead to Standard Model (SM) predictions for a_μ deviating from the BNL E821 experimental result [10] by $\sim 3.2 - 3.6\sigma$ [11–18]. In contrast, evaluating $[a_\mu]_{had}^{LO}$ using τ decay data in place of isovector electroproduction data [19–24] yields a SM prediction for a_μ differing from experiment by only $\sim 1.9 - 2.4\sigma$ [11, 12, 15–18]. Use of the τ decay data requires that a number of small IB corrections to the CVC relation be taken into account. These corrections have been extensively studied in Refs. [16, 25–29] and are believed to be well understood. We denote these corrections collectively by $[\delta a_\mu]_{had}^{LO}$ and focus in what follows

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on the particular contribution arising from $\rho - \omega$ mixing, $[\delta a_\mu]^{LO}_{\text{had;mix}}$, which is defined explicitly in Ref. [1].

An important observation made in Ref. [30] was that the generic structure of the $\rho - \omega$ interference contribution to $F_\pi(s)$ introduces strong fit-parameter-sensitive cancellations, and hence significant model dependence, into the integral corresponding to $[\delta a_\mu]^{LO}_{had;mix}$. Our analysis thus employs a range of models for $F_\pi(s)$, all having some basis in phenomenology. These are the Kuhn-Santamaria (KS) model [32], the Hidden Local Symmetry (HLS) model [33, 34], the Gounaris-Sakurai (GS) model [35], and a modified version of the GP/CEN model [26, 36]. (Detailed descriptions of the models can be found in section II of Ref. [1].) Refs. [1, 30] show that it is necessary to consider such a range of models if one wishes to properly assess the model dependence of $[\delta a_\mu]^{LO}_{had;mix}$, and, from this, the uncertainty in the $\pi\pi$ contribution to the τ decay-based estimates of $[a_\mu]^{LO}_{\text{had}}$.

Shortly after the publication of Ref. [1] the BaBar collaboration released the data corresponding to its measurement of the $e^+e^- \to \pi^+\pi^-(\gamma)$ cross-section, using the initial-state radiation method, from threshold to 3 GeV [2, 3]. Compared to the electroproduction data sets described and used in Ref. [1] (CMD-2 [4, 5], SND [8, 9], and KLOE [6, 7]) the BaBar data offers considerably increased statistics, including 15 data points in the interference region (770-800 MeV), as well as generally lower statistical and systematic errors. The BaBar data distinguishes itself from its predecessors, however, in that the value of a_μ computed using it as the source of the $\pi\pi$ contribution to $[a_\mu]^{LO}_{\text{had}}$ more closely corresponds to the experimental and τ decay based values, deviating from the experimental value by only 2.4σ [37].

As before, we perform fits to the BaBar data set using the models indicated above. Although the BaBar data extends up to 3 GeV, only the low-energy part of this data is relevant to analyzing $\rho - \omega$ mixing. We, therefore, limit our analysis to the maximum e^+e^- center-of-mass energy of 970 MeV employed in our previous analysis. The results quoted below for $[\delta a_\mu]^{LO}_{had;mix}$ are insensitive to modest changes in this choice of endpoint. All results correspond to the bare form factor (i.e. with the effects of vacuum polarization removed). Details of the fit procedure, including all input values, are unchanged from Ref. [1]. Fit results for each model are shown in Table I. The fit parameters are the ρ mass and width, m_ρ and Γ_ρ, the complex coefficient of the ω contribution, δ, the coefficient of the ρ' term, β, and the HLS model parameter, a_{HLS}. A blank entry indicates that a fit parameter is inapplicable to that particular model. For the GP/CEN$^+$ and GP/CEN^{++} models, the effective value of Γ_ρ is shown in brackets to highlight that it is in fact $\delta\Gamma_\rho$, an offset from the nominal chiral effective theory ρ width, which is the actual fit parameter.

