Estimation of LHC and ILC Capabilities for Precision Higgs Boson Coupling Measurements

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ABSTRACT

This paper discusses some aspects of the estimates of Higgs boson coupling sensitivities for LHC and ILC presented at the Snowmass 2013 meeting. I estimate the measurement accuracies underlying the CMS presentation to Snowmass. I present new fits for the ILC capabilities. I present some joints fits to prospective LHC and ILC data that demonstrates the synergy of the High-Luminosity LHC and ILC programs.

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1 Introduction

The evidence is accumulating that the resonance discovered by ATLAS and CMS at 125 GeV [1,2] is a Higgs boson. Both the intrinsic mystery surrounding this particle and the key role of the Higgs sector in elementary particle physics make it an important goal to understand the properties of this particle as accurately as possible.

The goals for the study of the couplings of the Higgs boson are discussed in detail in the Snowmass Higgs working group report [3]. There are two sides to the story. On one hand, the idea of the Standard Model that electroweak symmetry is broken by a single complex doublet of scalar fields has no compelling foundation. It is just the simplest choice among a large number of options. On the other hand, if there are additional particles in addition to the lightest Higgs boson but these particles are heavy, at mass M, the Decoupling Theorem [4] tells us that the lightest Higgs has the properties predicted by the Standard Model up to corrections of order m_h^2/M^2. At present, the properties of the 125 GeV resonance agree with those of the Standard Model Higgs boson to about 30% accuracy. This does not yet test the hypothesis of a single Higgs doublet. To discover new structure in the Higgs sector, we need to look for effects at the 5% level. To discover such effects, we need experiments that can explore the landscape of Higgs boson couplings in a model-independent way with accuracies at the 1% level [3].

One of the goals of the Snowmass 2013 study is to understand the accuracy with which the couplings of the Higgs boson will be measured at the future stages of the LHC and at other future colliders. An important question is the ultimate capability of the LHC experiments. The ATLAS and CMS experiments provided estimates of this capability for 300 fb^{-14} and 3000 fb^{-1} data sets at 14 TeV in their White Papers [5–7].

The CMS White Paper gave estimates that were more optimistic and also took account of possible future improvements in our ability to extract information from the LHC data. However, the results of this paper were not presented in a way that makes it straightforward to evaluate the capability of CMS to discover or exclude specific theoretical models or, indeed, to carry out any fits other than those specifically included in the paper.

The first purpose of this note is to suggest a way to remedy this difficulty. I provide a model of the CMS analysis in term of measurements accuracies for a list of Higgs boson process that can be considered as independent measurements. The second purpose of this note is to carry out fits involving the new estimates of ILC capabilities presented at Snowmass [8]. With both sets of inputs in hand, I then carry out some fits for the combined capabilities not yet included in the Snowmass documentation.

This paper is organized as follows: In Section 2, I present my methodology for

estimating Higgs boson coupling accuracies. In Section 3, I present an interpretation of the CMS results in this framework. In Section 4, I discuss the effect on the LHC fits of including invisible modes of Higgs decay. In Section 5, I carry out a 10-parameter fit similar to that proposed in [9] to quantify the accuracies of model-independent ILC Higgs coupling determinations at the various ILC stages. In Section 6, I present joint fits that make use of LHC and ILC results. In Section 7, I give some editorial comments.

The opinions expressed in this paper are strictly my own. They should not be mistaken for opinions of the Snowmass Higgs working group or opinions of the Snowmass Energy Frontier conveners.

2   Methodology

In this paper, I will parametrize deviations of the couplings of the 125 GeV Higgs boson as

$$\kappa_A = g(hA\overline{A})/(SM) \ , \tag{1}$$

where $g(hA\overline{A})$ is the coupling of the Higgs boson to the $A\overline{A}$ final state defined on the Higgs mass shell and, always in this paper, (SM) indicates the Standard Model expectation. I will treat the Higgs boson in the narrow resonance approximation. This is a very good approximation, since the Standard Model expectation for the width of the Higgs boson is about 4 MeV.

Couplings induced at the loop level in the Standard Model may receive contributions from the heavy particles of the Standard Model, and also from new particles that are not yet known. In this analysis, I will consider the couplings of the Higgs boson to gg and $\gamma\gamma$ to be parametrized by κ values that are independent of those for t and W, which give the largest contributions to the purely Standard Model loop effects. In this paper, I will ignore the minor modes $h \to Z\gamma$ and $h \to \mu^+\mu^-$.

Total cross sections and ratios of branching ratios have a simple dependence on the κ_A, for example,

$$\sigma(e^+e^- \to Zh)/(SM) = \kappa_Z^2 \ , \qquad \frac{BR(h \to ZZ^*)/(SM)}{BR(h \to \gamma\gamma)/(SM)} = \kappa_Z^2/\kappa_\gamma^2 \ . \tag{2}$$

However, collider experiments more typically measure the rate for a complete process of Higgs production and decay to a particular final state. The ratio of this rate to the Standard Model expectation is given by

$$\sigma(A\overline{A} \to h)BR(h \to B\overline{B})/(SM) = \frac{\kappa_A^2 \kappa_B^2}{\kappa_h^2} \ , \tag{3}$$

where κ_h is the scale factor for the Higgs total width. Within the Standard Model,

$$\kappa_h^2 = \sum_C \kappa_C^2 BR(h \to C\overline{C})|_{SM} . \tag{4}$$

In this paper, I will also consider the possibility of decay modes not included in the Standard Model. These include *invisible* decays, for example, the decay of the Higgs boson to a pair of dark matter particles that cannot be seen by a collider detector. Other exotic modes of Higgs decay, to visible light particles or to long-lived particles outside the Standard Model, are also possible. In this context, it is important to distinguish *invisible* decays—which can be measured at the LHC or the ILC in processes in which the Higgs is produced along with a W or Z boson or with jets tagging vector boson fusion—from *unrecognizable* decays—for which there is no strategy for observing the Higgs decay mode above background. At the LHC, the decay $h \to c\bar{c}$ is *unrecognizable*. It is easy to imagine exotic decays, for example, to multiple jets, that also could not be discovered by the LHC experiments. The decay $h \to gg$ cannot be observed at the LHC, but I will treat it as an observable decay, because the $h \to gg$ decay width is directly proportional to the gg fusion production cross section and is thus directly constrained by LHC Higgs measurements. The full formula for κ_h^2 is

$$\kappa_h^2 = \left(\sum_C \kappa_C^2 BR(h \to C\overline{C})|_{SM} \right) / (1 - BR_{inv} - BR_{exotic} - BR_{unr}) , \tag{5}$$

where C runs over observable Standard Model modes of Higgs decay, BR_{inv} is the branching ratio to invisible modes, BR_{exotic} is the branchig ratio to exotic, detectable modes, and BR_{unr} is the branching ratio to unrecognizable modes.

