

# QCD on the Light-Front

## A Systematic Approach to Hadron Physics

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Light-Front Hamiltonian theory, derived from the quantization of the QCD Lagrangian at fixed light-front time  $x^+ = x^0 + x^3$ , provides a rigorous frame-independent framework for solving non-perturbative QCD. The eigenvalues of the light-front QCD Hamiltonian  $H_{LF}$  predict the hadronic mass spectrum, and the corresponding eigensolutions provide the light-front wavefunctions which describe hadron structure, providing a direct connection to the QCD Lagrangian. In the semiclassical approximation the valence Fock-state wavefunctions of the light-front QCD Hamiltonian satisfy a single-variable relativistic equation of motion, analogous to the nonrelativistic radial Schr odinger equation, with an effective confining potential  $U$  which systematically incorporates the effects of higher quark and gluon Fock states. Remarkably, the potential  $U$  has a unique form of a harmonic oscillator potential if one requires that the chiral QCD action remains conformally invariant. A mass gap and the color confinement scale also arises when one extends the formalism of de Alfaro, Fubini and Furlan to light-front Hamiltonian theory. In the case of mesons, the valence Fock-state wavefunctions of  $H_{LF}$  for zero quark mass satisfy a single-variable relativistic equation of motion in the invariant variable  $\zeta^2 = b_\perp^2 x(1-x)$ , which is conjugate to the invariant mass squared  $M_{q\bar{q}}^2$ . The result is a nonperturbative relativistic light-front quantum mechanical wave equation which incorporates color confinement and other essential spectroscopic and dynamical features of hadron physics, including a massless pion for zero quark mass and linear Regge trajectories  $M^2(n, L, S) = 4\kappa^2(n+L+S/2)$  with the same slope in the radial quantum number  $n$  and orbital angular momentum  $L$ . Only one mass parameter  $\kappa$  appears. The corresponding light-front Dirac equation provides a dynamical and spectroscopic model of nucleons. The same light-front equations arise from the holographic mapping of the soft-wall model modification of AdS<sub>5</sub> space with a unique dilaton profile to QCD (3+1) at fixed light-front time. Light-front holography thus provides a precise relation between the bound-state amplitudes in the fifth dimension of AdS space and the boost-invariant light-front wavefunctions describing the internal structure of hadrons in physical space-time. We also discuss the implications of the underlying conformal template of QCD for renormalization scale-setting and the implications of light-front quantization for the value of the cosmological constant.

### I. INTRODUCTION

The remarkable advantages of using light-front time  $x^+ = x^0 + x^3$  (the ‘‘front form’’) to quantize a quantum field theory instead of the standard time  $x^0$  (the ‘‘instant form’’) were first demonstrated by Dirac [1]. As Dirac showed, the front form has the maximum number of kinematic generators of the Lorentz group, including the boost operator. Thus the description of a hadron at fixed  $x^+$  is independent of the observer’s Lorentz frame, making it the natural formalism for addressing dynamical processes in quantum chromodynamics. In contrast, the boost of a wavefunction of a hadron at fixed ‘instant’ time  $x^0$  is a difficult nonperturbative problem [2]. An extensive review of light-front quantization is given in Ref. [3].

The quantization of QCD at fixed light-front (LF) time – light-front quantization – provides a first-principles method for solving nonperturbative QCD. Given the Lagrangian, one can compute the LF Hamiltonian  $H_{LF}$  in terms of the independent quark and gluon fields. The eigenvalues of  $H_{LF}$  determine the mass-squared values of both the discrete and continuum hadronic spectra. The eigensolutions  $|\Psi_H\rangle$  projected on the free  $n$ -parton Fock state  $\langle n|\Psi_H\rangle$  determine the LF wavefunctions  $\psi_{n/H}(x_i, \vec{k}_{\perp i}, \lambda_i)$  where the  $x_i = \frac{k_i^+}{P^+} = \frac{k_i^0 + k_i^3}{P^0 + P^3}$ , with  $\sum_{i=1}^n x_i = 1$ , are the light-front moment fractions. The eigenstates are defined at fixed  $x^+$  within the causal horizon, so that causality is maintained without normal-ordering. In fact, light-front physics is a fully relativistic field theory, but its structure is similar to non-relativistic atomic physics, and the resulting bound-state equations can be formulated as relativistic Schr odinger-like equations at equal light-front time.

Given the frame-independent light-front wavefunctions (LFWFs)  $\psi_{n/H}$ , one can compute a large range of hadronic observables, starting with form factors, structure functions, generalized parton distributions, Wigner distributions, etc., as illustrated in Fig. 1. For example, the ‘‘handbag’’ contribution [4] to the  $E$  and  $H$  generalized parton distributions for deeply virtual Compton scattering can be computed from the overlap of LFWFs, automatically satisfy the known sum rules. In the case of deep-inelastic lepton-proton scattering  $\ell p \rightarrow \ell' X$  the lepton scatters at

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fixed  $x^+$  on any quark in any one of the proton's Fock states. The higher Fock states of a proton such as  $|uudQ\bar{Q}\rangle$  are the source of the sea-quark distributions, including contributions “intrinsic” to the proton structure [5]. One can also analyze and prove factorization and evolution equations for exclusive processes [6–8]. Analogous light-front methods can be applied to inclusive process where more than one parton from an initial- or final-state hadron are involved [9].

A measurement in the front form is analogous to taking a flash picture. The image in the resulting photograph records the state of the object as the front of the light wave from the flash illuminates it; in effect, this is a measurement within the spacelike causal horizon  $\Delta x_\mu^2 \leq 0$ . Similarly, measurements such as deep inelastic lepton-hadron scattering  $\ell H \rightarrow \ell' X$ , determine the LF wavefunction and structure of the target hadron  $H$  at fixed light-front time. For example, the BFKL Regge behavior of structure functions can be demonstrated [10] from the behavior of LFWFs at small  $x$ . The final-state interactions of the struck quark on the spectators quarks produce “lensing effects” such as the ‘Sivers’ effect [11], the pseudo- $T$ -odd correlation  $\vec{S} \cdot \vec{q} \times \vec{p}_q$  between the proton polarization and the virtual photon-quark scattering plane. Such lensing effects are leading twist; i.e. they are not power-law suppressed [12]. The Sivers correlation from initial-state lensing in the Drell-Yan process has the opposite sign [13–15]. The lensing interactions can be considered as properties of the LFWFs augmented by a Wilson line [16]. The existence of “lensing effects” at leading twist, such as the  $T$ -odd “Sivers effect” in spin-dependent semi-inclusive deep-inelastic scattering, was first demonstrated using LF methods [12].

The physics of diffractive deep inelastic scattering and other hard processes where the projectile hadron remains intact, such as  $\gamma^* p \rightarrow X + p'$ , is also most easily analysed using LF QCD [17]. LF quantization thus provides a distinction [18] between static (the square of LFWFs) distributions versus non-universal dynamic structure functions, such as the Sivers single-spin correlation and diffractive deep inelastic scattering which involve final state interactions. The origin of nuclear shadowing and flavor-dependent anti-shadowing also becomes explicit [19]. QCD properties such as “color transparency” [20], the “hidden color” of the deuteron LFWF [21], and the existence of intrinsic heavy quarks in the LFWFs of light hadrons [5, 22] can be derived from the structure of hadronic LFWFs. It is also possible to compute jet hadronization at the amplitude level from first principles from the LFWFs [23].

In principle, one can solve nonperturbative QCD by diagonalizing the light-front QCD Hamiltonian  $H_{LF}$  directly using the “discretized light-cone quantization” (DLCQ) method [3] which imposes periodic boundary conditions to discretize the  $k^+$  and  $k_\perp$  momenta, or the Hamiltonian transverse lattice formulation introduced in refs. [24–26]. The hadronic spectra and light-front wavefunctions are then obtained from the eigenvalues and eigenfunctions of the Heisenberg problem  $H_{LF}|\psi\rangle = M^2|\psi\rangle$ , an infinite set of coupled integral equations for the light-front components  $\psi_n = \langle n|\psi\rangle$  in a Fock expansion [3]. The DLCQ method has been applied successfully in lower space-time dimensions [3], such as QCD(1+1) [27]. For example, one can compute the complete spectrum of meson and baryon states in  $QCD(1+1)$  and their LF wavefunctions, for any number of colors, flavors, and quark masses by matrix diagonalization using DLCQ [28]. It has also been applied successfully to a range of 1+1 string theory problems by Hellerman and Polchinski [29, 30]. Unlike lattice gauge theory, the DLCQ method is relativistic, has no fermion-doubling, is formulated in Minkowski space, and is independent of the hadron's momentum  $P^+$  and  $P_\perp$ .

