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Geometric Representation of Fundamental Particles' Masses – Supplemental Material

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SUPPLEMENT A

TABLE 1: Table of the masses. Select particle counts are given at the end of some columns. L, S, C & B are the angular momentum, strangeness, charm, and bottom quantum numbers. Gravity was not considered so the zero mass, J=2 graviton was not included. Data is taken from C. Patrignani et al. (Particle Data Group), Chin. Phys. C **40**, 100001 (October 2016).

Particle	**Group**	**Mass** (MeV)	**Lifetime** (s)	**Spin** -	**L** -	**S** -	**C** -	**B** -
Electron	Leptons	0.51099893	1.45×10^{34}	1/2	0	0	0	0
Muon		105.658372	$< 2 \times 10^{-6}$	1/2	0	0	0	0
Tau		1776.82	$< 0.3 \times 10^{-12}$	1/2	0	0	0	0
e-neutrino		0.000002	$> 10^{10}$	1/2	0	0	0	0
μ-neutrino		0.19		1/2	0	0	0	0
τ-neutrino		18.2		1/2	0	0	0	0
Gamma γ	Gauge & Other bosons	$< 1 \times 10^{-24}$		1	0	0	0	0
gluon g		0		1	0	0	0	0
Axion A^o		$< 0.010 \times 10^{-6}$		0	0	0	0	0
Higgs H^o		126000		0	0	0	0	0
$W^{\pm}$		80398		1	0	0	0	0
Z^o		91188		1	0	0	0	0
pion $\pi^{\pm}$	Light, Unflavored Mesons(B=C=S=0)	139.570		0	0	0	0	0
pion π^o	$\pi^o \to 2\gamma$, $\pi^+ \to \mu^+\nu$, etc.	134.977		0	0	0	0	0
eta η		547.862		0	0	0	0	0
$f_0(600)^{\pm}$		600		0	1	0	0	0
rho $\rho(770)^{\pm}$		775.1		1	0	0	0	0
$\rho(770)^o$		775.5		1	0	0	0	0
omega $\omega(782)$		782.65		1	0	0	0	0
$\eta'(958)$		957.78		0	0	0	0	0
$f_0(980)$		980		0	1	0	0	0
$a_0(984.7)$		984.7		0	1	0	0	0
phi $\phi(1020)$		1019.46		1	0	0	0	0
$h_1(1170)$		1170		1	1	0	0	0
$b_1(1235)$		1229.5		1	1	0	0	0
$a_1(1260)$		1230		1	1	0	0	0
$f_2(1270)$		1275.1		2	1	0	0	0
$f_1(1285)$		1281.8		1	1	0	0	0
$\eta(1295)$		1294		0	0	0	0	0
$\pi(1300)$		1300		0	0	0	0	0
$a_2(1320)$		1318.3		2	1	0	0	0
$f_0(1370)$		1200-1500		0	1	0	0	0
$h_1(1380)$		1386		1	1	0	0	0
$\pi_1(1400)$		1351		1	0	0	0	0
$\eta(1405)$		1408.8		0	0	0	0	0
$f_1(1420)$		1426.4		1	1	0	0	0
$\omega(1420)$		1400-1450		1	0	0	0	0
$f_2(1430)$		1430		2	1	0	0	0
$a_0(1450)$		1474		0	1	0	0	0
$\rho(1450)$		1465		1	0	0	0	0
$\eta(1475)$		1476		0	0	0	0	0
$f_0(1500)$		1505		0	1	0	0	0
$f_1(1510)$		1518		1	1	0	0	0
$f_2'(1525)$		1525		2	1	0	0	0

