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FIRST OBSERVATION OF THE DECAY $\pi^+ \to e^+e^-e^+\nu$ AND A DETERMINATION OF THE FORM FACTORS F_V , F_A AND R

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The radiative pion decay $\pi^+ \to e^+e^-e^+\nu$ has been observed for the first time with a branching ratio of $(3.4\pm0.5)\times10^{-9}$. From the kinematical distribution of 79 events information on the form factors was extracted. The vector form factor F_V was found to have the same sign as the pion decay constant f_π. For the axial-vector form factors the following values have been measured: $\gamma = F_A/F_V = 0.7 \pm 0.5$ and $\xi = R/F_V = 2.3 \pm 0.6$. This value of γ agrees well with the small positive value of (0.53 ± 0.06) and excludes the large negative value obtained in $\pi^+ \to e^+\nu\gamma$ experiments.

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1. INTRODUCTION

The physical interest in the radiative pion decay $\pi^+ \to e^+e^-e^+\nu$ arises twofold. First the process $\pi^+ \to e^+e^-e^+\nu$ has never been observed up to now. An upper limit on the branching ratio was established[1] in 1976 to be 4.8×10^{-9}. Second in contrast to the ordinary pion decay $\pi^+ \to \mu^+\nu$ and the pion beta decay $\pi^+ \to \pi^0e^+\nu$, the two radiative decays $\pi^+ \to e^+e^-e^+\nu$ and $\pi^+ \to e^+\nu\gamma$ carry information on both the vector- and axial-vector weak hadronic currents. While measurements of $\pi^+ \to e^+\nu\gamma$ allow the determination of $|F_A/F_V|$, the process $\pi^+ \to e^+e^-e^+\nu$ can unambiguously reveal the signs and values of F_V, F_A and R. This can be achieved by using only the kinematical distribution of the observed events or by comparing the theoretical with the experimental E_ν–spectra. For this one does not depend on the absolute number of measured decays and is therefore much less subject to systematical errors.

2. THEORY

The decay amplitude for the process $\pi^+ \to e^+e^-e^+\nu$ contains the following terms:[2]

(*a*) an axial-vector term, describing the inner Bremsstrahlung from the electron (IB_e);

(*b*) an analogous term for the inner Bremsstrahlung from the pion (IB_π), which consists of a contact term and a pole term;

(*c*) several structure dependent terms, which arise from both the vector (SD_V) and the axial-vector (SD_A) weak currents.

Both IB terms (a) and (b) are calculable and can be parametrized by the pion decay constant f_π, which is well known from the ordinary decay $\pi^+ \to \mu^+\nu$: $f_\pi = 128$ MeV.

After separating the IB part from the decay amplitude we are left with the interesting SD terms, which can be reduced to the following expression:

$$M_{SD} \sim \epsilon^\alpha\{F_V\epsilon_{\alpha\beta\gamma\delta}k^\gamma q^\delta + F_A(Q_\alpha k_\beta - Qkg_{\alpha\beta}) + R(k_\alpha k_\beta - k^2g_{\alpha\beta})\} \times J^\beta_{e+\nu}$$

where

$$\epsilon^\alpha = \left(\frac{e}{k^2}\right) u_{e^-}\gamma^\alpha v_{e^+}$$

represents the internal γ converting into e^+e^-, and $J^\beta_{e+\nu}$ is the weak V-A current. Here q, k are the momenta of the pion and the internal γ, respectively, and $Q = q - k$.

In this way the terms SD_V and SD_A are parametrized by the vector form factor F_V and the two axial-vector form factors F_A and R. Another convention, which is frequently used in the literature, is $\gamma = F_A/F_V$ and $\xi = R/F_V$. One point to notice is that in the case of a real photon, as in $\pi^+ \to e^+\nu\gamma$, the term with the coefficient R vanishes. The q^2 and k^2–dependences of the form factors have been neglected on the level of less than about 3%.