The appearance of κ_h in the formula (3) couples all of the κ_A into the interpretation of any single rate measurement. This the major difficulty to be overcome in making a model-independent interpretation of the rate measurements in terms of Higgs couplings. Special difficulties arise at hadron colliders, because, in the Standard Model, the decay $h \to b\bar{b}$ account for over 50% of the total width and, at the same time, this decay is exceptionally difficult to observe above the hadron collider backgrounds.

This paper will be concerned with estimating the expected errors on the parameters κ_A in future accelerator programs. The methodology of this paper will be very simple, even naive, but I hope that its transparency will useful for further investigations. I consider a set of 10 parameters:

$$\kappa_W , \ \kappa_Z , \ \kappa_b , \ \kappa_g , \ \kappa_\gamma , \ \kappa_\tau , \ \kappa_c , \ \kappa_t , \ BR_{inv} , \ BR_{unr} . \tag{6}$$

Note that I assume for simplicity that the Higgs boson has no detectable exotic decay modes. For each fit, I give a prescription that defines the 10 parameters in terms of an

3

underlying set. I specify a set of input measurements that constrain these underlying parameters. Each measurement is considered to be a strictly independent piece of data, centered on the SM expectation with a Gaussian error distribution. The error is be determined by adding statistical and systematic errors in quadrature. By multiplying together the Gaussians and applying other needed constraints (for example, $BR_{inv} \geq 0$), I obtain a likelihood function for the underlying κ_A values. I explore this function using the VEGAS integrator [10], histogram the relevant variables, and quote the $1\,\sigma$ confidence intervals in each variable.

The systematic errors quoted include the theoretical errors necessary to extract the rate (for example, the uncertainties in the SM expectation for the cross sections for the relevant production processes) but not the uncertainty in the computation of the SM value of $g(h A \overline{A})$. At the ILC, at least, the Higgs partial widths are extracted in a model-independent way, and then these values can be directly compared to SM calculations. The uncertainties in the SM values of Higgs partial widths, as they are currently quoted [11], are dominated by the uncertainties in input parameters such as α_s and m_b. These are expected to improve greatly over the time scale relevant to these projections [12]. It is worth emphasizing that both experiment and theory must improve for the values of the Higgs partial widths in order to make tests of these couplings with errors at the 1% level.

3   Interpretation of the CMS results

In principle, it is straightforward to use the method described in the previous section to produce estimates of the Higgs coupling uncertainties from future collider programs. One must write down a list of input measurements, estimate the error for each, construct the likelihood function described in the previous section, and measure its properties. Unfortunately, the presentations to Snowmass from the LHC collaborations [5–7] do not provide the information needed for such an analysis.

ATLAS has presented its estimates for its capabilities for Higgs measurements in a very explicit way, including as numerical tables. The errors presented at Snowmass [5] were conservative in a way that disappointed many participants in the Snowmass study. For example, the error on the rate for Higgs production and decay to WW was projected not to change from 300 fb^{-1} to 3000 fb^{-1}. Some of this conservatism is removed in the more recent projections given in [6]. But, more importantly, neither of these ATLAS papers includes estimates for the capability to measure $h \to b\overline{b}$, which, as noted above, plays a central role in any global fit. It is not possible to estimate the uncertainty in individual Higgs boson couplings without that information.

CMS presented a less conservative set of estimates, using two scenarios. In Scenario 1, current systematic and theoretical errors were used. In Scenario 2, theoretical

errors were halved and systematic errors were assumed to decrease as the square root of the integrated luminosity. The estimates under Scenario 2 were meant to express the most optimistic estimates for the measurement of Higgs couplings, especially at high luminosity. One might understand them as estimating the *opportunity* that high-luminosity running of the LHC will make available. However, CMS did not provide the input measurement errors but, rather, only the results of some global fits. It is not possible to use this information to carry out other potentially interesting fits or to carry out joint fits with expected results from other facilities.

For this reason, it is interesting to propose an interpretation of the CMS results in terms of a set of errors on a minimal set of measurements, assumed to be independent, that reproduces their fit results. In Table 1, I give my proposed interpretation of the results, along with the ATLAS estimates, from [6], for comparison. The estimates given are coherent, in the sense that the accuracy improves systematically from Scenario 1 at 300 fb^{-1} to Scenario 2 at 3000 fb^{-1}, but never with a large step. By the definition of Scenario 2, the theoretical errors for Scenario 2 at 3000 fb^{-1} are identical to those at 300 fb^{-1} and the experimental errors decrease by $\sqrt{10}$. I have taken the theory errors to be the errors on the total production cross section as given by the LHC Higgs Cross Section Working Group [13–15]. It is not possible to obtain the fit results presented by CMS with theory errors as large as those quoted by ATLAS in [6].

In Table 2, I compare the results obtained by applying my naive fitting method to the numbers in Table 1 for the fits discussed in the CMS paper [7] with the results presented in that paper. Three types of data are presented. The first is an error for the determination of the overall rate μ into the various final states. I obtain these by combining the results for reactions with the same final state in Table 1. The second is the result of a 7-parameter fit taking $\kappa_c = \kappa_t$ and $BR_{inv} = BR_{unr} = 0$. Finally, I obtain a limit on the invisible branching ratio from an 8-parameter fit with BR_{inv} taken nonzero and with the restriction

$$\kappa_W, \kappa_Z < 1. \tag{7}$$

This latter constraint [16] is a minimally model-dependent approach to fixing the Higgs width that is often used in fits to Higgs couplings at hadron colliders [9,17–19]. This follows the methodology used by CMS, as explained in the Snowmass Higgs working group report [3]. Following the prescription given there, the quoted uncertainties on BR_{inv} do not include the direct constraint from measurement of $pp \to Zh$. This weak constraint, however, has only a small effect on the fit.