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$f_2(1565)$	1562		2	1	0	0	0
$\rho(1570)$	1570		1	0	0	0	0
$h_1(1595)$	1594		1	1	0	0	0
$\pi_1(1600)$	1662		1	0	0	0	0
$f_2(1640)$	1639		2	1	0	0	0
$a_1(1640)$	1647		1	1	0	0	0
$\eta_2(1645)$	1617		2	2	0	0	0
$\omega(1650)$	1670		1	2	0	0	0
$\omega_3(1670)$	1667		3	2	0	0	0
$\pi_2(1670)$	1672.2		2	2	0	0	0
$\phi(1680)$	1680		1	0	0	0	0
$\rho_3(1690)$	1688.8		3	2	0	0	0
$\rho(1700)$	1720		1	2	0	0	0
$a_2(1700)$	1732		2	1	0	0	0
$f_0(1710)$	1724		0	1	0	0	0
$\eta(1760)$	1756		0	0	0	0	0
$\pi(1800)$	1816		0	0	0	0	0
$f_2(1810)$	1815		2	1	0	0	0
$\phi_3(1850)$	1854		3	2	0	0	0
$\eta_2(1870)$	1842		2	2	0	0	0
$\pi_2(1880)$	1895		2	2	0	0	0
$\rho(1900)$	1897.8		1	0	0	0	0
$f_2(1910)$	1903		2	1	0	0	0
$f_2(1950)$	1944		2	1	0	0	0
$\rho_3(1990)$	1982(14)		3	2	0	0	0
$f_2(2010)$	2011		2	1	0	0	0
$f_0(2020)$	1992		0	1	0	0	0
$a_4(2040)$	2001		4	3	0	0	0
$f_4(2050)$	2018		4	3	0	0	0
$\pi_2(2100)$	2090		2	3	0	0	0
$f_0(2100)$	2103		0	1	0	0	0
$f_2(2150)$	2156		2	1	0	0	0
$\rho(2150)$	2149		1	0	0	0	0
$\phi(2170)$	2175		1	0	0	0	0
$f_0(2200)$	2189		0	1	0	0	0
$f_J(2220)$	2231.1		4	4	0	0	0
$\eta(2225)$	2226		0	0	0	0	0
$\rho_3(2250)$	2260(20)		3	2	0	0	0
$f_2(2300)$	2297		2	1	0	0	0
$f_4(2300)$	2320(60)		4	3	0	0	0
$f_0(2330)$	2325.5		0	1	0	0	0
$f_2(2340)$	2339		2	1	0	0	0
$\rho_5(2350)$	2330		5	4	0	0	0
$a_6(2450)$	2450		6	5	0	0	0
$f_6(2510)$	2465		6	5	0	0	0
77	**89**						
Strange Mesons (C=0=B, S=±1)							
Kaons $K^{\pm}$= usbar,subar	493.677	1.24x10-8	0	0	1	0	0
K^0 = dsbar	497.614	5.12x10-8	0	0	1	0	0
$K^{*0}(800)$	672		0	1	1	0	0
K*(892)	891.66		1	0	1	0	0
$K_1(1270)$	1272		1	1	1	0	0
$K_1(1400)$	1403		1	1	1	0	0
K*(1410)	1414(15)		1	0	1	0	0
$K_0^*(1430)$	1425(50)		0	1	1	0	0
$K_2^*(1430)^{\pm}$	1425.6(1.5)		2	1	1	0	0
K*(1630)	1629		1	1	1	0	0
K*(1680)	1717(27)		1	2	1	0	0
$K_2(1770)$	1773(8)		2	1	1	0	0
$K_3^*(1780)$	1776(7)		3	2	1	0	0
$K_2(1820)$	1816(13)		2	2	1	0	0
$K_0^*(1950)$	1945		0	2	1	0	0
$K_2^*(1980)$	1973		2	3	1	0	0

