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Minimal Universal Extra Dimensions in `CalcHEP/CompHEP`

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Abstract. We present an implementation of the model of minimal universal extra dimensions (MUED) in `CalcHEP/CompHEP`. We include all level-1 and level-2 Kaluza-Klein (KK) particles outside the Higgs sector. The mass spectrum is automatically calculated at one loop in terms of the two input parameters in MUED: the inverse radius R^{-1} of the extra dimension and the cut-off scale of the model Λ. We implement both the KK number conserving and the KK number violating interactions of the KK particles. We also account for the proper running of the gauge coupling constants above the electroweak scale. The implementation has been extensively cross-checked against known analytical results in the literature and numerical results from other programs. Our files are publicly available and can be used to perform various automated calculations within the MUED model.

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1. Introduction

The Standard Model (SM) of particle physics has been successfully verified by experiment at low energies. Nevertheless, even if the Higgs boson is discovered, the SM will still be considered to be an incomplete theory, as it fails to provide the long-sought missing link between Einstein's General Relativity and Quantum Mechanics. The leading candidate for a quantum theory of gravity, string theory, typically posits the existence of several new ingredients, which are absent in the SM: new spatial dimensions, a symmetry between bosons and fermions (supersymmetry), as well as new gauge interactions. All of these new ingredients are manifestly present at the Planck scale, but it is not at all clear which of them survive down to low energies. Traditionally, supersymmetry and extra gauge interactions have attracted the most attention, and their consequences for collider phenomenology have been extensively studied [1,2]. Within the last 10 years or so, there has been a resurgence of interest in models with extra spatial dimensions, whose presence might be revealed in high energy collider experiments such as the Tevatron at Fermilab, the Large Hadron Collider (LHC) at CERN, or the proposed International Linear Collider (ILC). By now a whole plethora of extra-dimensional models have been described and studied to various extent in the literature. Roughly speaking, they can all be classified according to the following two criteria:

- How many and which of the SM particles can access the extra dimensions (the bulk). The two extremes here are provided by the "large" extra dimension models (also known as ADD, after the initials of their original proponents) [3], in which only gravity can enter into the bulk, and the Universal Extra Dimensions (UED) models [4], in which *all* SM particles are allowed to propagate in the bulk.
- What is the metric of the bulk. It can be flat (e.g. in UED), or warped [5].

In this paper, we shall concentrate on the simplest case of a single flat extra dimension, which is accessible to the full SM particle content [4] (see Refs. [6–11] for the case of two universal extra dimensions). This particular scenario has recently been studied in relation to collider phenomenology [12–29], indirect low-energy constraints [30–43], dark matter [44–66] and cosmology [67–73]. It is therefore of great interest to have an implementation of the Minimal UED model (reviewed below in Section 2) in the most popular general purpose computer programs for collider and astroparticle phenomenology. The main goal of this paper is to present one such implementation, suitable for either CalcHEP [74] or CompHEP [75]. There are several advantages of choosing CalcHEP and CompHEP for this purpose:

- CalcHEP and CompHEP can be used for parton-level event generation, preserving the full spin correlations in both production and decay.
- CalcHEP and CompHEP can be easily interfaced [76] to a general purpose event generator such as PYTHIA [77] for the simulation of fragmentation, hadronization and showering.
- CalcHEP and CompHEP can be easily interfaced with a dark matter program such as micrOMEGAs [78] for the calculation of the relic density and detection rates of a generic dark matter candidate.
- The implementation of new models is very straightforward and user-friendly, as we shall demonstrate below with the example of Minimal UED.

The paper is organized as follows. In Section 2 we first review the Minimal UED model (MUED), introducing the relevant new particles, couplings and interactions. In Section 3 we explain how those were incorporated in CalcHEP and CompHEP. Throughout the paper we assume that the readers are already familiar with these programs, so that we only need to explain the additional *.mdl model files related to our UED implementation‡. In Section 4 we discuss how the implementation can be used to study the collider phenomenology of MUED and show some illustrative results. In the Appendices we list some more technical results which may be useful to some readers. For example, Appendix A contains the five-dimensional UED Lagrangian and Appendix C contains the resulting Feynman rules for the level 1 KK particles after compactification.

2. The Minimal UED Model

2.1. KK decomposition

The five-dimensional (5D) UED model [4] is simply the Standard Model placed in an extra dimension compactified on an S_1/Z_2 orbifold, as shown in Fig. 1. Let us label the usual $3+1$ space-time dimensions with x^μ, $\mu = 0, 1, 2, 3$, reserving the coordinate y for the extra dimension. In order to end up with chiral fermions in 4 dimensions and to project out

‡ Our implementation was originally developed for the Second MC4BSM workshop in Princeton, March 24-27, 2007. Since then, the Minimal UED model has been partially implemented in PYTHIA [79], and more fully in CalcHEP, MadGraph, PYTHIA or Sherpa through FeynRules [80, 81].

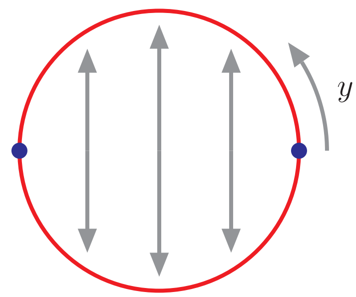

Figure 1. The S_1/Z_2 compactification of a single extra dimension on a circle with opposite points identified, as indicated by the grey arrows. The blue dots represent the fixed (boundary) points and y is the coordinate along the extra dimension.

unwanted gauge degrees of freedom, one typically imposes an additional symmetry, thus creating a manifold with boundaries. For example, in the case of the S_1/Z_2 orbifold shown in Fig. 1, one identifies the opposite points on the circle, which creates two fixed points, denoted with the blue dots. Any 5-dimensional field can now be assigned a definite parity with respect to the orbifold projection $\mathcal{P}_5 : y \to -y$. For example, consider a generic scalar field $\phi(x, y)$. An even scalar field $\phi^+(x, y)$ is expanded in Kaluza-Klein (KK) modes as

$$\phi^+(x,y) = \frac{1}{\sqrt{\pi R}}\,\phi_0^+(x) + \frac{2}{\sqrt{\pi R}}\sum_{n=1}^{\infty}\phi_n^+(x)\cos\frac{ny}{R}\,, \tag{1}$$

and obeys Neumann boundary conditions at the two fixed points:

$$\left(\frac{\partial\phi^+(x,y)}{\partial y}\right)_{y=0} = \left(\frac{\partial\phi^+(x,y)}{\partial y}\right)_{y=\pi R} = 0. \tag{2}$$

Here x is the usual 4-dimensional spacetime coordinate x^μ, R is the size of the extra dimension and n labels the KK-level. The SM modes correspond to $n = 0$. In contrast, the KK decomposition of an odd scalar field

$$\phi^-(x,y) = \frac{2}{\sqrt{\pi R}}\sum_{n=1}^{\infty}\phi_n^-(x)\sin\frac{ny}{R}\,, \tag{3}$$

is missing a zero mode ($n = 0$) and obeys Dirichlet boundary conditions

$$\phi^-(x,0) = \phi^-(x,\pi R) = 0. \tag{4}$$

One can similarly assign a definite $\mathcal{P}_5$ parity to each component of a gauge field $A_M(x, y)$, $M = 0, 1, 2, 3, 5$. The usual $3+1$ components A_μ, $\mu = 0, 1, 2, 3$, are chosen to be even, which ensures the presence of the SM gauge fields $A^0_\mu(x)$ at the $n = 0$ level, while the extra-dimensional component A_5 is taken to be odd. The corresponding KK expansions of the 5-dimensional gauge fields are given by

$$A_\mu(x,y) = \frac{1}{\sqrt{\pi R}}\left\{A^0_\mu(x) + \sqrt{2}\sum_{n=1}^{\infty} A^n_\mu(x)\cos(\frac{ny}{R})\right\}, \tag{5}$$

$$A_5(x,y) = \sqrt{\frac{2}{\pi R}}\sum_{n=1}^{\infty} A^n_5(x)\sin(\frac{ny}{R}). \tag{6}$$

At the two fixed points $y = 0$ and $y = \pi R$, the components $A_\mu(x,y)$ ($A_5(x,y)$) obey Neumann (Dirichlet) boundary conditions analogous to eq. (2) (eq. (4)).

The KK decomposition of a fermion is rather interesting. Since there is no chirality in 5 dimensions, the KK modes of the SM fermions come in vectorlike pairs, i.e. there is a left-handed *and* a right-handed KK mode for each SM chiral fermion. For example, the $SU(2)_W$-singlet chiral fermions $\psi^0_R(x)$ of the SM (which happen to be all right-handed) are obtained from the following decomposition

$$\psi^+_R(x,y) = \frac{1}{\sqrt{2\pi R}}\psi^0_R(x) + \frac{1}{\sqrt{\pi R}}\sum_{n=1}^{\infty}\psi^n_R(x)\cos\frac{ny}{R}, \tag{7}$$

$$\psi^-_R(x,y) = \frac{1}{\sqrt{\pi R}}\sum_{n=1}^{\infty}\psi^n_L(x)\sin\frac{ny}{R}, \tag{8}$$

where upon compactification, the two KK fermions $\psi^n_R(x)$ and $\psi^n_L(x)$ at any given KK level n pair up to give a Dirac fermion of mass $\frac{n}{R}$. Similarly, the $SU(2)_W$-doublet SM fermions $\Psi^0_L(x)$ (which happen to be left-handed) arise from

$$\Psi^+_L(x,y) = \frac{1}{\sqrt{2\pi R}}\Psi^0_L(x) + \frac{1}{\sqrt{\pi R}}\sum_{n=1}^{\infty}\Psi^n_L(x)\cos\frac{ny}{R}, \tag{9}$$

$$\Psi^-_L(x,y) = \frac{1}{\sqrt{\pi R}}\sum_{n=1}^{\infty}\Psi^n_R(x)\sin\frac{ny}{R}, \tag{10}$$

where the massive Dirac fermion at each n is now formed from $\Psi^n_L(x)$ and $\Psi^n_R(x)$.