The CVC hypothesis connects F_V with the π^0 lifetime and gives a firm prediction of $|F_V| = 0.0255$. The sign of F_V has as of now not yet been determined. The PCAC hypothesis relates R with the electromagnetic radius of the pion $R = (m_\pi/3) f_\pi < r_\pi^2 >$. With $< r_\pi^2 >= 0.45 fm^2$, this predicts a value of $\xi = (R/F_V) = 2.7$. The calculation of F_A is model dependent. Relativistic quark models, using a calculated $< r_\pi^2 >$ value, obtain[3] a $\gamma = \xi/2 = 1$. Replacing $< r_\pi^2 >$ by the measured number, the value of γ increases somewhat to 1.4. Very small values for γ are being predicted by a modified bag model[4] as well as by a nonperturbative QCD–calculation.[5] A recent calculation using a chiral-symmetric Lagrangian[6] yields a value of $\gamma = 0.43$.

3. EXPERIMENTAL SITUATION

Up to now experimental information on γ has been collected from the decay $\pi^+ \to e^+\nu\gamma$. In this process the structure dependent contribution can be separated into a term (SD^+) which is proportional to $(1+\gamma)^2$ and a term $(SD^-) \sim (1-\gamma)^2$. Experimental problems arise in the attempt to measure (SD^-), since in the kinematically accessible regions the rate is dominated by either SD^+, IB or accidental coincidences. Consequently these experiments are sensitive to SD^+ and therefore to $(1+\gamma)^2$, resulting in two solutions for the value of γ:[7] a positive value around 0.5 and a negative value around −2.4 (see Table 1). Although the negative value cannot be completely excluded, the more recent experiments (see Ref.[7]) prefer the positive value to be correct.

Table 1. Previous experimental values of γ

γ_1	γ_2	References[7]
0.26±0.5	−1.98±0.7	Depommier 63
0.50±0.12	−2.43±0.12 [a]	Stetz 78
0.40±0.07 [a]	−2.36±0.07	Perroud 84
0.53±0.05 [a]	−2.49±0.05	Ruegger 85
0.25±0.12 [a]	(−2.13±0.15)	Piilonen 86

[a]indicates the preferred value.

4. THE MEASUREMENT

In our experiment done at SIN, positive pions of 95 MeV/c momentum were degraded and stopped at a rate of $1.5 \times 10^6 s^{-1}$ in a cone shaped target made of twelve 1 mm thick plastic scintillators. Two additional scintillators served as beam monitors. Charged particles were measured with the magnetic spectrometer **SINDRUM**. Its main components are a normal conducting magnet, producing a field of 0.33 T in a volume of 75 cm ø × 110 cm, and five cylindrical multiwire proportional chambers. They are concentrically arranged around the target and surrounded by a hodoscope, made of 48 scintillator strips. The apparatus is described in detail in Ref. 8. A total of $(3.8 \pm 0.7) \times 10^{12} \pi^+$ were stopped in the target. An example of an $\pi^+ \to e^+e^-e^+\nu$ candidate event is shown in Fig. 1 in the $r - \phi$ and in the $r - z$ projection.

5. DATA REDUCTION AND ANALYSIS

The online and offline data reduction was done similarly to the previous muon decay experiments [8] with the additional requirement of a π stop signal in the target counter where the decay vertex was located. That means that we already online required the identification of an $e^+e^-e^+$ triple within a time window of 12 ns, having a total $p_{transverse} \leq 25$ MeV/c.