Solving the eigenvalue problem using DLCQ is a formidable computational task for a non-abelian quantum field theory in four-dimensional space-time because of the large number of independent variables. Consequently, alternative methods and approximations are necessary to better understand the nature of relativistic bound-states in the strong-coupling regime. One of the most promising methods for solving nonperturbative (3+1) QCD is the “Basis Light-Front Quantization” (BLFQ) method [31]. In the BLFQ method one constructs a complete orthonormal basis of eigenstates based on the eigensolutions of the effective light-front Schrödinger equation derived from light-front holography, in the spirit of the nuclear shell model. Matrix diagonalization for BLFQ should converge more rapidly than DLCQ since the basis states have a mass spectrum close to the observed hadronic spectrum.

As we shall discuss here, light-front quantized field theory in physical 3 + 1 space-time has a holographic dual with the dynamics of theories in five-dimensional anti-de Sitter (AdS) space [32], giving important insight into the nature of color confinement in QCD. Light-front holography – the duality between the front form and classical gravity based on the isometries of AdS<sub>5</sub> space, provides a new method for determining the eigenstates of the QCD LF Hamiltonian in the strongly coupled regime. In the case of mesons, for example, the valence Fock-state wavefunctions of  $H_{LF}$  for zero quark mass satisfy a single-variable relativistic equation of motion in the invariant variable  $\zeta^2 = b_\perp^2 x(1-x)$ , which is conjugate to the invariant mass squared  $M_{q\bar{q}}^2$ . The effective confining potential  $U(\zeta^2)$  in this frame-independent “light-front Schrödinger equation” systematically incorporates the effects of higher quark and gluon Fock states [33]. The hadron mass scale – its “mass gap” – is generated in a novel way. Remarkably, the potential  $U(\zeta^2)$  has a unique form of a harmonic oscillator potential if one requires that the chiral QCD action remains conformally invariant [33]. The result is a nonperturbative relativistic light-front quantum mechanical wave equation which incorporates color confinement and other essential spectroscopic and dynamical features of hadron physics.

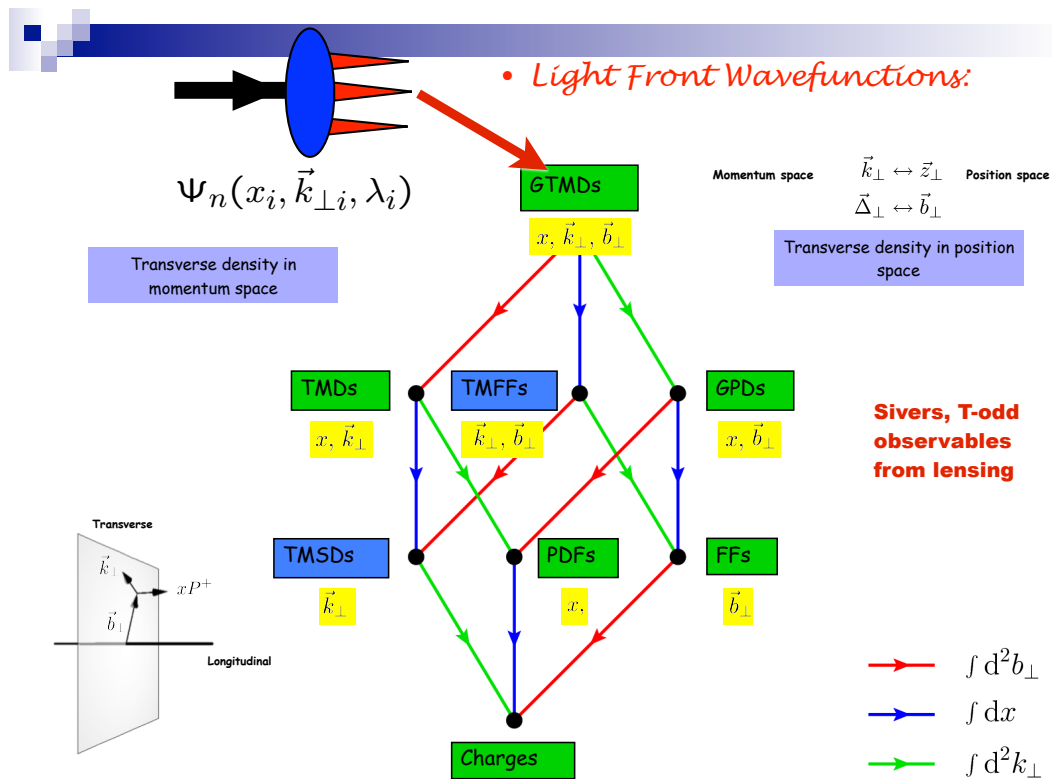


FIG. 1: Examples of hadronic observables, including Wigner functions and  $T$ -odd observables which are based on overlaps of light-front wavefunctions. Adopted from a figure by F. Lorce and B. Pasquini. [34]

## II. WHAT IS THE ORIGIN OF THE QCD MASS SCALE?

If one sets the masses of the quarks to zero, no mass scale appears explicitly in the QCD Lagrangian. The classical theory thus displays invariance under both scale (dilatation) and special conformal transformations [35]. Nevertheless, the quantum theory built upon this conformal template displays color confinement, a mass gap, as well as asymptotic freedom. A fundamental question is thus how does the mass scale which determines the masses of the light-quark hadrons, the range of color confinement, and the running of the coupling appears in QCD?

A hint to the origin of the mass scale in nominally conformal theories was given in 1976 in a remarkable paper by V. de Alfaro, S. Fubini and G. Furlan (dAFF) [36] in the context of one-dimensional quantum mechanics. They showed that the mass scale which breaks dilatation invariance can appear in the equations of motion without violating the conformal invariance of the action. In fact, this is only possible if the resulting potential has the form of a confining harmonic oscillator, and the transformed time variable  $\tau$  that appears in the confining theory has a limited range.

In this contribution we will review how the application of the dAFF procedure, together with light-front quantum mechanics and light-front holographic mapping, leads to a new analytic approximation to QCD – a light-front Hamiltonian and corresponding one-dimensional light-front Schrödinger and Dirac equations which are frame-independent, relativistic, and reproduce crucial features of the spectroscopy and dynamics of the light-quark hadrons. The predictions of the LF equations of motion include a zero-mass pion in the chiral  $m_q \rightarrow 0$  limit, and linear Regge trajectories  $M^2(n, L) \propto n + L$  with the same slope in the radial quantum number  $n$  (the number of nodes) and  $L = \max |L^z|$ , the internal orbital angular momentum. In fact, we will also show that the effective confinement potential which appears in the LF equations of motion is unique if we require that the corresponding one-dimensional effective action which encodes the chiral symmetry of QCD remains conformally invariant.

## III. APPLICATIONS OF LIGHT-FRONT HAMILTONIAN THEORY

Computing hadronic matrix elements of currents is particularly simple in the light-front, since space-like current matrix elements can be written as an overlap of frame-independent light-front wave functions as in the Drell-Yan-

West formula [37–39]. If the virtual space-like photon has  $q^+ = 0$ , only processes with the same number of initial and final partons are allowed. One can also prove fundamental theorems for relativistic quantum field theories using the front form, including: the cluster decomposition theorem [40], and the vanishing of the anomalous gravitomagnetic moment for any Fock state of a hadron [41]. The contribution to the hadron’s anomalous gravitomagnetic moment vanishes for each Fock state - a crucial test of the consistency of the front-form formalism. One also can show that a nonzero anomalous magnetic moment of a bound state requires nonzero angular momentum of the constituents [39]. In contrast, if one uses ordinary fixed time  $x^0$  the hadronic states must be boosted from the hadron’s rest frame to a moving frame – an intractable dynamical problem. In fact, the boost of a composite system at fixed time  $x^0$  is only known at weak binding [2, 42]. Moreover, form factors at fixed instant time  $x^0$  require computing off-diagonal matrix elements as well as the contributions of currents arising from fluctuations of the vacuum in the initial state which connect to the hadron wavefunction in the final state. Thus the knowledge of the wave functions of hadronic eigenstates alone is not sufficient to compute covariant current matrix elements in the usual instant-form framework.