The uncertainties in the total cross sections reported in [13,14] are large for gg fusion and associated production of the Higgs with $t\bar{t}$ but quite small for vector boson fusion and for associated production with W and Z. This can be seen in the theory errors reported for CMS in Table 1. As the luminosity increases, it is possible to decrease the theory error in output Higgs couplings by relying increasingly on

300 fb^{-1} :

Observable	ATLAS	CMS-1	CMS-2
$\sigma(gg) \cdot BR(\gamma\gamma)$	$12 \oplus 19$	$6 \oplus 12.3$	$3 \oplus 6.2$
$\sigma(WW) \cdot BR(\gamma\gamma)$	$47 \oplus 15$	$20 \oplus 2.4$	$14 \oplus 1.2$
$\sigma(gg) \cdot BR(WW)$	$8 \oplus 18$	$6 \oplus 12.3$	$5 \oplus 6.2$
$\sigma(WW) \cdot BR(WW)$	$20 \oplus 8$	$35 \oplus 2.4$	$28 \oplus 1.2$
$\sigma(gg) \cdot BR(ZZ)$	$6 \oplus 11$	$7 \oplus 12.3$	$5 \oplus 6.2$
$\sigma(WW) \cdot BR(ZZ)$	$31 \oplus 13$	$12 \oplus 2.4$	$10 \oplus 1.2$
$\sigma(gg) \cdot BR(\tau\tau)$	——	$13 \oplus 12.3$	$6 \oplus 6.2$
$\sigma(WW) \cdot BR(\tau\tau)$	$16 \oplus 15$	$16 \oplus 2.4$	$9 \oplus 1.2$
$\sigma(Wh) \cdot BR(b\bar{b})$	——	$17 \oplus 3.8$	$14 \oplus 1.7$
$\sigma(t\bar{t}h) \cdot BR(b\bar{b})$	——	$60 \oplus 11.7$	$50 \oplus 5.9$
$\sigma(t\bar{t}h) \cdot BR(\gamma\gamma)$	$54 \oplus 10$	$40 \oplus 11.7$	$38 \oplus 5.9$
$\sigma(Zh) \cdot BR(invis)$	——	$16 \oplus 4.3$	$11 \oplus 2.2$

3000 fb^{-1} :

Observable	ATLAS-HL	CMS-HL-1	CMS-HL-2
$\sigma(gg) \cdot BR(\gamma\gamma)$	$5 \oplus 19$	$4 \oplus 12.3$	$0.9 \oplus 6.2$
$\sigma(WW) \cdot BR(\gamma\gamma)$	$15 \oplus 15$	$10 \oplus 2.4$	$4.4 \oplus 1.2$
$\sigma(gg) \cdot BR(WW)$	$5 \oplus 18$	$6 \oplus 12.3$	$1.6 \oplus 6.2$
$\sigma(WW) \cdot BR(WW)$	$9 \oplus 8$	$24 \oplus 2.4$	$8.9 \oplus 1.2$
$\sigma(gg) \cdot BR(ZZ)$	$4 \oplus 11$	$4 \oplus 12.3$	$1.6 \oplus 6.2$
$\sigma(WW) \cdot BR(ZZ)$	$16 \oplus 13$	$7 \oplus 12.3$	$1.9 \oplus 6.2$
$\sigma(WW) \cdot BR(\tau\tau)$	$12 \oplus 15$	$8 \oplus 2.4$	$2.8 \oplus 1.2$
$\sigma(Wh) \cdot BR(b\bar{b})$	——	$8 \oplus 3.8$	$4.4 \oplus 1.7$
$\sigma(t\bar{t}h) \cdot BR(b\bar{b})$	——	$35 \oplus 11.7$	$16 \oplus 5.9$
$\sigma(t\bar{t}h) \cdot BR(\gamma\gamma)$	$17 \oplus 12$	$28 \oplus 11.7$	$12 \oplus 5.9$
$\sigma(Zh) \cdot BR(invis)$	——	$10 \oplus 4.3$	$3.5 \oplus 2.2$

Table 1: Error estimates for measurement of Higgs boson processes at the LHC. All numbers are given as 1 σ uncertainties, in %. Errors are given in the form (experiment)⊕(theory), where (theory) is an error on the theory used to extract the rate. These errors are added in quadrature in the analysis. The first three columns give estimates for 14 TeV with 300 fb^{-1}; the second three columns gives estimates for 14 TeV and 3000 fb^{-1}. The columns for ATLAS give numbers presented in [6]. The columns for CMS are my own estimates, justified only by the results of the fits shown in Table 2. CMS-1 denotes Scenario 1; CMS-2 denotes Scenario 2.

μ values	300 fb^{-1}		3000 fb^{-1}	
	CMS	here	CMS	here
$\gamma\gamma$	[6 , 12]	[6.2 , 11.3]	[4 , 8]	[3.7 , 8.0]
WW	[6 , 11]	[7.6 , 12.7]	[4 , 7]	[5.2 , 11.9]
ZZ	[7 , 11]	[6.2 , 12.7]	[4 , 7]	[3.0 , 7.0]
$b\bar{b}$	[11 , 14]	[13.6 , 16.7]	[5 , 7]	[4.7 , 8.6]
$\tau^+\tau^-$	[8 , 14]	[6.2 , 12.0]	[5 , 8]	[2.8 , 7.2]
invis.	[11 , 17]	[11.2 , 16.6]	[4 , 11]	[4.1 , 10.9]
κ values	300 fb^{-1}		3000 fb^{-1}	
	CMS	here	CMS	here
γ	[5 , 7]	[5.7 , 9.0]	[2 , 5]	[2.9 , 6.5]
W	[4 , 6]	[4.2 , 5.4]	[2 , 5]	[1.6 , 3.3]
Z	[4 , 6]	[5.7 , 8.5]	[2 , 4]	[2.8 , 6.3]
g	[6 , 8]	[4.9 , 6.9]	[3 , 5]	[2.3 , 4.8]
b	[10 , 13]	[11.4 , 14.9]	[4 , 7]	[4.2 , 8.5]
t	[14 , 15]	[17.3 , 20.5]	[6 , 8]	[5.7 , 12.9]
τ	[6 , 8]	[5.8 , 9.5]	[2 , 5]	[2.7 , 6.5]
inv.	[8 , 11]	[6.3 , 8.0]	[4 , 7]	[2.0 , 4.0]

Table 2: Comparison of the results of fits with the inputs in Table 1 to the fit results given in [7]. All numbers are given as 1 σ uncertainties, in %. In expressions in brackets, the first entry is for Scenario 2, the second is for Scenario 1.

measurements of Higgs production in these latter two modes. The evolution of the μ and κ accuracies reported by CMS from 300 fb^{-1} to 3000 fb^{-1} reflects increasing reliance at higher luminosity on the vector boson fusion production mode. It is very important to note this special role of vector boson fusion in any considerations of the experimental program of the High-Luminosity LHC. The model I have presented captures that this evolution to higher accuracies, at least in a qualitative way.