There were two serious sources of background events: One source is e^+e^- pairs in accidental coincidence with a stopping π^+ and a Michel e^+. These pairs mainly originated from $\mu^+ \to e^+\nu\bar{\nu}$ decays followed by Bhabha scattering of the e^+ in the target: $e^+e^- \to e^+e^-$. The other source is prompt $\pi^+ \to e^+\nu\gamma$ decays, where the γ converted externally in the target $\gamma \to e^+e^-$. Both types were strongly suppressed by requiring the opening angle of the e^+e^- pairs (θ) to be at least 18°. Events with $18° \leq \theta \leq 32°$ were also discarded if $(E_{e^+} + E_{e^-})$ or $E_{e_2^+} \leq 56$ MeV/c². The mass spectra of the prompt and accidental events before and after these cuts are shown in Fig. 2. Note that the shape of the π-mass peak, containing 79 events, agrees very well with the Monte Carlo prediction. These events yield a lifetime of $\tau_\pi = (26 \pm 4)$ ns.

To extract the form factor values, these 79 events were analyzed applying the maximum likelihood method. Since only one background event is expected within the π-mass peak the likelihood function was defined as:

$$\mathcal{L}(F_V, F_A, R) = \prod_{i=1}^{79} p_i(F_V, F_A, R)$$

$$p_i(F_V, F_A, R) = \frac{|M(F_V, F_A, R, \mathrm{x}_i)|^2}{\int |M(F_V, F_A, R, \mathrm{x})|^2 d\mathrm{x}}$$