The gauge-invariant meson and baryon distribution amplitudes  $\phi_H(x_i)$  which control hard exclusive and direct reactions are the valence LFWFs integrated over transverse momentum at fixed  $x_i = k^+/P^+$ . The ERBL evolution of distribution amplitudes and the factorization theorems for hard exclusive processes are derived using LF theory [7, 8].

Quantization in the light-front provides the rigorous field-theoretical realization of the intuitive ideas of the parton model [43, 44] which is formulated at fixed  $t$  in the infinite-momentum frame (IMF) [45, 46]. The same results are obtained in the front form for any frame; e.g., the structure functions and other probabilistic parton distributions measured in deep inelastic scattering are obtained from the squares of the boost invariant LFWFs, the eigensolution of the light-front Hamiltonian. The familiar kinematic variable  $x_{bj}$  of deep inelastic scattering becomes identified with the LF fraction at small  $x$ . The IMF prescription thus allows one to formally use the acausal instant form, but it is awkward, since it is frame-dependent, requiring an unphysical “infinite” boost of a composite system.

Because of Wick’s theorem, light-front time-ordered perturbation theory is equivalent to covariant Feynman perturbation theory. The higher order calculation of the electron anomalous moment at order  $\alpha^3$  and the “alternating denominator method” for renormalizing LF perturbation theory is given in Ref. [47]. LF Hamiltonian perturbation theory provides a simple method for deriving analytic forms for the analog of Parke-Taylor amplitudes [48] where each particle spin  $S^z$  is quantized in the LF  $z$  direction. The gluonic  $g^6$  amplitude  $T(-1 - 1 \rightarrow +1 + 1 + 1 + 1 + 1)$  requires  $\Delta L^z = 8$ ; it thus must vanish at tree level since each three-gluon vertex has  $\Delta L^z = \pm 1$ . However, the order  $g^8$  one-loop amplitude can be nonzero. Stasto and Cruz-Santiago [49] have shown that the cluster properties [50] of LF time-ordered perturbation theory, together with  $J^z$  conservation, can also be used to elegantly derive the Parke-Taylor rules for multi-gluon scattering amplitudes.

The counting-rule [51] behavior of structure functions at large  $x$  and Bloom-Gilman duality have also been derived in light-front QCD as well as from holographic QCD [52]. The instantaneous fermion interaction in LF quantization provides a simple derivation of the  $J = 0$  fixed pole contribution to deeply virtual Compton scattering [53], i.e., the  $e_q^2 s^0 F(t)$  contribution to the DVCS amplitude which is independent of photon energy and virtuality. LF quantization also can provide a method to implement jet hadronization at the amplitude level [23].

Angular momentum  $J^z = L^z + S^z$  is conserved at every interaction vertex and for every LF Fock State in the Front Form. The basic physics of the angular momentum composition of hadrons is thus expressed most simply using the front form. If one chooses light-cone gauge  $A^+ = 0$ , then gauge particles have positive metric and physical polarization  $S^z = \pm 1$ . It is also possible to quantize QCD in Feynman gauge in the light front [54].

In a LF Fock state with  $n$  constituents there are  $n - 1$  independent relative orbital angular momentum components  $L_i^z$  so that  $\sum_{i=1}^{n-1} L_i^z = L^z$ . For example, the  $|e^+e^- \rangle$  Fock state of positronium has one relative angular momentum of the electron relative to the positron. This agrees with the usual convention used in atomic physics. In fact, the total sum  $\sum_{i=1}^n L_i^z$  vanishes because of the vanishing of the anomalous gravitomagnetic moment [41].

Information on the spin composition of the proton is obtained from measurements of deep inelastic lepton scattering on a polarized proton target. As we discuss below, the proton’s LF wavefunctions are predicted in detail from AdS/QCD in the quark-diquark picture via the LF Dirac equation obtained from LF Holography using a unique confinement potential. One predicts equal probability for the struck quark to have  $S^z = \pm 1/2$ , and thus the proton spin is carried on the average by quark orbital angular momentum  $\langle L^z \rangle = J^z = \pm 1/2$ , not quark spin. As Burkardt has emphasized [55], the effects of lensing in deep inelastic lepton scattering also impacts the observed proton spin composition.

Light-front quantization is thus the natural framework for interpreting measurements like deep inelastic scattering in terms of the nonperturbative relativistic bound-state structure of hadrons in quantum chromodynamics. The LFWFs of hadrons provide a direct connection between observables and the QCD Lagrangian. Moreover, the formalism is rigorous, relativistic and frame-independent.

#### IV. THE LIGHT-FRONT VACUUM

It is conventional to define the vacuum in quantum field theory as the lowest energy eigenstate of the instant-form Hamiltonian. Such an eigenstate is defined at a single time  $x^0$  over all space  $\vec{x}$ . It is thus acausal and frame-dependent. The instant-form vacuum thus must be normal-ordered in order to avoid violations of causality when computing correlators and other matrix elements. In contrast, in the front form, the vacuum state is defined as the eigenstate of lowest invariant mass  $M$ . It is defined at fixed light-front time  $x^+ = x^0 + x^3$  over all  $x^- = x^0 - x^3$  and  $\vec{x}_\perp$ , the extent of space that can be observed within the speed of light. It is frame-independent and only requires information within the causal horizon.

Since all particles have positive  $k^+ = k^0 + k^z > 0$  and + momentum is conserved in the front form, the usual vacuum bubbles are kinematically forbidden in the front form. In fact the LF vacuum for QED, QCD, and even the Higgs Standard Model is trivial up to possible zero modes – backgrounds with zero four-momentum. In this sense it is already normal-ordered. In the case of the Higgs theory, the usual Higgs vacuum expectation value is replaced by a classical  $k^+ = 0$  background zero-mode field which is not sensed by the energy momentum tensor [56]. The phenomenology of the Higgs theory is unchanged.

There are thus no quark or gluon vacuum condensates in the LF vacuum – as first noted by Casher and Susskind [57]; the corresponding physics is contained within the LFWFs themselves [58–62], thus eliminating a major contribution to the cosmological constant. In the light-front formulation of quantum field theory, phenomena such as the GMOR relation – usually associated with condensates in the instant form vacuum – are properties of the hadronic LF wavefunctions themselves. An exact Bethe-Salpeter analysis [63] shows that the quantity that appears in the Gell-Mann-Oakes-Renner (GMOR) relation [64] relation is the matrix element  $\langle 0 | \bar{\psi} \gamma_5 \psi | \pi \rangle$  for the pion to couple locally to the vacuum via a pseudoscalar operator – not a vacuum expectation value  $\langle 0 | \bar{\psi} \psi | 0 \rangle$ . In the front-form  $\langle 0 | \bar{\psi} \gamma_5 \psi | \pi \rangle$  involves the pion LF Fock state with parallel  $q$  and  $\bar{q}$  spin and  $L^z = \pm 1$ . This second pion Fock state automatically appears when the quarks are massive.

The simple structure of the light-front vacuum thus allows an unambiguous definition of the partonic content of a hadron in QCD. The frame-independent causal front-form vacuum is a good match to the “void” – the observed universe without luminous matter. Thus it is natural in the front form to obtain zero cosmological constant from quantum field theory. For a recent review see Ref. [65].

#### V. THE CONFORMAL SYMMETRY TEMPLATE

In the case of perturbative QCD (pQCD), the running coupling  $\alpha_s(Q^2)$  becomes constant in the limit of zero  $\beta$ -function and zero quark mass, and conformal symmetry becomes manifest. In fact, the renormalization scale uncertainty in pQCD predictions can be eliminated by using the Principle of Maximum Conformality (PMC) [66]. Using the PMC/BLM procedure [67], all non-conformal contributions in the perturbative expansion series are summed into the running coupling by shifting the renormalization scale in  $\alpha_s$  from its initial value, and one obtains unique, scale-fixed, scheme-independent predictions at any finite order. One can also introduce a generalization of conventional dimensional regularization, the  $\mathcal{R}_\delta$  schemes. For example, when one generalizes the MSBar scheme by subtracting  $\ln 4\pi - \gamma_E - \delta$  instead of just  $\ln 4\pi - \gamma_E$  the new terms generated in the pQCD series proportional to  $\delta$  expose the  $\beta$  terms and thus the renormalization scheme dependence. Thus the  $\mathcal{R}_\delta$  schemes uncover the renormalization scheme and scale ambiguities of pQCD predictions, exposes the general pattern of nonconformal terms, and allows one to systematically determine the argument of the running coupling order by order in pQCD in a form which can be readily automatized [68, 69]. The resulting PMC scales and finite-order PMC predictions are to high accuracy independent of the choice of initial renormalization scale. For example, PMC scale-setting leads to a scheme-independent pQCD prediction [70] for the top-quark forward-backward asymmetry which is within one  $\sigma$  of the Tevatron measurements. The PMC procedure also provides scale-fixed, scheme-independent commensurate scale relations [71] between observables which are based on the underlying conformal behavior of QCD, such as the generalized Crewther relation [72]. The PMC satisfies all of the principles of the renormalization group: reflectivity, symmetry, and transitivity, and it thus eliminates an unnecessary source of systematic error in pQCD predictions [73].