From eqs. (7-10) we see that there exist *left-handed* KK modes $\psi^n_L(x)$, which are associated with the *right-handed* SM fermions $\psi^0_R(x)$ and vice versa — there are *right-handed* KK modes $\Psi^n_R(x)$, which go along with the *left-handed* SM fermions $\Psi^0_L(x)$. This often leads to some confusion in the literature when it comes to the labelling of fermion KK partners. It should be understood that the chiral index (L or R) of a KK mode fermion refers to the chirality of its SM partner. Here we shall also utilize an alternative convention, introduced in [82], where the KK fermions are identified by their $SU(2)_W$ quantum numbers

Table 1. Fermion content of the Minimal UED model. $SU(2)_W$-doublets ($SU(2)_W$-singlets) are denoted with capital (lowercase) letters. KK modes carry a KK index n, and for simplicity we omit the index "0" for the SM zero modes.

$SU(2)_W$ representations	SM mode	KK modes
Quark doublet	$q_L(x) = \begin{pmatrix} U_L(x) \\ D_L(x) \end{pmatrix}$	$Q_L^n(x) = \begin{pmatrix} U_L^n(x) \\ D_L^n(x) \end{pmatrix}, \; Q_R^n(x) = \begin{pmatrix} U_R^n(x) \\ D_R^n(x) \end{pmatrix}$
Lepton doublet	$L_L(x) = \begin{pmatrix} \nu_L(x) \\ E_L(x) \end{pmatrix}$	$L_L^n(x) = \begin{pmatrix} \nu_L^n(x) \\ E_L^n(x) \end{pmatrix}, \; L_R^n(x) = \begin{pmatrix} \nu_R^n(x) \\ E_R^n(x) \end{pmatrix}$
Quark Singlet	$u_R(x)$	$u_R^n(x)$, $u_L^n(x)$
Quark Singlet	$d_R(x)$	$d_R^n(x)$, $d_L^n(x)$
Lepton Singlet	$e_R(x)$	$e_R^n(x)$, $e_L^n(x)$

instead: $SU(2)_W$-doublets ($SU(2)_W$-singlets) are denoted with capital (lowercase) letters. This convention was already employed in eqs. (7-10) as well. With those conventions, the fermion content of the Minimal UED model is listed in Table 1.

Finally, notice that the geometry in Fig. 1 is still invariant under the interchange of the two fixed points. The corresponding Z_2 symmetry is the celebrated KK parity and will be a symmetry of the Lagrangian as long as one continues to treat the two boundary points in a symmetric fashion.

2.2. KK mass spectrum

At tree level, the mass m_n of any KK mode at the n-th KK level is given by

$$m_n^2 = \frac{n^2}{R^2} + m_0^2 \,, \tag{11}$$

where R is the radius of the extra dimension as illustrated in Fig. 1, and m_0 is the mass of the corresponding SM particle (zero mode). The resulting mass spectrum for the first KK level is shown in Fig. 2a for $R^{-1} = 500$ GeV, and can be seen to be highly degenerate. In fact, several of the lightest $n = 1$ KK modes have no allowed decays and are absolutely stable.

However, this drastic conclusion is completely reversed, once radiative corrections are taken into account [82]. First, the mass spectrum gets renormalized by bulk interactions, which are uniquely fixed in terms of the SM gauge and Yukawa couplings, and thus contain no new parameters beyond those already appearing in the SM. At the same time, the KK masses also receive contributions from terms localized on the boundary points (the two blue dots in

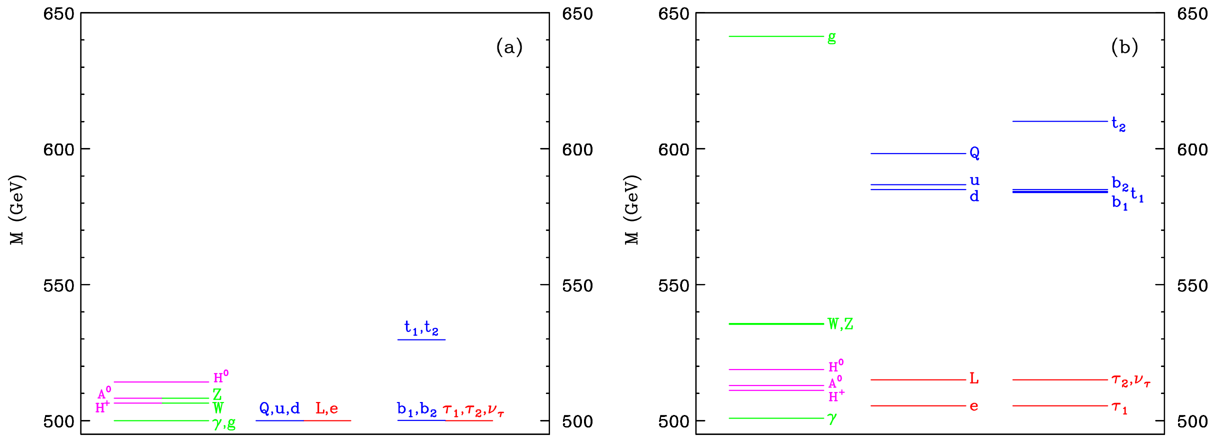

Figure 2. The spectrum of the first KK level at (a) tree level and (b) one-loop, for $R^{-1} = 500$ GeV, $\Lambda R = 20$, $m_h = 120$ GeV, and assuming vanishing boundary terms at the cut-off scale Λ. (From Ref. [82].)

Fig. 1). The coefficients of the boundary terms are in principle new free parameters of the theory. The Minimal UED model makes the ansatz that all boundary terms simultaneously vanish at some high scale $\Lambda > R^{-1}$. The boundary terms are then regenerated at lower scales through RGE running, and lead to additional corrections to the KK mass spectrum [82]. The resulting one-loop corrected mass spectrum is shown in Fig. 2b. The mass splittings among the different $n = 1$ KK modes are now sufficiently large to allow prompt cascade decays to the lightest KK particle (LKP). For the parameter values shown in the figure, the LKP turns out to be§ the KK "photon" γ_1, although at larger m_h the LKP can also be the charged KK Higgs boson $H_1^{\pm}$ [23].

The mass eigenstates of the KK photon γ_n and the KK Z-boson Z_n are mixtures of the corresponding interaction eigenstates: the KK mode B_n of the hypercharge gauge boson and the KK mode W_n^3 of the neutral $SU(2)_W$ gauge boson. The mixing angle θ_n is obtained by diagonalizing the mass matrix in the (B_n, W_n^3) basis

$$\begin{pmatrix} \frac{n^2}{R^2} + \frac{1}{4}g_1^2 v^2 + \hat{\delta} m_{B_n}^2 & \frac{1}{4}g_1 g_2 v^2 \\ \frac{1}{4}g_1 g_2 v^2 & \frac{n^2}{R^2} + \frac{1}{4}g_2^2 v^2 + \hat{\delta} m_{W_n^3}^2 \end{pmatrix}, \tag{12}$$

where g_1 (g_2) is the hypercharge (weak) gauge coupling, $v = 246$ GeV is the vev of the SM Higgs boson, and $\hat{\delta}$ represents the total one-loop correction, including both bulk (δ) and

§ Strictly speaking, the true LKP in Fig. 2b is the KK graviton $\mathcal{G}_1$ (not shown). However, due to its extremely weak couplings, $\mathcal{G}_1$ is irrelevant for collider phenomenology. For its astrophysical implications, see [83].

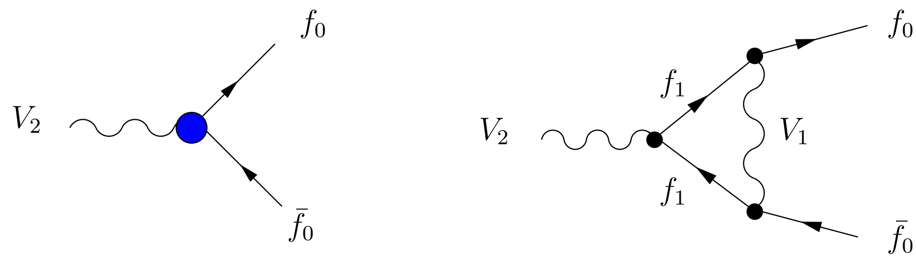

Figure 3. The effective $\bar{f}_0 V_2^\mu f_0$ KK-number violating coupling on the left is generated at one loop order from the one loop diagram on the right.

boundary ($\bar{\delta}$) contributions [82]:

$$\hat{\delta} m_{V_n}^2 \equiv \delta m_{V_n}^2 + \bar{\delta} m_{V_n}^2 \, . \tag{13}$$

Note that for $n \geq 1$ the KK mixing angle θ_n is in general different from the zero-mode (Weinberg) angle $\theta_0 \equiv \theta_W$ in the SM. For typical values of R^{-1} and Λ, $\theta_n \ll \theta_W$, and the neutral gauge boson KK mass eigenstates become approximately aligned with the corresponding interaction eigenstates: $\gamma_n \approx B_n$ and $Z_n \approx W_n^3$ for $n \geq 1$. This approximation will be used in our MUED implementation described below in Section 3.