Comparing the results of Table I with those of Tables I-IV in Ref. [1], we see that the BaBar data yields a ρ width larger by 1-7 MeV (depending on the specific data set and model) and a reduced ρ-ω mixing phase. Reasonable χ^2/dof results are obtained despite the reduced scale of statistical errors in the BaBar data relative to the other data sets.

The values obtained for $[\delta a_\mu]^{LO}_{had;mix}$ using the BaBar data and for each of the models considered are shown in Table II, along with the values from the data sets used in Ref. [1]. The latter are included for ease of comparison. The BaBar data yields somewhat larger central values, along with reduced errors. However, as before [1, 30], the variation in the values of $[\delta a_\mu]^{LO}_{had;mix}$ across the various models is greater than the experimental

TABLE I: Results of fits to the BaBar 2009 data.

Parameter	KS	HLS	GS	GP/CEN$^+$	GP/CEN^{++}
m_ρ (MeV)	772.11±0.30	773.48±0.29	774.29±0.30	775.87±0.29	775.87±0.29
Γ_ρ (MeV)	147.56±0.54	149.68±0.57	149.87± 0.57	(148.66)	(148.65)
$\delta\Gamma_\rho$ (MeV)	-	-	-	1.33±0.46	1.32± 0.46
$\|\delta\|$ (10^{-3})	1.89±0.03	1.99±0.03	1.96±0.03	2.30±0. 03	1.98± 0.03
Arg(δ) (deg)	9.4 ± 1.2	10.3 ± 1.1	10.3 ± 1.1	10.8 ± 1.1	10.8 ± 1.1
β	-0.152±0.002	-	-0.088±0.002	-	-
a_{HLS}	-	2.3989±0.007	-	-	-
χ^2/dof	392/250	320/250	322/250	413/251	414/251

TABLE II: $[\delta a_\mu]^{LO}_{had;mix} \times 10^{10}$ for the models discussed in the text and the CMD-2, SND, and KLOE $e^+e^- \to \pi^+\pi^-$ cross-sections.

Experiment	KS	HLS	GS	GP/CEN$^+$	GP/CEN^{++}
CMD-2(94)	3.8 ± 0.6	4.0 ± 0.6	2.0 ± 0.5	2.0 ± 0.5	1.8 ± 0.4
CMD-2(98)	4.0 ± 0.6	4.6 ± 0.6	2.5 ± 0.5	2.2 ± 0.4	2.1 ± 0.4
SND	4.2 ± 0.4	4.3 ± 0.4	2.2 ± 0.3	1.9 ± 0.3	1.7 ± 0.3
KLOE(02)	(2.2 ± 0.6)	4.2 ± 0.7	2.2 ± 0.6	(0.5 ± 0.8)	(0.3 ± 0.8)
BaBar(09)	5.0 ± 0.2	5.0 ± 0.2	2.9 ± 0.2	2.6 ± 0.2	2.4 ± 0.2

uncertainty produced by any single model. In arriving at a final assessment of our results for $[\delta a_\mu]^{LO}_{had;mix}$, we have adopted the view that, since all the models considered have a reasonable basis in phenomenology, all results corresponding to a given data set and given model which produce an acceptable quality fit are to be included in the assessment. (Those entries in brackets in Table II correspond to poor quality fits and are not included in our final result.) We thus first perform a weighted average over all experiments for each separate model, and then take the average (half the difference) of the maximum and minimum values allowed by the resulting error intervals for the different models to define our central values (model-dependence-induced uncertainties). The updated combined assessment, now including the BaBar results, is

$$[\delta a_\mu]^{LO}_{had;mix} = (3.5 \pm 1.5_{model} \pm 0.2_{data}) \times 10^{-10}. \tag{1}$$

The central value has increased by 0.4×10^{-10} and the data error has decreased by 0.1×10^{-10} compared to the value reported in [1].