#### VI. LIGHT-FRONT HOLOGRAPHY

An important analysis tool for QCD is Anti-de Sitter space in five dimensions. In particular, AdS<sub>5</sub> provides a remarkable geometric representation of the conformal group which underlies the conformal symmetry of classical QCD. A simple way to obtain confinement and discrete normalizable modes is to truncate AdS space with the introduction of a sharp cut-off in the infrared region of AdS space, as in the “hard-wall” model [74], where one considers a slice of

AdS space,  $0 \leq z \leq z_0$ , and imposes boundary conditions on the fields at the IR border  $z_0 \sim 1/\Lambda_{\text{QCD}}$ . As first shown by Polchinski and Strassler [74], the modified AdS space, provides a derivation of dimensional counting rules [75, 76] in QCD for the leading power-law fall-off of hard scattering beyond the perturbative regime. The modified theory generates the point-like hard behavior expected from QCD, instead of the soft behavior characteristic of extended objects [74]. The physical states in AdS space are represented by normalizable modes  $\Phi_P(x, z) = e^{iP \cdot x} \Phi(z)$ , with plane waves along Minkowski coordinates  $x^\mu$  and a profile function  $\Phi(z)$  along the holographic coordinate  $z$ . The hadronic invariant mass  $P_\mu P^\mu = M^2$  is found by solving the eigenvalue problem for the AdS wave equation.

One can also modify AdS space by using a dilaton factor in the AdS action  $e^{\varphi(z)}$  to introduce the QCD confinement scale. To a first semiclassical approximation, light-front QCD is formally equivalent to the equations of motion on a fixed gravitational background asymptotic to AdS<sub>5</sub>. In fact, the introduction of a dilaton profile is equivalent to a modification of the AdS metric, even for arbitrary spin [77], but it is left largely unspecified. However, we shall show that if one imposes the requirement that the action of the corresponding one-dimensional effective theory remains conformal invariant, then the dilaton profile  $\varphi(z) \propto z^s$  is constrained to have the specific power  $s = 2$ , a remarkable result which follows from the dAFF construction of conformally invariant quantum mechanics [78]. A related argument is given in Ref. [79]. The quadratic form  $\varphi(z) = \pm \kappa^2 z^2$  immediately leads to linear Regge trajectories [80] in the hadron mass squared.

“Light-Front Holography” refers to the remarkable fact that dynamics in AdS space in five dimensions is dual to a semiclassical approximation to Hamiltonian theory in physical 3+1 space-time quantized at fixed light-front time [32]. The correspondence between AdS and QCD, which was originally motivated by the AdS/CFT correspondence between gravity on a higher-dimensional space and conformal field theories in physical space-time [81], has its most explicit and simplest realization as a direct holographic mapping to light-front Hamiltonian theory [32]. For example, the equation of motion for mesons on the light-front has exactly the same single-variable form as the AdS equation of motion; one can then interpret the AdS fifth dimension variable  $z$  in terms of the physical variable  $\zeta$ , representing the invariant separation of the  $q$  and  $\bar{q}$  at fixed light-front time. There is a precise connection between the quantities that enter the fifth dimensional AdS space and the physical variables of LF theory. The AdS mass parameter  $\mu R$  maps to the LF orbital angular momentum. The formulae for electromagnetic [82] and gravitational [83] form factors in AdS space map to the exact Drell-Yan-West formulae in light-front QCD [84–86].

The light-front holographic principle provides a precise relation between the bound-state amplitudes in AdS space and the boost-invariant LF wavefunctions describing the internal structure of hadrons in physical space-time (See Fig. 2). The resulting valence Fock-state wavefunctions satisfy a single-variable relativistic equation of motion analogous to the eigensolutions of the nonrelativistic radial Schrödinger equation. The quadratic dependence in the effective quark-antiquark potential  $U(\zeta^2, J) = \kappa^4 \zeta^2 + 2\kappa^2(J - 1)$  is determined uniquely from conformal invariance. The constant term  $2\kappa^2(J - 1) = 2\kappa^2(S + L - 1)$  is fixed by the duality between AdS and LF quantization for spin- $J$  states, a correspondence which follows specifically from the separation of kinematics and dynamics on the light-front [77]. The LF potential thus has a specific power dependence—in effect, it is a light-front harmonic oscillator potential. It is confining and reproduces the observed linear Regge behavior of the light-quark hadron spectrum in both the orbital angular momentum  $L$  and the radial node number  $n$ . The pion is predicted to be massless in the chiral limit [87] - the positive contributions to  $m_\pi^2$  from the LF potential and kinetic energy are cancelled by the constant term in  $U(\zeta^2, J)$  for  $J = 0$ . This holds for the positive sign of the dilaton profile  $\varphi(z) = \kappa^2 z^2$ . The LF dynamics retains conformal invariance of the action despite the presence of a fundamental mass scale. The constant term in the LF potential  $U(\zeta^2, J)$  derived from LF Holography is essential; the masslessness of the pion and the separate dependence on  $J$  and  $L$  are consequences of the potential derived from the holographic LF duality with AdS for general  $J$  and  $L$  [77, 78]. Thus the light-front holographic approach provides an analytic frame-independent first approximation to the color-confining dynamics, spectroscopy, and excitation spectra of the relativistic light-quark bound states of QCD. It is systematically improvable in full QCD using the basis light-front quantization (BLFQ) method [31] and other methods.

We now give an example of light-front holographic mapping for the specific case of the elastic pion form factor. In the higher-dimensional gravity theory, the hadronic transition amplitude corresponds to the coupling of an external electromagnetic field  $A^M(x, z)$ , for a photon propagating in AdS space, with an extended field  $\Phi_P(x, z)$  describing a meson in AdS is [82]

$$\int d^4x dz \sqrt{g} A^M(x, z) \Phi_{P'}^*(x, z) \overleftrightarrow{\partial}_M \Phi_P(x, z) \sim (2\pi)^4 \delta^4(P' - P - q) \epsilon_\mu (P + P')^\mu F_M(q^2), \quad (1)$$

where the coordinates of AdS<sub>5</sub> are the Minkowski coordinates  $x^\mu$  and  $z$  labeled  $x^M = (x^\mu, z)$ , with  $M, N = 1, \dots, 5$ , and  $g$  is the determinant of the metric tensor. The expression on the right-hand side of (1) represents the space-like QCD electromagnetic transition amplitude in physical space-time  $\langle P' | J^\mu(0) | P \rangle = (P + P')^\mu F_M(q^2)$ . It is the EM matrix element of the quark current  $J^\mu = \sum_q e_q \bar{q} \gamma^\mu q$ , and represents a local coupling to pointlike constituents. Although

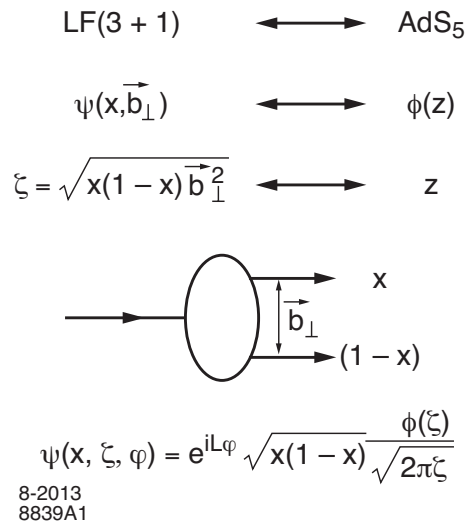


FIG. 2: Light-Front Holography: Mapping between the hadronic wavefunctions of the Anti-de Sitter approach and eigensolutions of the light-front Hamiltonian theory derived from the equality of LF and AdS formula for EM and gravitational current matrix elements and their identical equations of motion.

the expressions for the transition amplitudes look very different, one can show that a precise mapping of the matrix elements can be carried out at fixed light-front time [84, 85].