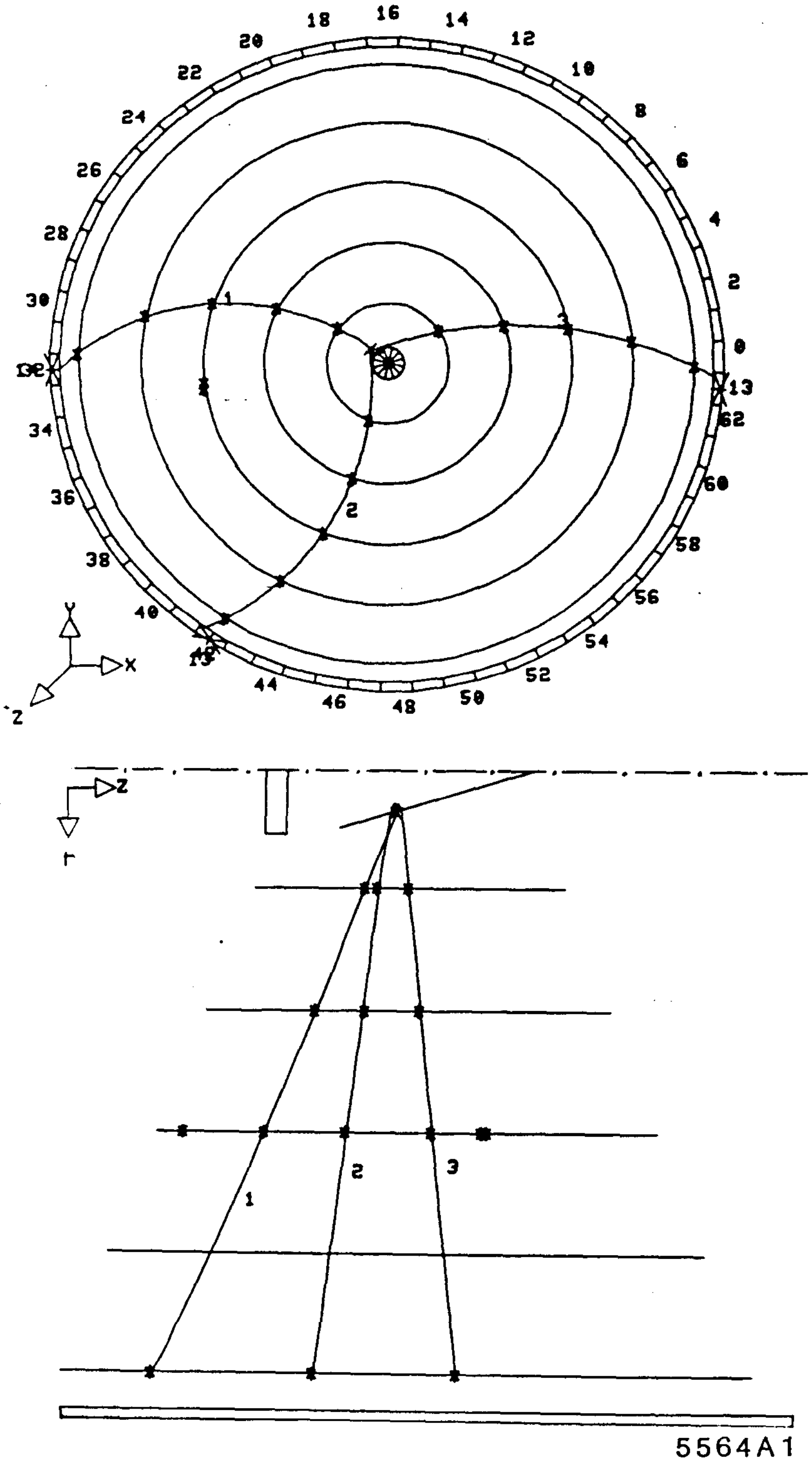

Fig. 1. A candidate $\pi^+ \rightarrow e^+e^-e^+\nu$ event, shown in the $r-\phi$ (top) and in the $r-z$ (bottom) projection. The lines indicate a fit to the three charged particle tracks.