There are some defects in the agreement of my model with the CMS results. The most serious is the constraint on the invisible modes of Higgs decay, which is significantly stronger in my fit than that reported in [7]. This may be the result of my treating correlated theoretical errors as uncorrelated, which stiffens the global pattern of the constraints. The CMS analysis also uses a much larger number of input measurements, with correspondingly larger errors, and takes proper account of the correlations among these errors. Such a treatment is beyond the level of my interpretation. Nevertheless, I hope that the information that I have given in Table 1 will suffice for the purpose of estimating Higgs capabilities for experiments that will be carried out in the future.

4 Fits including invisible modes

The coupling accuracies listed in Table 2 come mainly from a fit in which only SM modes of Higgs decay are taken into account. Only the uncertainties quoted for the invisible branching fraction are based on a fit that allows the Higgs to decay invisibly. It is interesting to perform another fit to understand how the possible presence of invisible modes of Higgs decay affects the capabilities of the LHC experiments.

As noted above, there are two types of "invisible" modes of Higgs decay at the LHC. In Section 2, I distinguished between *invisible* modes of Higgs decay (which can actually be measured directly at the LHC) from *unrecognizable* modes of Higgs decay, which are hidden by Standard Model backgrounds.

To take account of both possibilities, we can fit the inputs given in Table 1 with a model in which $\kappa_c = 0$ but all of the other 9 parameters in (6) are allowed to float. This fit includes the constraint on Zh production with the Higgs decaying invisibly. The decay of the Higgs to $c\bar{c}$ is an unrecognizable mode in the sense of the previous section, so I omit this variable, as explained in the definitions of the parameters of (5). To keep the total width of the Higgs boson from running to large values, we must impose the condition $\kappa_W, \kappa_Z < 1$. The prescription is close to the one proposed in [9] to measure the ability of colliders to perform model-independent determinations of the Higgs couplings.

The results of the fit are shown in Table 3. The table compares these results to those of the 7-parameter fit including only Standard Model decays discussed in the previous section.

It is noteworthy that the coupling errors in the 9-parameter fit are typically smaller than those in the 7-parameter fit, despite the fact that the 9-parameter fit contains extra unconstrained degrees of freedom. This is the effect of imposing the condition $\kappa_W, \kappa_Z < 1$. A fit with this condition imposed but with no allowance for invisible or unrecognized modes gives similar results for the uncertainties in the κ_A for Standard Model decay modes.

5 Estimates of coupling accuracy for ILC

Using a similar methodology, it is straightforward to produce estimates of the accuracy of the Higgs boson couplings that will be obtained at the various stages of the ILC. The ILC Higgs White Paper [8] reviews and improves the Higgs boson coupling analyses reported in the ILC TDR [20]. This paper also emphasizes that the long-term program of the ILC will lead to further improvements in Higgs coupling measurements. Whereas the LHC Higgs coupling determinations become dominated

300 fb^{-1}	Scenario 2		Scenario 1	
	7-param	9-param	7-param	9-param
γ	5.7	4.3	9.0	7.3
W	4.2	3.5	5.4	4.6
Z	5.7	5.0	8.5	6.6
g	4.9	4.1	6.9	6.3
b	11.4	7.6	14.9	10.2
t	17.3	17.3	20.5	20.6
τ	5.8	4.4	9.5	7.7
invis.	—	4.6	—	6.1
3000 fb^{-1}	Scenario 2		Scenario 1	
	7-param	9-param	7-param	9-param
γ	2.9	2.4	6.5	5.3
W	1.6	1.2	3.3	2.3
Z	2.8	2.2	6.3	4.3
g	2.3	2.0	4.8	4.4
b	4.2	2.9	8.5	6.0
t	5.7	5.6	12.9	12.8
τ	2.7	2.2	6.5	5.1
invis.	—	1.5	—	3.2

Table 3: Comparison of the results for Higgs coupling uncertainties, in %, without and with allowance for invisible and unrecognized Higgs decays, as defined in the text. The analyses are performed, respectively, with 7-parameter fits including only Standard model decay modes and constrained 9-parameter fits including invisible and unrecognizable modes, as described in the text. All numbers are computed with the inputs in Table 1 and are given as 1 σ uncertainties, in %. The lines labeled *invis* refer to invisible modes of Higgs boson decay.

	250	500	250up	500up	1000	1000up
Energy (GeV)	250	500	250	500	1000	1000
Luminosity (fb^{-1})	250	500	1150	1600	1000	2500

Table 4: The ILC program envisioned in [8]. The stages are carried out sequentially, each one adding the data set given in the column.

by systematic errors in the High-Luminosity LHC era, the ILC measurements are always dominated by statistical errors and so improve continually with larger data samples. Running at any e^+e^- center of mass energy makes a contribution, especially because the cross section for WW fusion production of Higgs grows with energy. The program set out in [8] includes the programs of ILC running at 250 and 500 GeV discussed in the ILC TDR, the anticipated energy upgrade to 1000 GeV, and also a set of luminosity upgrades for which the strategies are mapped out in the TDR. With the somewhat conservative luminosity projections of the TDR, this program would require 18 Snowmass years (18×10^7 sec), comparable to the expected running period of the LHC. With insights gained from the experience of operating the ILC, this time could be shortened or more integrated luminosity could be obtained.