The form factor is computed in the light front formalism from the matrix elements of the plus current  $J^+$  in order to avoid coupling to Fock states with different numbers of constituents and is given by the Drell-Yan-West expression. The form factor can be conveniently written in impact space as a sum of overlap of LFWFs of the  $j = 1, 2, \dots, n-1$  spectator constituents [88]

$$F_M(q^2) = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2 \mathbf{b}_{\perp j} \exp\left(i \mathbf{q}_{\perp} \cdot \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j}\right) |\psi_{n/M}(x_j, \mathbf{b}_{\perp j})|^2, \quad (2)$$

corresponding to a change of transverse momentum  $x_j \mathbf{q}_{\perp}$  for each of the  $n-1$  spectators with  $\sum_{i=1}^n \mathbf{b}_{\perp i} = 0$ . The formula is exact if the sum is over all Fock states  $n$ .

For simplicity, consider a two-parton bound-state. The  $q\bar{q}$  LF Fock state wavefunction for a meson can be written as

$$\psi(x, \zeta, \varphi) = e^{iL\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}, \quad (3)$$

thus factoring the longitudinal,  $X(x)$ , transverse  $\phi(\zeta)$  and angular dependence  $\varphi$ . If both expressions for the form factor are to be identical for arbitrary values of  $Q$ , we obtain  $\phi(\zeta) = (\zeta/R)^{3/2} \Phi(\zeta)$  and  $X(x) = \sqrt{x(1-x)}$  [84], where we identify the transverse impact LF variable  $\zeta$  with the holographic variable  $z$ ,  $z \rightarrow \zeta = \sqrt{x(1-x)} |\mathbf{b}_{\perp}|$ , where  $x$  is the longitudinal momentum fraction and  $b_{\perp}$  is the transverse-impact distance between the quark and antiquark. Extension of the results to arbitrary  $n$  follows from the  $x$ -weighted definition of the transverse impact variable of the  $n-1$  spectator system given in Ref. [84]. Identical results follow from mapping the matrix elements of the energy-momentum tensor [86].

## VII. THE LIGHT-FRONT SCHRÖDINGER EQUATION: A SEMICLASSICAL APPROXIMATION TO QCD

It is advantageous to reduce the full multiparticle eigenvalue problem of the LF Hamiltonian to an effective light-front Schrödinger equation which acts on the valence sector LF wavefunction and determines each eigensolution separately [89]. In contrast, diagonalizing the LF Hamiltonian yields all eigensolutions simultaneously, a complex task. The central problem then becomes the derivation of the effective interaction  $U$  which acts only on the valence sector of the theory and has, by definition, the same eigenvalue spectrum as the initial Hamiltonian problem. In order

to carry out this program one must systematically express the higher Fock components as functionals of the lower ones. This method has the advantage that the Fock space is not truncated, and the symmetries of the Lagrangian are preserved [89].

A hadron has four-momentum  $P = (P^-, P^+, \mathbf{P}_\perp)$ ,  $P^\pm = P^0 \pm P^3$  and invariant mass  $P^2 = M^2$ . The generators  $P = (P^-, P^+, \vec{P}_\perp)$  are constructed canonically from the QCD Lagrangian by quantizing the system on the light-front at fixed LF time  $x^+$ ,  $x^\pm = x^0 \pm x^3$  [3]. The LF Hamiltonian  $P^-$  generates the LF time evolution with respect to the LF time  $x^+$ , whereas the LF longitudinal  $P^+$  and transverse momentum  $\vec{P}_\perp$  are kinematical generators.

In the limit of zero quark masses the longitudinal modes decouple from the invariant LF Hamiltonian equation  $H_{LF}|\phi\rangle = M^2|\phi\rangle$ , with  $H_{LF} = P_\mu P^\mu = P^- P^+ - \mathbf{P}_\perp^2$ . The result is a relativistic and frame-independent light-front wave equation for  $\phi$  [32] (See Fig. 3)

$$\left[ -\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta^2, J) \right] \phi_{n,J,L}(\zeta^2) = M^2 \phi_{n,J,L}(\zeta^2). \quad (4)$$

This equation describes the spectrum of mesons as a function of  $n$ , the number of nodes in  $\zeta$ , the total angular momentum  $J$ , which represent the maximum value of  $|J^z|$ ,  $J = \max |J^z|$ , and the internal orbital angular momentum of the constituents  $L = \max |L^z|$ . The variable  $z$  of AdS space is identified with the LF boost-invariant transverse-impact variable  $\zeta$  [84], thus giving the holographic variable a precise definition in LF QCD [32, 84]. For a two-parton bound state  $\zeta^2 = x(1-x)b_\perp^2$ .

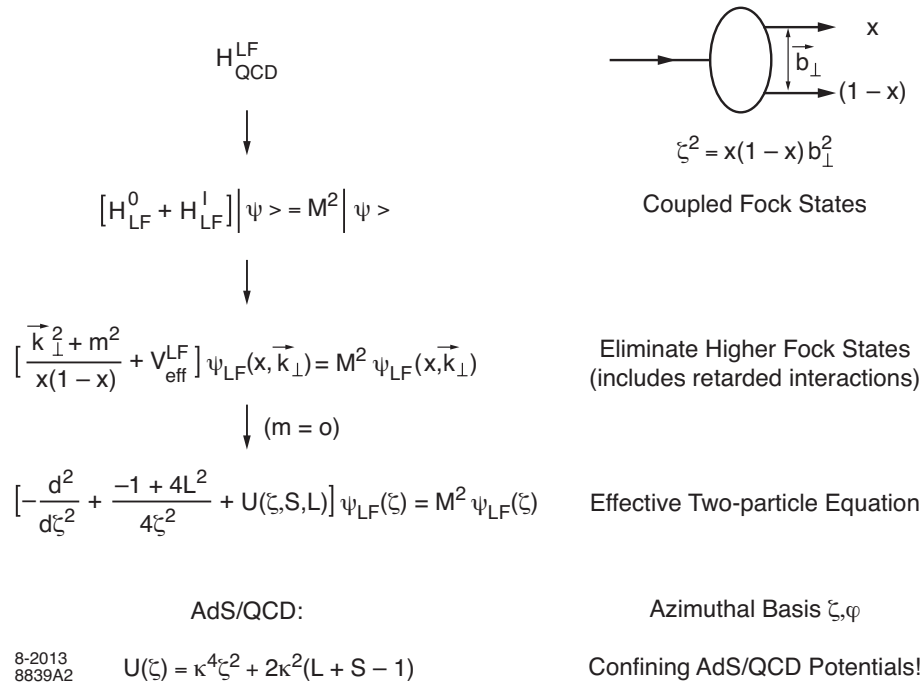


FIG. 3: Reduction of the QCD light-front Hamiltonian to an effective  $q\bar{q}$  bound state equation. The potential is determined from spin- $J$  representations on  $\text{AdS}_5$  space. The harmonic oscillator form of  $U(\zeta^2)$  is determined by the requirement that the action remain conformally invariant.

In the exact QCD theory the potential in the the Light-Front Schrödinger equation (4) is determined from the two-particle irreducible (2PI)  $q\bar{q} \rightarrow q\bar{q}$  Greens' function. In particular, the reduction from higher Fock states in the intermediate states leads to an effective interaction  $U(\zeta^2, J)$  for the valence  $|q\bar{q}\rangle$  Fock state [89]. A related approach for determining the valence light-front wavefunction and studying the effects of higher Fock states without truncation has been given in Ref. [90].

Unlike ordinary instant-time quantization, the light-front Hamiltonian equations of motion are frame independent; remarkably, they have a structure which matches exactly the eigenmode equations in AdS space. This makes a direct connection of QCD with AdS methods possible. In fact, one can derive the light-front holographic duality of AdS by starting from the light-front Hamiltonian equations of motion for a relativistic bound-state system in physical space-time [32].



### VIII. EFFECTIVE CONFINEMENT FROM THE GAUGE/GRAVITY CORRESPONDENCE

Recently we have derived wave equations for hadrons with arbitrary spin  $J$  starting from an effective action in AdS space [77]. An essential element is the mapping of the higher-dimensional equations to the LF Hamiltonian equation found in Ref. [32]. This procedure allows a clear distinction between the kinematical and dynamical aspects of the LF holographic approach to hadron physics. Accordingly, the non-trivial geometry of pure AdS space encodes the kinematics, and the additional deformations of AdS encode the dynamics, including confinement [77].

