

Supersymmetry in the Double-Heavy Hadronic Spectrum

Marina Nielsen

*Instituto de Física, Universidade de São Paulo, Rua do Matão,
Travessa R187, 05508-090 São Paulo, São Paulo, Brazil and
SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94309, USA**

Stanley J. Brodsky

SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94309, USA†

Guy F. de Téramond

Universidad de Costa Rica, 11501 San Pedro de Montes de Oca, Costa Rica‡

Hans Günter Dosch

Institut für Theoretische Physik, Philosophenweg 16, 69120 Heidelberg, Germany§

Fernando S. Navarra

*Instituto de Física, Universidade de São Paulo, Rua do Matão,
Travessa R187, 05508-090 São Paulo, São Paulo, Brazil¶*

Liping Zou

*Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou, China***

(Dated: May 30, 2018)

Relativistic light-front bound-state equations for double-heavy mesons, baryons and tetraquarks are constructed in the framework of supersymmetric light front holographic QCD. Although heavy quark masses strongly break conformal symmetry, supersymmetry and the holographic embedding of semiclassical light-front dynamics still holds. The theory, derived from five-dimensional anti-de Sitter space, predicts that the form of the confining potential in the light-front Hamiltonian is harmonic even for heavy quarks. Therefore, the basic underlying supersymmetric mechanism, which transforms meson-baryon and baryon-tetraquark wave functions into each other, can also be applied to the double-heavy sector; one can then successfully relate the masses of the double-heavy mesons to double-heavy baryons and tetraquarks. The dependence of the confining potential on the hadron mass scale agrees completely with the one derived in heavy light systems from heavy quark symmetry. We also make predictions for higher excitations of the charmonium and bottomonium states. In particular, the remarkable equality of the Regge slopes in the orbital angular momentum, L , and the principal quantum number, n , is predicted to remain valid.

I. INTRODUCTION

Light front holographic QCD (LFHQCD) leads to a remarkable semiclassical approximation to QCD [1–3]. The basis of LFHQCD is the Maldacena conjecture (or the “holographic principle”) [4], which states the equivalence of a five dimensional classical gravity theory with a four dimensional quantum field theory. The five dimensional classical theory has a non-Euclidean geometry, the so called Anti-de-Sitter (AdS) metric. The resulting four dimensional quantum field theory is a quantum gauge theory, like QCD, but instead of having $N_c = 3$ colours, it has $N_c \rightarrow \infty$. It has also conformal symmetry and it is supersymmetric. This means that to each fermion field there exists also a bosonic field with properties governed by a supersymmetry.

This superconformal quantum gauge theory with infinitely many colours is not QCD. To consider QCD in LFHQCD one chooses a bottom-up approach: one modifies the five dimensional classical theory in such a way to obtain, from this modified theory and the holographic embedding, realistic features of hadron physics which are not apparent in the QCD Lagrangian, such as confinement and the appearance of a mass scale. In a series of articles [5–7] it was shown how the implementation of superconformal symmetry [8–11] of the semiclassical theory, as expressed by holographic

* mnielsen@if.usp.br

† sjbth@slac.stanford.edu

‡ gdt@asterix.cnet.cr

§ h.g.dosch@thphys.uni-heidelberg.de

¶ navarra@if.usp.br

** zoulp@impcas.ac.cn

light front bound-state equations, completely fixes the necessary modifications of the AdS₅ Lagrangian for mesons and baryons. Although heavy quark masses break conformal symmetry, the presence of a heavy mass does not need to also break supersymmetry, since it can stem from the dynamics of color confinement. Indeed, as was shown in Refs. [12, 13], supersymmetric relations between the meson and baryon masses still hold to a good approximation even for heavy-light, *i.e.*, charm and bottom, hadrons. In the limit of massless quarks one has an universal scale (fixed for instance by one hadronic mass) and for massive quarks, one has also the quark masses as parameters. This SuSyLFHQCD leads to remarkable relations which connect meson, baryon and tetraquark spectroscopy [7, 14, 15].

In this work we will show that supersymmetric relations between double-heavy mesons, baryons, and tetraquarks can still be derived from the supersymmetric algebra even though conformal invariance is explicitly broken by the heavy quark masses. We emphasize that the supersymmetric relations which are derived from supersymmetric quantum mechanics are not based on supersymmetric Quantum Field Theory in which QCD is embedded; instead, they are based on the fact that the supercharges of the supersymmetric algebra relate the eigenstates of mesons, baryons and tetraquarks in a Hilbert space in which the light-front (LF) Hamiltonian acts. This could be considered as a realisation of supersymmetric quantum mechanics [16]. These relations are possible since in the light-front holographic approach the baryon must be described by the wave function of a quark and a diquark-cluster, and the tetraquark must be described by the wave function of a diquark-cluster and an antidiquark-cluster. This clustering is purely kinematical, it does not imply that the diquark cluster forms a tightly bound system; on the contrary, the results of the form factor analysis [17] show that the cluster is of the usual hadronic size and must be resolved [18]. The properties of the supercharges predict specific constraints between mesons and baryons, and between baryons and tetraquarks superpartners, in agreement with measurements across the entire hadronic spectrum, including the double-heavy sector [14].

This paper is organized as follows: In Sec. II we give a brief review of the LF Hamiltonian from supersymmetric quantum mechanics. In Sec. III we extend our approach to systems containing double-heavy, charm or bottom, quarks. We compare our predictions with experiment in Sec. IV, and in Sec. V we present our conclusions.

II. SUPERSYMMETRIC LIGHT FRONT HAMILTONIAN

In the framework of supersymmetric quantum mechanics [16], the LF Hamiltonian [6, 7, 12, 13] can be written in terms of two fermionic generators, the supercharges, Q and Q^\dagger , which satisfy anticommutations relations:

$$\{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0. \quad (1)$$

The Hamiltonian

$$H = \{Q, Q^\dagger\}, \quad (2)$$

commutes with these fermionic generators: $[Q, H] = [Q^\dagger, H] = 0$. Its minimal realization in matrix notation is

$$Q = \begin{pmatrix} 0 & q \\ 0 & 0 \end{pmatrix}, \quad Q^\dagger = \begin{pmatrix} 0 & 0 \\ q^\dagger & 0 \end{pmatrix}, \quad (3)$$

with

$$q = -\frac{d}{d\zeta} + \frac{f}{\zeta} + V(\zeta), \quad q^\dagger = \frac{d}{d\zeta} + \frac{f}{\zeta} + V(\zeta), \quad (4)$$

where ζ has dimension of length. The resulting Hamiltonian

$$H = \{Q, Q^\dagger\} = \begin{pmatrix} -\frac{d^2}{d\zeta^2} + \frac{4(f+1/2)^2-1}{4\zeta^2} + U_1(\zeta) & 0 \\ 0 & -\frac{d^2}{d\zeta^2} + \frac{4(f-1/2)^2-1}{4\zeta^2} + U_2(\zeta) \end{pmatrix}, \quad (5)$$

where

$$U_1(\zeta) = V^2(\zeta) - V'(\zeta) + \frac{2f}{\zeta}V(\zeta), \quad (6)$$

$$U_2(\zeta) = V^2(\zeta) + V'(\zeta) + \frac{2f}{\zeta}V(\zeta), \quad (7)$$

can be identified with a semiclassical approximation to the QCD LF Hamiltonian of mesons, $H_M = H_{11}$, and baryons, $H_B = H_{22}$.