$K_4^*(2045)$	2045(9)		4	3	1	0	0
$K_2(2250)$	2247		2	3	1	0	0
$K_3(2320)$	2324		3	3	1	0	0
$K_5^*(2380)$	2382		5	4	1	0	0
$K_4(2500)$	2490		4	3	1	0	0
21	**111**						
Charmed Mesons (C=±1)							
$D^{\pm}$ =cdbar, dcbar	1869.62	1.04x10-12	0	0	0	1	0
D^0 =cubar	1864.84	0.41x10-12	0	0	0	1	0
$D^*(2007)^0$	2006.97		1	0	0	1	0
$D^*(2010)^{\pm}$	2010.27		1	0	0	1	0
$D_0^*(2400)^0$	2318		0	1	0	1	0
$D_0^*(2400)^{\pm}$	2403		0	1	0	1	0
$D_1(2420)^0$	2421.4		1	1	0	1	0
$D_1(2420)^{\pm}$	2423.4		0	?	0	1	0
$D_1(2430)^0$	2427		1	1	0	1	0
$D_2^*(2460)^0$	2461.1		2	1	0	1	0
$D_2^*(2460)^{\pm}$	2465.4		2	?	0	1	0
11	**122**						
Charmed Strange Mesons (C=S=±1)							
$D_S^{\pm}$ +=csbar, -=scbar	1968.49	0.50x10-12	0	0	1	1	0
$D_S^{*\pm}$	2112.3		1	0	1	1	0
$D_{S0}^*(2317)^{\pm}$	2317.8		0	1	1	1	0
$D_{S1}(2460)^{\pm}$	2459.6		1	1	1	1	0
$D_{S1}(2536)^{\pm}$	2535.35		1	1	1	1	0
$D_{S2}(2573)^{\pm}$	2572.6		2	1	1	1	0
$D_{S1}(2700)^{\pm}$	2708.3		1	2	1	1	0
7	**129**						
Bottom Mesons (B=±1)							
$B^{\pm}$ +=ubbar, -=bubar	5279.15	1.65x10-12	0	0	0	0	0
B^0 0 =dbbar	5279.53	1.53x10-12	0	0	0	0	0
B^*	5325.1		1	0	0	0	0
$B_1(5721)^+$	5725.9		1	1	0	0	0
$B_1(5721)^0$	5726.0		1	1	0	0	0
$B_2^*(5747)^+$	5737.2		2	1	0	0	0
$B_2^*(5747)^0$	5739.5		2	1	0	0	0
7	**136**						
Bottom, Strange Mesons (B=±1 S=∓1)							
B_S^0 =sbbar	5366.3	1.47x10-12	0	0	-1	0	0
B_S^*	5412.8		1	0	1	0	0
$B_{S1}(5830)^0$	5829.4		1	1	1	0	0
$B_{S2}(5840)^0$	5839.7		2	1	1	0	0
$B_{SJ}(5850)^1$	5853		1	1	1	0	0
5	**141**						
Bottom, Charmed Mesons (B=C=±1)							
B_C^+ =cbbar	6275.1	0.507x10-12	0	0	-1	0	0
1	**142**						
Charmed Mesons - flavorless charmonium (ccbar)							
η_C(1S) =ccbar	2980.3		0	0	0	0	0
η_C(2S)	3637		0	0	0	0	0
J/ψ(1S)	3096.916		1	0	0	0	0
J/ψ(2S)	3686.09		1	0	0	0	0
χ_{C0}(1P)	3414.75		0	1	0	0	0
χ_{C1}(1P)	3510.66		1	1	0	0	0
h_C(1P)	3525.93		1	1	0	0	0
χ_{C2}(1P)	3556.2		2	1	0	0	0
η_C(2S)	3639.2		0	1	0	0	0
ψ(2S)	3686.097		1	1	0	0	0
ψ(3770)	3773.13		1	1	0	0	0
ψ(3823)	3822.2		2	1	0	0	0
X(3872)	3871.69		1	2	0	0	0
X(3900)	3886.6		1	2	0	0	0
X(3915)	3918.4		2	2	0	0	0