In this paper, I will consider the ILC program as progressing along the line given in Table 4. At each successive stage, the new measurements obtained from the data sets shown in the Table are added to all previous ILC measurements. I will consider the ILC stages as being carried out in the order given in Table 4. This is a slightly different order from that assumed in [8]. It allows the full program at 250 GeV and 500 GeV to be completed in parallel with the construction needed for ILC collisions at 1000 GeV. The program through the 500up stage would require 12 Snowmass years of data-taking, again assuming that our current understanding of the operation of a linear collider does not improve after years of running the ILC.

Data from the LHC are not included in the fits described in this section. Joint fits to LHC and ILC data are considered in the next section.

The fits to ILC data includes measurement of the total cross section $\sigma(e^+e^- \to Zh)$, and independent measurements of $BR(h \to b\bar{b})$ and $\sigma(e^+e^- \to \nu\bar{\nu}h)BR(h \to b\bar{b})$, from which the total cross section for the WW fusion process can be extracted. The first two of these measurements are enabled by observing the decaying Z boson that tags Higgs boson production in $e^+e^- \to Zh$. These reactions allow the total width of the Higgs boson to be constrained in a model-independent way. Then we can include the possibility of invisible Higgs decays into the fit without any need for the restriction (7).

It should be noted that the fit used here assumes more information than is included in the corresponding multi-parameter fits performed in [3], where the total width of

	250	500	500up	1000	1000up
W	4.6	0.46	0.22	0.19	0.15
Z	0.78	0.50	0.23	0.22	0.22
g	6.1	2.0	0.96	0.79	0.60
γ	18.8	8.6	4.0	2.9	1.9
b	4.7	0.97	0.46	0.39	0.32
c	6.4	2.6	1.2	0.98	0.72
τ	5.2	2.0	0.89	0.79	0.65
invis.	0.54	0.52	0.22	0.22	0.21

Table 5: Comparision of the results for Higgs coupling uncertainties, in %, from ILC at its various stages, based on estimates of σ and $\sigma \times BR$ accuracy given in [8]. The results are based on a 10-parameter fit defined in the text. The stages of the ILC program are those defined in Table 4.

the Higgs is taken to be an additional free parameter. I assume that it is possible use the tagged Higgs decays from $e^+e^- \rightarrow Zh$ to put an experimental bound on exotic Higgs decay modes equal to the direct experimental constraint that will be placed on invisible decay modes. Since an exotic decay that is not invisible has an observable component, this is a quite conservative assumption. I will discuss this assumption further in Section 7.

My estimates will then be based on a 10-parameter fit with all 10 parameters in (6) free to vary under the constraints given by the ILC measurements. The underlying values are assumed to be those of the Standard Model. The constraint (7) is *not* applied. The measurement accuracies assumed are those given in Tables 5.4 and 5.5 of [8], with exotic modes assumed to have, independently, the same upper limit as the invisible modes.

My results for the uncertainties in ILC Higgs coupling determination at the various ILC stages are given in Table 5. These uncertainties are smaller than those estimated in the reports [3] and [8] because I take into account that the experiments will search for exotic Higgs decays and, if these are not present, will present strong upper limits. Graphical comparison of the uncertainties from my analysis with those estimated by CMS in [7] are shown for the WW and ZZ couplings in Fig. 1, for the $b\bar{b}$ and $\tau^+\tau^-$ couplings in Fig. 2, and the invisible and $\gamma\gamma$ couplings in Fig. 3. All of these estimates except for the $\gamma\gamma$ case show a steady progression to smaller errors with increasing statistics that quickly reach projections of sub-1% accuracy. The conclusion that the error on the $\gamma\gamma$ coupling does not achieve high accuracy will be reconsidered in the next section.

The capabilities of the ILC at 1000 GeV for direct measurements of the $ht\bar{t}$ coupling and the Higgs self-coupling do not enter the analysis I have presented here.

They are nevertheless impressive. Those measurements are described in detail in [8].

6   Joint fits to LHC and ILC data

It is an interesting exercise to fit ILC data in isolation, but a more realistic projection would take into account the fact that the LHC results for 3000 fb^{-1} will be known at the time of all except possibly the earliest stages of the ILC program. The figures shown in the previous section emphasize the competition between e^+e^- and pp colliders, but there is also the possibility of synergy.

It is not possible carry out joint fits for the ILC and CMS data from the information provided in [7]. However, my interpretation of the CMS results as a set of uncertainties for $\sigma \cdot BR$ measurements makes it possible to estimate the improvement in the complete picture of Higgs couplings that will come from combining the ILC and LHC results. Performing the 10-parameter fit described in the previous section with the additional input of the projected CMS results, I find the uncertainties given in Table 6. For each entry in this table, the first column gives the estimated uncertainty from Table 5 and the third column gives the estimated uncertainty when the data from the ILC program, to this stage, is combined with the data from the LHC program with 300 fb^1 assuming the CMS Scenerio 2 errors.

When we compare these pairs of numbers, it is apparent that the main effect of this combination is a dramatic improvement of the uncertainty on the Higgs coupling to $\gamma\gamma$. This impact is clarified if we take a different approach. In [5] and [6], the ATLAS Collaboration presents projected uncertainties on ratios of Higgs branching ratios. In many cases, these ratios of branching ratios have substantial theoretical and modelling errors. For example, in the measurement of $BR(WW^*)/BR(ZZ^*)$, a jet veto is used for the identification of WW^* events but not for ZZ^* events. However, there is one ratio that should be almost completely free of theoretical errors. This is the ratio $BR(\gamma\gamma)/BR(ZZ^*)$. For both the $\gamma\gamma$ and the ZZ^* final states, the dominant contribution to the measurement comes from gg production. But also, more importantly, both of these final states allow the Higgs boson to be completely reconstructed, so that it is possible to tailor the measurement in such a way that the Higgs production dynamics is identical for the two samples being compared. In [5], ATLAS claims that, with 3000 fb^{-1}, this measurement could be made with an uncertainty of 2.9%, with *no* theoretical component. In [6], this estimate is revised to an experimental component of 3.6% plus a theoretical component of 13.8% (1.8% and 7.9%, respectively, in the ratio of couplings $\lambda_{Z\gamma}$). In my opinion, the inclusion of this theoretical component is certainly an error [21].