In the LFHQCD approach the LF potential is derived from the AdS₅ action from the mapping of the AdS equations to the light front for arbitrary spin [2, 3, 15]. As has been shown in [13], the form of the LF potential (6) is only compatible with the one derived from an arbitrary dilaton profile in the meson Lagrangian, if

$$V(\zeta) = \lambda_Q \zeta. \quad (8)$$

This signifies that, even in the absence of conformal symmetry, the special form of the light front potential for massless quarks persists, provided that the holographic embedding is possible; namely, that the separation of the dynamical and kinematical variables also persists, at least to a good approximation, in the presence of heavy quark masses [13]. This can be understood if, to first order, the transverse dynamics is unchanged, and consequently the transverse LF wave function (LFWF) is also unchanged to first order [19]. In this case the confinement scale λ_Q takes the place of the confinement scale of massless quarks, $\lambda = \kappa^2$, but depends, however, on the mass of the heavy quark as expected from Heavy Quark Effective Theory (HQET) [20]. The LF potentials, $U_M(\zeta)$ and $U_B(\zeta)$, are derived from (6) and (7) respectively,

$$U_M(\zeta) = \lambda_Q^2 \zeta^2 + 2 \lambda_Q (f - 1/2), \quad (9)$$

$$U_B(\zeta) = \lambda_Q^2 \zeta^2 + 2 \lambda_Q (f + 1/2), \quad (10)$$

with the light-front orbital angular momentum, L , for baryons and mesons related by $L_B + \frac{1}{2} = L_M - \frac{1}{2} = f$.

The addition of a constant term to the Hamiltonian (5) does not violate supersymmetry and, in the following, we will use the Hamiltonian [7]

$$H_{S,m_q} = \{Q, Q^\dagger\} + (2\lambda_Q S + \Delta M^2[m_1, \dots, m_N]) \mathbf{I}, \quad (11)$$

where S depends on the internal quark spin and the term $\Delta M^2[m_1, \dots, m_N]$ is the correction for the quark masses given by Eq. (31) in the next section. This term differs by the additional light quark mass present in baryons or tetraquarks and, therefore, slightly breaks supersymmetry.

The Hamiltonian (11) acts on the 4-plet [7, 14, 15]

$$|\phi^{LF}\rangle = \begin{pmatrix} \phi_M(L_M = L_B + 1) & \psi^-(L_B + 1) \\ \psi^+(L_B) & \phi_T(L_T = L_B) \end{pmatrix}, \quad (12)$$

with $H_{S,m_q}|\phi^{LF}\rangle = M_{S,m_q}^2|\phi^{LF}\rangle$. The resulting expressions for the squared masses of the mesons, baryons and tetraquarks are [7, 14]:

$$\text{Mesons: } M_M^2 = 4\lambda_Q(n + L_M + \frac{S_M}{2}) + \Delta M^2[m_1, m_2], \quad (13)$$

$$\text{Baryons: } M_B^2 = 4\lambda_Q(n + L_B + \frac{S_D}{2} + 1) + \Delta M^2[m_1, m_2, m_3], \quad (14)$$

$$\text{Tetraquarks: } M_T^2 = 4\lambda_Q(n + L_T + \frac{S_T}{2} + 1) + \Delta M^2[m_1, m_2, m_3, m_4], \quad (15)$$

where S_M is the meson spin, S_D is the lowest possible value of the diquark cluster spin of the baryons, while S_T is the total tetraquark spin. The different values of the mass corrections, ΔM^2 , on the supermultiplet break supersymmetry explicitly to order m_q^2/m_Q^2 , where we label respectively by q and Q the light and heavy quark masses. These equations show that the excitation spectra of meson, baryon and tetraquark bound states lie on linear Regge trajectories with identical slopes in the radial, n , and orbital, L , quantum numbers. Mesons with L_M and S_M are the superpartners of baryons with $L_B = L_M - 1$ and whose diquark has $S_D = S_M$. Analogously, baryons with L_B and diquark with S_D are the superpartners of tetraquarks with $L_T = L_B$, and $S_T = S_D$. The relation $S_T = S_D$ implies that one of the diquarks in the tetraquark always has spin zero [14].

III. QUARKONIUM AS A RELATIVISTIC BOUND STATE ON THE LIGHT FRONT

A system consisting of two light quarks, or one light and one heavy quark, is relativistic. On the other hand, a system consisting of two heavy quarks is close to the non-relativistic case. However, the front form (FF) formulation (light front dynamics) of the theory of interacting particles is applicable to nonrelativistic as well as relativistic constituents. Therefore, quarkonia can be naturally treated as a relativistic bound state in the LF formulation, as done for instance

in [21, 22]. In Refs. [21, 22] a one gluon exchange interaction, in addition to a hyperfine-splitting contribution [21], or to a longitudinal confining potential [22], was added to the holographic potential to describe the double-heavy mesonic states. Since it was shown in Ref. [23] that a linear confining potential in the instant form of dynamics agrees with a quadratic confining potential in the FF of dynamics, it seems natural to extend the formulation developed in [3, 5–7] to a system with two heavy constituents.

In the LF form, the mass for a meson with two massive constituents in momentum space is given by [1, 3, 15]:

$$M^2 = \int_0^1 dx \int d^2 k_\perp \left(\frac{1}{x(1-x)} \vec{k}_\perp^2 + \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \right) |\tilde{\psi}(x, \vec{k}_\perp)|^2 + \text{interactions}, \quad (16)$$

where $\tilde{\psi}(x, \vec{k}_\perp)$ is the LFWF of two constituents with relative momentum \vec{k}_\perp and longitudinal momentum fractions $x_1 = x$, $x_2 = (1-x)$. For a system with two heavy quarks, $m_1 = m_2 = m_Q$, Eq.(16) can be written as:

$$M^2 = \int_0^1 dx \int \frac{d^2 k_\perp}{16\pi^3} \left(\frac{\vec{k}_\perp^2 + m_Q^2}{x(1-x)} \right) |\tilde{\psi}(x, \vec{k}_\perp)|^2 + \text{interactions}. \quad (17)$$

By using the Fourier transform of $\psi(x, \vec{b}_\perp)$:

$$\tilde{\psi}(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2 b_\perp e^{-i\vec{k}_\perp \cdot \vec{b}_\perp} \psi(x, \vec{b}_\perp), \quad (18)$$

in Eq. (17) we obtain:

$$M^2 = \int_0^1 dx \int d^2 b_\perp \psi^*(x, \vec{b}_\perp) \left(\frac{-\vec{\partial}_{b_\perp}^2 + m_Q^2}{x(1-x)} \right) \psi(x, \vec{b}_\perp) + \text{interactions}, \quad (19)$$

with normalization

$$\int_0^1 dx \int d^2 \vec{b}_\perp |\psi(x, \vec{b}_\perp)|^2 = \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} |\tilde{\psi}(x, \vec{k}_\perp)|^2 = 1. \quad (20)$$