2.3. KK interactions

The bulk interactions of the KK modes are already fixed by the SM. The 5D MUED Lagrangian is a straightforward generalization of the SM Lagrangian to 5 dimensions, as discussed in Appendix A. Upon compactification, integrating over the extra-dimensional coordinate y, one recovers the bulk interactions among the various KK modes and their SM counterparts (see Appendix C). Since translational invariance holds in the bulk, all these bulk interactions conserve both KK number and KK parity.

However, as already alluded to in the previous subsection, there may also exist "boundary" interactions localized on the fixed points in Fig. 1. They do not respect translational invariance and therefore break KK number by even units. Such interactions may already appear at the scale Λ, being generated by the new physics which is the ultraviolet completion of UED. In the Minimal UED version, one makes the assumption that no such terms are present at the scale Λ. Even so, upon renormalization to lower energy scales, boundary terms are radiatively generated from bulk interactions. This is illustrated in Fig. 3, where we show how an effective coupling between a level 2 KK gauge boson V_2 and two SM

Table 2. Boundary interactions involving level 2 KK gauge bosons and two SM fermions. Here I_3 is the fermion isospin and Y_L (Y_R) is the hypercharge of a left-handed (right-handed) SM fermion. In the case of top quarks, one has to include in $\bar{\delta}(m_{f_2})$ the additional corrections proportional to the top Yukawa coupling h_t: $\bar{\delta}_{h_t} m_{T_n}$ and $\bar{\delta}_{h_t} m_{t_n}$, respectively (see [82] for details).

$n=2$ KK boson	$n=0$ SM fermion	Vertex
$U(1)_Y$ gauge boson	Lepton	$ig_1\gamma^\mu \frac{1}{\sqrt{2}}\frac{1}{16\pi^2}\ln\left(\frac{\Lambda}{\mu}\right)^2 \left[\frac{Y_L}{2}P_L\left(\frac{31}{24}g_1^2+\frac{27}{8}g_2^2\right) + \frac{Y_R}{2}P_R\left(\frac{14}{3}g_1^2\right)\right]$
B_2	Quark (up)	$ig_1\gamma^\mu \frac{1}{\sqrt{2}}\frac{1}{16\pi^2}\ln\left(\frac{\Lambda}{\mu}\right)^2 \left[\frac{Y_L}{2}P_L\left(\frac{7}{24}g_1^2+\frac{27}{8}g_2^2+6g_3^2\right) + \frac{Y_R}{2}P_R\left(\frac{13}{6}g_1^2+6g_3^2\right)\right]$
	Quark (down)	$ig_1\gamma^\mu \frac{1}{\sqrt{2}}\frac{1}{16\pi^2}\ln\left(\frac{\Lambda}{\mu}\right)^2 \left[\frac{Y_L}{2}P_L\left(\frac{7}{24}g_1^2+\frac{27}{8}g_2^2+6g_3^2\right) + \frac{Y_R}{2}P_R\left(\frac{2}{3}g_1^2+6g_3^2\right)\right]$
$SU(2)_W$ gauge boson	Lepton	$iI_3 g_2\gamma^\mu \frac{1}{\sqrt{2}}\frac{1}{16\pi^2}\ln\left(\frac{\Lambda}{\mu}\right)^2 P_L\left[\frac{9}{8}g_1^2-\frac{33}{8}g_2^2\right]$
Z_2	Quark	$iI_3 g_2\gamma^\mu \frac{1}{\sqrt{2}}\frac{1}{16\pi^2}\ln\left(\frac{\Lambda}{\mu}\right)^2 P_L\left[\frac{1}{8}g_1^2-\frac{33}{8}g_2^2+6g_3^2\right]$
$SU(2)_W$ gauge boson	Lepton	$i\frac{g_2}{\sqrt{2}}\gamma^\mu \frac{1}{\sqrt{2}}\frac{1}{16\pi^2}\ln\left(\frac{\Lambda}{\mu}\right)^2 P_L\left[\frac{9}{8}g_1^2-\frac{33}{8}g_2^2\right]$
W_2	Quark	$i\frac{g_2}{\sqrt{2}}\gamma^\mu \frac{1}{\sqrt{2}}\frac{1}{16\pi^2}\ln\left(\frac{\Lambda}{\mu}\right)^2 P_L\left[\frac{1}{8}g_1^2-\frac{33}{8}g_2^2+6g_3^2\right]$
$SU(3)_c$ gauge boson	Quark (up)	$ig_3\frac{\lambda^A}{2}\gamma^\mu \frac{1}{\sqrt{2}}\frac{1}{16\pi^2}\ln\left(\frac{\Lambda}{\mu}\right)^2 \left[P_L\left(\frac{1}{8}g_1^2+\frac{27}{8}g_2^2-\frac{11}{2}g_3^2\right) + P_R\left(2g_1^2-\frac{11}{2}g_3^2\right)\right]$
G_2	Quark (down)	$ig_3\frac{\lambda^A}{2}\gamma^\mu \frac{1}{\sqrt{2}}\frac{1}{16\pi^2}\ln\left(\frac{\Lambda}{\mu}\right)^2 \left[P_L\left(\frac{1}{8}g_1^2+\frac{27}{8}g_2^2-\frac{11}{2}g_3^2\right) + P_R\left(\frac{1}{2}g_1^2-\frac{11}{2}g_3^2\right)\right]$

fermions is generated at one loop from a diagram with level 1 KK particles running in the loop. This effective coupling

$$-i\frac{g}{\sqrt{2}}\left(\frac{\bar{\delta}m_{A_2}^2}{m_2^2}-2\frac{\bar{\delta}m_{f_2}}{m_2}\right)\bar{\psi}_0\gamma^\mu T^a P_+\psi_0 A_{2\mu}$$

can be expressed in terms of the boundary contributions $\bar{\delta}m_n$ (see eq. (13)) to the one-loop mass corrections [82]. The explicit form of this effective coupling is summarized in Table 2 for each different type of level 2 KK gauge boson and for the various possible SM fermion pairs.

3. Model files

Having reviewed the MUED model, we are now in a position to describe its implementation in `CalcHEP` and `CompHEP`. Each one of these programs gives its users an opportunity to incorporate new physics in the already existing framework of the SM, MSSM, etc. To

Table 3. KK gauge bosons.

Name	A	A+	2*spin	mass	width	color
G^1_μ	KG	KG	2	MKG	wKG	8
B^1_μ	B1	B1	2	MB1	0	1
Z^1_μ	Z1	Z1	2	MZ1	wZ1	1
W^1_μ	$\sim W+$	$\sim W-$	2	MW1	wW1	1
G^2_μ	$\sim G2$	$\sim G2$	2	MKG2	wKG2	8
B^2_μ	B2	B2	2	MB2	wB2	1
Z^2_μ	Z2	Z2	2	MZ2	wZ2	1
W^2_μ	$\sim W2$	$\sim w2$	2	MW2	wW2	1

this end, one must simply supply an updated version of the four model files defining a given physics scenario in `CalcHEP` and `CompHEP`: `prtclsN.mdl`, `varsN.mdl`, `funcN.mdl` and `lgrngN.mdl`, where `N` stands for the numerical label of the physics scenario in the model menu of `CalcHEP` and `CompHEP`. We shall now discuss each one of those files, which are available from `http://home.fnal.gov/~kckong/mued/`.

3.1. Particles

New particles are defined in the `prtclsN.mdl` model file. We incorporate the $n = 1$ and $n = 2$ KK modes of the gauge bosons (see Table 3), leptons (see Table 4) and quarks (see Table 5). In Tables 3-5 the KK number is represented by a superscript $n = 1$ or $n = 2$, while the subscript is either the Lorentz index (μ) of the vector particles in Table 3 or the chirality index of the fermion particles in Tables 4 and 5. We remind the reader that all KK fermions are vectorlike and the chirality index refers to the chirality of their SM counterparts. The corresponding masses and widths of the KK fermions in Tables 4 and 5 carry "D" or "S" to indicate their nature, $SU(2)_W$-doublet or $SU(2)_W$-singlet, respectively. The new particles listed in Tables 3-5 are in addition to the usual SM particles which are not shown here.

3.2. Variables

The input parameters for any given physics scenario are defined in the `varsN.mdl` model file. In principle, MUED has only two additional input parameters beyond the SM: the radius R of the extra dimension and the cut-off scale Λ. For convenience, we use the inverse radius R^{-1} and the number of KK levels ΛR which can fit below the scale Λ. R^{-1} has dimensions

Table 4. KK leptons.

