SLAC-PUB-16973

POTENTIAL PERFORMANCE LIMIT OF STORAGE RINGS *

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Abstract

The next generation of storage ring light sources will have significantly higher performance as multi-bend achromat cell structures are made practical with strong quadrupole and sextupole magnets. In principle the natural emittance can be made ever smaller with stronger magnets and larger rings until it reaches the true diffraction limit for hard X-rays. By considering the scaling laws of linear optics, intra-beam scattering, and nonlinear beam dynamics of storage rings and technical challenges, we explore the potential performance limit of future storage rings.

INTRODUCTION

The success of MAX-IV lattice design [1, 2] has introduced a revolution in the storage ring light source community. Since 2011, there have been a wave of storage ring upgrade or new storage ring proposals with lattice designs based on the multi-bend achromat (MBA) cell structure. The horizontal emittances of the new designs are typically reduced from the existing ring of the same size and beam energy by a factor of 10~50.

The substantial reduction of horizontal emittance comes from the ability to pack many more focused optics dipoles (i.e., dipoles located at waists of both horizontal beta and dispersion), which was made possible by the use of high gradient quadrupole magnets. For a given cell-type, the natural emittance scales with beam energy and cell bending angle θ_c by

$$\epsilon_x \propto \gamma^2\theta_c^3 \propto \frac{E^2}{N_c^3}, \tag{1}$$

where E is beam energy, $N_c = \frac{2\pi}{\theta_c}$ the number of cells. It seems, as more cells are added to the circumference, the emittance will be ever decreasing.

However, for a low emittance beam with low or medium beam energy, intra-beam scattering (IBS) is known to increase its momentum spread and emittance. How does IBS affect the path to reach the desired low emittance goal?

In reaching lower emittance, the dispersion function will naturally decrease, which in turn requires stronger sextupoles to correct chromaticities. Stronger and more sextupoles will cause more severe challenges in nonlinear beam dynamics. Will the dynamic aperture and momentum aperture become so small to prevent future storage rings to reach the desired performance?

The goal of emittance reduction is to achieve high brightness and high transverse coherence for undulator radiation. For the ideal case, the electron beam is matched to the photon beam in both transverse planes, i.e, $\beta_x = \beta_y = \sigma_r/\sigma_{r'}$, where the Σ's are rms photon beam sizes, $\sigma_r\sigma_{r'} = \frac{\lambda}{4\pi}$ is the single photon emittance and λ is photon wavelength. We also assume a round electron beam through full linear coupling, i.e., with $\epsilon_x = \epsilon_y = \frac{1}{2}\epsilon_n$, where ϵ_n is the storage ring natural emittance. Then the transverse coherence is given by

$$f_\perp \equiv \frac{\sigma_r^2\sigma_{r'}^2}{\Sigma_x\Sigma_{x'}\Sigma_y\Sigma_{y'}} = \left(1+\frac{2\pi\epsilon_n}{\lambda}\right)^{-2}. \tag{2}$$

We assume a future storage ring is aimed at delivering high transverse coherence for a 100 keV photon beam and the required natural emittance is 1 pm.

SCALING OF LINEAR OPTICS

To estimate the storage ring performance limit, it is necessary to have a quantitative rule that properly scales with ring circumference, beam energy, and the maximum quadrupole gradient, $B_1 = \frac{dB_y}{dx}$. The maximum quadrupole gradient is a measure of the technological limitation, which is closely related to the vacuum subsystem, aperture requirement, and the impedance budget. The maximum gradient is about 20~25 T/m for typical third generation light sources, 43 T/m for MAX-IV, and up to 100 T/m for recent designs.

We notice that the linear optics features remain unchanged if the element lengths and quadrupole strengths are scaled while keeping $\sqrt{K}L$ (where $K_1 = \frac{B_1}{B\rho}$) constant. In this case, the phase advance at any point remains the same. In fact, the linear lattice functions of a lattice cell with length L can be scaled as follows

$$\hat{s} = \frac{s}{L}, \quad \hat{\eta} = \frac{\eta}{L\theta_c}, \quad \hat{\beta}_{x,y} = \frac{\beta_{x,y}}{L}, \tag{3}$$

$$\hat{h} = \frac{hL}{\theta_c} = \frac{L}{\rho\theta_c}, \quad \hat{k} = KL^2, \tag{4}$$

where η is dispersion, $\beta_{x,y}$ are beta function, $h = \frac{1}{\rho}$ is the curvature of radius of bending magnets, and all $\hat{\cdot}$ quantities are dimensionless. Then the equations for $\beta_{x,y}$ and η becomes