We introduce the invariant impact variable $\zeta = \sqrt{x(1-x)}|\vec{b}_\perp|$, which is precisely mapped to the coordinate z of AdS space [1] by the relation $\zeta = z$. In terms of ζ we write the LFWF ψ as

$$\psi(x, \zeta, \varphi) = e^{iL\varphi} \chi(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}, \quad (21)$$

where we have factored out the longitudinal and orbital dependence from the LFWF ψ . From (20) the normalization of the transverse and longitudinal modes is given by

$$\langle \phi | \phi \rangle = \int_0^\infty d\zeta \phi^2(\zeta) = 1, \quad (22)$$

$$\langle \chi | \chi \rangle = \int_0^1 dx \frac{\chi^2(x)}{x(1-x)} = 1. \quad (23)$$

Using (21) we obtain

$$M^2 = \int d\zeta \phi^*(\zeta) \sqrt{\zeta} \left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta} \frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta \phi^*(\zeta) U(\zeta) \phi(\zeta) + \Delta M_Q^2, \quad (24)$$

where

$$\Delta M_Q^2 = m_Q^2 \int_0^1 \frac{dx}{x^2(1-x)^2} \chi^2(x), \quad (25)$$

The longitudinal function $\chi(x) \rightarrow \sqrt{x(1-x)}$ in the limit of zero quark masses [3, 24].

In deriving (24) we have assumed that separation of transverse and longitudinal dynamics is a good approximation, even in the presence of heavy quark masses, and that the effective potential, U , only depends on the transverse invariant variable ζ .

Therefore, also in the case of double-heavy quarks and strongly broken conformal invariance, the confinement potential U has the same quadratic form as the one dictated by the conformal algebra. The LF effective transverse potential can still be obtained from holography and is given by Eq. (9), at the scale λ_Q , namely $U(\zeta) = \lambda_Q^2 \zeta^2 + 2\lambda_Q(L_M - 1)$. Since the eigenvalues of the LF Hamiltonian

$$H = \left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta} \frac{d}{d\zeta} + \frac{L_M^2}{\zeta^2} + \lambda_Q^2 \zeta^2 + 2\lambda_Q(L_M - 1) \right), \quad (26)$$

are $4\lambda_Q^2(n + L_M)$, we obtain from (24)

$$M_M^2 = 4\lambda_Q^2(n + L_M) + \Delta M_Q^2, \quad (27)$$

for a spinless double-heavy meson. Extension of this result to mesons with internal spin and to baryons and tetraquarks is carried out using the procedures described in Sec. II, if the supersymmetric connection between mesons, baryons and tetraquark bound states holds also for double-heavy quarks. The masses of the double-heavy states are thus given by Eqs. (13), (14), (15).

To actually compute ΔM in (24) we need to know the longitudinal component of the LFWF $\chi(x)$, which is determined by the holographic mapping only for massless quarks [24]. To this end we follow the procedure introduced in [25], in the framework of the holographic soft-wall model [26], for the LFWF of a meson bound state with massive constituents. This procedure amounts to the change

$$\frac{\vec{k}_\perp^2}{x(1-x)} \rightarrow \frac{\vec{k}_\perp^2 + m_Q^2}{x(1-x)}, \quad (28)$$

in the exponential factor in the LFWF in momentum space; the LFWF in impact space then follows from the Fourier transform (18). In particular, for the $n = L = S = 0$ meson bound states one obtains for (21) [3, 15]

$$\psi(x, \vec{b}_\perp) = N_m \sqrt{\frac{\lambda_Q}{\pi}} \sqrt{x(1-x)} e^{-\frac{\lambda_Q}{2} x(1-x) b_\perp^2} e^{-m_Q^2/(2\lambda_Q x(1-x))}, \quad (29)$$

with

$$N_m^2 = \frac{1}{\int_0^1 dx e^{-m_Q^2/(\lambda_Q x(1-x))}}, \quad (30)$$

The mass correction, ΔM^2 , in (13) or (24) is given by

$$\Delta M^2[m_Q, m_Q] = m_Q^2 N_m^2 \int_0^1 dx \frac{e^{-m_Q^2/(\lambda_Q x(1-x))}}{x(1-x)}. \quad (31)$$

Since we are also interested in baryons and tetraquarks, which have more than two constituents, one has to form two clusters with N_a constituents each and to introduce the effective x values and transverse separations [15] :

$$x_a^{eff} = \sum_{i=1}^{N_a} x_i, \quad \vec{b}_{\perp,a}^{eff} = \frac{1}{x_a} \sum_{i=1}^{N_a} x_i \vec{b}_{\perp,i}, \quad a = 1, 2. \quad (32)$$

The resulting light front variable ζ occurring in the wave function is

$$\zeta = \sqrt{x_1^{eff} x_2^{eff}} \left| \vec{b}_{\perp,1}^{eff} - \vec{b}_{\perp,2}^{eff} \right|. \quad (33)$$

For n constituents the mass correction is then given by [7]:

$$\Delta M^2[m_1, \dots, m_n] = \frac{\lambda_Q^2}{F} \frac{dF[\lambda_Q]}{d\lambda_Q} \quad (34)$$

with $F[\lambda_Q] = \int_0^1 \dots \int_0^1 e^{-\frac{1}{\lambda_Q} \sum_{i=1}^n m_i^2/x_i}$.

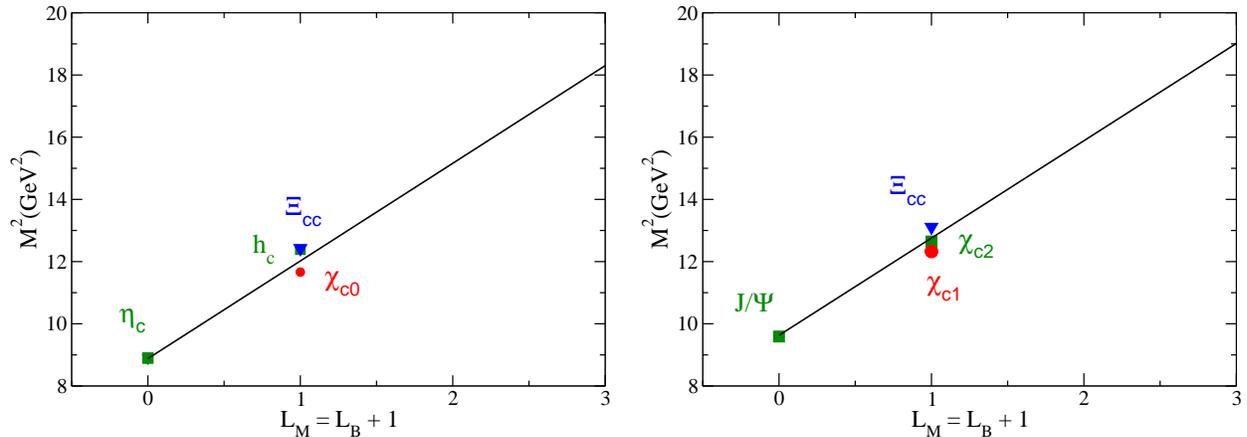


FIG. 1. Double charm mesons (shown as green squares) baryons (shown as blue triangles) and tetraquarks (shown as red circles). The solid lines are the trajectories fit from (13). Hadron masses are taken from PDG [27]. In the left hand side figure we show states with $S_M = S_D = S_T = 0$. In the right hand side figure we show states with $S_M = S_D = S_T = 1$.