χ_{C2}(2P)	3927.2		2	1	0	0	0
X(4020)	4024.1		?	1	0	0	0
ψ(4040)	4039		1	3	0	0	0
X(4140)	4146.9		?	2	0	0	0
ψ(4160)	4191(5)		1	3	0	0	0
X(4260)	4251(8)		1		0	0	0
X(4360)	4346(6)		1		0	0	0
ψ(4415)	4421(4)		1		0	0	0
X(4430)$^{\pm}$	4478		1		0	0	0
X(4660)	4643(9)		1		0	0	0
25	**167**						
Bottom Mesons (bbbar)							
η_b(1S) =bbbar	9399.0		0	0	0	0	0
Υ(1S)	9460.3		1	0	0	0	0
χ_{b0}(1P)	9859.44		0	1	0	0	0
χ_{b1}(1P)	9892.78		1	1	0	0	0
h_b(1P)	9899.3		1	1	0	0	0
χ_{b2}(1P)	9912.21		2	1	0	0	0
Υ(2S)	10023.26		1	0	0	0	0
Υ(1D)	10161.1		2	0	0	0	0
χ_{b0}(2P)	10232.5		0	1	0	0	0
χ_{b1}(2P)	10255.46		1	1	0	0	0
χ_{b2}(2P)	10268.65		2	1	0	0	0
χ_{b1}(3P)	10512.1		1	1	0	0	0
Υ(3S)	10355.2		1	0	0	0	0
Υ(4S)	10579.4		1	0	0	0	0
X(10610)$^{\pm}$	10607.2		1		0	0	0
X(10610)^{0}	10609		1		0	0	0
Υ(10860)	10891		1	0	0	0	0
Υ(11020)	10987.5		1	0	0	0	0
18	**185**						
N Baryons			Spin	L	S	C	B
proton =uud	938.272046	$> 1x10^{32}$ yr	0.5	0	0	0	0
neutron =udd	939.565379	885.7	0.5	0	0	0	0
N(1440)	1440		0.5	0	0	0	0
N(1520)	1515		1.5	1	0	0	0
N(1535)	1535		0.5	1	0	0	0
N(1650)	1655		0.5	1	0	0	0
N(1675)	1675		2.5	1	0	0	0
N(1680)	1685		2.5	2	0	0	0
N(1700)	1700		1.5	1	0	0	0
N(1710)	1710		0.5	0	0	0	0
N(1720)	1720		1.5	2	0	0	0
N(1875)	1875(20)		1.5	2	0	0	0
N(1900)	1900		1.5	2	0	0	0
N(1905)	1900		2.5	2	0	0	0
N(1910)			0.5	2	0	0	0
N(1920)			1.5	2	0	0	0
N(1950)			3.5	2	0	0	0
N(2000)			2.5	2	0	0	0
N(2190)	2190		3.5	3	0	0	0
N(2200)	2200		2.5	3	0	0	0
N(2220)	2250		4.5	4	0	0	0
N(2250)	2275		4.5	3	0	0	0
N(2600)	2600		5.5	5	0	0	0
N(2700)	2700		6.5	6	0	0	0
24	**209**						
Delta (Δ) Baryons (S=0, I=3/2)							
Δ(1232)(Δ^{++}=uuu, Δ^{0}=udd)	1232		1.5	0	0	0	0
Δ(1600)	1600		1.5	0	0	0	0
Δ(1620)	1630		0.5	1	0	0	0
Δ(1700)	1700		1.5	1	0	0	0
Δ(1905)	1890		2.5	2	0	0	0

$\Delta(1910)$	1910		0.5	2	0	0	0
$\Delta(1920)$	1920		1.5	1	0	0	0
$\Delta(1930)$	1960		2.5	1	0	0	0
$\Delta(1950)$	1940		3.5	1	0	0	0
$\Delta(1950)$	1935		3.5	2	0	0	0
$\Delta(2400)$	2400		4.5	3	0	0	0
$\Delta(2420)$	2420		5.5	4	0	0	0
$\Delta(2950)$	2950		7.5	6	0	0	0
13	**222**						
Lambda (Λ) Baryons (S=-1, I=0)							
Λ^0 = uds	1115.683	2.63x10-10	0.5	1	-1	0	0
$\Lambda(1405)$	1406.5		0.5	0	-1	0	0
$\Lambda(1520)$	1519.5		1.5				
$\Lambda(1600)$	1600		0.5				
$\Lambda(1670)$	1670		0.5				
$\Lambda(1690)$	1690		1.5				
$\Lambda(1800)$	1800		0.5				
$\Lambda(1810)$	1810		0.5				
$\Lambda(1820)$	1820		2.5				
$\Lambda(1830)$	1830		2.5				
$\Lambda(1890)$	1890		1.5				
$\Lambda(2100)$	2100		3.5				
$\Lambda(2110)$	2110		2.5				
$\Lambda(2325)$	2325		1.5				
$\Lambda(2350)$	2350		4.5				
$\Lambda(2585)$	2585						
16	**238**						
Sigma (Σ) Baryons (S=-1, I=1)							
Σ^+ = uus	1189.37	0.80x10-10	0.5		-1	0	S+=uus
Σ^0 = uds	1192.642	7.4x10-20	0.5		-1	0	uds
Σ^- = dds	1197.449	1.48x10-10	0.5		-1	0	dds
$\Sigma(1385)^+$	1382.8		1.5				
$\Sigma(1385)^0$	1383.7		1.5				
$\Sigma_a(1385)^-$	1387.2		1.5				
$\Sigma(1620)^+$	1620		0.5				
$\Sigma(1660)^+$	1660		0.5				
$\Sigma(1670)^-$	1670		1.5				
$\Sigma(1750)^-$	1750		0.5				
$\Sigma(1775)^-$	1775		2.5				
$\Sigma(1915)^+$	1915		2.5				
$\Sigma(1940)^-$	1940		1.5				
$\Sigma(2030)^+$	2030		3.5				
$\Sigma(2250)$	2250		2.5				
15	**253**						
Xi Ξ Baryons (S=-2, I=1/2)							
Ξ^0 = uss	1314.86	2.90x10-10	0.5		-2	0	
Ξ^- = dss	1321.71	1.64x10-10	0.5		-2	0	
$\Xi(1530)$	1531.8		1.5				
$\Xi(1690)$	1690		0.5				
$\Xi(1820)$	1823		1.5				
$\Xi(1950)$	1950		0.5				
$\Xi(2030)$	2025		2.5				
$\Xi(2120)$	2120						
$\Xi(2250)$	2250		1.5				
$\Xi(2370)$	2370						
10	**263**						
Omega(Ω) Baryons (S=-3, I=0)							
Ω^- = sss	1672.45	0.82x10-10	1.5		-3	0	
$\Omega(2250)^-$	2252		0.5				
$\Omega(2380)^-$	2380		2.5				
$\Omega(2470)^-$	2474						
4	**267**						
Charmed Baryons (C=+1, I=0)							