A spin- $J$  field in AdS $_{d+1}$  is represented by a rank  $J$  tensor field  $\Phi_{M_1 \dots M_J}$ , which is totally symmetric in all its indices. In presence of a dilaton background field  $\varphi(z)$  the effective action is [77]

$$S_{\text{eff}} = \int d^d x dz \sqrt{|g|} e^{\varphi(z)} g^{N_1 N'_1} \dots g^{N_J N'_J} \left( g^{MM'} D_M \Phi_{N_1 \dots N_J}^* D_{M'} \Phi_{N'_1 \dots N'_J} - \mu_{\text{eff}}^2(z) \Phi_{N_1 \dots N_J}^* \Phi_{N'_1 \dots N'_J} \right), \quad (5)$$

where  $D_M$  is the covariant derivative which includes parallel transport. The effective mass  $\mu_{\text{eff}}(z)$ , which encodes kinematical aspects of the problem, is an *a priori* unknown function, but the additional symmetry breaking due to its  $z$ -dependence allows a clear separation of kinematical and dynamical effects [77]. The dilaton background field  $\varphi(z)$  in (5) introduces an energy scale in the five-dimensional AdS action, thus breaking conformal invariance. It vanishes in the conformal ultraviolet limit  $z \rightarrow 0$ .

A physical hadron has plane-wave solutions and polarization indices along the  $3 + 1$  physical coordinates  $\Phi_P(x, z)_{\nu_1 \dots \nu_J} = e^{iP \cdot x} \Phi_J(z) \epsilon_{\nu_1 \dots \nu_J}(P)$ , with four-momentum  $P_\mu$  and invariant hadronic mass  $P_\mu P^\mu = M^2$ . All other components vanish identically. The wave equations for hadronic modes follow from the Euler-Lagrange equation for tensors orthogonal to the holographic coordinate  $z$ ,  $\Phi_{z N_2 \dots N_J} = 0$ . Terms in the action which are linear in tensor fields, with one or more indices along the holographic direction,  $\Phi_{z N_2 \dots N_J}$ , give us the kinematical constraints required to eliminate the lower-spin states [77]. Upon variation with respect to  $\hat{\Phi}_{\nu_1 \dots \nu_J}^*$ , we find the equation of motion [77]

$$\left[ -\frac{z^{d-1-2J}}{e^{\varphi(z)}} \partial_z \left( \frac{e^{\varphi(z)}}{z^{d-1-2J}} \partial_z \right) + \frac{(mR)^2}{z^2} \right] \Phi_J = M^2 \Phi_J, \quad (6)$$

with  $(mR)^2 = (\mu_{\text{eff}}(z)R)^2 - Jz\varphi'(z) + J(d-J+1)$ , which is the result found in Refs. [32, 91] by rescaling the wave equation for a scalar field. Similar results were found in Ref. [92]. Upon variation with respect to  $\hat{\Phi}_{N_1 \dots N_J}^*$  we find the kinematical constraints which eliminate lower spin states from the symmetric field tensor [77]

$$\eta^{\mu\nu} P_\mu \epsilon_{\nu\nu_2 \dots \nu_J}(P) = 0, \quad \eta^{\mu\nu} \epsilon_{\mu\nu\nu_3 \dots \nu_J}(P) = 0. \quad (7)$$

Upon the substitution of the holographic variable  $z$  by the LF invariant variable  $\zeta$  and replacing  $\Phi_J(z) = (R/z)^{J-(d-1)/2} e^{-\varphi(z)/2} \phi_J(z)$  in (6), we find for  $d = 4$  the LF wave equation (4) with effective potential [93]

$$U(\zeta^2, J) = \frac{1}{2} \varphi''(\zeta^2) + \frac{1}{4} \varphi'(\zeta^2)^2 + \frac{2J-3}{2\zeta} \varphi'(\zeta^2), \quad (8)$$

provided that the AdS mass  $m$  in (6) is related to the internal orbital angular momentum  $L = \max|L^z|$  and the total angular momentum  $J^z = L^z + S^z$  according to  $(mR)^2 = -(2-J)^2 + L^2$ . The critical value  $L = 0$  corresponds to the lowest possible stable solution, the ground state of the LF Hamiltonian. For  $J = 0$  the five dimensional mass  $m$  is related to the orbital momentum of the hadronic bound state by  $(mR)^2 = -4 + L^2$  and thus  $(mR)^2 \geq -4$ . The quantum mechanical stability condition  $L^2 \geq 0$  is thus equivalent to the Breitenlohner-Freedman stability bound in AdS [94].

The effective interaction  $U(\zeta^2, J)$  is instantaneous in LF time and acts on the lowest state of the LF Hamiltonian. This equation describes the spectrum of mesons as a function of  $n$ , the number of nodes in  $\zeta^2$ , the internal orbital angular momentum  $L = L^z$ , and the total angular momentum  $J = J^z$ , with  $J^z = L^z + S^z$  the sum of the orbital angular momentum of the constituents and their internal spins. The SO(2) Casimir  $L^2$  corresponds to the group of rotations in the transverse LF plane. The LF wave equation is the relativistic frame-independent front-form analog of the non-relativistic radial Schrödinger equation for muonium and other hydrogenic atoms in presence of an instantaneous Coulomb potential. The LF harmonic oscillator potential could in fact emerge from the exact QCD formulation when one includes contributions from the effective potential  $U$  which are due to the exchange of two connected gluons; *i.e.*, ‘‘H’’ diagrams [95]. We notice that  $U$  becomes complex for an excited state since a denominator can vanish; this gives a complex eigenvalue and the decay width. The multi gluon exchange diagrams also could be connected to the Isgur-Paton flux-tube model of confinement; the collision of flux tubes could give rise to the ridge phenomena recently observed in high energy  $pp$  collisions at RHIC [96].

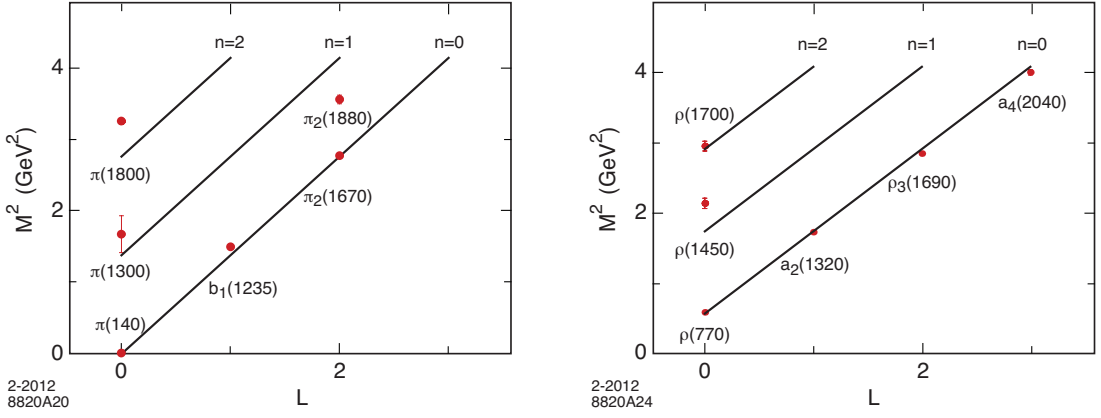


FIG. 4:  $I = 1$  parent and daughter Regge trajectories for the  $\pi$ -meson family (left) with  $\kappa = 0.59$  GeV; and the  $\rho$ -meson family (right) with  $\kappa = 0.54$  GeV.

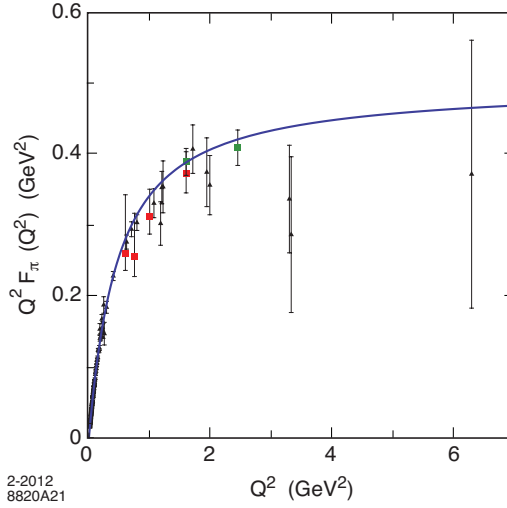


FIG. 5: Light-front holographic prediction for the space-like pion form factor.