IV. COMPARISON WITH EXPERIMENT

A. Mass spectrum

In Fig. 1 we show data [27] for double-charm mesons and baryons superpartners. We also include in this figure the tetraquark candidates, χ_{c0} and χ_{c1} , for the superpartners of the baryons, as discussed in [14]. The squared masses are plotted against $L_M = L_B + 1$; mesons, baryons and tetraquarks with the same abscissa are then predicted to have the same mass. The lightest meson has angular momentum zero and, therefore, does not have a supersymmetric baryon partner [12]. The solid lines in these figures are the trajectories fit from (13). The $L_M = 1$ state in the $\eta_c(2984)$ family is the $h_c(3525)$ and the $L_M = 1$ state in the $J/\psi(3096)$ family is the $\chi_{c2}(3556)$. The baryonic superpartner of the meson $h_c(3525)$ is the Ξ_{cc} state with quantum numbers $J^P = \frac{1}{2}^+$. There are two candidates for this state: the $\Xi_{cc}^{LHCb}(3620)$ observed in 2017 by the LHCb collaboration [28] and the $\Xi_{cc}^{SELEX}(3520)$ state reported by the SELEX collaboration in 2002 [29, 30]; both masses are well within the uncertainties of our model in the $h_c(3525)$ and $\chi_{c2}(3556)$ mass range. For additional interpretation concerning the Ξ_{cc} states see Ref. [31]. The Ξ_{cc}^{SELEX} and Ξ_{cc}^{LHCb} are the baryonic superpartners of the $h_c(3525)$ and $\chi_{c2}(3556)$ mesonic states. The tetraquark candidates for the superpartners of the baryonic states $\Xi_{cc}^{SELEX}(3520)$ and $\Xi_{cc}^{LHCb}(3620)$ are the scalar, $J^{PC} = 0^{++}$, $\chi_{c0}(3415)$, and the axial, $J^{PC} = 1^{++}$, $\chi_{c1}(3510)$, states, respectively. As tetraquark states, $\chi_{c0}(3415)$ has $L_T = S_T = 0$ and $\chi_{c1}(3510)$ has $L_T = 0$ and $S_T = 1$. As pointed out in Sec. II, one of the diquarks in the tetraquark has always spin zero. See Ref. [14] for more details.

In Fig. 2 we show data [27] for double-beauty mesons and tetraquark candidates [14]. The $L_M = 1$ state in the $\eta_b(9400)$ family is the $h_b(9900)$ and the $L_M = 1$ state in the $\Upsilon(9460)$ family is the $\chi_{b2}(9910)$. There is still no experimental observation of double beauty baryons. The predicted mass for Ξ_{bb} from this supersymmetric approach is $M_{\Xi_{bb}} = (9.90 \pm 0.05)$ GeV for both J^P assignments [14]. The tetraquark candidates for the $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$ baryonic states are the scalar, $J^{PC} = 0^{++}$, $\chi_{b0}(9860)$, and the axial, $J^{PC} = 1^{++}$, $\chi_{b1}(9893)$, states respectively.

Unfortunately the data for double-heavy hadrons are sparse and one cannot really test the predicted linear trajectories. However, the excellent agreement between the masses of the superpartners in the double-heavy-quark sector supports this attempt. From the fits for the different trajectories we arrive at the values for $\sqrt{\lambda_Q}$ shown in Table I. In this table we also include the values for $\sqrt{\lambda_Q}$ obtained for light [6], light-strange [12], one-charm and one-beauty [13] states.

In [13] it was shown that for heavy-light mesons consistency with HQET [20] requires that the confining scale, λ_Q , has for heavy quark masses, to be proportional to the mass of the heavy meson:

$$\sqrt{\lambda_Q} = C \sqrt{M_M}, \quad (35)$$

where C is a constant with dimension $[\text{mass}^{1/2}]$.

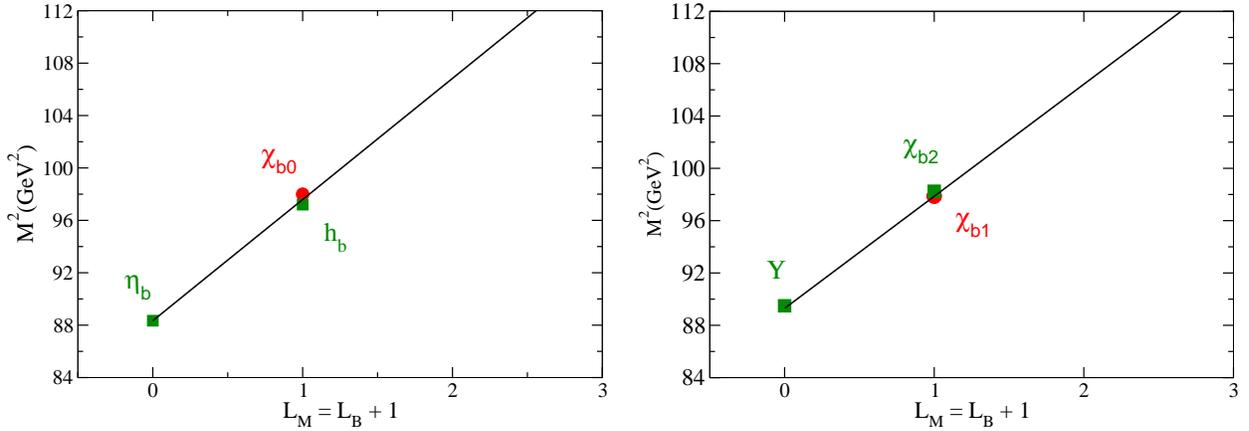


FIG. 2. Same as in Fig. 1 for double beauty hadrons.

Meson	M_M (GeV)	$\sqrt{\lambda_Q}$ (GeV)	Ref.
π	0.14	0.57 ± 0.03	[6]
K	0.50	0.57 ± 0.03	[12]
D	1.87	0.71 ± 0.04	[13]
η_c	2.98	0.90 ± 0.04	this work
B	5.28	1.1 ± 0.1	[13]
η_b	9.40	1.49 ± 0.03	this work

TABLE I. The fitted value of $\sqrt{\lambda_Q}$ for different meson trajectories as a function of the mass of the lowest meson state on the trajectory.