Lambda$_C$ Λ_C^+ = udc	2286.46	0.20x10-12	0.5	0	0	
$\Lambda_C(2595)^+$	2595.4		0.5			
$\Lambda_C(2625)^+$	2628.1		1.5			
$\Lambda_C(2765)^+$	2766.6					
$\Lambda_C(2880)^+$	2881.53		2.5			
$\Lambda_C(2940)^+$	2939.3					
$\Sigma_C(2455)^{++}$ =uuc	2454.02		0.5			
$\Sigma_C(2455)^+$ =udc	2452.9		0.5			
$\Sigma_C(2455)^0$ =ddc	2453.76		0.5			
$\Sigma_C(2520)^{++}$ =uuc	2518.4		1.5			
$\Sigma_C(2520)^+$	2517.5		1.5			
$\Sigma_C(2520)^0$	2518		1.5			
$\Sigma_C(2800)^{++}$	2801					
$\Sigma_C(2800)^+$	2792					
$\Sigma_C(2800)^0$	2802					
15	**282**					
Xi Ξ Baryons (C=+1, I=0.5)						
Ξ_c^+ =usc	2467.9	0.44x10-12	0.5	-1	1	0
Ξ_c^0 =dsc	2471	0.11x10-12	0.5	-1	1	0
$\Xi_C'^+$	2575.7		0.5			
$\Xi_C'^0$	2578		0.5			
$\Xi_C(2645)^+$	2646.6		1.5			
$\Xi_C(2645)^0$	2646.1		1.5			
$\Xi_C(2790)^+$	2789.2		0.5			
$\Xi_C(2790)^0$	2791.9		0.5			
$\Xi_C(2815)^+$	2816.5		1.5			
$\Xi_C(2815)^0$	2819.6		1.5			
$\Xi_C(2970)^+$	2970.7		1.5			
$\Xi_C(2970)^0$	2968.0		1.5			
$\Xi_C(3055)^+$	3055.1		1.5			
$\Xi_C(3080)^+$	3077		1.5			
$\Xi_C(3080)^0$	3079.9		1.5			
Ω_C^0 =ssc	2695.2	0.69x10-13	0.5			
$\Omega_C^0(2770)$	2768.3		1.5			
17	**299**					
Bottom Baryons (B=-1)						
Λ_b^0 = udb	5620.2	1.38x10-12	0.5	0	0	
$\Lambda_b(5912)^0$	5912.11	1.38x10-12	0.5	0	0	
$\Lambda_b(5920)^0$	5919.81	1.38x10-12	1.5	0	0	
Σ_b^+	5811.3		0.5	0	0	-1
Σ_b^-	5815.2		0.5			
Σ_b^{*+}	5832.1		1.5	0	0	-1
Σ_b^{*-}	5835.2		1.5	0	0	-1
Ξ_b^0 = usb	5791.9	1.46x10-12	0.5	-1	0	
Ξ_b^-	5794.5	1.56x10-12	0.5	-1	0	
$\Xi_b'(5935)^-$	5935.02	1.56x10-12	0.5	-1	0	
$\Xi_b(5945)^0$	5948.9	1.56x10-12	1.5	-1	0	
$\Xi_b^*(5955)^-$	5955.33	1.56x10-12	1.5	-1	0	
Ω_b^- = ssb	6046.4		0.5			
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SUPPLEMENT B