Name	A	A+	2*spin	mass	width	color
e_L^1	$\sim eL$	$\sim EL$	1	DMe	wDe1	1
μ_L^1	$\sim mL$	$\sim ML$	1	DMm	wDe2	1
τ_L^1	$\sim tL$	$\sim TL$	1	DMt	wDe3	1
e_R^1	$\sim eR$	$\sim ER$	1	SMe	wSe1	1
μ_R^1	$\sim mR$	$\sim MR$	1	SMm	wSe2	1
τ_R^1	$\sim tR$	$\sim TR$	1	SMt	wSe3	1
ν_e^1	$\sim n1$	$\sim N1$	1	DMen	wDn1	1
ν_μ^1	$\sim n2$	$\sim N2$	1	DMmn	wDn2	1
ν_τ^1	$\sim n3$	$\sim N3$	1	DMtn	wDn3	1
e_L^2	$\sim le$	$\sim lE$	1	DMe2	wDe12	1
μ_L^2	$\sim lm$	$\sim lM$	1	DMm2	wDe22	1
τ_L^2	$\sim lt$	$\sim lT$	1	DMt2	wDe32	1
e_R^2	$\sim re$	$\sim rE$	1	SMe2	wSe12	1
μ_R^2	$\sim rm$	$\sim rM$	1	SMm2	wSe22	1
τ_R^2	$\sim rt$	$\sim rT$	1	SMt2	wSe32	1
ν_e^2	$\sim en$	$\sim eN$	1	DMen2	wDn12	1
ν_μ^2	$\sim mn$	$\sim mN$	1	DMmn2	wDn22	1
ν_τ^2	$\sim tn$	$\sim tN$	1	DMtn2	wDn32	1

of GeV, while ΛR is dimensionless. Our additions to the `varsN.mdl` model file are listed in Table 6. As seen from the table, we also include several other variables of interest. `RG` is used to turn on and off the running of coupling constants, while `scaleN` is the renormalization scale μ at which the couplings are evaluated. The remaining parameters in Table 6 are some useful numerical constants related to the RGE running of the gauge couplings (see Section 3.5).

3.3. Constraints

The `funcN.mdl` model file is reserved for variables which are not numerical inputs, but are instead computed in terms of the parameters already defined in the `varsN.mdl` model file. In our case, we use `funcN.mdl` to supply the masses and two-body decay widths of the KK particles introduced in Section 3.1. Therefore they are automatically computed by `CalcHEP/CompHEP` at the beginning of each numerical session. The masses for all KK particles are evaluated based on the 1-loop formulas of Ref. [82] and we have also made

Table 5. KK quarks.

Name	A	A+	2*spin	mass	width	color
u_L^1	Du	DU	1	DMu	wDu	3
d_L^1	Dd	DD	1	DMd	wDd	3
c_L^1	Dc	DC	1	DMc	wDc	3
s_L^1	Ds	DS	1	DMs	wDs	3
t_L^1	Dt	DT	1	DMtop	wDt	3
b_L^1	Db	DB	1	DMb	wDb	3
u_R^1	Su	SU	1	SMu	wSu	3
d_R^1	Sd	SD	1	SMd	wSd	3
c_R^1	Sc	SC	1	SMc	wSc	3
s_R^1	Ss	SS	1	SMs	wSs	3
t_R^1	St	ST	1	SMtop	wSt	3
b_R^1	Sb	SB	1	SMb	wSb	3
u_L^2	$\sim Du$	$\sim DU$	1	DMu2	wDu2	3
d_L^2	$\sim Dd$	$\sim DD$	1	DMd2	wDd2	3
c_L^2	$\sim Dc$	$\sim DC$	1	DMc2	wDc2	3
s_L^2	$\sim Ds$	$\sim DS$	1	DMs2	wDs2	3
t_L^2	$\sim Dt$	$\sim DT$	1	DMtop2	wDt2	3
b_L^2	$\sim Db$	$\sim DB$	1	DMb2	wDb2	3
u_R^2	$\sim Su$	$\sim SU$	1	SMu2	wSu2	3
d_R^2	$\sim Sd$	$\sim SD$	1	SMd2	wSd2	3
c_R^2	$\sim Sc$	$\sim SC$	1	SMc2	wSc2	3
s_R^2	$\sim Ss$	$\sim SS$	1	SMs2	wSs2	3
t_R^2	$\sim St$	$\sim ST$	1	SMtop2	wSt2	3
b_R^2	$\sim Sb$	$\sim SB$	1	SMb2	wSb2	3

numerical cross-checks with the results from the private code used in Ref. [82]. Our formulas for the widths have been derived analytically and cross-checked with `CalcHEP`/`CompHEP` (see Section 4). A partial list of 2 body decay widths can be found in [14,15,20] and our formulas agree with their expressions. In the older versions of `CalcHEP`/`CompHEP`, defining the widths as constraints was very convenient in our implementation, since one did not have to launch a separate numerical session for their calculation, and then enter their numerical values as input parameters. However, the more recent versions of `CalcHEP` and `CompHEP` allow for the automatic calculation of the particle widths on the fly, using the interactions defined in the `lgrngN.mdl` model file. Our implementation thus allows for backward compatibility with

Table 6. Parameters added to the `varsN.mdl` model file.

Parameters	Default values	Symbols	Comments
Rinv	500	R^{-1}	Inverse radius of the extra dimension
LR	20	ΛR	The number of KK levels below Λ
RG	1		1 turn on the running of the coupling constants 0 turn off the running of the coupling constants
scaleN	2	n	Renormalization scale, $\mu = \frac{n}{R}$ n=2 can be used for KK level 1 pair production or level 2 single production n=4 can be used for KK level 2 pair production
cb1	6.8333	b_1	$\frac{41}{6}$, The coefficient of the SM β-function for $U(1)_Y$
cb2	-3.16667	b_2	$-\frac{19}{6}$, The coefficient of the SM β-function for $SU(2)_W$
cb3	-7	b_3	-7, The coefficient of the SM β-function for $SU(3)_c$
cb1t	6.8333	$\tilde{b}_1$	$\frac{41}{6}$, The coefficient of the KK β-function for $U(1)_Y$
cb2t	-2.83333	$\tilde{b}_2$	$-\frac{17}{6}$, The coefficient of the KK β-function for $SU(2)_W$
cb3t	-6.5	$\tilde{b}_3$	$-\frac{13}{2}$, The coefficient of the KK β-function for $SU(3)_c$
c1MZ	98.4151	α_1^{-1}	$\alpha_1^{-1}(\mu = M_Z)$
c2MZ	29.5846	α_2^{-1}	$\alpha_2^{-1}(\mu = M_Z)$
c3MZ	8.53244	α_3^{-1}	$\alpha_3^{-1}(\mu = M_Z)$

older versions of `CalcHEP`/`CompHEP`.

3.4. Interactions

The new interactions of the KK particles of Section 3.1 are added to the `lgrngN.mdl` model file. We include the usual bulk interactions, as well as the KK number violating boundary interactions listed in Table 2 [82]. Since the Weinberg angle θ_n for any $n \geq 1$ is small [82], we ignore the mixing among the neutral KK gauge bosons. Thus the KK-photon γ_n is identical to the hypercharge gauge boson B_n and the KK Z-boson Z_n is identical to the neutral $SU(2)_W$ gauge boson W_n^3. We also ignore the mixing between $SU(2)_W$-doublet and $SU(2)_W$-singlet KK fermions.

Our `lgrngN.mdl` model file includes all interactions of level-1 and level-2 KK particles except for the KK Higgs bosons. The phenomenology of the KK Higgs bosons is very model dependent, depending on the value of the SM Higgs mass m_h and the bulk Higgs mass term (see [82] for details). Therefore we omit any interactions involving KK Higgs bosons ‖.

‖ The collider phenomenology of the KK Higgs bosons has been discussed in [23, 24, 29].

The UED Lagrangian can be easily derived as shown in Appendix C. Here we only point out how to deal with 4-point interactions involving KK gluons, since this case requires special treatment when implemented in `CalcHEP/CompHEP`.