$$\frac{1}{2}\hat{\beta}_x'' + \hat{k}_x\hat{\beta}_x - \frac{1}{\hat{\beta}_x}\left(1+\frac{1}{4}(\hat{\beta}_x')^2\right) = 0, \tag{5}$$

$$\frac{1}{2}\hat{\beta}_y'' + \hat{k}_y\hat{\beta}_y - \frac{1}{\hat{\beta}_y}\left(1+\frac{1}{4}(\hat{\beta}_y')^2\right) = 0, \tag{6}$$

$$\hat{\eta}'' + \hat{k}_x\hat{\eta} = \hat{h}, \tag{7}$$

where $\hat{k}_x = \hat{k} + \hat{h}^2\theta_c^2 \approx \hat{k}$, $\hat{k}_y = -\hat{k}$, $'$ and $''$ refer to first and second order derivatives w.r.t. $\hat{s}$, respectively. The curvature of bending radius is also normalized such that

$$\int_0^1 \hat{h}(\hat{s})d\hat{s} = 1. \tag{8}$$

* Work supported by DOE Contract No. DE-AC02-76SF00515

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This material is based upon work supported by the U.S. Department of Energy, Office of Science, under Contract No. DE-AC02-76SF00515.

Table 1: Scaling parameters for selected rings.

Ring	E GeV	C km	ϵ_{nat} nm	max B_1 T/m	F_n $\frac{\text{nm}\cdot(\text{T}\cdot\text{km})^{1.5}}{\text{GeV}^{3.5}}$
ALS	1.9	0.197	2.0	22	0.166
SPEAR3	3	0.234	9.8	22	0.277
APS	7	1.110	2.5	19	0.307
MAX-IV	3	0.528	0.24	43	0.213
ALS-U	2	0.200	0.109	105	0.083
APS-U	6	1.110	0.041	85	0.082

It is worth noting that the weak focusing effect from bending magnets for a large ring can be neglected.

The maximum quadrupole strength requirement becomes

$$|\hat{k}| \leq \hat{k}_{\text{m}} \equiv K_{\text{m}} L^2 = \frac{eB_{1,\text{m}} L^2}{\gamma mc}, \tag{9}$$

where $B_{1,\text{m}}$ is the maximum quadrupole gradient and m is electron rest mass.

Dispersion invariant and radiation integrals can be similarly defined in the dimensionless parameters. The emittance and momentum spread will be

$$\epsilon_{\text{nat}} = \mathcal{F}_\epsilon C_q \gamma^2 \theta_c^3, \qquad \sigma_\delta^2 = \mathcal{F}_\delta C_q \gamma^2 \frac{\theta_c}{L}, \tag{10}$$

where $\mathcal{F}_\epsilon$ and $\mathcal{F}_\delta$ are form factors.

The normalized cell can be applied for rings with various size, beam energy, and maximum quadrupole strengths. For a ring of circumference C, we have

$$\theta_c = \frac{2\pi L}{C} = \frac{2\pi}{C}\left(\frac{\hat{k}_m \gamma mc}{eB_1}\right)^{1/2}, \tag{11}$$

which can be inserted to Eq. (10) to yield [3]

$$\epsilon_{\text{nat}} = \mathcal{F}_\epsilon C_q \left(\frac{\hat{k}_m mc}{e}\right)^{1.5} \frac{\gamma^{3.5}(2\pi)^3}{B_{1,m}^{1.5} C^3} = F_n \frac{E^{3.5}}{B_{1,m}^{1.5} C^3}, \tag{12}$$

where $F_n \propto \mathcal{F}_\epsilon \hat{k}_m^{1.5}$. With large $\hat{k}_m$, more focused dipole can be packed in one cell and hence $\mathcal{F}_\epsilon$ will be reduced. The net effect is included in the form factor F_n. Parameters of some existing and proposed rings are listed in Table 1 to illustrate the above scaling law.

For the APS-U [7], the emittance is reduced by a factor of 61, of which $(6/7)^{3.5} = 0.58$ is from beam energy decrease, $(19/85)^{1.5} = 0.106$ from the quadrupole gradient increase, and the remaining factor is from the form factor F_n reduction, which represents the increase of efficiency of the lattice cell type toward lower emittance.

