SLAC-PUB-4519
January 1988
T/E

Chiral symmetry and the spin of the proton*

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ABSTRACT

Recent EMC data on the spin-dependent proton structure function suggest that very little of the proton spin is due to the helicity of its constituent quarks. We argue that, at leading order in the $1/N_c$ expansion, none of the proton spin would be carried by quarks in the chiral limit where $m_q = 0$. This result is derived in the Skyrme model, which is also used to estimate quark contribution to the proton spin when chiral symmetry and $SU(3)$ are broken: this contribution turns out to be small. Therefore, even in the real world most of the proton spin is due to gluons and/or orbital angular momentum, as suggested by the EMC. We mention other experiments to test this suggestion.

Submitted to *Physics Letters B*

⋆ Work supported by the Department of Energy, contract DE–AC03–76SF00515.
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There are two approximate descriptions of hadrons which have met with considerable success: chiral symmetry and the naive non-relativistic quark model (NQM). Chiral symmetry[1] is solidly based on the known symmetries of QCD and describes qualitatively the properties and low energy interactions of pseudoscalar mesons. The non-relativistic quark model[2] for heavy quarks is soundly based on QCD, but it is also apparently successful at predicting the properties and couplings of states containing light quarks, notably baryons. These can, however, also be described in the Skyrme model,[3] where the baryons appear as classical soliton solutions in an effective chiral Lagrangian for large N_c QCD.

Recently, the European Muon Collaboration (EMC) published[4] measurements of the proton's polarized electroproduction structure function $g_1^p(x)$ which overlap those previously available[5] and extend them to smaller values of $x \equiv -q^2/2p \cdot q$. On the basis of these measurements, the EMC quotes

$$\int_0^1 dx g_1^p(x, \langle -q^2 \rangle = 10.7 \text{ GeV}^2) = 0.114 \pm 0.012 \pm 0.026 \quad . \tag{1}$$

This value can be compared with an old prediction[6] from the NQM

$$\int_0^1 dx g_1^p(x) = 0.19 \tag{2}$$

which was based on flavor $SU(3)$ and on the assumption that the axial current matrix element $\langle p| \bar{s}\gamma_\mu\gamma_5 s |p\rangle \equiv \Delta s \cdot \Sigma_\mu(p) = 0$.‡ This assumption has the physical meaning that the $\bar{s}s$ pairs in a polarized proton carry no net spin, and is automatic in the NQM, which does not even contain a sea of $\bar{q}q$ pairs. The seeming contradiction between experiment (1) and theory (2) was not apparent in the earlier data[5] on $g_1^p(x)$ because of uncertainties at low values of x.

‡ Here $\Sigma_\mu(p)$ is the proton spin, and we will define Δq ($q = u, d$) analogously to Δs.

As we review below, the EMC data suggest[7] that the flavor-singlet axial current matrix element

$$\begin{aligned} \langle p| A^0_\mu |p\rangle &\equiv \sqrt{2/3}\, \langle p| \left(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s\right) |p\rangle \\ &= \sqrt{2/3}(\Delta u + \Delta d + \Delta s)\cdot\Sigma_\mu(p) \simeq 0 \end{aligned} \tag{3}$$

indicating[7] that (within errors) none of the proton spin is carried by the helicities of its constituent quarks. Note that A^0_μ is renormalized, since it has a 2-loop anomalous dimension[8], related to the axial anomaly[9]. The multiplicative nature of the renormalization means that if $\Delta u + \Delta d + \Delta s = 0$ at some renormalization scale q, it follows that $\Delta u + \Delta d + \Delta s = 0$ at all q. It has been suggested[10] that $\langle p| A^0_\mu |p\rangle$ may be non-zero, as predicted by the NQM, at small q and evolve to become small at large q. In the following we argue that this is not the case.