The Lagrangian for the quartic interactions with KK gluons is the following

$$\begin{aligned}\mathcal{L}_4 = & -\frac{1}{4}g_3^2 f^{abc}f^{ade}G_\mu^{0,b}G_\nu^{0,c}G^{0,d\mu}G^{0,e\nu} - \frac{g_3^2}{2}f^{abc}f^{ade}G_\mu^{0,d}G_\nu^{0,e}G^{1,b\mu}G^{1,c\nu} \\ & -\frac{g_3^2}{4}\left(f^{abc}(G_\mu^{0,b}G_\nu^{1,c}+G_\nu^{0,c}G_\mu^{1,b})\right)^2 - \frac{1}{4}\cdot\frac{3}{2}g_3^2 f^{abc}f^{ade}G_\mu^{1,b}G_\nu^{1,c}G^{1,d\mu}G^{1,e\nu}. \quad (14)\end{aligned}$$

The color structure of these 4-point interactions cannot be directly written down in `CalcHEP/CompHEP` format. Hence, to implement this vertex in `CalcHEP/CompHEP`, we use the following trick. We introduce three auxiliary tensor fields $t^a_{\mu\nu}$, $s^a_{\mu\nu}$ and $u^a_{\mu\nu}$ in the same way as the original `CalcHEP/CompHEP` approach for SM gluons. Then one can rewrite the Lagrangian as

$$\begin{aligned}\mathcal{L} = & -\frac{1}{2}t^a_{\mu\nu}t^{a\mu\nu} + i\frac{g_3}{\sqrt{2}}t^a_{\mu\nu}f^{abc}G^{0b\mu}G^{0c\nu} + i\frac{g_3}{\sqrt{2}}t^a_{\mu\nu}f^{abc}G^{1b\mu}G^{1c\nu} \\ & -\frac{1}{2}s^a_{\mu\nu}s^{a\mu\nu} + i\frac{g_3}{2}s^a_{\mu\nu}f^{abc}G^{1b\mu}G^{1c\nu} \\ & -\frac{1}{2}u^a_{\mu\nu}u^{a\mu\nu} + i\frac{g_3}{\sqrt{2}}u^a_{\mu\nu}f^{abc}\left(G^{0b\mu}G^{1c\nu}+G^{1b\mu}G^{0c\nu}\right) \\ = & -\frac{1}{2}\left(t^a_{\mu\nu} - ig_3\frac{1}{\sqrt{2}}f^{abc}G_\mu^{0b}G_\nu^{0c} - ig_3\frac{1}{\sqrt{2}}f^{abc}G_\mu^{1b}G_\nu^{1c}\right)^2 \qquad (15) \\ & -\frac{1}{4}g_3^2 f^{abc}f^{ade}\left(G_\mu^{0b}G_\nu^{0c}+G_\mu^{1b}G_\nu^{1c}\right)\left(G^{0d\mu}G^{0e\nu}+G^{1d\mu}G^{1e\nu}\right) \\ & -\frac{1}{2}\left(s^a_{\mu\nu} - ig_3\frac{1}{2}f^{abc}G_\mu^{1b}G_\nu^{1c}\right)^2 - \frac{1}{8}g_3^2 f^{abc}f^{ade}G_\mu^{1b}G_\nu^{1c}G^{1d\mu}G^{1e\nu} \\ & -\frac{1}{2}\left(u^a_{\mu\nu} - ig_3\frac{1}{\sqrt{2}}f^{abc}\left(G_\mu^{0b}G_\nu^{1c}+G_\mu^{1b}G_\nu^{0c}\right)\right)^2 \\ & -\frac{1}{4}g_3^2\left(f^{abc}\left(G_\mu^{0b}G_\nu^{1c}+G_\mu^{1b}G_\nu^{0c}\right)\right)^2,\end{aligned}$$

It is easy to show that the functional integration over the three auxiliary tensor fields reproduces the 4-gluon interactions (14).

3.5. Running of the coupling constants

Due to the additional contributions from the KK modes to the beta functions, the gauge couplings run faster in theories with extra dimensions. The RGE for $\alpha_i \equiv \frac{g_i^2}{4\pi}$ is given by [84]

$$\frac{d\alpha_i^{-1}}{dt} = -\frac{b_i - \tilde{b}_i}{2\pi} - \frac{\tilde{b}_i X_\delta}{2\pi}\left(\frac{\mu}{\mu_0}\right)^\delta, \qquad (16)$$

where δ is the number of extra dimensions, μ_0 is some reference energy scale, $X_\delta = \frac{2\pi^{\delta/2}}{\delta\Gamma(\delta/2)}$,

$$(b_1, b_2, b_3) = \left(\frac{41}{6}, -\frac{19}{6}, -7\right) \tag{17}$$

are the SM beta function coefficients, while

$$\left(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3\right) = \left(\frac{41}{6}, -\frac{17}{6}, -\frac{13}{2}\right) \tag{18}$$

correspond to the contributions of the Kaluza-Klein states at each massive KK excitation level [85, 86]. The solution to (16) becomes

$$\alpha_i^{-1} = \alpha_i^{-1}(M_Z) - \frac{b_i}{2\pi} \ln \frac{\mu}{M_Z} + \frac{\tilde{b}_i}{2\pi} \ln \frac{\mu}{\mu_0} - \frac{\tilde{b}_i X_\delta}{2\pi\delta} \left[\left(\frac{\mu}{\mu_0}\right)^\delta - 1 \right] . \tag{19}$$

The effect of the RGE running (19) can be accounted for by setting the `RG` parameter in Table 6 to 1 and choosing the appropriate renormalization scale via `scaleN`.

4. Discussion

4.1. Code validation

In general, the availability of `CalcHEP/CompHEP` model files opens the door to a number of applications related to collider phenomenology and dark matter searches. Each such individual study contributes to the validation of the code. Further consistency checks are provided by comparing to existing analytical and/or numerical results in the literature.

- For starters, we have compared the KK mass spectrum calculated with our implementation to the results shown in Fig. 2, which were obtained independently in Ref. [82]. Using identical inputs, and neglecting the running of the gauge couplings (as was done in [82]), we found perfect agreement.
- The interaction vertices of Appendix C can be independently derived with the automated tool `LanHEP` [87]. We checked some of the more technically challenging cases (especially the self-interactions of gauge bosons) and also found agreement.
- To minimize the possibility of typing mistakes, we computed analytically the cross-sections for a selected number of simple scattering processes, and compared to the analytical expressions derived by `CalcHEP/CompHEP`.
- We have similarly checked that the KK particle widths calculated from our analytical expressions agree with those computed with `CalcHEP/CompHEP` by means of our MUED implementation.

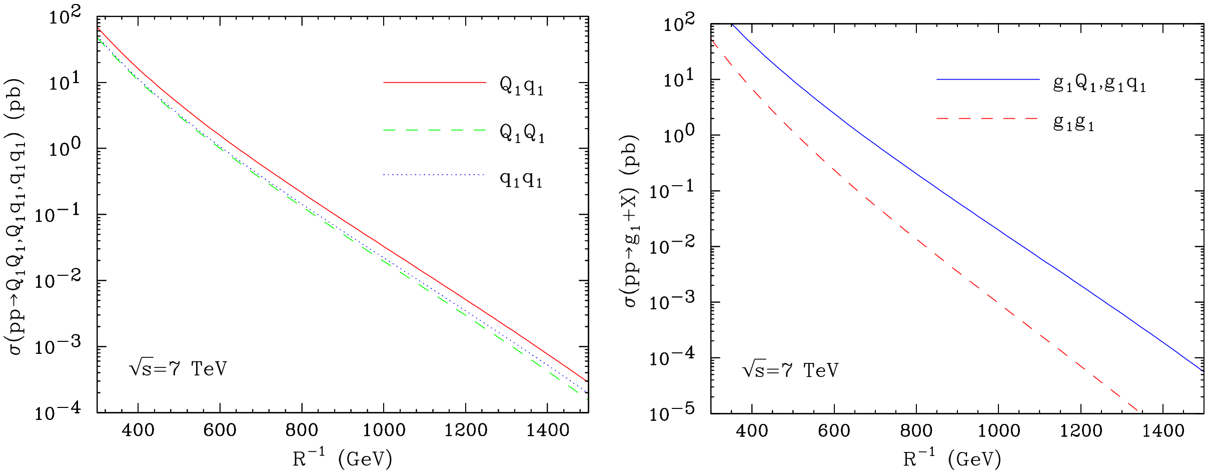

Figure 4. Strong production of $n = 1$ KK particles at the LHC for $\sqrt{s} = 7$ TeV: (a) KK-quark pair production; (b) KK-quark/KK-gluon associated production and KK-gluon pair production. The cross sections have been summed over all quark flavors and also include charge-conjugated contributions such as $Q_1\bar{q}_1$, $\bar{Q}_1 q_1$, $g_1\bar{Q}_1$, etc. We use CTEQ6L parton distributions [91] and choose the scale of the strong coupling constant α_s to be equal to the parton level center of mass energy.

- Our analytic formulas for decay widths agree with the expressions given in [14, 15, 20].
- Our implementation was used for the analytic calculation of *all* (co)annihilation cross-sections of level 1 KK particles [59] and the results were in complete agreement with [44, 58].
- Our model files have already been used for various collider studies [17, 19–22, 26, 59, 88–90]. One example is shown in Fig. 4, which shows the strong production cross-section of level 1 KK particles at the imminent LHC energy of 7 TeV.
- We have compared results for various production cross-sections in MUED to those in published papers [12, 13] and find agreement.
- Our model files were also cross-checked against the known analytical expressions for various invariant mass distributions [18, 92, 93].
- Our model files have also been tested by other groups, for example in creating Pythia_UED [79, 94, 95], which implemented the matrix elements for certain processes in PYTHIA [77]. Another extensive comparison to an independent MUED implementation via FeynRules was done in [81].

4.2. Outlook

Moving forward, it is important to be mindful of the limitations of our implementation. First of all, it is still Minimal UED, and the spectrum is quite constrained, given in terms of only 2 parameters: R^{-1} and Λ. If a signal consistent with UED is discovered at the LHC or the Tevatron, one would like to start testing the data with a more general UED framework, which allows for the presence of arbitrary boundary terms at the scale Λ. Work along these lines has already started and a beta version of the corresponding UED model files is available from the authors upon request.