INTRA-BEAM SCATTERING

With IBS, the beam emittance and momentum spread will grow as beam current increases. The IBS growth rates of emittance momentum spread are related to beam energy and beam sizes. Using the high energy approximation [4, 5], the horizontal emittance growth rate is found to be

$$\frac{1}{T_x} \propto \frac{N_b \mathcal{H}_x}{\gamma^3 \epsilon^{5/2} \sigma_z (\beta_x \beta_y)^{1/4}} \propto \frac{N_b B_{1,m}^{2.5} C^6}{\gamma^{11.5}}, \tag{13}$$

where N_b is the number of electrons in the bunch, we used $\mathcal{H}_x \propto L\theta_c^2$ and

$$\sigma_z = \frac{\alpha T_0 E/e}{|dV_{\text{rf}}/dt|}\sigma_\delta \propto \frac{f(q)}{\delta_{\max}\sqrt{q}}\gamma\theta_c^{1/2} \propto \gamma\theta_c^{1/2}, \tag{14}$$

where $f(q) = \left(\sqrt{q^2-1} - \cos^{-1}\frac{1}{q}\right)^{1/2}$, and we assumed the over voltage factor $q = V_{\text{rf}}/U_0$ is chosen to keep the factor before $\gamma\theta_c^{1/2}$ constant. Note that since the number of dipoles for a fixed cell type is $N_d \propto N_c = \frac{2\pi}{\theta_c}$, the empirical scaling law of in Ref. [9] becomes

$$\frac{1}{T_x} \propto \frac{N_d^{5.5}}{E^8}, \rightarrow \quad \frac{1}{T_x} \propto \frac{C^{5.5} B_{1,m}^{2.75}}{\gamma^{10.75}}$$

The difference from Eq. (13) is not big and could come from different scaling assumptions, e.g., that of the RF parameters.

In choosing parameters to minimize the IBS effect it is reasonable to assume that at the equilibrium the IBS growth rate is equal to the radiation damping rate, which scales as

$$\frac{1}{\tau_x} \propto \gamma^3 L \propto \gamma^{3.5} B_{1,m}^{-0.5}. \tag{15}$$

At this condition the emittance doubles from IBS effects. Therefore we should require

$$\frac{B_{1,m} C^2}{\gamma^5} = \frac{B_{10,m} C_0^2}{\gamma_0^5} = \text{const}, \tag{16}$$

where subscript "0" indicates a known case when $T_x = \tau_x$ for the same cell type.

Applying Eq. (16) to Eq. (12), we obtain the scaling law for achieving low emittance for the IBS dominated case,

$$\epsilon_{\text{nat}} = \epsilon_{\text{nat},0}\left(\frac{E_0}{E}\right)^4, \tag{17}$$

while keeping Eq. (16) satisfied, where $\epsilon_{\text{nat},0}$ and E_0 are the emittance and beam energy for the known case when IBS growth rate is equal to radiation damping rate.

The use of damping wigglers [4, 6] is not considered in the above scaling analysis. Damping wigglers require a lot of straight section space and its contribution to emittance reduction saturates due to the intrinsic dispersion. Most new ring design do not include damping wigglers. Nonetheless, the damping wiggler effects would impose only a small change to the above scaling law.

Ref. [9] reported a 7-BA based lattice (TeV-USR) for a ring of $C = 6.21$ km. At 9 GeV the natural emittance is $\epsilon_{\text{nat}} = 3.0$ pm and the emittance roughly doubles by IBS at a beam current of 200 mA with full coupling. This gives us a reference point for scaling with Eq. (17) (see Figure 1).

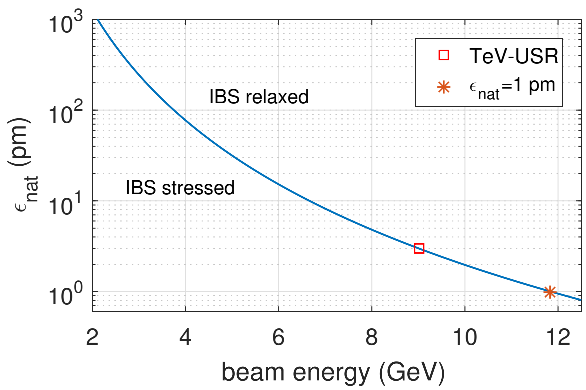

Figure 1: Emittance scaling for IBS limited ring with critical beam energy.

To scale the TeV-USR design to reach a natural emittance of 1 pm, and to only allow the IBS to double the emittance for a 200 mA beam, the beam energy needs to increase to 11.8 GeV. The combined factors of $B_{1,m}C^2$ should increase by a factor of 3.87. If no increase of $B_{1,m}$ can be made, the storage ring size needs to increase by a factor of 1.97, to 12.2 km.