Our goal here is to demonstrate that given the solution of Eq.(6) $[\psi_N^{(\mathrm{Masson})}(\theta)]$ in its polynomial representation (in terms of its zeroes), we can determine the effective normalized potential responsible for the Masson's dynamics. Since the Legendre Polynomials form a complete orthonormal set

$$\int_0^\pi d\theta \sin\theta P_n(\cos\theta) P_{n'}(\cos\theta) = \frac{2}{2n+1}\delta_{n,n'},$$

we may expand both the solution function $\psi_N^{(\mathrm{Masson})}(\theta)$ and the normalized potential $V_N(\theta)$ in terms of the Legendre functions, namely

$$\psi_N^{(\mathrm{Masson})}(\theta) = \Pi_{\nu=1}^{N}(1-\theta/\theta_\nu) = \sum_{n=0}^{\infty} a_n P_n(\cos\theta)$$

and

$$V_N(\theta) = \sum_{n=0}^{\infty} \alpha_n P_n(\cos\theta).$$

Explicitly, our goal is to establish α_n in terms of a_n. Substituting $\sum_{n=0}^{\infty} a_n P_n(\cos\theta)$ in Eq.6 we get

$$\sum_{n=0}^{\infty} a_n \left[-n(n+1) + V_N(\theta)\right] P_n(\cos\theta) = 0.$$

Next we substitute the expression for $V_N(\theta)$

$$\sum_{n=0}^{\infty} a_n \left[-n(n+1) + \sum_{m=0}^{\infty} \alpha_m P_m(\cos\theta)\right] P_n(\cos\theta) = 0.$$

Note that the normalization coefficient of the solution can not affect the normalized potential (α_n). Our last step is to multiply the last equation by $P_{n'}(\cos\theta)$ and employ the orthogonality condition

$$\sum_{n,m=0}^{\infty} \alpha_m \int_0^{\pi} d\theta \sin\theta P_m(\cos\theta) P_n(\cos\theta) P_{n'}(\cos\theta) = a_{n'} \frac{2n'(n'+1)}{2n'+1}.$$

In the text [Eq.(7)] this expression is given in matrix and vector form with the corresponding definitions.

[1] Rev. Part. Prop., Phys. Lett. B, Vol. **667/1** (1-1340) (2008); Rev. Part. Phys., J. Phys. G, Vol. **37**, 7A(1-1422) (2010) and Chin. Phys. C, **40**, 100001 (2016). The Table of Masses can be accessed in Supplement A. In Supplement B we show that the zeroes of the solution $[\psi_N^{(\mathrm{Masson})}(\theta)]$ determine up to a constant its polynomial representation.

[2] We take the "rest mass" as simply the "mass" – a relativistic invariant with neither "rest" nor subscript "0" attached. The *observed* masses [1] are understood to be greater than the bare masses due to self interaction contributions. The Axion was included but not the graviton because we did not consider gravity even though neither of these has yet been observed.

[3] Our definition is from *The Encylopedia of Mathematics.*

[4] P. A. M. Dirac, "The Quantum Theory of the Electron", Proc. Roy. Soc.**A:** Math., Phys. & Eng. Sci.**117,** 610 (1928).

[5] Stanley J. Brodsky, "Supersymmetric Meson-Baryon Properties of QCD from Light-Front Holography and Superconformal Algebra", Presented at Light Cone 2016, Sept., 2016, University of Lisbon, Portugal. See also: SLAC-PUB-16874.

[6] Sommerfeld A. , *Electrodynamics* in *Lectures on Theoretical Physics*, (Academic Press, New York, 1952), pp. 278.

[7] Richard L. Liboff and Michael Wong, "Quasi-Chaotic Property of Prime-Number Sequence", Int. J. Theo. Phys. **37,** 3109-3117 (1998).

[8] Large (1 km across) isolated clouds of positrons may have been observed recently but with short lifetimes (0.2 s); J.R.Dwyer et al., J. Plasma Physics **81**, 475810405 (2015).

[9] B.P. Abbott et al., GW170104: Observation of a 50-Solar-Mass Binary Black Hole Coalescence at Redshift 0.2, Phys. Rev. Lett., PRL **118** 221101 (2017).