The purpose of this paper is to show that approximate chiral symmetry combined with the $1/N_c$ expansion favors the result (3) at all values of q, predicting a value of $\int_0^1 dx g_1^p(x)$ which is different from the NQM value (2), but in good agreement with the EMC result (1). The Skyrme model[3] is an example of a chiral model that successfully predicts (3). We first discuss the behavior of $g_1^p(x)$ at small x, and give a heuristic argument why $\langle p| \bar{s}\gamma_\mu\gamma_5 s |p\rangle < 0$ in any chiral model where the proton has a surrounding cloud of pseudoscalar mesons. Then we use large N_c arguments in the chiral limit ($m_{u,d,s} \to 0$) to argue that $\Delta u + \Delta d + \Delta s \simeq 0$ in a world with massless quarks. Thus in this limit all the spin of the proton would be carried by gluons and/or orbital angular momentum. The result (3) is derived explicitly in the Skyrme model.[3] We also use the Skyrme model as a tool to predict what happens in the real world with massive quarks. We show that if the η and η' mesons only have orthogonal mass mixing, then (3) remains true, whilst a simple *Ansatz* for kinetic mixing, introduced[11] to fit the correct ratio of pseudoscalar decay constants f_K/f_π, results in $\Delta u + \Delta d + \Delta s = -0.18$, which is also consistent with the EMC experiment. We conclude by mentioning experiments to test whether the spin of the proton is in fact carried by gluons and/or by orbital angular momentum.

We first discuss the interpretation of the EMC data.[4] As was already mentioned, they agree with previous measurements[5] where their values of x overlap. At smaller values of x, the EMC data look as if $g_1^p(x) \sim x^\delta$, where $\delta \sim 0.1$, as expected on the basis of Regge arguments.[12] According to the standard lore, $g_1^p(x) \sim x^{-\alpha_i(0)}$ as $x \to 0$ and should be dominated by the $a_1(1270)$, $f_1(1285)$ and $f_1(1420)$ trajectories. It has been suggested[12] that $\alpha_{a_1}(0) \simeq -0.14$ ($\simeq \alpha_{f_1}(0)$?), whereas if these trajectories have the same slopes as the well-studied ρ, ω and ϕ trajectories, their intercepts $\alpha(0) \sim -\frac{1}{2}$, yielding $g_1^p(x) \sim x^{\frac{1}{2}}$. The arguments[13] for Regge behavior of the leading-twist structure functions are not rigorous, and require an interchange of limits. However, data[14] on $F_3(x)$ are consistent with the Regge prediction, as is the trend of the EMC data.[4] Therefore we see no reason to doubt the extrapolation of $g_1^p(x)$ to $x = 0$ used by the EMC, and we accept their value for the sum rule (1). Standard operator product expansion arguments[15] tell us that (1) amounts to a measurement of

$$\begin{aligned} \langle p| \sum_q Q_q^2 \bar{q}\gamma_\mu\gamma_5 q\,|p\rangle &= \langle p|\left[\frac{4}{9}\bar{u}\gamma_\mu\gamma_5 u + \frac{1}{9}\left(\bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s\right)\right]|p\rangle \\ &= \left(\frac{4}{9}\Delta u + \frac{1}{9}\Delta d + \frac{1}{9}\Delta s\right)\cdot \Sigma_\mu(p) \end{aligned} \tag{4}$$

namely that

$$\frac{1}{2}\left(\frac{4}{9}\Delta u + \frac{1}{9}\Delta d + \frac{1}{9}\Delta s\right) = 0.114 \pm 0.012 \pm 0.026 \tag{5}$$

Two other combinations of the Δq can be inferred from other data: neutron decay and flavor $SU(2)$ tell us that

$$\Delta u - \Delta d = g_A = 1.25 \tag{6}$$

whilst hyperon decay[16] and flavor $SU(3)$ tell us that

$$\frac{1}{\sqrt{3}}\Big(\Delta u + \Delta d - 2\Delta s\Big) = 0.39 \tag{7}$$

Now one can solve the three simultaneous equations [(4),(6),(7)] to determine independently the different Δq.[7] Incorporating perturbative QCD corrections[17,18]

$$\Delta u = 0.74 \pm 0.08, \qquad \Delta d = -0.51 \pm 0.08, \qquad \Delta s = -0.23 \pm 0.08 \tag{8}$$