In Fig. 3 we show the values of λ_Q for the π , K , D , η_c , B and η_b meson families as a function of the meson mass M_M . For the light quarks we are far away from the heavy quark limit result (35). It is remarkable that the simple functional dependence (35) derived in the heavy quark limit works very well for all heavy states, including double-heavy states. This shows universal behavior for all heavy states, including the double-heavy and the heavy-light states. In contrast, HQET is applicable only for states with only one heavy quark. Fitting the results in Table I for $M_M \geq 1.87$ GeV with Eq. (35) one finds

$$C = (0.49 \pm 0.02) \text{ GeV}^{1/2}. \quad (36)$$

This value agrees, within the errors, with the value obtained in [13] from heavy-light hadrons.

B. Excitation energies of heavy mesons

We can use Eq. (35) with (36) to address a longstanding puzzle in the quarkonia spectrum [32]: Why are the excitation energies of the heavy mesons approximately independent of the heavy quark mass?

From Eq. (13) one sees that $\Delta M^2[m_1, m_2] \equiv M_0^2$ is the mass of the lowest meson state on the trajectory with $n = L_M = S_M = 0$. Therefore one can write:

$$M_M^2[n, L_M, S_M] = M_0^2 \left(1 + \frac{4\lambda_Q}{M_0^2} (n + L_M + S_M/2) \right) = M_0^2 \left(1 + \frac{4C^2}{M_0} (n + L_M + S_M/2) \right), \quad (37)$$

where, from (35) and (36), $C^2 = \frac{\lambda_Q}{M_0} \sim 0.24$ GeV. Thus, for heavy quark masses:

$$M_M[n, L_M, S_M] \approx M_0 + 2C^2(n + L_M + S_M/2), \quad (38)$$

which implies

$$M_M(1, L_M, S_M) - M_M(0, L_M, S_M) = 2C^2 \sim 480 \text{ MeV}. \quad (39)$$

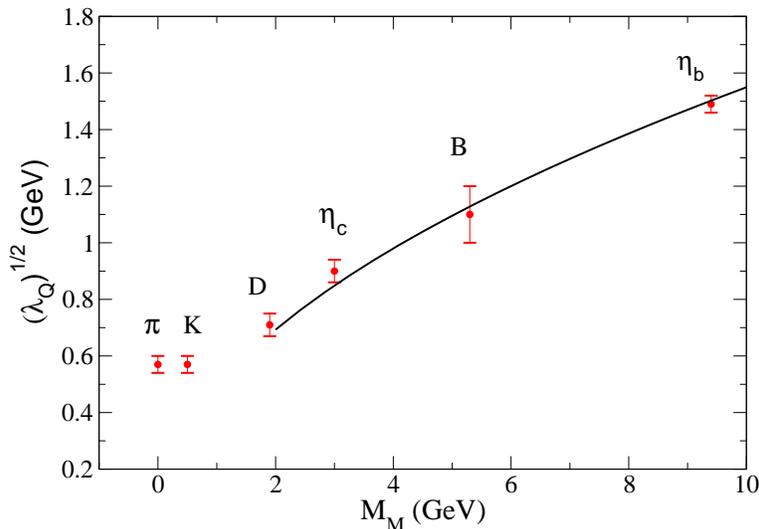


FIG. 3. Fitted values of $\sqrt{\lambda_Q}$ as a function of the mass of the lowest meson state on the trajectory. The solid line is the fit from Eq. (35).

The prediction in (39) shows that the excitation energies of the heavy mesons are indeed independent of the heavy quark mass. The experimental mass differences are consistent within the expected model uncertainties, as a first order approximation to the QCD theory. As a matter of fact, one would expect that gluon exchange would play an important role for small size states.

C. Predictions for higher excitations of charmonium and bottomonium

Meson			Baryon			Tetraquark		
q -cont	J^P	Name	q -cont	J^P	Name	q -cont	J^P	Name
$\bar{b}c$	0^-	$B_c(6275)$	—	—	—	—	—	—
$\bar{b}c$	1^+	$B_{c1}(\sim 6750)$	$[bq]c$	$(1/2)^+$	$\Xi_{cb}(\sim 6750)$	$[bq][\bar{c}\bar{q}]$	0^+	$B_{c0}(\sim 6750)$

TABLE II. Quantum number assignment for the B_c trajectory and baryonic and tetraquark superpartners.

The relation in Eq. (35) together with (36) also allows us to determine the value: $\sqrt{\lambda_Q} = (1.23 \pm 0.05)$ GeV, for the $B_c(6275)$, a $J^P = 0^-$ state. Using this value of $\sqrt{\lambda_Q}$, we can also predict the masses for the mesons on the $B_c(6275)$ trajectory. The prediction for the B_{c1} ($J^P = 1^+$) mass is

$$M_{B_{c1}} = (6.75 \pm 0.10) \text{ GeV}. \quad (40)$$

Therefore, from supersymmetry we predict a similar mass for its baryonic superpartner, Ξ_{cb} , and for the tetraquark superpartner, B_{c0} ($J^P = 0^+$). We show these predictions in Table II.

Our prediction for the B_{c1} mass is in excellent agreement with the recent lattice estimate: $M_{B_{c1}} = (6.726 \pm 0.016)$ GeV [33]. We show in Table III other model predictions for the baryonic superpartner of the B_{c1} meson, the Ξ_{cb} state. Comparing the numbers in Table III, we can see that our predictions are in good agreement with most of the previous model predictions.

$M_{\Xi_{cb}}$ (GeV)	6.86 ± 0.28	6.933	6.75 ± 0.05	6.72 ± 0.20	6.92 ± 0.13	6.75 ± 0.10
Ref.	[34]	[35]	[36]	[37]	[38]	this work

TABLE III. Predictions for the Ξ_{cb} mass.

In our approach the B_{c0} state is considered as the tetraquark superpartner of the baryon Ξ_{cb} . Other predictions for the mass of the B_{c0} state, also considered as four-quark state, are shown in the first three columns in Table IV. In the

case of B_{c0} our prediction is somewhat smaller than most of the previous predictions, but still in agreement within the errors. There is also a prediction, from lattice gauge theory, for the B_{c0} mass. However, in the lattice calculation the B_{c0} is considered as a $c\bar{b}$ state. The predicted mass is included in the last column in Table IV and it is still in agreement with our prediction within the errors.

$M_{B_{c0}}$ (GeV)	7.15 ± 0.05	6.97 ± 0.19	6.77 ± 0.11	6.75 ± 0.10	6.690 ± 0.016
Ref.	[39]	[40]	[41]	this work	[33]

TABLE IV. Predictions for the B_{c0} mass.

We can also use the information extracted from the fits in Figs. 1 and 2 to predict the masses of higher orbital excitations of the charmonium and bottomonium states. We show our predictions in Table V, where we also show predictions based on different models.

charmonium						
state	$^{2S+1}L_J$	this work	[42]	[43]	[44]	[45]
η_{c2}	1D_2	3.90 ± 0.09	3.662	3.802	3.799	3.796
ψ_3	3D_3	3.93 ± 0.11	3.770	3.843	3.806	3.799
bottomonium						
state	$^{2S+1}L_J$	this work	[42]	[43]	[46]	[47]
η_{b2}	1D_2	10.30 ± 0.10	10.068	10.166	10.148	10.163
Υ_3	3D_3	10.32 ± 0.09	10.140	10.177	10.155	10.170

TABLE V. Mass spectrum of the predicted charmonium and bottomonium orbital excited states. All mass values are in GeV.