Motivated by these considerations, I have carried out the 10-parameter fit to ILC results combined with the single result from the High-Luminoisity LHC that the ratio
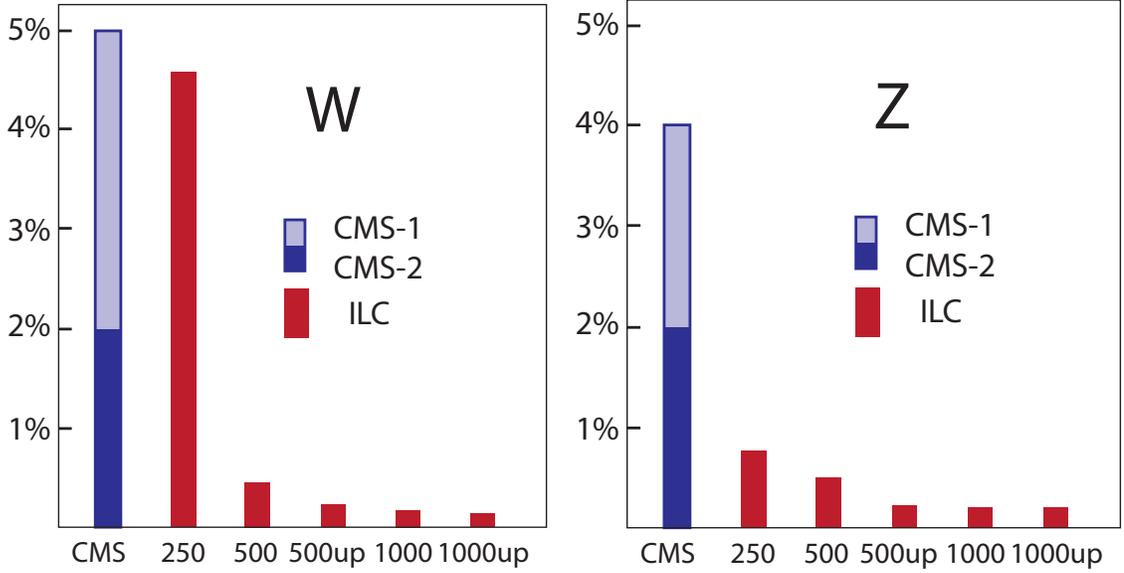
Figure 1: Estimates of the ILC measurement accuracies for the Higgs boson couplings to WW and ZZ. These estimates are based on the 10-parameter fit described in the text. The successive entries correspond to the stages of the ILC program shown in Table 4. The CMS Scenario 1 and Scenario 2 estimates are shown on the left.
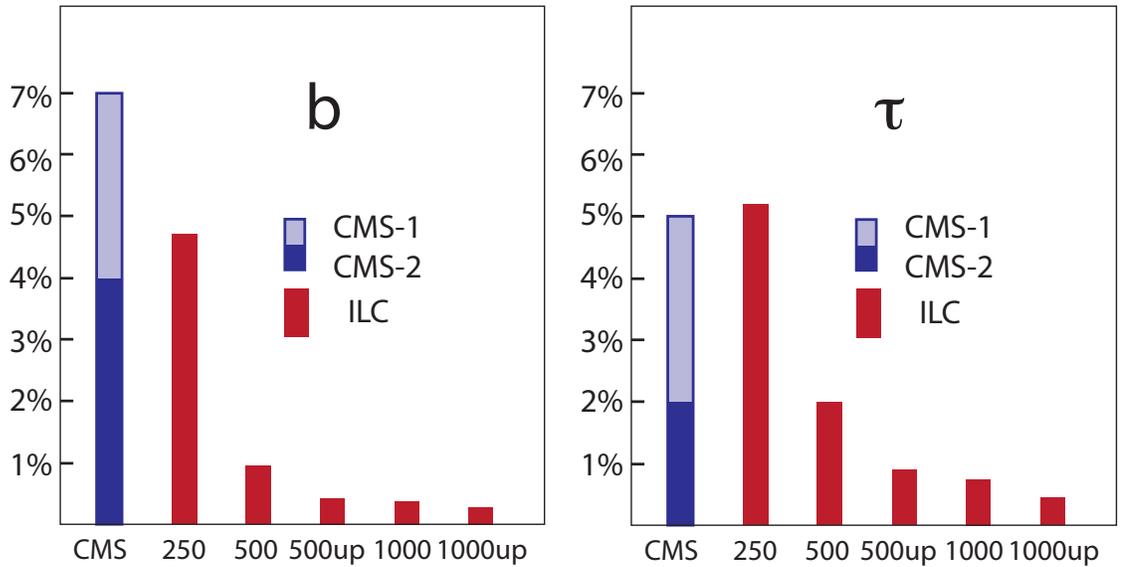

Figure 2: Estimates of the ILC measurement accuracies for the Higgs boson couplings to $b\bar{b}$ and $\tau^+\tau^-$. These estimates are based on the 10-parameter fit described in the text. The successive entries correspond to the stages of the ILC program shown in Table 4. The CMS Scenario 1 and Scenario 2 estimates are shown on the left.
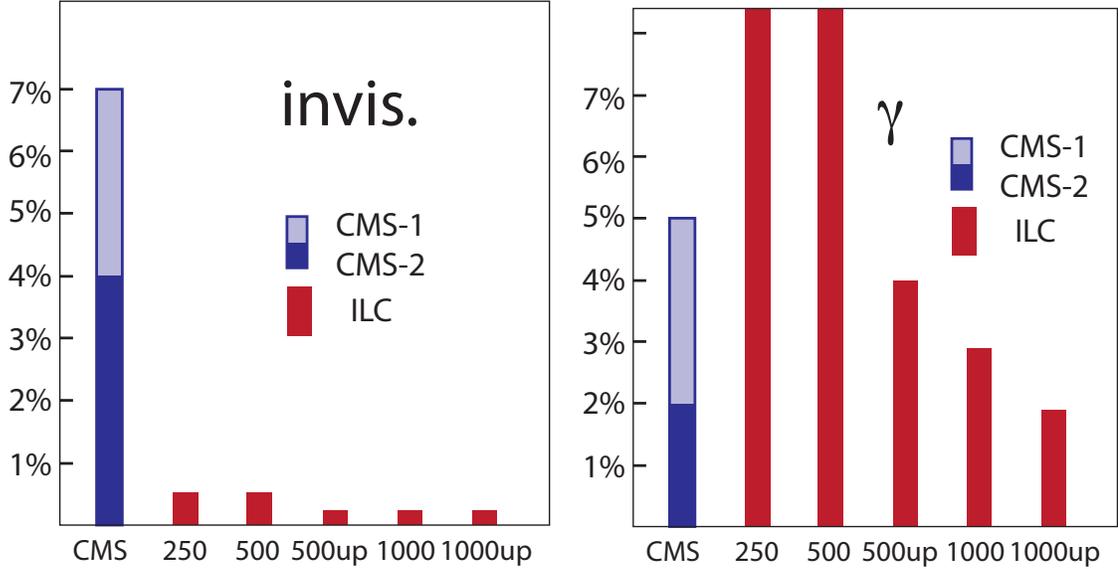
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Figure 3: Estimates of the ILC measurement accuracies for the Higgs boson couplings to invisible modes and to $\gamma\gamma$. These estimates are based on the 10-parameter fit described in the text. The successive entries correspond to the stages of the ILC program shown in Table 4. The CMS Scenario 1 and Scenario 2 estimates are shown on the left.