It is striking that $\Delta s < 0^\star$ in contrast to the NQM expectation $\Delta s = 0$,† and that[17]

$$\Delta u + \Delta d + \Delta s = 0.00 \pm 0.24 \tag{9}$$

We will see shortly how these values may be understood in the context of chiral symmetry. Since in the quark-parton model the Δq are interpreted as the quark contributions to the proton spin, i.e. its helicity in the infinite momentum frame (at equal light-cone time):

$$\Delta q = \int_0^1 dx \Big[q_\uparrow(x) + \bar{q}_\uparrow(x) - q_\downarrow(x) - \bar{q}_\downarrow(x)\Big] \tag{10}$$

the EMC result (1) via the determination (8) tells us that the net $\bar{s}s$ helicity is anticorrelated with the proton spin in such a way that the total net contribution of the quarks to the proton spin is consistent (9) with zero. The full proton spin must be made up by gluons and orbital angular momentum: $\frac{1}{2}\sum_q \Delta q + \Delta G + \langle L_z\rangle = \frac{1}{2}$.

$\star$ We have checked that the constraints of Regge behavior at small x and the observed size of the $\bar{s}s$ sea do not exclude $|\Delta s|$ as large as in Eq. (8). If one combines[4] the EMC data with the previous data [5] on $g_1^p(x)$, the values of the Δq are essentially unchanged, whilst the errors are decreased by factors ~ 2.

† The alternative estimate in Ref. 4, which assumes $\Delta s = 0$, conflicts with the confirmed success of flavor $SU(3)$ in describing hyperon decays[16], and should be disregarded.

Should it have surprised us that $\Delta s < 0$? Analyses of low-energy meson-nucleon scattering and σ-terms have previously been used[19] to argue that

$$\frac{\langle p|\,\bar{s}s\,|p\rangle}{\langle p|\,(\bar{u}u+\bar{d}d+\bar{s}s)\,|p\rangle} \simeq 0.21 \tag{11}$$

implying that the proton does contain a substantial $\bar{s}s$ sea, unlike the NQM. In fact, the ratio (11) is reproduced[19] in the Skyrme model,[3] which yields

$$\frac{\langle p|\,\bar{s}s\,|p\rangle}{\langle p|\,(\bar{u}u+\bar{d}d+\bar{s}s)\,|p\rangle} = \frac{7}{30} \approx 0.23 \tag{12}$$

Since $\langle p|\,\bar{u}u\,|p\rangle$ and $\langle p|\,\bar{d}d\,|p\rangle$ are both $\mathcal{O}(N_c)$, equation (12) implies that $\langle p|\,\bar{s}s\,|p\rangle = \mathcal{O}(N_c)$ also. Given that the proton contains a large $\bar{s}s$ sea,‡ is this sea likely to be polarized? The following heuristic argument may serve as an intuitive mnemonic that $\Delta s < 0$ in the chiral limit. Let us think about the proton wave function starting with the 3-quark Fock-space component $|uud\rangle$, with $\Delta u > 0$ and $\Delta d < 0$ but $\Delta u + \Delta d > 0$, as suggested by the NQM.§ Let us now add to the wave-function a component containing a $\bar{q}q$ pair: $|uud\bar{q}q\rangle$, and further assume that at least some of the time the $\bar{q}$ will pair with one of the valence u or d quarks to form a pseudoscalar meson. (The pseudoscalars are massless in the chiral limit, and many phenomenological models postulate meson clouds around nucleons.) If we focus on the $|uud\bar{s}s\rangle$ component of the proton, the $\bar{s}$ may combine with u or d to form a K^+ or K^0 meson. Since the quark and antiquark spins in pseudoscalar mesons are anticorrelated, the fact that $\Delta u + \Delta d > 0$ means that the $\bar{s}$ must make a net negative contribution to Δs. There is no obvious reason why this contribution should be exactly cancelled by the s quark, so we are left with the suggestion that $\Delta s < 0$ as observed (8). We stress here that this argument should be thought of as a useful mnemonic only, indicating why $\Delta s = 0$ is not sacred.