D. Model uncertainties

It should be noted that the errors quoted in our results in Tables III, IV and V were obtained considering only the uncertainty in the value of λ_Q and in the quark masses. These errors are probably underestimated, therefore our uncertainties should be considered as lower limits.

From Table V we can see that our predictions, in the case of charmonia, are in agreement with most of the other predictions, considering the errors. It is very interesting to notice that our prediction for the mass of the Ψ_3 state is in excellent agreement with the mass of the first radial excitation of the χ_{c2} (3556): the $n = 1$ χ_{c2} (3927) state, and in a good agreement (within the error) with the $n = 2$ and $L_M = 0$ ψ (4039) state. According to Eq. (13), states with $n = 0$ and $L_M = 2$ should have the same mass as the states with $n = 1$ and $L_M = 1$ or $n = 2$ and $L_M = 0$, if they have the same S_M , as the case of the ψ_3 , χ_{c2} (3927) and ψ (4039) states respectively.

In the case of bottomonia our predictions are somewhat higher than previous model calculations. However, as in the case of Ψ_3 discussed above, our prediction for the mass of the Υ_3 state is in a good agreement with the mass of the first radial excitation of the χ_{b2} (9910): the $n = 1$ χ_{b2} (10270) state, and in excellent agreement with the $n = 2$ and $L_M = 0$ Υ (10355) state. Unfortunately, there are no other observed radial excited states ($n = 1$) to be compared with the other predictions in Table V. We urge the experimentalists to make an effort to measure the masses of the predicted states. The two examples discussed above show that, even in the case of the double-heavy states, the Regge slope is the same in both n and L quantum numbers, as predicted by the SuSyLFHQCD.

Since λ_Q and ΔM^2 , in Eq. (13), are determined from the fits to the Regge trajectories to the different meson families, we can use these values in (31) to estimate the effective heavy quark masses. We obtain $m_c = (1.52 \pm 0.07)$ GeV and $m_b = (4.63 \pm 0.04)$ GeV. Using the same procedure, the heavy quark masses obtained from heavy-light systems in [13] were $m_c = 1.55$ GeV and $m_b = 4.922$ GeV, indicating the inherent uncertainties of the model.

E. Decay constants

The decay constant f_M of a pseudoscalar meson is the coupling of the hadron to its current. In a bound state model for mesons it is related to the value of the LF wave function at the origin [48].

$$f_M = \sqrt{\frac{2N_C}{\pi}} \int_0^1 dx \psi(x, \vec{b}_\perp = 0). \quad (41)$$

Using the result in (29) one has for a meson with two constituents of equal mass m_Q :

$$f_M = N_m \frac{\sqrt{2N_C \lambda_Q}}{\pi} \int_0^1 dx \sqrt{x(1-x)} e^{-m_Q^2/(2\lambda_Q x(1-x))}, \quad (42)$$

with (see (30)):

$$N_m^2 = \frac{1}{\int_0^1 dx e^{-m_Q^2/(\lambda_Q x(1-x))}}. \quad (43)$$

For the particular case of spin projection zero, the radiative vector meson decay constant is also given by Eq. (42) [21, 49].

The values of the quark masses and λ_Q determined in the previous section can be used to evaluate the meson decay constant in (42). In the limit of very large quark masses the integrals in (42) and (43) can also be approximately evaluated analytically by the saddle point method, since in the limit $m_Q \rightarrow \infty$ the function $e^{-m_Q^2/(2\lambda_Q x(1-x))}$ is very sharply peaked at $x = \frac{1}{2}$. Introducing $z^2 = m_Q^2/\lambda_Q$, one obtains: $N_m = e^{2z^2} \frac{2\sqrt{z}}{\pi^{1/4}}$, and

$$\int_0^1 dx \sqrt{x(1-x)} e^{-z^2/(2x(1-x))} = \frac{e^{-2z^2}}{2z} \sqrt{\frac{\pi}{8}}. \quad (44)$$

This leads to the asymptotic value of the decay constant:

$$f_M = \sqrt{\frac{3}{4}} \pi^{-3/4} \frac{\sqrt{\lambda_Q}}{\sqrt{m_Q}} \lambda_Q^{1/4} \approx (0.178 \text{ GeV}^{3/4}) M_M^{1/4}, \quad (45)$$

where we have used $M_M \sim 2 m_Q$ and $C = \frac{\sqrt{\lambda_Q}}{\sqrt{M_M}} \approx 0.49 \text{ GeV}^{1/2}$, see (35) and (36). LFHQCD therefore predicts an increase in the decay constant with the meson mass.

Meson	f_M [MeV] this work	$f_V^{(exp)}/e_V$ [MeV] Ref. [27]	\hat{e}_V
ρ	160	221 ± 5	$\frac{1}{\sqrt{2}}$
ω	160	196 ± 3	$\frac{1}{3\sqrt{2}}$
ϕ	161	229 ± 4	$\frac{1}{3}$
J/ψ	228	416 ± 5	$\frac{2}{3}$
Υ	299	715 ± 5	$\frac{1}{3}$

TABLE VI. Decay constants for different vector mesons, The value f_M is determined from (42), the experimental values in the third column are obtained from the radiative constant by dividing through the effective quark charge \hat{e}_V .

The radiative decay constants of neutral vector mesons are, up to the charge factor and radiative corrections, the general decay constants [50]. In Table VI we display the results of (42) together with the observed values, obtained from the electromagnetic decay constant divided by the effective quark charge \hat{e}_V . To show the tendency we have also included the results for the light vector mesons.

As shown in Table VI, the increase of the decay constant with meson mass is indeed observed, but the theoretical increase is much too slow. The agreement of the coupling constants obtained here with the experimental values is poor, as in other similar approaches [21, 49]. This suggests that, despite the fact that the eigenvalues obtained in this SuSyLFHQCD are in good agreement with the experimental values, the wave functions are too simple to convey all the complexity of the quarkonium states. Even for the light mesons the decay constant comes out to small, as can be seen in Table VI. The increasing discrepancy suggests that special effects play a role in heavy quarkonia. A probable cause is the color-Coulomb attraction, since its effect increases with increasing mass and correspondingly smaller radius of the quarkonium.

In [21] the LF wave functions were modified by introducing a phenomenological longitudinal term, with a new dimensional parameter which scales as $\sqrt{m_Q}$, while keeping the dilaton parameter fixed. However, such modification of the wave function did not improve the agreement of the decay constants with the experimental values. In [51–53] the wave function proposed in [21] was modified by considering a helicity-dependent holographic wavefunction. With

this modification a better agreement, of the decay constants with data, is obtained in the case of light and heavy-light mesons. However, the authors of [51–53] have not studied heavy-heavy mesons. In [54] the quarkonium decay constants were evaluated directly from the two point correlator function, calculated at some finite value, $z = z_0$, of the radial coordinate of AdS_5 space. This corresponds to introducing a new energy scale: $1/z_0$, in the model that leads to a better agreement with data. We expect to further investigate the origin of this discrepancy.