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Appendix A. UED Lagrangian in 5 dimensions

The Lagrangian for the 5-dimensional UED model is written as

$$\mathcal{L}_{Gauge} = \int_0^{\pi R} dy \left\{ -\frac{1}{4} B_{MN} B^{MN} - \frac{1}{4} W^a_{MN} W^{aMN} - \frac{1}{4} G^A_{MN} G^{AMN} \right\} , \quad (A.1)$$

$$\mathcal{L}_{GF} = \int_0^{\pi R} dy \left\{ -\frac{1}{2\xi} \left(\partial^\mu B_\mu - \xi \partial_5 B_5\right)^2 - \frac{1}{2\xi} \left(\partial^\mu W^a_\mu - \xi \partial_5 W^a_5\right)^2 - \frac{1}{2\xi} \left(\partial^\mu G^A_\mu - \xi \partial_5 G^G_5\right)^2 \right\} , \quad (A.2)$$

$$\mathcal{L}_{Leptons} = \int_0^{\pi R} dy \left\{ i\bar{L}(x,y)\Gamma^M D_M L(x,y) + i\bar{e}(x,y)\Gamma^M D_M e(x,y) \right\} , \quad (A.3)$$

$$\mathcal{L}_{Quarks} = \int_0^{\pi R} dy \left\{ i\bar{Q}(x,y)\Gamma^M D_M Q(x,y) + i\bar{u}(x,y)\Gamma^M D_M u(x,y) + i\bar{d}(x,y)\Gamma^M D_M d(x,y) \right\} , \quad (A.4)$$

$$\mathcal{L}_{Yukawa} = \int_0^{\pi R} dy \left\{ \lambda_u \bar{Q}(x,y) u(x,y) i\tau^2 H^*(x,y) + \lambda_d \bar{Q}(x,y) d(x,y) H(x,y) + \lambda_e \bar{L}(x,y) e(x,y) H(x,y) \right\} , \quad (A.5)$$

$$\mathcal{L}_{Higgs} = \int_0^{\pi R} dy \left[\left(D_M H(x,y)\right)^\dagger \left(D^M H(x,y)\right) + \mu^2 H^\dagger(x,y) H(x,y) - \lambda \left(H^\dagger(x,y) H(x,y)\right)^2 \right] , \quad (A.6)$$

in terms of 5-dimensional fields decomposed as discussed in Section 2.1:

$$\begin{aligned}
H(x,y) &= \frac{1}{\sqrt{\pi R}} \left\{ H(x) + \sqrt{2} \sum_{n=1}^{\infty} H_n(x) \cos(\frac{ny}{R}) \right\} , \\
B_\mu(x,y) &= \frac{1}{\sqrt{\pi R}} \left\{ B^0_\mu(x) + \sqrt{2} \sum_{n=1}^{\infty} B^n_\mu(x) \cos(\frac{ny}{R}) \right\} , \\
B_5(x,y) &= \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} B^n_5(x) \sin(\frac{ny}{R}) , \\
W_\mu(x,y) &= \frac{1}{\sqrt{\pi R}} \left\{ W^0_\mu(x) + \sqrt{2} \sum_{n=1}^{\infty} W^n_\mu(x) \cos(\frac{ny}{R}) \right\} , \\
W_5(x,y) &= \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} W^n_5(x) \sin(\frac{ny}{R}) , \\
G_\mu(x,y) &= \frac{1}{\sqrt{\pi R}} \left\{ G^0_\mu(x) + \sqrt{2} \sum_{n=1}^{\infty} G^n_\mu(x) \cos(\frac{ny}{R}) \right\} , \qquad (A.7) \\
G_5(x,y) &= \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} G^n_5(x) \sin(\frac{ny}{R}) , \\
Q(x,y) &= \frac{1}{\sqrt{\pi R}} \left\{ q_L(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[P_L Q^n_L(x) \cos(\frac{ny}{R}) + P_R Q^n_R(x) \sin(\frac{ny}{R}) \right] \right\} ,
\end{aligned}$$

$$\begin{aligned}
u(x,y) &= \frac{1}{\sqrt{\pi R}}\left\{u_R(x)+\sqrt{2}\sum_{n=1}^{\infty}\left[P_R u_R^n(x)\cos(\frac{ny}{R})+P_L u_L^n(x)\sin(\frac{ny}{R})\right]\right\}\,,\\
d(x,y) &= \frac{1}{\sqrt{\pi R}}\left\{d_R(x)+\sqrt{2}\sum_{n=1}^{\infty}\left[P_R d_R^n(x)\cos(\frac{ny}{R})+P_L d_L^n(x)\sin(\frac{ny}{R})\right]\right\}\,,\\
L(x,y) &= \frac{1}{\sqrt{\pi R}}\left\{L_L(x)+\sqrt{2}\sum_{n=1}^{\infty}\left[P_L L_L^n(x)\cos(\frac{ny}{R})+P_R L_R^n(x)\sin(\frac{ny}{R})\right]\right\}\,,\\
e(x,y) &= \frac{1}{\sqrt{\pi R}}\left\{e_R(x)+\sqrt{2}\sum_{n=1}^{\infty}\left[P_R e_R^n(x)\cos(\frac{ny}{R})+P_L e_L^n(x)\sin(\frac{ny}{R})\right]\right\}\,.
\end{aligned}$$

Here $H(x,y)$ is the 5D Higgs scalar field and $(B_\mu(x,y), B_5(x,y))$, $(W_\mu(x,y), W_5(x,y))$ and $(G_\mu(x,y), G_5(x,y))$ are the 5D gauge fields B_M, W_M and G_M for $U(1)_Y$, $SU(2)_W$ and $SU(3)_c$, respectively. The 5D index M runs over $M=\mu,5$, where $\mu=0,1,2,3$. The $SU(2)_W$ and $SU(3)_c$ gauge fields are

$$\begin{aligned}
W_M &\equiv W_M^a\frac{\tau^a}{2}\,,\\
G_M &\equiv G_M^A\frac{\lambda^A}{2}\,,
\end{aligned}$$

where τ^a, $a=1,2,3$, are the usual Pauli matrices and λ^A, $A=1,2,...,8$, are the usual Gell-Mann matrices. The 5D field strength tensors for $U(1)_Y$, $SU(2)_W$ and $SU(3)_c$ are defined as follows

$$\begin{aligned}
B_{MN} &= \partial_M B_N - \partial_N B_M\,,\\
W_{MN}^a &= \partial_M W_N^a - \partial_N W_M^a + g_2^{(5)}\epsilon^{abc}W_M^b W_N^c\,,\\
G_{MN}^A &= \partial_M G_N^A - \partial_N G_M^A + g_3^{(5)} f^{ABC} G_M^B G_N^C\,,
\end{aligned} \tag{A.8}$$

where ϵ^{abc} and f^{ABC} are the structure constants for $SU(2)_W$ and $SU(3)_c$, respectively. The parameter ξ in (A.2) is the gauge fixing parameter in the generalized R_ξ gauge.

The 5-dimensional (4-dimensional) gauge couplings are denoted by $g_i^{(5)}$ (g_i), where $i=1,2,3$ stands for $U(1)_Y$, $SU(2)_W$ and $SU(3)_c$, correspondingly. The two types of couplings are related by

$$g_i = \frac{g_i^{(5)}}{\sqrt{\pi R}}\,. \tag{A.9}$$

Finally, $Q(x,y)$ and $L(x,y)$ are the $SU(2)_W$-doublet fermions from Table 1, while $u(x,y)$, $d(x,y)$ and $e(x,y)$ are the corresponding $SU(2)_W$-singlet fermions from Table 1. $P_{L,R}=\frac{1\mp\gamma^5}{2}$ are the 4D chiral projectors in terms of the usual γ_5 matrix. The gamma matrices in 5D

$$\Gamma^M = (\gamma^\mu, i\gamma^5)\,, \tag{A.10}$$

satisfy the Dirac-Clifford algebra

$$\{\Gamma^M, \Gamma^N\} = 2g^{MN} , \tag{A.11}$$

where g^{MN} is the 5D metric

$$g_{MN} = \begin{pmatrix} g_{\mu\nu} & 0 \\ 0 & -1 \end{pmatrix} , \tag{A.12}$$

and $g^{\mu\nu} = (+ - --)$ is the usual 4D metric.

The covariant derivatives act on 5D fields as follows

$$\begin{aligned} D_M Q(x,y) &= \left(\partial_M + ig_3^{(5)} G_M + ig_2^{(5)} W_M + i\frac{Y_Q}{2} g_1^{(5)} B_M\right) Q(x,y) , \\ D_M u(x,y) &= \left(\partial_M + ig_3^{(5)} G_M + i\frac{Y_u}{2} g_1^{(5)} B_M\right) u(x,y) , \\ D_M d(x,y) &= \left(\partial_M + ig_3^{(5)} G_M + i\frac{Y_d}{2} g_1^{(5)} B_M\right) d(x,y) , \\ D_M L(x,y) &= \left(\partial_M + ig_2^{(5)} W_M + i\frac{Y_L}{2} g_1^{(5)} B_M\right) L(x,y) , \\ D_M e(x,y) &= \left(\partial_M + i\frac{Y_e}{2} g_1^{(5)} B_M\right) e(x,y) , \end{aligned} \tag{A.13}$$

where the fermion hypercharges are $Y_Q = \frac{1}{3}$, $Y_u = \frac{4}{3}$, $Y_d = -\frac{2}{3}$, $Y_L = -1$ and $Y_e = -2$.