The correspondence between the LF and AdS equations thus determines the effective confining interaction  $U$  in terms of the infrared behavior of AdS space and gives the holographic variable  $z$  a kinematical interpretation. The identification of the orbital angular momentum is also a key element of our description of the internal structure of hadrons using holographic principles.

The dilaton profile  $\exp(\pm\kappa^2 z^2)$  leads to linear Regge trajectories [80]. For the confining solution  $\varphi = \exp(\kappa^2 z^2)$  the effective potential is  $U(\zeta^2, J) = \kappa^4 \zeta^2 + 2\kappa^2(J - 1)$  leads to eigenvalues  $M_{n,J,L}^2 = 4\kappa^2(n + \frac{J+L}{2})$ , with a string Regge form  $M^2 \sim n + L$ . A detailed discussion of the light meson and baryon spectrum, as well as the elastic and transition form factors of the light hadrons using LF holographic methods, is given in Ref. [91]. As an example, the spectral predictions for the  $J = L + S$  light pseudoscalar and vector meson states are compared with experimental data in Fig. 4 for the positive sign dilaton model.

The predictions of the resulting LF Schrödinger and Dirac equations for hadron light-quark spectroscopy and form factors for  $m_q = 0$  and  $\kappa \simeq 0.5$  GeV are shown in Figs. 4-7 for a dilaton profile  $\varphi(z) = \kappa^2 z^2$ . A detailed discussion of the computations is given in Ref. [91].

## IX. UNIQUENESS OF THE CONFINING POTENTIAL

If one starts with a dilaton profile  $e^{\varphi(z)}$  with  $\varphi \propto z^s$ , the existence of a massless pion in the limit of massless quarks determines uniquely the value  $s = 2$ . To show this, one can use the stationarity of bound-state energies with

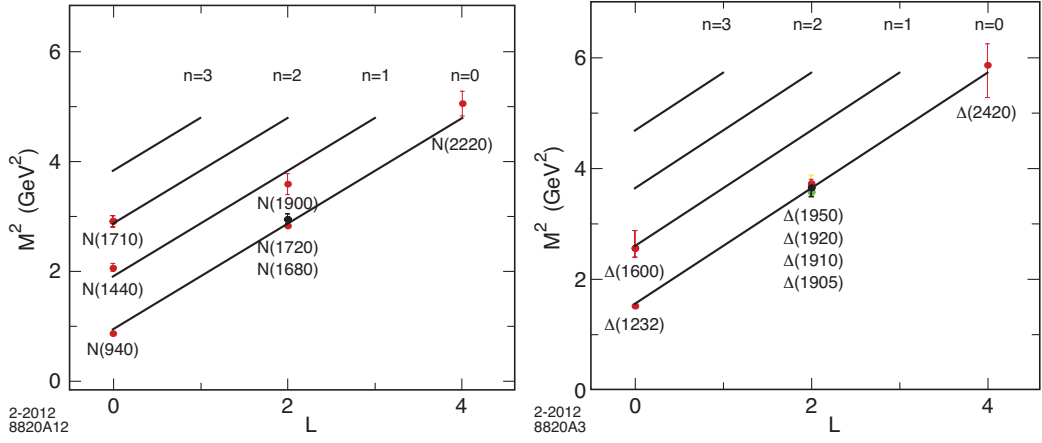


FIG. 6: Light front holographic predictions of the light-front Dirac equation for the nucleon spectrum. Orbital and radial excitations for the positive-parity sector are shown for the  $N$  (left) and  $\Delta$  (right) for  $\kappa = 0.49$  GeV and  $\kappa = 0.51$  GeV respectively. All confirmed positive and negative-parity resonances from PDG 2012 are well accounted using the procedure described in [91].

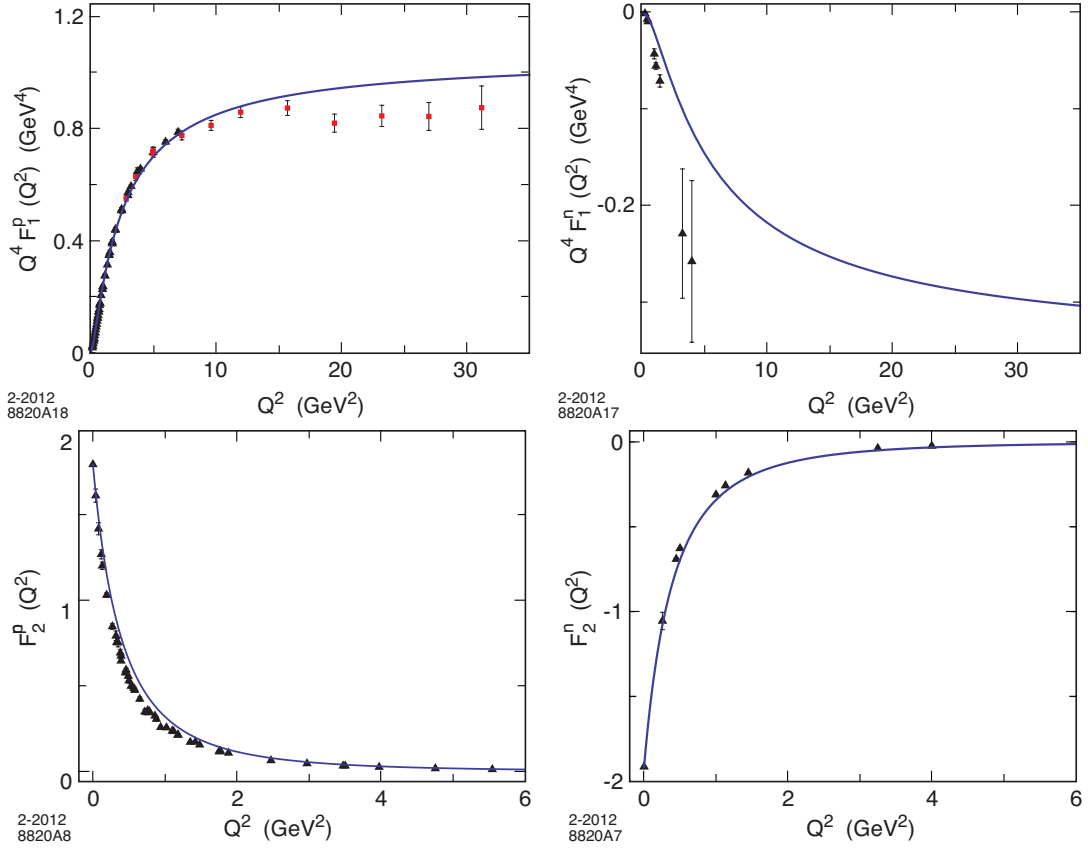


FIG. 7: Light-front holographic predictions for the nucleon form factors normalized to their static values.

respect to variation of parameters. More generally, the effective theory should incorporate the fundamental conformal symmetry of the four-dimensional classical QCD Lagrangian in the limit of massless quarks. To this end we study the invariance properties of a one-dimensional field theory under the full conformal group following the dAFF construction of Hamiltonian operators described in Ref. [36].

One starts with the one-dimensional action

$$S = \frac{1}{2} \int dt \left( \dot{Q}^2 - \frac{g}{Q^2} \right), \quad (9)$$

which is invariant under conformal transformations in the variable  $t$ . In addition to the Hamiltonian  $H_t(Q, \dot{Q}) = \frac{1}{2} \left( \dot{Q}^2 + \frac{g}{Q^2} \right)$  there are two more invariants of motion for this field theory, namely the dilation operator  $D$  and  $K$ , corresponding to the special conformal transformations in  $t$ . Specifically, if one introduces the new variable  $\tau$  defined through  $d\tau = dt/(u + vt + wt^2)$  and the rescaled fields  $q(\tau) = Q(t)/(u + vt + wt^2)^{1/2}$ , it then follows that the operator  $G = uH_t + vD + wK$  generates the quantum mechanical evolution in  $\tau$  [36]. The Hamiltonian corresponding to the operator  $G$ , which introduces the mass scale, is a linear combination of the old Hamiltonian  $H_t$ ,  $D$ , the generator of dilations, and  $K$ , the generator of special conformal transformations.