‡ Though see ref. 20 for a critique of this conclusion.

§ The data for $g_1^p(x)$ indicate a strong positive correlation at large x, where valence quarks dominate. This agrees qualitatively with the expectation[21] that $\Delta u > 0$ in the valence uud Fock-space component of the proton.

One can also argue directly that $\Delta s < 0$ at leading order in the $1/N_c$ expansion, by considering the N_c dependences of $\langle p| A^0_\mu |p\rangle \propto \Delta u + \Delta d + \Delta s$ and of $\langle p| A^{3,8}_\mu |p\rangle \propto \Delta u - \Delta d, \quad \Delta u + \Delta d - 2\Delta s$. It is well known[3] that $g_A \propto \langle p| A^3_\mu |p\rangle \sim \mathcal{O}(N_c)$. It is also known[22,11] that in the Skyrme model the ratio F/D is a pure number, 5/9, independent of N_c. Using this and the fact that $g_A = \Delta u - \Delta d = F + D$ one can deduce that the combination inferred from hyperon decay, namely $\Delta u + \Delta d - 2\Delta s = 3F - D$, must also scale like N_c. On the other hand, both in the large-N_c quark model and in the Skyrme model (to be discussed later) $\langle p| A^0_\mu |p\rangle \propto \Delta u + \Delta d + \Delta s \leq \mathcal{O}(N_c^0)$, This means that Δu, Δd and Δs are all $\mathcal{O}(N_c)$ and that $\Delta s < 0$, in contrast to the prediction based on the NQM. We will next go one step further, and argue that $\langle p| A^0_\mu |p\rangle \propto \Delta u + \Delta d + \Delta s = \mathcal{O}(1/N_c)$, in the limit of chiral symmetry.

We use the Skyrme model,[3] which provides an accurate description of baryons to leading order in N_c, the number of colors. The baryons are solitons of the effective chiral Lagrangian

$$\begin{aligned}\mathcal{L} =& \frac{f_\phi^2}{16}\left\{\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) - \frac{8v}{f_\phi^2}\mathrm{Tr}\left(m_q(U + U^\dagger - 2)\right) - \frac{A}{N_c}(-i\ln\det U)^2\right\} \\ &+ \frac{1}{32e^2}\mathrm{Tr}\left(\left[U_{\mu R}, U_{\nu R}\right]\left[U^\mu_R, U^\nu_R\right]\right) + N_c\mathcal{L}_{WZ}\end{aligned} \tag{13}$$

where $U_{\mu L} \equiv U^\dagger \partial_\mu U$, $U_{\mu R} \equiv \partial_\mu U U^\dagger$ and the 3×3 unitary matrix U may be parametrized as

$$U(x) = \exp\left[\left(\frac{2i}{f_\phi}\right)\sum_{i=0}^{8}\lambda_i\phi_i(x)\right] \tag{14}$$

where $\{\phi_i \equiv \eta_0,\ \underset{\sim}{\pi},\ K_a,\ \eta_8\}$, f_ϕ is the pseudoscalar nonet decay constant, $\lambda_0 \equiv \sqrt{2/3}\,diag(1,1,1)$, $m_q = diag\,(m_u, m_d, m_s)$ is the quark mass matrix, $v = -\langle 0|\,\bar{u}u\,|0\rangle = -\langle 0|\,\bar{d}d\,|0\rangle = -\langle 0|\,\bar{s}s\,|0\rangle$, A is the non-perturbative quantity[23] that appears at leading non-trivial order in the $1/N_c$ expansion, e is determined by fitting static properties of the baryons,[3] and the Wess-Zumino part of the

Lagrangian is given in refs. 3, 11. Note that in (13) the flavor-singlet η_0 decouples from the other mesons, a result which is only valid to leading order in $1/N_c$.