It is now a rather straightforward but tedious exercise to substitute the expansions (A.7) into the 5D Lagrangians (A.1-A.6) and perform the integration over y with the help of the orthonormality relations listed in Appendix B. The resulting Feynman rules in terms of 4-dimensional fields are listed in Appendix C.

Appendix B. Orthonormality Relations

The following orthonormality relations can be used in the process of compactifying the 5-dimensional Lagrangian listed in Appendix A.

$$\begin{aligned}
\int_0^{\pi R} dy \cos(\frac{my}{R}) \cos(\frac{ny}{R}) &= \frac{\pi R}{2}\delta_{m,n}\,, \\
\int_0^{\pi R} dy \sin(\frac{my}{R}) \sin(\frac{ny}{R}) &= \frac{\pi R}{2}\delta_{m,n}\,, \\
\int_0^{\pi R} dy \cos(\frac{my}{R}) \cos(\frac{ny}{R}) \cos(\frac{ly}{R}) &= \frac{\pi R}{4}\Delta^1_{mnl}\,, \\
\int_0^{\pi R} dy \cos(\frac{my}{R}) \cos(\frac{ny}{R}) \cos(\frac{ly}{R}) \cos(\frac{ky}{R}) &= \frac{\pi R}{8}\Delta^2_{mnlk}\,, \\
\int_0^{\pi R} dy \sin(\frac{my}{R}) \sin(\frac{ny}{R}) \sin(\frac{ly}{R}) \sin(\frac{ky}{R}) &= \frac{\pi R}{8}\Delta^3_{mnlk}\,, \\
\int_0^{\pi R} dy \sin(\frac{my}{R}) \sin(\frac{ny}{R}) \cos(\frac{ly}{R}) &= \frac{\pi R}{4}\Delta^4_{mnl}\,, \\
\int_0^{\pi R} dy \sin(\frac{my}{R}) \sin(\frac{ny}{R}) \cos(\frac{ly}{R}) \cos(\frac{ky}{R}) &= \frac{\pi R}{8}\Delta^5_{mnlk}\,, \\
\int_0^{\pi R} dy \cos(\frac{my}{R}) \sin(\frac{ny}{R}) &= 0\,, \\
\int_0^{\pi R} dy \sin(\frac{my}{R}) \sin(\frac{ny}{R}) \sin(\frac{ly}{R}) &= 0\,, \\
\int_0^{\pi R} dy \sin(\frac{my}{R}) \cos(\frac{ny}{R}) \cos(\frac{ly}{R}) &= 0\,, \\
\int_0^{\pi R} dy \sin(\frac{my}{R}) \cos(\frac{ny}{R}) \cos(\frac{ly}{R}) \cos(\frac{ky}{R}) &= 0\,, \\
\int_0^{\pi R} dy \sin(\frac{my}{R}) \sin(\frac{ny}{R}) \sin(\frac{ly}{R}) \cos(\frac{ky}{R}) &= 0\,,
\end{aligned} \tag{B.1}$$

where the Δ symbols are defined as

$$\Delta^1_{mnl} = \delta_{l,m+n} + \delta_{n,l+m} + \delta_{m,l+n}\,, \tag{B.2}$$

$$\begin{aligned}\Delta^2_{mnlk} = {} & \delta_{k,l+m+n} + \delta_{l,m+n+k} + \delta_{m,n+k+l} + \delta_{n,k+l+m} \\ & + \delta_{k+m,l+n} + \delta_{k+l,m+n} + \delta_{k+n,l+m}\,,\end{aligned} \tag{B.3}$$

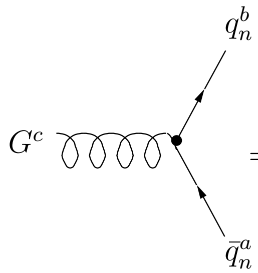
$$\begin{aligned}\Delta^3_{mnlk} = {} & -\delta_{k,l+m+n} - \delta_{l,m+n+k} - \delta_{m,n+k+l} - \delta_{n,k+l+m} \\ & + \delta_{k+l,m+n} + \delta_{k+m,l+n} + \delta_{k+n,l+m}\,,\end{aligned} \tag{B.4}$$

$$\Delta^4_{mnl} = -\delta_{l,m+n} + \delta_{n,l+m} + \delta_{m,l+n}\,, \tag{B.5}$$

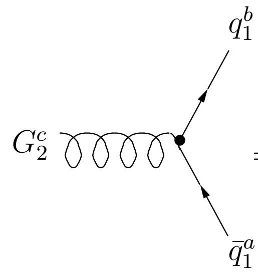
$$\begin{aligned}\Delta^5_{mnlk} = {} & -\delta_{k,l+m+n} - \delta_{l,m+n+k} + \delta_{m,n+k+l} + \delta_{n,k+l+m} \\ & - \delta_{k+l,m+n} + \delta_{k+m,l+n} + \delta_{k+n,l+m}\,.\end{aligned} \tag{B.6}$$

Appendix C. Feynman Rules

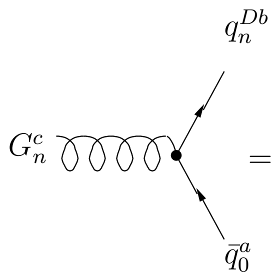
Here we list some of the KK-number conserving vertices with subscripts 'n' standing for the KK-level, which are obtained after compactifying the 5-dimensional Lagrangian of Appendix A with the help of the orthonormality relations of Appendix B. For KK-number violating (but still KK-parity conserving) vertices, refer to Fig. 3 and Table 2.



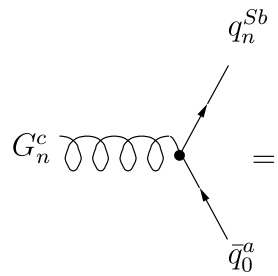
$= -ig_3\gamma^\mu T^c_{ba}$



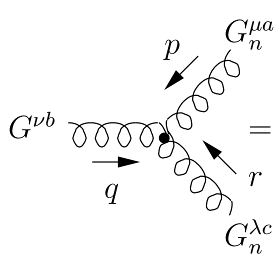
$= -i\frac{g_3}{\sqrt{2}}\gamma^\mu T^c_{ba}$



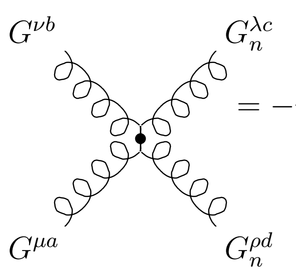
$= -ig_3\gamma^\mu T^c_{ba} P_L$



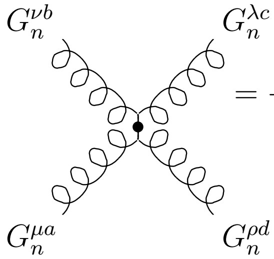
$= -ig_3\gamma^\mu T^c_{ba} P_R$



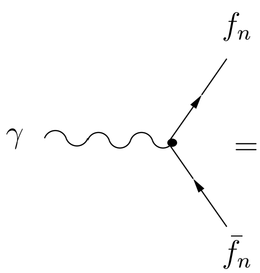
$= ig_3 f^{abc}\left[(p-q)_\lambda g_{\mu\nu} + (q-r)_\mu g_{\lambda\nu} + (r-p)_\nu g_{\lambda\mu}\right]$



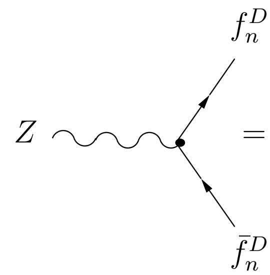
$$= -ig_3^2\Big[f^{abe}f^{cde}(g_{\lambda\nu}g_{\mu\rho} - g_{\lambda\rho}g_{\mu\nu}) + f^{acd}f^{bde}(g_{\lambda\mu}g_{\nu\rho} - g_{\lambda\rho}g_{\mu\nu}) + f^{ade}f^{bce}(g_{\lambda\mu}g_{\nu\rho} - g_{\lambda\nu}g_{\mu\rho})\Big]$$



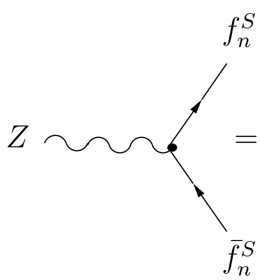
$$= -i\tfrac{3}{2}g_3^2\Big[f^{abe}f^{cde}(g_{\lambda\nu}g_{\mu\rho} - g_{\lambda\rho}g_{\mu\nu}) + f^{acd}f^{bde}(g_{\lambda\mu}g_{\nu\rho} - g_{\lambda\rho}g_{\mu\nu}) + f^{ade}f^{bce}(g_{\lambda\mu}g_{\nu\rho} - g_{\lambda\nu}g_{\mu\rho})\Big]$$



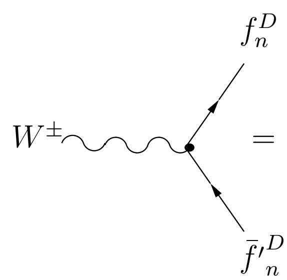
$$= -iQ_f e\gamma^\mu$$



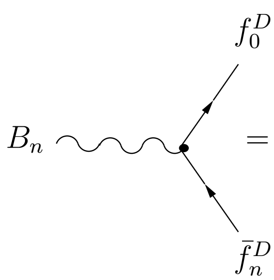
$$= -i\tfrac{g_2}{\cos\theta_W}c_L\gamma^\mu$$