We also note that at 4.5 GeV, the natural emittance should be 48 pm. The PEP-X design has a lower natural emittance of 29 pm and its IBS growth is at the critical condition (when emittance doubles from IBS) at the same current [4]. The difference is because of the use of damping wigglers of a total length of 90 m which helps in both reducing natural emittance and suppressing IBS.

SCALING OF NONLINEAR DYNAMICS

When the number of cells is increased to reduce emittance, the dispersion function is naturally reduced. But the chromaticity contribution from the cell does not change. The sextupole strengths required to correct the chromaticities have to increase, according to

$$\Delta\xi = \beta\eta[K_2L_2] \propto L^2\theta_c[K_2L_2], \tag{18}$$

where $[K_2L_2]$ is the integrated sextupole strength. Stronger sextupoles will cause more severe challenges in nonlinear beam dynamics by limiting the dynamic aperture (DA) and local momentum aperture (LMA).

Although generally storage ring nonlinear dynamics is complicated to predict, some optimistic estimates can be made. For example, we can assume the lattice cell is designed so that there is little nonlinear interaction between cells to the second order (e.g, Ref. [4]). The remaining effects are the linear addition of contributions to the resonance driving terms (RDTs) from the cells. Hence, the sextupole driven tune shifts with amplitude from RDTs h_{22000}, h_{11110} and h_{00220} will scale as

$$\frac{d\nu}{dJ} \propto N_c[K_2L_2]^2 \propto (\Delta\xi)^2\left(\frac{B_{1,m}}{E}\right)^{3.5}C^3. \tag{19}$$

If we assume such tune shifts are the limiting factors (tune shifts are usually limited to <~ 0.2) for DA and LMA, the dynamic acceptance will be

$$J_{\mathrm{DA}} \equiv \frac{A^2}{\beta} \approx \frac{\Delta\nu_{\mathrm{lim}}}{d\nu/dJ} \propto \frac{E^{3.5}}{B_{1,m}^{3.5}C^3} \propto \frac{\epsilon_{\mathrm{nat}}}{B_{1,m}^2}, \tag{20}$$

and the dynamic aperture will thus scale as [3]

$$A \propto \frac{\sqrt{\beta\epsilon_{\mathrm{nat}}}}{B_{1,m}} \propto \frac{\epsilon_{\mathrm{nat}}^{1/2}E^{1/4}}{B_{1,m}^{5/4}}. \tag{21}$$

We note that when the maximum quadrupole gradient is increased for emittance reduction, there is an additionally penalty factor of $1/B_{1,m}^2$in the dynamic acceptance. This is because with increased $B_{1,m}$, the cell length decreases and hence both beta function and dispersion scales down, forcing sextupole strengths to go up.

SCALING OF ORBIT AND OPTICS ERRORS

Random misalignment and magnetic field errors cause orbit and optics distortions, which can be statistically estimated. The rms orbit errors scales with

$$\langle\Delta x\rangle_{\mathrm{rms}} \propto \sqrt{N}\beta\langle K_1Lu\rangle_{\mathrm{rms}} \propto \sqrt{N} \propto \frac{E^{1/3}}{\epsilon_{\mathrm{nat}}^{1/6}}, \tag{22}$$

where u is the rms misalignment offset and in the last step we used Eq. (12). Similarly, the rms beta beat scales with

$$\langle\frac{\Delta\beta}{\beta}\rangle_{\mathrm{rms}} \propto \sqrt{N}\beta\langle\Delta K_1L\rangle_{\mathrm{rms}} \propto \sqrt{N} \propto \frac{E^{1/3}}{\epsilon_{\mathrm{nat}}^{1/6}}. \tag{23}$$

For the IBS dominated critical case (Eq. (17) satisfied), the scaling rule becomes

$$\langle\Delta x\rangle_{\mathrm{rms}} \propto \frac{1}{\epsilon_{\mathrm{nat}}^{1/4}}, \qquad \langle\frac{\Delta\beta}{\beta}\rangle_{\mathrm{rms}} \propto \frac{1}{\epsilon_{\mathrm{nat}}^{1/4}}. \tag{24}$$

These are weak dependence on the natural emittance.

SUMMARY

By normalizing the linear optics of a lattice cell with cell length and using it for scaling, we obtained a scaling rule of natural emittance as a function of beam energy, ring circumference, and maximum quadrupole gradient. Including the scaling property of the IBS effect, we derived an optimal strategy to choose beam energy for a certain low emittance design target. For the optimistic assumption that the nonlinear dynamics effects of the cells are only linearly additive, we obtained a scaling rule of dynamic aperture vs. emittance. Scaling properties of rms orbit and optics distortions by random alignment and field errors are also obtained.

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