Noether's theorem applied to the Lagrangian (13) yields the chiral $U(3)$ currents:

$$J_\mu^{iR} = i\frac{f_\phi^2}{8}\mathrm{Tr}\left(\frac{\lambda_i}{2}U_{\nu R}\right) - \frac{i}{8e^2}\mathrm{Tr}\left(\left[\frac{\lambda_i}{2}, U_{\nu R}\right]\left[U_{\mu R}, U_{\nu R}\right]\right) \\ + (\mathrm{WZ\ term}) \tag{15}$$

and similarly for J_μ^{iL}. We see from equation (15) that the axial currents $A^i\mu \equiv J^{iL}\mu - J_\mu^{iR} = -(f_\phi/2)\partial_\mu\pi_i + \ldots$ so that their matrix elements are dominated by pseudoscalar meson poles at leading order in the $1/N_c$ expansion. Initially, we set $m_q = 0$ in (13), so as to be in the chiral limit where $m_{\phi_i} = 0$ $(i = 1, 2, \ldots, 8)$ but $m_{\eta_0} = \sqrt{A/N_c}$. In this limit, the baryons B are solitons,[3]

$$U(\mathbf{x}) = VU_0V^\dagger \tag{16}$$

where $U_0 = \exp\left[2i\boldsymbol{\tau}\cdot\mathbf{x}F(|\mathbf{x}|)\right]$ takes values in an $SU(2)$ subgroup of $SU(3)$, and V is an $SU(3)$ rotation matrix.[3,11] Axial current matrix elements at zero momentum transfer $\langle B|\, A_\mu^a\,|B\rangle$ can be obtained from an explicit expression for the space integral of the axial current[3,11]

$$\int d^3x A_i^a \propto \mathrm{Tr}[\lambda_i V^{-1}\lambda_a V] \tag{17}$$

For the ninth current the λ_a matrix is replaced by the identity matrix λ_0 in the right-hand side of Eq. (17), which therefore vanishes identically. The decoupling of the ninth axial current can also be seen through the use of generalized Goldberger-Treiman relations for the pseudoscalar meson couplings $g_{\phi_i BB}$, which are proportional to the leading non-trivial terms in $\hat{U}(\mathbf{x}) \equiv U(\mathbf{x}) - \mathbf{1}$ at large distances in the nucleon tail:[3]

$$g_{\phi_i BB} \propto \mathrm{Tr}(\lambda_i\hat{U}(\mathbf{x})) = \mathrm{Tr}(\lambda_i V\hat{U}_0V^\dagger) \tag{18}$$

It is clear from equation (18) that $g_{\eta_0 BB} = 0$, and hence $\langle B|\, A_\mu^0\,|B\rangle = 0$, because

$\mathrm{Tr}(V\hat{U}_0 V^\dagger) = \mathrm{Tr}(\hat{U}_0) = 0$. This zero was to be expected, because the baryonic soliton's existence reflects the fact that $\Pi_3\,(SU(2)) = \mathbb{Z}$, whereas $\Pi_3\,(U(1)) = 0$. Therefore, the soliton contains $\phi_i\,(i = 1, \ldots 8)$ components. The ninth pseudoscalar η_0, corresponding to the $U(1)$ in $U(3)/SU(3)$, has no non-trivial soliton configuration. Moreover, since the η_0 decouples from the other mesons at leading order in $1/N_c$, the baryonic soliton contains no non-trivial η_0 component. However, at next order in $1/N_c$, one should add to the Lagrangian (13) terms which couple the η_0 to the other mesons, and the modified soliton solution may then generate $\eta_0(\mathbf{x}) \neq 0$ and hence $g_{\eta_0 BB} \neq 0$ at next order in $1/N_c$.