V. SUMMARY AND CONCLUSIONS

In this paper we have described the consequences of extending semiclassical light-front bound-state equations to double-heavy quark systems. The approach is based on supersymmetric light front holographic QCD. The supersymmetry relates wave functions of mesons to baryons and of baryons to tetraquarks: this approach is not based on a supersymmetric quantum field theory but on supersymmetric quantum mechanics [16].

We have shown that the mass spectra of double-charmed and double-beauty mesons are compatible with the linear Regge trajectories given in Eq. (13). In particular, the remarkable equality of the Regge slopes in both, orbital angular momentum L and principal quantum number n is predicted to remain valid, even for hadrons containing double-heavy quarks. From the determination of the Regge slope from these trajectories, we have shown that this parameter follows the same relation (35) as obtained in heavy-light systems from heavy quark symmetry [13]. This relation also explains an old puzzle in quarkonia physics: why the excitation energies are approximately independent of the heavy meson mass [32]. The relation in (35) allowed us to make predictions for several double-heavy states, shown in Eq. (40) and in Tables III, IV and V. Our predictions are in good agreement with other model predictions in the case of charmonium, and in a fair agreement in the case of bottomonium.

We have also evaluated the radiative decay constant of the vector states J/ψ and Υ . The poor agreement with the experimental values of the decay constants shows that, although the eigenvalues obtained in this SuSyLFHQCD are in good agreement with the experimental values, the wave functions used are too simple to express all the complexity of the quarkonium states.

We have shown how supersymmetry, together with light-front holography, leads to connections between double-heavy mesons, baryons and tetraquarks, thus providing new perspectives for hadron spectroscopy and QCD. We emphasize that measurements of additional states in the double-heavy quarks sector will test our predictions.

ACKNOWLEDGMENTS

S.J.B. is supported by the Department of Energy, contract DE-AC02-76SF00515. M.N. is supported by FAPESP process# 2017/07278-5. SLAC-PUB-17259.

-
- [1] G. F. de Teramond and S. J. Brodsky, Light-front holography: A first approximation to QCD, *Phys. Rev. Lett.* **102**, 081601 (2009) [arXiv:0809.4899 [hep-ph]].
 - [2] G. F. de Teramond, H. G. Dosch and S. J. Brodsky, Kinematical and dynamical aspects of higher-spin bound-state equations in holographic QCD, *Phys. Rev. D* **87**, 075005 (2013) [arXiv:1301.1651 [hep-ph]].
 - [3] S. J. Brodsky, G. F. de Teramond, H. G. Dosch and J. Erlich, Light-front holographic QCD and emerging confinement, *Phys. Rept.* **584**, 1 (2015) [arXiv:1407.8131 [hep-ph]].
 - [4] J. M. Maldacena, The Large N limit of superconformal field theories and supergravity, *Int. J. Theor. Phys.* **38**, 1113 (1999)[*Adv. Theor. Math. Phys.* **2**, 231 (1998)] [hep-th/9711200].
 - [5] G. F. de Teramond, H. G. Dosch and S. J. Brodsky, Baryon spectrum from superconformal quantum mechanics and its light-front holographic embedding, *Phys. Rev. D* **91**, 045040 (2015) [arXiv:1411.5243 [hep-ph]].
 - [6] H. G. Dosch, G. F. de Teramond and S. J. Brodsky, Superconformal baryon-meson symmetry and light-front holographic QCD, *Phys. Rev. D* **91**, 085016 (2015) [arXiv:1501.00959 [hep-th]].
 - [7] S. J. Brodsky, G. F. de Teramond, H. G. Dosch and C. Lorcé, Universal effective hadron dynamics from superconformal algebra, *Phys. Lett. B* **759**, 171 (2016) [arXiv:1604.06746 [hep-ph]].
 - [8] V. P. Akulov and A. I. Pashnev, Quantum superconformal model in (1,2) space, *Teor. Mat. Fiz.* **56**, 344 (1983).
 - [9] S. Fubini and E. Rabinovici, Superconformal quantum mechanics, *Nucl. Phys. B* **245**, 17 (1984).
 - [10] V. de Alfaro, S. Fubini and G. Furlan, Conformal invariance in quantum mechanics, *Nuovo Cim. A* **34**, 569 (1976).
 - [11] R. Haag, J. T. Lopuszanski and M. Sohnius, All possible generators of supersymmetries of the S Matrix, *Nucl. Phys. B* **88**, 257 (1975).
 - [12] H. G. Dosch, G. F. de Teramond and S. J. Brodsky, Supersymmetry across the light and heavy-light hadronic spectrum, *Phys. Rev. D* **92**, 074010 (2015) arXiv:1504.05112 [hep-ph]].