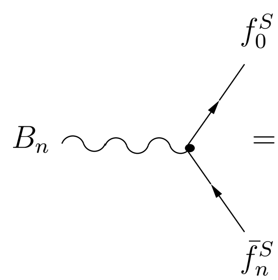
$$= -i\tfrac{g_2}{\cos\theta_W}c_R\gamma^\mu$$



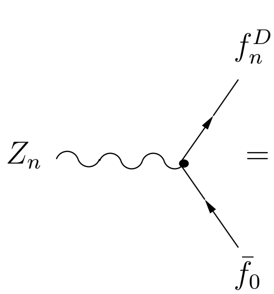
$$= -i\tfrac{g_2}{\sqrt{2}}\gamma^\mu V_{ff'}$$



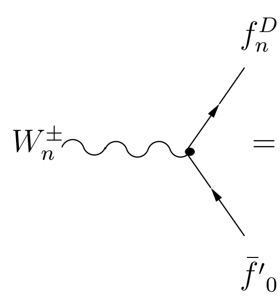
$$= -i\tfrac{Y}{2}g_1\gamma^\mu P_L$$



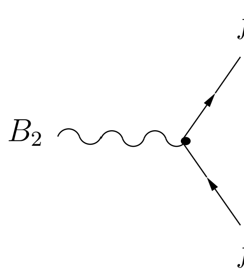
$$= -i\tfrac{Y}{2}g_1\gamma^\mu P_R$$



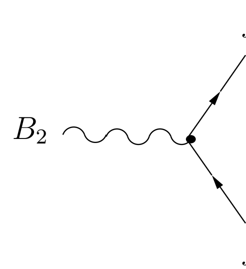
$$= -iI_3 g_2\gamma^\mu P_L$$



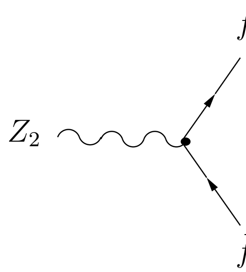
$$= -i\tfrac{g_2}{\sqrt{2}}\gamma^\mu P_L V_{ff'}$$



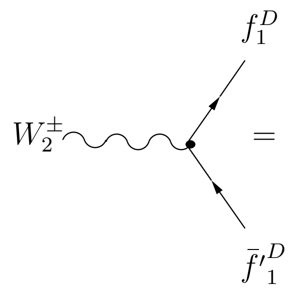
$= i\frac{Y}{2}\frac{g_1}{\sqrt{2}}\gamma^\mu\gamma^5$



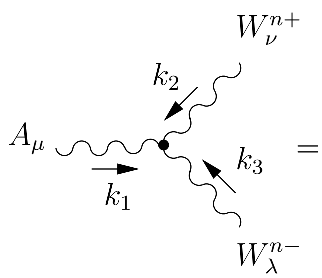
$= -i\frac{Y}{2}\frac{g_1}{\sqrt{2}}\gamma^\mu\gamma^5$



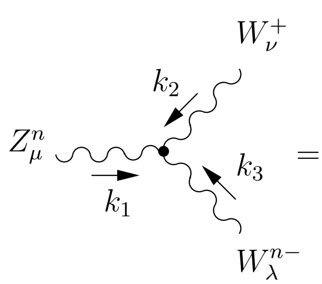
$= iI_3\frac{g_2}{\sqrt{2}}\gamma^\mu\gamma^5$



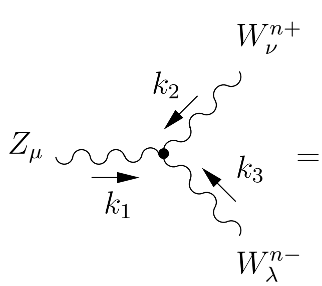
$= -i\frac{g_2}{2}\gamma^\mu P_L V_{ff'}$



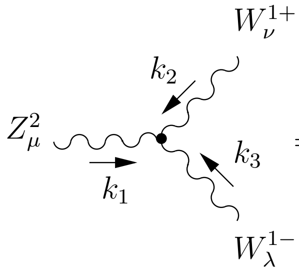
$= -ie[(k_1 - k_2)g_{\mu\nu} + (k_2 - k_3)g_{\nu\lambda} + (k_3 - k_1)g_{\lambda\mu}]$



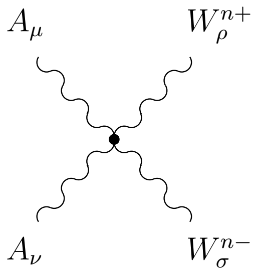
$= -ig_2[(k_1 - k_2)g_{\mu\nu} + (k_2 - k_3)g_{\nu\lambda} + (k_3 - k_1)g_{\lambda\mu}]$



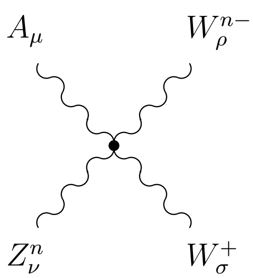
$= -ig_2\cos\theta_W[(k_1 - k_2)g_{\mu\nu} + (k_2 - k_3)g_{\nu\lambda} + (k_3 - k_1)g_{\lambda\mu}]$



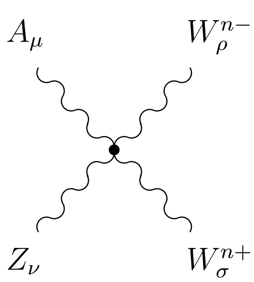
$$= -i\frac{g_2}{\sqrt{2}}\cos\theta_W[(k_1 - k_2)g_{\mu\nu} + (k_2 - k_3)g_{\nu\lambda} + (k_3 - k_1)g_{\lambda\mu}]$$



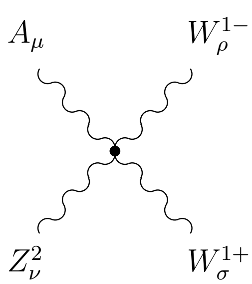
$$= -ie^2(2g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho})$$



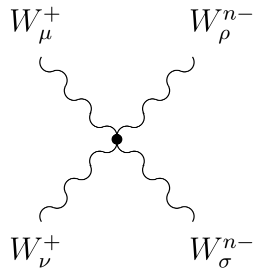
$$= -i\frac{e^2}{\sin\theta_W}(2g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho})$$



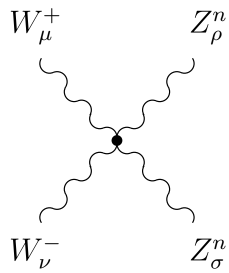
$$= -i\frac{\cos\theta_W e^2}{\sin\theta_W}(2g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho})$$



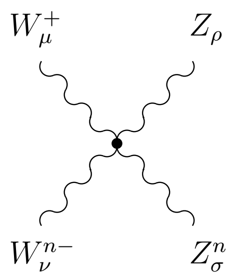
$$= -i\frac{1}{\sqrt{2}}\frac{e^2}{\sin\theta_W}(2g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho})$$



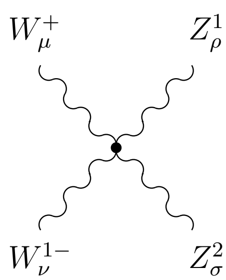
$$= ig_2^2(2g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho})$$



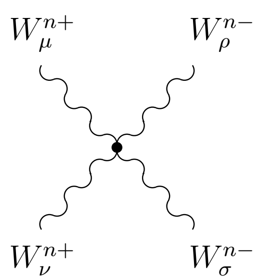
$$= -ig_2^2(2g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho})$$



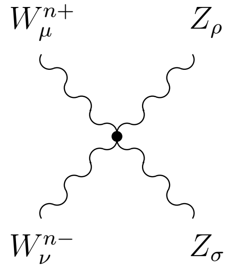
$$= -i\cos\theta_W g_2^2(2g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho})$$



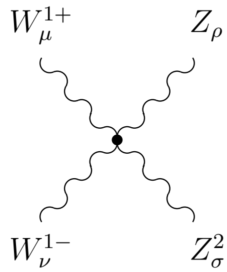
$$= -i\tfrac{1}{\sqrt{2}}\tfrac{e^2}{\sin\theta_W}(2g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho})$$



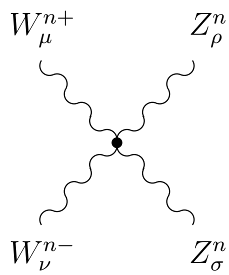
$$= i\tfrac{3}{2}g_2^2(2g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho})$$



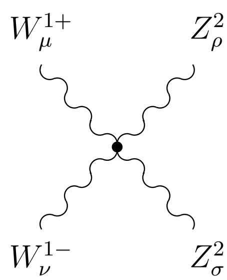
$$= -i\cos^2\theta_W g_2^2(2g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho})$$



$$= -i\tfrac{1}{\sqrt{2}}\cos\theta_W g_2^2(2g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho})$$



$$= -i\tfrac{3}{2}g_2^2(2g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho})$$



$$= -i\tfrac{1}{2}g_2^2(2g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho})$$

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