Thus the Skyrme model indicates that $\langle p|\, A^0_\mu\, |p\rangle = \mathcal{O}(1/N_c)$ in a chirally symmetric world. This and the multiplicative nature of the renormalization of A^0_μ means that $\langle p|\, A^0_\mu\, |p\rangle$ is small at all renormalization scales q. The same result holds for the matrix element of A^0_μ in any other baryon, and in nuclei.

We now explore the effects of chiral symmetry breaking in the chiral Lagrangian (13) which appear when $m_q \neq 0$. As we will see later, $\langle p|\, A^0_\mu\, |p\rangle$ still vanishes at leading order in $1/N_c$ unless we also introduce $SU(3)$ symmetry breaking in the kinetic terms for the pseudoscalars, which are anyway required to accommodate the ratio $f_K/f_\pi \neq 1$ of the K and π decay constants.* Neglecting isospin violation, first order $SU(3)$ breaking in the kinetic terms may be parametrized by[11]

$$\Delta\mathcal{L}_K = \epsilon\frac{f_\phi^2}{16}\,\mathrm{Tr}\left[\frac{\lambda_8}{2}\left(U^\dagger U_{\mu L} U^{\mu L} + U^\dagger U_{\mu R} U^{\mu R}\right)\right] \tag{19}$$

Expanding (19), we obtain the bilinear terms

$$\begin{aligned}
-\frac{\epsilon}{2}\Bigg[&\frac{1}{\sqrt{3}}\partial_\mu\pi_i\partial^\mu\pi^i - \frac{1}{2\sqrt{3}}\partial_\mu K_a\partial^\mu K_a - \frac{1}{\sqrt{3}}\partial_\mu\eta_8\partial^\mu\eta_8 \\
&+ \sqrt{\frac{2}{3}}\left(\partial_\mu\eta_0\partial^\mu\eta_8 + \partial_\mu\eta_8\partial^\mu\eta_0\right)\Bigg]
\end{aligned} \tag{20}$$

$\star$ Effects of higher order in $1/N_c$, including chiral loops,[24] could also give $\langle p|\, A^0_\mu\, |p\rangle \neq 0$. We are not able to estimate these, since the treatment[23] of the η' that we use, and the Skyrme model, give an accurate description of the physics to leading order in $1/N_c$ only.

so that the total kinetic terms are

$$\frac{1}{2}(1-\epsilon\sqrt{3})\partial_\mu\pi_i\partial^\mu\pi_i+\frac{1}{2}(1+\frac{\epsilon}{2\sqrt{3}})\partial_\mu K_a\partial^\mu K_a$$

$$+\tfrac{1}{2}\left(\,\partial_\mu\eta_8\,,\quad \partial_\mu\eta_0\,\right)\begin{pmatrix}1+\frac{\epsilon}{\sqrt{3}} & -\sqrt{\frac{2}{3}}\epsilon\\ -\sqrt{\frac{2}{3}}\epsilon & 1\end{pmatrix}\begin{pmatrix}\partial_\mu\eta_8\\ \partial_\mu\eta_0\end{pmatrix} \tag{21}$$

Making field redefinitions to leading order in ϵ we find

$$\begin{aligned} f_\pi &= f_\phi\left(1-\frac{\epsilon}{2\sqrt{3}}\right);\\ f_K &= f_\phi\left(1+\frac{\epsilon}{4\sqrt{3}}\right). \end{aligned} \tag{22}$$

and hence to leading order in ϵ

$$\frac{f_K}{f_\pi}=1+\frac{\sqrt{3}}{4}\epsilon\,. \tag{23}$$

Taking $f_K/f_\pi \simeq 1.2$ from experiment we obtain

$$\epsilon \simeq 0.46 \tag{24}$$

Making a non-orthogonal transformation, the $(\partial_\mu\eta_0, \partial_\mu\eta_8)$ terms in (21) can be diagonalized to become

$$=\frac{1}{2}(\partial_\mu\eta\partial^\mu\eta+\partial_\mu\eta'\partial^\mu\eta') \tag{25}$$

where

$$\eta \equiv (1+\epsilon/2\sqrt{3})\eta_8 + a\epsilon\eta_0, \qquad \eta' = b\epsilon\eta_8+\eta_0\,. \tag{26}$$