of branching ratios $BR(\gamma\gamma)/BR(ZZ^*)$ is measured to 3.6%. These are the results given in the second column for each entry in Table 6. The combined results in this case are comparable to those obtained from the combination with the full set of CMS projections.

The revised estimates for the uncertainties in the Higgs coupling to $\gamma\gamma$ from the various ILC stages are displayed in Fig. 4. The eventual error on the $\gamma\gamma$ coupling is somewhat better than 1.8% in the 500 GeV ILC era and becomes significantly better, even below 1%, using the statistics from the WW fusion reaction at the ILC in the 1000 GeV era. In comparing the results from CMS and the combined ILC/LHC analysis, it is important to remember that the former is based on a model-dependent fit while the latter is model-independent and dominated by statistical errors.

7 Editorial comments

A number of aspects of this analysis deserve further comment:

1. If we compare the ATLAS and CMS projections of Higgs rate measurement accuracies side by side, as is done in Table 1, it is difficult not to conclude that

250	ILC	w BR	w CMS-2		ILC	w BR	w CMS-2
W	4.6	4.6	1.4	Z	0.78	0.77	0.57
g	6.1	6.0	2.0	γ	18.8	2.0	2.0
b	4.7	4.5	1.8	c	6.4	6.3	4.6
τ	5.2	5.0	1.6	invis.	0.54	0.54	0.52
500	ILC	w BR	w CMS-2		ILC	w BR	w CMS-2
W	0.46	0.46	0.43	Z	0.50	0.50	0.47
g	2.0	2.0	1.4	γ	8.6	1.8	1.9
b	0.97	0.96	0.80	c	2.6	2.6	2.5
τ	1.9	1.9	1.3	invis.	0.52	0.52	0.51
500up	ILC	w BR	w CMS-2		ILC	w BR	w CMS-2
W	0.22	0.22	0.21	Z	0.23	0.23	0.23
g	0.96	0.96	0.85	γ	4.0	1.7	0.9
b	0.46	0.46	0.43	c	1.2	1.2	1.2
τ	0.89	0.88	0.78	invis.	0.22	0.22	0.22
1000	ILC	w BR	w CMS-2		ILC	w BR	w CMS-2
W	0.19	0.19	0.19	Z	0.22	0.22	0.22
g	0.79	0.79	0.72	γ	2.9	1.6	0.89
b	0.39	0.39	0.37	c	0.98	0.97	0.98
τ	0.79	0.79	0.70	invis.	0.22	0.21	0.21
1000up	ILC	w BR	w CMS-2		ILC	w BR	w CMS-2
W	0.15	0.15	0.15	Z	0.22	0.22	0.21
g	0.60	0.56	0.56	γ	1.9	0.83	0.83
b	0.32	0.29		c	0.72	0.74	0.74
τ	0.65	0.60	0.60	invis.	0.21	0.21	0.21

Table 6: Comparision of the results for Higgs coupling uncertainties, in %, from data samples from the ILC combined with those from LHC. Each block of entries corresponds to an ILC stage. For each entry, corresponding to the measurement of a Higgs coupling at that ILC stage, the three columns represent: first, the entry in Table 5; second, the combination of this data set with an LHC measurement of $BR(\gamma\gamma)/BR(ZZ^*)$ at 3000 fb^{-1}; third, the combination of this data set with the results from the CMS analysis for 3000 fb^{-1} and Scenario 2, as represented in Table 1.
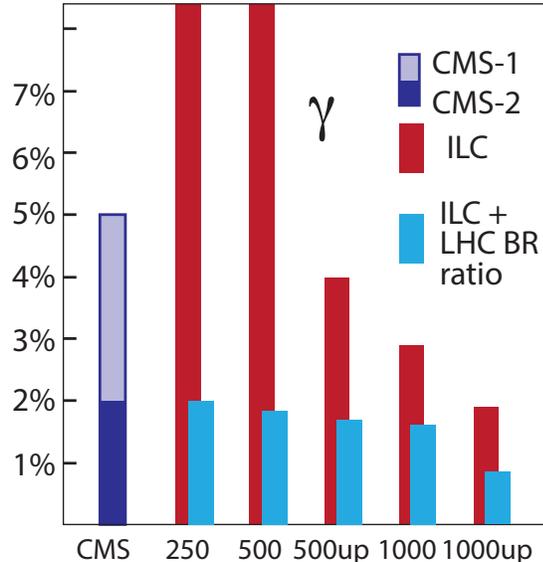
Figure 4: Estimates of the ILC measurement accuracies for the Higgs boson couplings to $\gamma\gamma$ when combined with the measurement of $BR(\gamma\gamma)/BR(ZZ^*)$ projected by ATLAS [6]. The successive entries correspond to the stages of the ILC program shown in Table 4. The CMS Scenario 1 and Scenario 2 estimates are shown on the left.

the CMS projections are quite aggressive, even for Scenario 1. One aspect of this comparison especially deserves comment.

There are three types of theoretical uncertainties that contribute to the uncertainty in Higgs rates. The first is the uncertainty in the total cross section. The second is the uncertainty in the probability of finding a particular event property used to search for the Higgs events (for example, a jet veto). The third is the modeling uncertainty involved in determining the background in a signal region by extrapolation from a control region. It is typical in LHC Higgs analyses that the Higgs contributes only 10% of the total number of events in the signal region. The rest is SM background that must be subtracted. To measure the Higgs rate to 5%, it is necessary to normalize the background to 0.5%. It is often assumed that data from a control region determines the background with precision that increases indefinitely with the statistics. But, at some level, the uncertainties from the model used for the extrapolation must be included.