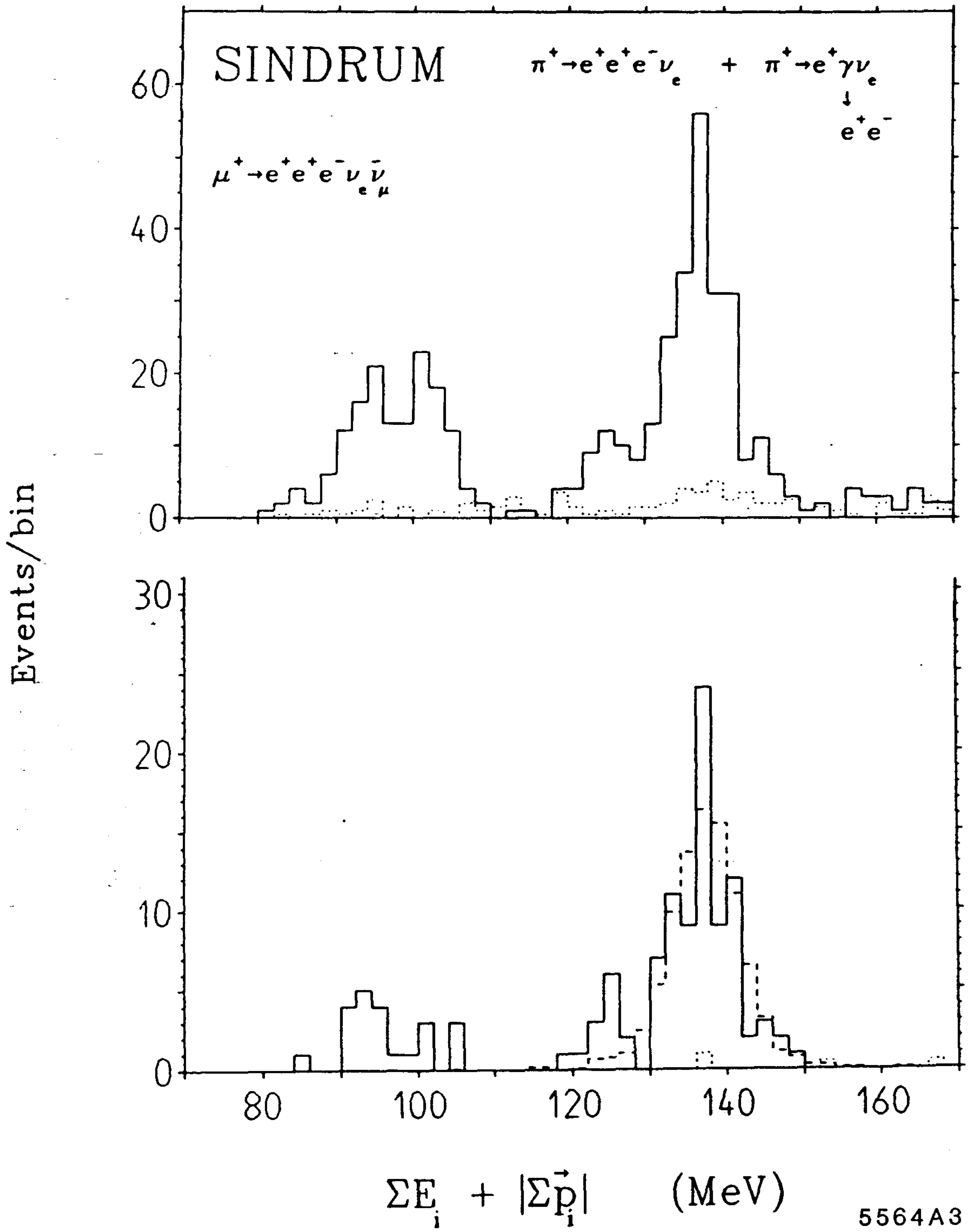

Fig. 2. Distribution of $\Sigma E_i + \Sigma|\mathbf{p}_i c|$ before and after the cut on the e^+e^- opening angle θ for prompt (solid line) and accidental (dotted line) $e^+e^-e^+$ events. The dashed line in (b) represents the Monte-Carlo generated $\pi^+ \to e^+e^-e^+\nu$ events.

where p_i is the probability density of observing the event i in the five dimensional phase space $\mathbf{x}_i$ as a function of F_V, F_A and R. M represents the $\pi^+ \to e^+e^-e^+\nu$ decay matrix element.

6. RESULTS AND DISCUSSION

The likelihood function $\mathcal{L}$ reached its maximum for the following values:

$$F_V = 0.029 \begin{Bmatrix} -0.014 \\ +0.019 \end{Bmatrix} \quad ;$$

$$F_A = 0.018 \begin{Bmatrix} -0.012 \\ +0.015 \end{Bmatrix} \quad ;$$

$$R = 0.063 \begin{Bmatrix} -0.016 \\ +0.026 \end{Bmatrix} \quad .$$

This directly determines the sign of F_V to be positive (sign of f_π is positive by convention) and its value to be in agreement with the CVC–prediction. In a second step F_V was held at the CVC–value of 0.0255, yielding a likelihood function $\mathcal{L}(\gamma, \xi)$ as shown in Fig. 3. The resulting form factor values then are :

$$\gamma = F_A/F_V = 0.7 \pm 0.5; \quad \xi = R/F_V = 2.3 \pm 0.6;$$

γ clearly agrees with the small positive values and thereby excludes the large negative values obtained in the $\pi^+ \to e^+\nu\gamma$ experiments.[7] The value of ξ confirms the PCAC prediction. It should be emphasized again, that the extraction of these values did *not* depend on the absolute number of stopped π's (N_π).

Inserting the observed number of π's, the experimental efficiency and N_π, the branching ratio for $\pi^+ \to e^+e^-e^+\nu$ was determined to be :

$$B\left(\frac{\pi^+ \to e^+e^-e^+\nu}{\pi^+ \to \mu^+\nu}\right) = (3.4 \pm 0.5) \times 10^{-9}$$

This value has been measured for the first time and is just slightly smaller than the previously established upper limit.[1]