One can show explicitly [36, 78] that a confinement length scale appears in the action when one expresses the action (9) in terms of the new time variable  $\tau$  and the new fields  $q(t)$ , without affecting its conformal invariance. Furthermore, for  $g \geq -1/4$  and  $4uw - v^2 > 0$  the corresponding Hamiltonian  $H_\tau(q, \dot{q}) = \frac{1}{2} \left( \dot{q}^2 + \frac{g}{q^2} + \frac{4uw - v^2}{4} q^2 \right)$  is a compact operator. Finally, we can transform back to the original field operator  $Q(t)$  in (9). We find

$$\begin{aligned} H_\tau(Q, \dot{Q}) &= \frac{1}{2}u \left( \dot{Q}^2 + \frac{g}{Q^2} \right) - \frac{1}{4}v \left( Q\dot{Q} + \dot{Q}Q \right) + \frac{1}{2}wQ^2 \\ &= uH_t + vD + wK, \end{aligned} \quad (10)$$

at  $t = 0$ . We thus recover the evolution operator  $G = uH_t + vD + wK$  which describes the evolution in the variable  $\tau$ , but expressed in terms of the original field  $Q$ .

The Schrödinger picture follows by identifying  $Q \rightarrow x$  and  $\dot{Q} \rightarrow -i \frac{d}{dx}$ . Then the evolution operator in the new time variable  $\tau$  is

$$H_\tau = \frac{1}{2}u \left( -\frac{d^2}{dx^2} + \frac{g}{x^2} \right) + \frac{i}{4}v \left( x \frac{d}{dx} + \frac{d}{dx} x \right) + \frac{1}{2}wx^2 \quad (11)$$

If we now compare the Hamiltonian (11) with the light-front wave equation (4) and identifying the variable  $x$  with the light-front invariant variable  $\zeta$ , we have to choose  $u = 2$ ,  $v = 0$  and relate the dimensionless constant  $g$  to the LF orbital angular momentum,  $g = L^2 - 1/4$ , in order to reproduce the light-front kinematics. Furthermore  $w = 2\lambda^2$  fixes the confining light-front potential to a quadratic  $\lambda^2 \zeta^2$  dependence. The mass scale brought in via  $w = 2\kappa^2$  then generates the confining mass scale  $\kappa$ . The dilaton is also unique:  $e^{\phi(z)} = e^{\kappa^2 z^2}$ , where  $z^2$  is matched to  $\zeta^2 = b_1^2 x(1-x)$  via LF holography. The spin- $J$  representations in AdS<sub>5</sub> [77] then leads uniquely to the LF confining potential  $U(\zeta^2) = \kappa^4 \zeta^2 - 2\kappa^2(J-1)$ .

The new time variable  $\tau$  is related to the variable  $t$  for the case  $uw > 0$ ,  $v = 0$  by

$$\tau = \frac{1}{\sqrt{uw}} \arctan \left( \sqrt{\frac{w}{u}} t \right), \quad (12)$$

*i.e.*,  $\tau$  has only a limited range. The finite range of invariant LF time  $\tau = x^+/P^+$  can be interpreted as a feature of the internal frame-independent LF time difference between the confined constituents in a bound state. For example, in the collision of two mesons, it would allow one to compute the LF time difference between the two possible quark-quark collisions [78].

## X. SUMMARY

The triple complementary connection of (a) AdS space, (b) its LF holographic dual, and (c) the relation to the algebra of the conformal group in one dimension, is characterized by a quadratic confinement LF potential, and thus a dilaton profile with the power  $z^s$ , with the unique power  $s = 2$ . In fact, for  $s = 2$  the mass of the  $J = L = n = 0$  pion is automatically zero in the chiral limit. The separate dependence on  $J$  and  $L$  leads to a mass ratio of the  $\rho$  and the  $a_1$  mesons which coincides with the result of the Weinberg sum rules [97]. One predicts linear Regge trajectories with the same slope in the relative orbital angular momentum  $L$  and the LF radial quantum number  $n$ . The AdS approach, however, goes beyond the purely group theoretical considerations of dAFF, since features such as the masslessness of the pion and the separate dependence on  $J$  and  $L$  are a consequence of the potential (8) derived from the duality with AdS for general high-spin representations. The pion distribution amplitude predicted in the nonperturbative domain is  $\phi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}$ , and the pion decay constant is  $f_\pi = \frac{\sqrt{3}}{8} \kappa$ .

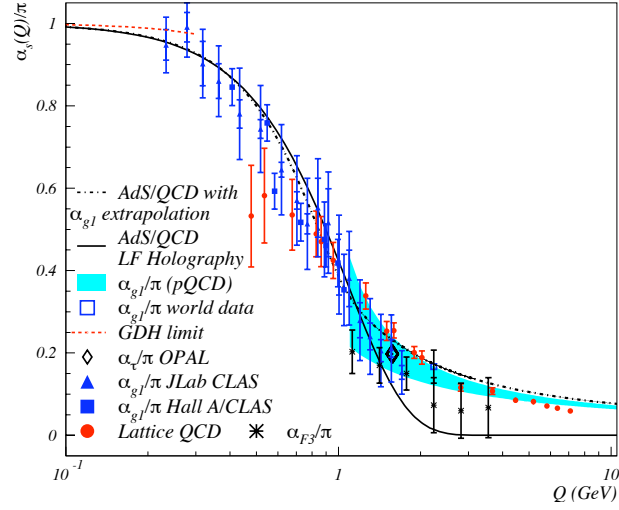


FIG. 8: Light-front holographic results for the QCD running coupling from Ref. [98] normalized to  $\alpha_s(0)/\pi = 1$ . The result is analytic, defined at all scales and exhibits an infrared fixed point.

The QCD mass scale  $\kappa$  in units of GeV has to be determined by one measurement; e.g., the pion decay constant  $f_\pi$ . All other masses and size parameters are then predicted. The running of the QCD coupling is predicted in the infrared region for  $Q^2 < 4\kappa^2$  to have the form  $\alpha_s(Q^2) \propto \exp\left(\frac{-Q^2}{4\kappa^2}\right)$ . As shown in Fig. 8, the result agrees with the shape of the effective charge defined from the Bjorken sum rule [98], displaying an infrared fixed point. In the nonperturbative domain soft gluons are in effect sublimated into the effective confining potential. Above this region, hard-gluon exchange becomes important, leading to asymptotic freedom. The scheme-dependent scale  $\Lambda_{QCD}$  that appears in the QCD running coupling in any given renormalization scheme such as  $\Lambda_{\overline{MS}}$  could be determined in terms of  $\kappa$ .

In our previous papers we have applied LF holography to baryon spectroscopy, space-like and time-like form factors, as well as transition amplitudes such as  $\gamma^* \gamma \rightarrow \pi^0$ ,  $\gamma^* N \rightarrow N^*$ , all based on essentially the single mass scale parameter  $\kappa$ . Many other applications have been presented in the literature, including recent results by Forshaw and Sandapen [99] for diffractive  $\rho$  electroproduction which are based on the light-front holographic prediction for the longitudinal  $\rho$  LFWF. Other recent applications include predictions for generalized parton distributions (GPDs) [100], and a model for nucleon and flavor form factors [101].

The treatment of the chiral limit in the LF holographic approach to strongly coupled QCD is substantially different from the standard approach based on chiral perturbation theory. In the conventional approach, spontaneous symmetry breaking by a non-vanishing chiral quark condensate  $\langle \bar{\psi}\psi \rangle$  plays the crucial role. In QCD sum rules [102]  $\langle \bar{\psi}\psi \rangle$  brings in non-perturbative elements into the perturbatively calculated spectral sum rules. It should be noted, however, that the definition of the condensate, even in lattice QCD necessitates a renormalization procedure for the operator product, and it is not a directly observable quantity. In contrast, in Bethe-Salpeter [103] and light-front analyses [65], the Gell Mann-Oakes-Renner relation [64] for  $m_\pi^2/m_q$  involves the decay matrix element  $\langle 0|\bar{\psi}\gamma_5\psi|\pi \rangle$  instead of  $\langle 0|\bar{\psi}\psi|0 \rangle$ .

In the color-confining light-front holographic model discussed here, the vanishing of the pion mass in the chiral limit, a phenomenon usually ascribed to spontaneous symmetry breaking of the chiral symmetry, is obtained specifically from the precise cancellation of the LF kinetic energy and LF potential energy terms for the quadratic confinement potential. This mechanism provides a viable alternative to the conventional description of nonperturbative QCD based on vacuum condensates, and it eliminates a major conflict of hadron physics with the empirical value for the cosmological constant [58, 59].

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