- [13] H. G. Dosch, G. F. de Teramond and S. J. Brodsky, Supersymmetry across the light and heavy-light hadronic spectrum II, *Phys. Rev. D* **95**, 034016 (2017) [arXiv:1612.02370 [hep-ph]].
- [14] M. Nielsen and S. J. Brodsky, Hadronic superpartners from superconformal and supersymmetric algebra, arXiv:1802.09652 [hep-ph].
- [15] L. Zou and H. G. Dosch, A very practical guide to light front holographic QCD, [arXiv:1801.00607 [hep-ph]].
- [16] E. Witten, Dynamical breaking of supersymmetry, *Nucl. Phys. B* **188**, 513 (1981).
- [17] R. S. Sufian, G. F. de Teramond, S. J. Brodsky, A. Deur and H. G. Dosch, Analysis of nucleon electromagnetic form factors from light-front holographic QCD : The spacelike region, *Phys. Rev. D* **95**, 014011 (2017) [arXiv:1609.06688 [hep-ph]].
- [18] S. J. Brodsky, C. R. Ji and G. P. Lepage, Quantum chromodynamic predictions for the deuteron form-factor, *Phys. Rev. Lett.* **51** (1983) 83.
- [19] G. F. de Teramond, S. J. Brodsky and H. G. Dosch, Light-front holography in QCD and hadronic physics, arXiv:1405.2451 [hep-ph].
- [20] N. Isgur and M. B. Wise, Spectroscopy with heavy quark symmetry, *Phys. Rev. Lett.* **66**, 1130 (1991).
- [21] T. Branz, T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, Light and heavy mesons in a soft-wall holographic approach, *Phys. Rev. D* **82**, 074022 (2010) [arXiv:1008.0268 [hep-ph]].
- [22] Y. Li, P. Maris and J. P. Vary, Quarkonium as a relativistic bound state on the light front, *Phys. Rev. D* **96**, 016022 (2017) [arXiv:1704.06968 [hep-ph]].
- [23] A. P. Trawiński, S. D. Glazek, S. J. Brodsky, G. F. de Teramond and H. G. Dosch, Effective confining potentials for QCD, *Phys. Rev. D* **90**, 074017 (2014) [arXiv:1403.5651 [hep-ph]].
- [24] S. J. Brodsky and G. F. de Teramond, Hadronic spectra and light-front wavefunctions in holographic QCD, *Phys. Rev. Lett.* **96**, 201601 (2006) [hep-ph/0602252].
- [25] S. J. Brodsky and G. F. de Teramond, AdS/CFT and light-front QCD, *Subnucl. Ser.* **45**, 139 (2009) [arXiv:0802.0514 [hep-ph]].
- [26] A. Karch, E. Katz, D. T. Son and M. A. Stephanov, Linear confinement and AdS/QCD, *Phys. Rev. D* **74**, 015005 (2006) [hep-ph/0602229].
- [27] C. Patrignani et al. (Particle Data Group), Review of particle physics, *Chin. Phys. C* **40**, 100001 (2016) and 2017 update.
- [28] R. Aaij *et al.* [LHCb Collaboration], Observation of the doubly charmed baryon Ξ_{cc}^{++} , *Phys. Rev. Lett.* **119**, 112001 (2017) [arXiv:1707.01621 [hep-ex]].
- [29] M. Mattson *et al.* [SELEX Collaboration], First observation of the doubly charmed baryon $\Xi^+(cc)$, *Phys. Rev. Lett.* **89**, 112001 (2002) [hep-ex/0208014].
- [30] A. Ocherashvili *et al.* [SELEX Collaboration], Confirmation of the double charm baryon $Xi^+(cc)(3520)$ via its decay to pD^+K^- , *Phys. Lett. B* **628**, 18 (2005) [hep-ex/0406033].
- [31] S. J. Brodsky, S. Groote and S. Koshkarev, Resolving the SELEX–LHCb double-charm baryon conflict: The impact of intrinsic heavy-quark hadroproduction and supersymmetric light-front holographic QCD, arXiv:1709.09903 [hep-ph].
- [32] C. Quigg and J. L. Rosner, Quantum mechanics with applications to quarkonium, *Phys. Rept.* **56**, 167 (1979).
- [33] N. Mathur, M. Padmanath and R. Lewis, Charmed-bottom mesons from lattice QCD, *PoS LATTICE* **2016**, 100 (2016) [arXiv:1611.04085 [hep-lat]].
- [34] E. Bagan, H. G. Dosch, P. Gosdzinsky, S. Narison and J. M. Richard, Hadrons with charm and beauty, *Z. Phys. C* **64**, 57 (1994) [hep-ph/9403208].
- [35] D. Ebert, R. N. Faustov, V. O. Galkin and A. P. Martynenko, Mass spectra of doubly heavy baryons in the relativistic quark model, *Phys. Rev. D* **66**, 014008 (2002) [hep-ph/0201217].
- [36] J. R. Zhang and M. Q. Huang, Doubly heavy baryons in QCD sum rules, *Phys. Rev. D* **78**, 094007 (2008) [arXiv:0810.5396 [hep-ph]].
- [37] T. M. Aliev, K. Azizi and M. Savci, Doubly Heavy Spin-1/2 Baryon Spectrum in QCD, *Nucl. Phys. A* **895**, 59 (2012) [arXiv:1205.2873 [hep-ph]].
- [38] M. Karliner and J. L. Rosner, Quark-level analogue of nuclear fusion with doubly-heavy baryons, *Nature* **551**, 89 (2017) [arXiv:1708.02547 [hep-ph]].
- [39] W. Chen, T. G. Steele and S. L. Zhu, Exotic open-flavor $bc\bar{q}\bar{q}$, $bc\bar{s}\bar{s}$ and $qc\bar{q}\bar{b}$, $sc\bar{s}\bar{b}$ tetraquark states, *Phys. Rev. D* **89**, 054037 (2014) [arXiv:1310.8337 [hep-ph]].
- [40] S. S. Agaev, K. Azizi and H. Sundu, Open charm-bottom scalar tetraquarks and their strong decays, *Phys. Rev. D* **95**, 034008 (2017) [arXiv:1611.00293 [hep-ph]].
- [41] R. M. Albuquerque, X. Liu and M. Nielsen, Exotic B_c -like molecules in QCD Sum Rules, *Phys. Lett. B* **718**, 492 (2012) [arXiv:1203.6569 [hep-ph]].
- [42] T. Bhavsar, M. Shah and P. C. Vinodkumar, Status of quarkonia-like negative and positive parity states in a relativistic confinement scheme, *Eur. Phys. J. C* **78**, no. 3, 227 (2018) [arXiv:1803.07249 [hep-ph]].
- [43] M. Shah, A. Parmar and P. C. Vinodkumar, Leptonic and digamma decay properties of S-wave quarkonia states, *Phys. Rev. D* **86**, 034015 (2012) [arXiv:1203.6184 [hep-ph]].
- [44] T. Barnes, S. Godfrey and E. S. Swanson, Higher charmonia, *Phys. Rev. D* **72**, 054026 (2005) [hep-ph/0505002].
- [45] B. Q. Li and K. T. Chao, Higher charmonia and X, Y, Z states with screened potential, *Phys. Rev. D* **79**, 094004 (2009) [arXiv:0903.5506 [hep-ph]].
- [46] S. Godfrey and K. Moats, Bottomonium mesons and strategies for their observation, *Phys. Rev. D* **92**, 054034 (2015) [arXiv:1507.00024 [hep-ph]].
- [47] J. Z. Wang, Z. F. Sun, X. Liu and T. Matsuki, Higher bottomonium zoo, arXiv:1802.04938 [hep-ph].
- [48] S. J. Brodsky and G. F. de Teramond, Light-front dynamics and AdS/QCD correspondence: The pion form Factor in the

- space- and time-like regions, Phys. Rev. D **77**, 056007 (2008) [arXiv:0707.3859 [hep-ph]].
- [49] A. Vega, I. Schmidt, T. Branz, T. Gutsche and V. E. Lyubovitskij, Meson wave function from holographic models, Phys. Rev. D **80**, 055014 (2009) [arXiv:0906.1220 [hep-ph]].
- [50] R. Barbieri *et al.*, Meson hyperfine splittings and leptonic decays, Phys. Lett. B **57**, 455 (1975); Meson masses and widths in a gauge theory with linear binding potential, Nucl.Phys. B **105**, 125 (1975).
- [51] Q. Chang, X. N. Li, X. Q. Li and F. Su, Decay constants of pseudoscalar and vector mesons with improved holographic wavefunction, arXiv:1805.00718 [hep-ph].
- [52] Q. Chang, S. Xu and L. Chen, Application of the light-front holographic wavefunction for heavy-light pseudoscalar meson in $B_{d,s} \rightarrow D_{d,s} P$ decays, Nucl. Phys. B **921**, 454 (2017) [arXiv:1805.02011 [hep-ph]].
- [53] M. Ahmady, C. Mondal and R. Sandapen, Dynamical spin effects in the holographic light-front wavefunctions of light pseudoscalar mesons, arXiv:1805.08911 [hep-ph].
- [54] N. R. F. Braga, M. A. Martin Contreras and S. Diles, Decay constants in soft wall AdS/QCD revisited, Phys. Lett. B **763**, 203 (2016) [arXiv:1507.04708 [hep-th]].