In the CMS Scenario 2, only the first of these three types of theoretical uncertainty is treated as an error that will be reduced by a factor $1/2$. The other uncertainties are put into a category that is decreased as $\sqrt{N}$, a factor of $1/11$ between the current LHC data set and the end of the HL-LHC running. This prescription seems to overstate the value of the large statistics that the HL-LHC will acquire.

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It is quite appropriate to take the CMS Scenario 2 estimates as a *goal* or an *opportunity* of the high-luminosity LHC stage. But, for the reason just explained, I believe that it is not appropriate to treat these as an *expectation*. Because of the dominance of systematic uncertainties, it is also not generally appropriate to combine projections from ATLAS and CMS. My own judgement is that HL-LHC can expect to reach the CMS Scenario 1 accuracies. This will already add considerably to our knowledge of the Higgs boson.

2. It is often commented that the LHC experiments can accurately measure ratios of Higgs branching ratios. However, detailed studies such as that in [6] find large theoretical errors for ratios of branching ratios. This reflects the different systems of cuts used to select events with different Higgs final states, which bring in theoretical errors from the second and third sources discussed above.

   Still, it is worth trying to create analyses in which theoretical errors cancel as much as possible. I believe that this is possible at least for the measurement of the ratio of branching ratios $BR(\gamma\gamma)/BR(ZZ^*)$, as I have discussed in Section 6. In view of the key importance of this measurement for the long-term program of Higgs coupling measurements, it would be valuable to define a precise protocol for extracting this quantity with minimal systematic errors.

3. In contrast to the projections for LHC, the ILC projections are obtained as the result of full simulation in the environment in which the measurements are expected to be made [8]. They are optimistic only in that the detectors described in the ILC TDR might be descoped as they move toward construction. On the other hand, some properties estimated for the TDR – in particular, the efficiency for collecting $\gamma\gamma$ events – are known not to be fully optimized. I believe that the ILC estimates *can* be treated as an expectation, and one that is likely to be surpassed with experience in operating the machine and the detectors.

   As is explained in [8], the ILC Higgs results depend on running the machine not only at 250 GeV but also at higher energies where WW fusion production becomes important. This is necessary to provide enough statistics for the high-precision determination of the Higgs width, which determines the overall scale of partial widths. The effect of this higher-energy running is seen most clearly in the left-hand panel of Fig. 1. In contrast, the hZZ coupling, represented by the right-hand panel, is determined with high precision already at 250 GeV by the measurement of $\sigma(e^+e^- \to hZ)$. Precision determination of the Higgs width at 250 GeV is possible in principle, but it requires a multi-ab^{-1} data set.

4. The treatment of the total Higgs boson width in making projections for e^+e^- colliders was controversial in the Snowmass Higgs study. Blondel, in particular, argued eloquently for treating the Higgs total width as a free parameter, to be determined by the fit. In his talk at the Seattle Energy Frontier meeting, he said that one should make "no assumption on the Higgs exotic decays ...

ILC stage :	500	1000	1000up
Higgs width uncertainty, from [3] :	5.0	4.6	2.5
Higgs width uncertainty, from this analysis :	1.8	1.3	0.6

Table 7: Comparison of the uncertainty on the Higgs boson width, in %, between the fits presented in the Snowmass Higgs working group report [3] and those given here.

thus making the fit truly model-independent and truly representative of the lepton-collider potential." [22].

However, this approach is incorrect in that it does not take full account of the information that will be available from e^+e^- experiments, especially those with tagged Higgs decays. For example, the fits in [3] that use this prescription quote an uncertainty on the Higgs total width of 5.0% at the ILC500 stage. The bulk of this uncertainty must come from the presence of undetected exotic decay modes. This should be compared to the results for the mode $h \to c\bar{c}$, which has a branching ratio of 3% in the SM and whose rate is expected to be measured to 5% (0.15% of the total width) at the same ILC stage. The assumption in my fit is that undetected exotic decay modes have an upper limit of 0.9% at this stage, similarly to the truly invisible modes.

In a tagged Higgs program, *all* decays of the Higgs boson register experimentally in some way, so it is possible to impose the constraint

$$\sum_i BR(h \to i) = 1 . \tag{8}$$

This constraint has a very powerful effect on the overall fit. In Table 7, I compare the uncertainties on the Higgs total width obtained by the Snowmass Higgs Working Group, which did not apply the constraint (8), to those obtained from my fits. I would claim that my fits are equally model-independent to those in [3] but simply use more of the available information.

To make further progress in understanding the full power of precision Higgs measurements at the ILC, it would be interesting to define a protocol that uses the power of the constraint (8) more directly. An example of such an analysis is the study of tau lepton decay branching ratios performed by the Mark II experiment in the late 1980's [23]. At the time, the sum of the measured branching ratios of the tau seemed to deviate from 1, possibly significantly. This was called the "tau 1-prong problem". The Mark II collaboration collected events in which one tau could be cleanly identified, then classified *all events* in this sample into a set of categories using the information from the opposite hemisphere. A key aspect of the analysis was that it defined more categories than there are SM decay channels, so the goodness of fit could test the hypothesis that there are no exotic tau decays [24]. The conclusion of the analysis, that small adjustments

were needed in a number of measured branching ratios, and that this eliminates the evidence for exotic tau decays, has stood up over the years. We can take advantage of this strategy to design an analysis that classifies all Higgs decay candidates at the ILC, so that the constraint (8) can be applied with maximum power.

8   Conclusions

The Snowmass 2013 study pointed out the importance of the precision measurement of the couplings of the newly discovered Higgs boson. The study emphasized that the High-Luminosity LHC program will dramatically improve our knowledge of these couplings, and that further qualitative improvements with important physics implications are expected from measurements at a lepton collider such as the ILC. The analysis presented in this paper sharpens some of the points made in that study while reaffirming its general conclusions. Now we patiently await the data.

As I was completing this paper, I received a paper on the same subject by Han, Liu, and Sayre [25]. That paper quotes significantly larger uncertainties for the ILC projections. These authors do not take into account most of the points made in Section 5 above.

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