From the diagonal terms we obtain the decay constants,

$$f_\eta = f_\phi(1+\epsilon/2\sqrt{3}), \qquad f_{\eta'}=f_\phi\,; \tag{27}$$

while the mixing term yields

$$a + b = -\sqrt{2/3}. \tag{28}$$

There is a residual ambiguity in the choice of (a, b) which corresponds to the freedom of performing an orthogonal transformation on Eq.(25). That ambiguity is removed by considering the mass matrix:

$$\begin{aligned} (\,\eta_8\,, \quad \eta_0\,) \begin{pmatrix} \frac{4}{3}m_K^2 & -\frac{2\sqrt{2}}{3}m_K^2 \\ -\frac{2\sqrt{2}}{3}m_K^2 & \frac{2}{3}m_K^2 + A/N_c \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix} \\ = m_\eta^2 \eta^2 + m_{\eta'}^2 {\eta'}^2 \end{aligned} \tag{29}$$

where we have neglected terms $\mathcal{O}(m_\pi^2/m_K^2)$ and $\mathcal{O}(\epsilon^2)$. Diagonalization of (29) yields

$$m_\eta^2 = \frac{\frac{4}{3}m_K^2 - m_{\eta'}^2 b^2 \epsilon^2}{1 + \epsilon/\sqrt{3}} \tag{30}$$

and

$$b\epsilon = -\frac{2\sqrt{2}}{3}\frac{m_K^2}{m_{\eta'}^2} = -0.25 \tag{31}$$

Thus we have $b = -0.55$, $a = -0.27$, and the $\eta - \eta'$ mixing angle $\theta = 14^0$, which is qualitatively consistent with phenomenological analyses.[25]

Armed with these results, we now estimate the coupling of the isoscalar axial current to the proton. To do this, we calculate the divergence of the corresponding matrix element in the proton state:

$$\langle p|\, \partial^\mu A_\mu^0 \,|p\rangle \simeq \sum_X \langle 0|\, \partial^\mu A_\mu^0 \,|X\rangle \, \frac{1}{m_X^2}\, g_{Xpp} \tag{32}$$

where X runs over all particles that can be created from the vacuum by $\partial^\mu A_\mu^0$ and g_{Xpp} is the X-proton-proton coupling. In the chiral limit, the only state created by

$\partial_\mu A^0_\mu$ is η_0, for which however $g_{\eta_0 pp} = 0$, and therefore in that limit $\langle p| A^0_\mu |p\rangle = 0$. When chiral symmetry and flavour $SU(3)$ are broken, the η contribution to the sum (32) is given by $\langle 0| \partial^\mu A^0_\mu |\eta\rangle = im^2_\eta (a\epsilon) f_\phi$ and $g_{\eta pp} = g_{\eta_8 pp}$, whilst the η' contribution is given by $\langle 0| \partial^\mu A^0_\mu |\eta'\rangle = im^2_{\eta'} f_\phi$ and $g_{\eta' pp} = (b\epsilon) g_{\eta_8 pp}$ to leading order in ϵ.* Hence

$$\frac{\langle p| A^0_\mu |p\rangle}{\langle p| A^8_\mu |p\rangle} = \frac{\langle p| \partial^\mu A^0_\mu |p\rangle}{\langle p| \partial^\mu A^8_\mu |p\rangle} = (a+b)\epsilon = -0.38 \tag{33}$$