The value shown in Table II obtained using BaBar data and the GS model is compatible with the GS result reported in Ref. [37]. The KS model result, however, is not, the KS and GS results for $[\delta a_\mu]^{LO}_{had;mix}$ differing significantly in Table II but being the same in Ref. [37]. The source of this apparent discrepancy is that two distinct 'KS' models have in fact been employed: the one we denoted KS above, and the alternate version

used in Ref. [37], which we call KS′. As discussed in Ref. [1] these two models differ in the s-dependence assumed for the ρ-ω mixing contribution to $F_\pi(s)$. We have confirmed that the alternate, KS′, form indeed yields results for $[\delta a_\mu]^{LO}_{had;mix}$ compatible with those of the GS model. In fact, it turns out that the presence or absence of the extra s/m_ω^2 factor (which is what distinguishes the KS and KS′ model forms) is also the key feature distinguishing those models which yield 'high' values of $[\delta a_\mu]^{LO}_{had;mix}$ (KS, HLS) from those which yield 'low' values (GS, GP/CEN^{++}). The data, in the narrow range of s over which $\rho - \omega$ interference is significant, is incapable of distinguishing between these differing s-dependences. While such differences have only a very small impact on the values of the model fit parameters, the presence or absence of the factor of s/m_ω^2 strongly affects the very close cancellation occurring in the weighted integral for $[\delta a_\mu]^{LO}_{had;mix}$. Since there is, at present, no compelling theoretical argument favouring one choice of s-dependence over the other in the interference region, we adopt the view that the unknown s-dependence of the mixing term must be treated as an additional source of uncertainty for $[\delta a_\mu]^{LO}_{had;mix}$. This uncertainty significantly increases the total error on $[a_\mu]^{LO}_{had;mix}$.

As explained in Refs. [1, 38], analysis of the electroproduction data in the interference region also allows one to extract the off-diagonal $\rho - \omega$ element of the vector meson self-energy matrix, $\Pi_{\rho\omega}(q^2)$, and the isospin-breaking coupling ratio $G \equiv g_{\omega_I \pi\pi}/g_{\rho_I \pi\pi}$, with $g_{\omega_I \pi\pi}$ and $g_{\rho_I \pi\pi}$ the isospin-pure $\pi\pi$ couplings of the ρ and ω mesons. $\Pi_{\rho\omega}(q^2)$ is of interest, for example, for meson-exchange models of IB in the NN interaction. The procedure for performing this determination has been described in detail in Refs. [1, 38].

The separation of mixing and direct $\omega \to \pi\pi$ contributions depends on the model used for the broad ρ contribution to $F_\pi(s)$. We report in Table III the results for ϕ (the Orsay phase), G, and $\tilde{T} \equiv \tilde{\Pi}_{\rho\omega}(m_\rho^2)/\hat{m}_\rho\Gamma_\rho$ (with $\tilde{\Pi}_{\rho\omega}$ the real part of $\Pi_{\rho\omega}$ and $\hat{m}_\rho$ the real part of the complex ρ pole position), obtained from the BaBar data set for the various models used. The one-sigma contours for G and $\tilde{T}$ are shown in Fig. 1. The corresponding contours for the CMD-2(98) and SND data sets are shown for comparison in Fig. 2. Readers are directed to Ref. [1] for full details.

TABLE III: Orsay phase and separated mixing and direct $\omega\pi\pi$ coupling parameters for the BaBar(09) data.

Parameter	KS	HLS	GS	GP/CEN$^+$	GP/CEN^{++}
ϕ (deg)	108 ± 1	108 ± 1	107 ± 1	107 ± 1	107 ± 1
G	0.028 ± 0.013	0.035 ± 0.013	0.036 ± 0.013	0.039 ± 0.015	0.040 ± 0.013
$\tilde{T}$	-0.037 ± 0.002	-0.038 ± 0.002	-0.038 ± 0.002	-0.0381 ± 0.0009	-0.038 ± 0.001