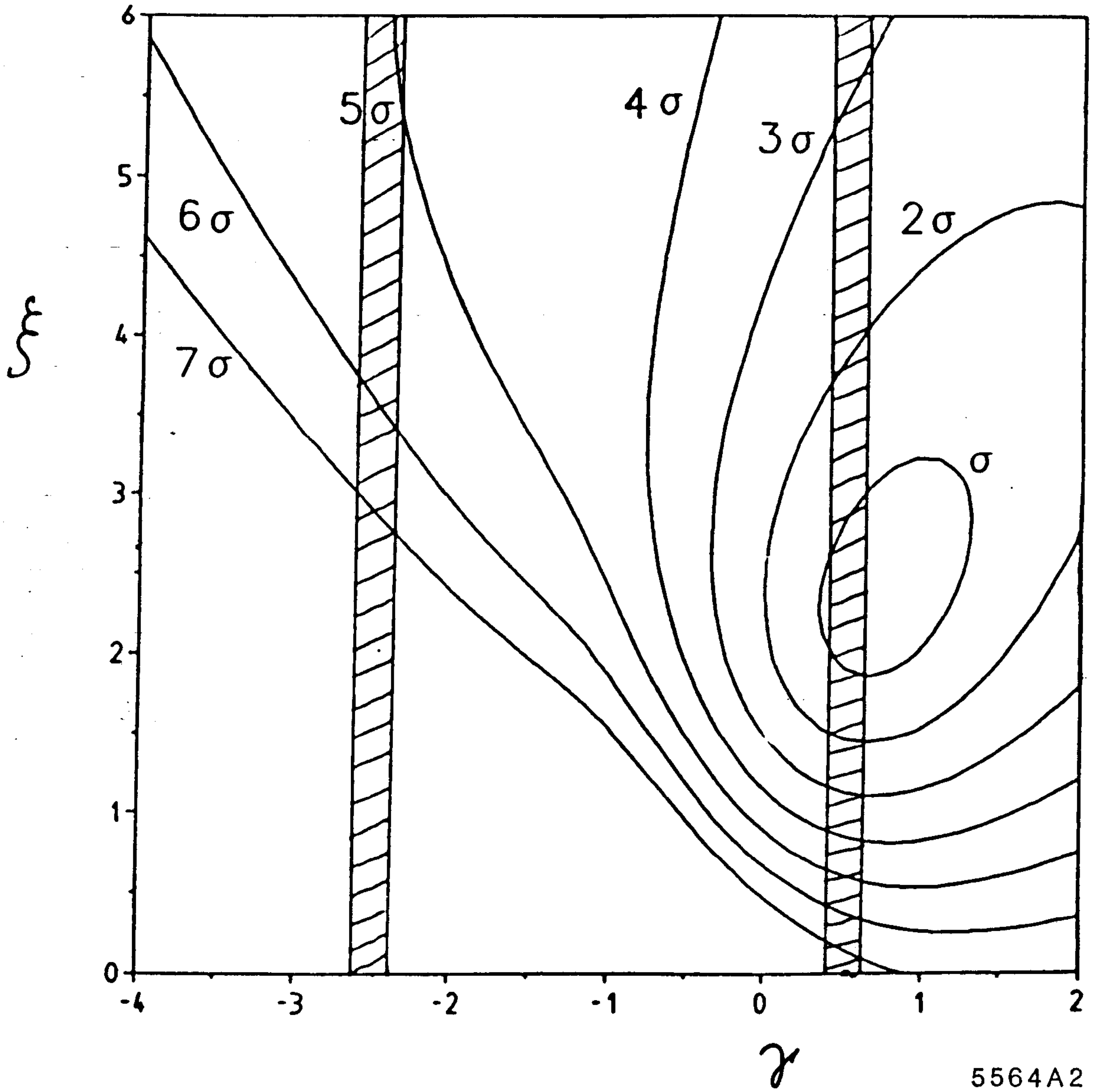

Fig. 3. Contour plot of the likelihood function $\mathcal{L}(\gamma, \xi)$. Indicated are the probability contours corresponding to up to seven standard deviations. The hashed bands mark the 2σ probability regions for γ from the $\pi^+ \to e^+\nu\gamma$ experiment Rüegger *et al.*[7]

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