to leading order in ϵ. This argument is more general than the Skyrme model, and also applies to any other model for baryons in which $g_{\eta_0 pp} = 0$ to order ϵ^0. Note that if there were no kinetic mixing for the η and η', so that they were related to η_0 and η_8 by a simple orthogonal rotation, we would have $a = -b$ and hence $\langle p| A^0_\mu |p\rangle = 0$,† as in the chiral limit. The prediction (33) can be compared with the NQM prediction[6]

$$\frac{\langle p| A^0_\mu |p\rangle}{\langle p| A^8_\mu |p\rangle} \equiv \frac{\sqrt{\frac{2}{3}}(\Delta u + \Delta d + \Delta s)}{\frac{1}{\sqrt{3}}(\Delta u + \Delta d - 2\Delta s)} = \sqrt{2} \tag{34}$$

Eq. (33), when combined with Eqs. (6) and (7), corresponds to the prediction that to leading order in N_c the quarks carry a fraction

$$\Delta u + \Delta d + \Delta s = -0.18 \tag{35}$$

of the proton spin, which is compatible with the EMC value (9).[4]

Clearly it is of great importance to confirm the EMC result (1) for $\int_0^1 dx g_1^p(x)$, and to measure also $\int_0^1 dx g_1^n(x)$ using polarized neutrons, so as to check the Bjorken

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$\star$ In this framework $g_{\eta' pp} = 0$ in the chiral limit, which becomes $|g_{\eta' pp}/g_{\eta pp}| = 0.25$ with realistic $m_q \neq 0$. Current determinations[26] of the pseudoscalar baryon are consistent with $g_{\eta' pp}$ being small, as predicted.

† This is a GIM-like cancellation.

sum rule.[15] The theoretical interest in new experiments to measure these quantities is enhanced by the fundamental information about chiral symmetry and its breaking that they provide. We also remind the reader of the astrophysical relevance of $\langle p| A^0_\mu |p\rangle$ to dark matter searches[7,17] and to axion couplings.[27] Assuming that the EMC measurement (1) is essentially correct, the next priority is to determine the origin of the bulk of the proton spin, which must be carried by gluons and/or orbital angular momentum: $\frac{1}{2}(\Delta u + \Delta d + \Delta s) + \Delta G + \langle L_z \rangle = \frac{1}{2}$. There are various possibilities for measuring ΔG, including the following:‡

(*a*) Measurement of J/ψ production and decay properties in deep inelastic muon scattering off polarized targets;[29]

(*b*) Measurements of charm distributions in deep inelastic scattering off a polarized target using dimuon events from $c(\bar{c}) \rightarrow \mu^+(\mu^-) + X$ decays;

(*c*) Hadronic jet asymmetries in polarized pp collisions;[30]

(*d*) Direct photon production at large p_T by polarized protons;[30]

(*e*) Hyperon production at large p_T in polarized pp collisions;[31] §

(*f*) Higher order effects in polarized ep collisions;[32]

(*g*) Drell-Yan l^+l^- production with polarized beams;[33]

(*h*) Large p_T hadron production in photoproduction off polarized targets.[34]

We have considered QCD in the chiral limit of zero current quark masses and large N_c, and demonstrated that in this limit the net contribution of quark helicities to the proton spin is zero. In this limit the proton spin is therefore due to the gluon polarization and/or to the orbital angular momentum of the partons. We have also shown that this result remains approximately unchanged for protons when the perturbation due to non-zero quark masses is included in an effective chiral Lagrangian. The net helicity carried by quarks in the proton remains small,

‡ For a review and other references on spin physics at short distances, see Ref. 28.

§ The fact that $\Delta s < 0$ suggests that there may be significant spin anticorrelation for hyperons produced by polarized protons, even at low p_T.

consistent with the recent EMC measurements[4] of the spin-dependent proton structure function.

ACKNOWLEDGEMENTS

One of us (J.E.) thanks the SLAC and LBL Theory Groups for their kind hospitality. We thank Mike Chanowitz, Fred Gilman, Vadim Kaplunovsky, Michael Peskin and Steven Sharpe for useful discussions.

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