The results of Table III should be compared to those in Tables VI to IX of Ref. [1]. It is immediately apparent that the lower statistical uncertainty of the BaBar data translates into much greater precision in the extracted value of $\tilde{T}$. Two further significant differences concern the central values of ϕ and G, which are both lower for the BaBar data compared to the other data sets. The BaBar data also significantly improves the significance of
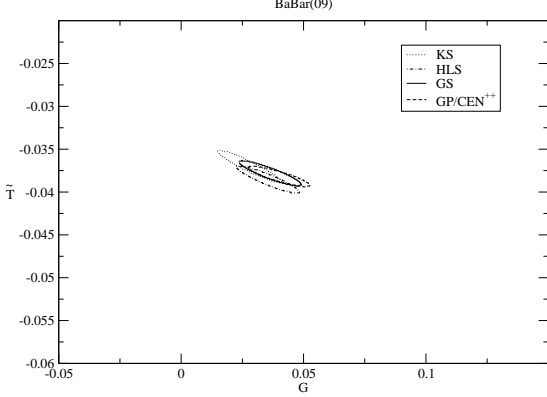
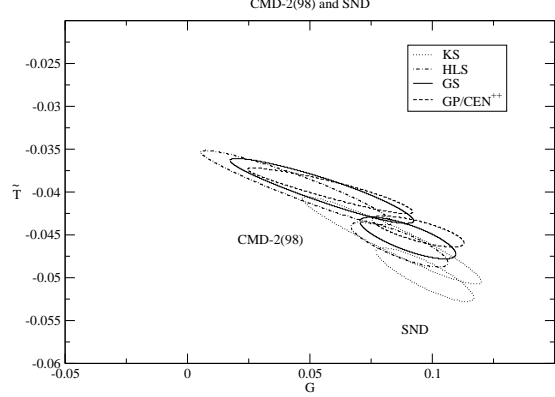FIG. 1: BaBar(09) G and $\tilde{T}$ one-sigma regions.

FIG. 2: CMD-2(98) and SND G and $\tilde{T}$ one-sigma regions.

the evidence for $G \neq 0$. In Ref. [1] we presented combined averages both including the KLOE data and excluding it. The high precision BaBar data now so dominates the combined averages that there is little distinction between the results obtained including or excluding KLOE; we thus present only the former in Table IV below.

TABLE IV: Combined averages including/excluding BaBar data.

	This work	Ref. [1]
	KLOE Included	
ϕ	$109.0° \pm 1.9°_{model} \pm 0.8°_{data}$	$113° \pm 4°_{model} \pm 2°_{data}$
$\tilde{T}$	$-0.041 \pm 0.003_{model} \pm 0.001_{data}$	$-0.044 \pm 0.006_{model} \pm 0.002_{data}$
G	$0.054 \pm 0.014_{model} \pm 0.010_{data}$	$G = 0.080 \pm 0.026_{model} \pm 0.015_{data}$

Note that the lower model dependence shown in the first column of Table IV reflects the dominance of the high-precision BaBar data over the other data sets in the averages, rather than any improved model consistency. The combined average for the complex-valued off-diagonal part of the physical $\rho - \omega$ self-energy matrix, $\Pi_{\rho\omega}(m_\rho^2)$, now including the BaBar data, is

$$\Pi_{\rho\omega}(m_\rho^2) = (-4620 \pm 220_{model} \pm 170_{data}) + (-6100 \pm 1800_{model} \pm 1110_{data})i \text{ MeV}^2. \quad (2)$$

In conclusion, we have updated the determination of $[\delta a_\mu]_{had;mix}^{LO}$ and the separation of $\rho - \omega$ interference in the $e^+ e^- \to \pi^+ \pi^-$ cross-sections into direct and mixing induced terms using the recently released BaBar ISR data. The main results are given in Eqs. 1 and 2, and in Table III. We conclude that, while not at present dominant, the model-dependence of $[\delta a_\mu]_{had;mix}^{LO}$ given in Eq. 1 will eventually represent a fundamental limitation on the use of τ data in the evaluation of a_μ.

Acknowledgments

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