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Multi-Scalar Models with a High Energy Scale*

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ABSTRACT

We study multi-Higgs models under the assumption that new physics exists at some high energy scale (Λ_{NP}). If we perform the minimally required fine-tuning in order to set the electroweak scale (Λ_{EW}), we find that the low-energy scalar spectrum is identical to that of the Standard Model with minimal Higgs content, up to corrections of order $\Lambda_{EW}^2/\Lambda_{NP}^2$. If, in addition, the model is supersymmetric down to a scale $\Lambda_{SB} << \Lambda_{NP}$, then the low-energy scalar spectrum is identical to that of the minimal supersymmetric extension of the Standard Model, up to corrections of order $\Lambda_{SB}^2/\Lambda_{NP}^2$. Additional light scalars, if they exist, would be a signature of additional low-energy symmetries. Flavor changing neutral currents and CP-violation effects due to Higgs exchange are also discussed. Finally, we clarify the Linde-Weinberg bound for the minimum Higgs mass in such models.

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1. Introduction

The simplest example of spontaneous breaking of the $SU(2)_L \otimes U(1)_Y$ symmetry by the Higgs mechanism requires the existence of, at least, pne fundamental scalar: the Higgs boson. Experimentally, the existence of such a scalar has not yet been confirmed. So far, the search for a Higgs boson has been limited to a rather low range of masses ($\leq 5\ GeV$) [1]. Forthcoming experiments at SLC, LEP, LEP-2 and finally at the SSC will gradually survey a range of masses up to the TeV scale PI-

Within the ***minimal*** Standard Model (SM) there is a single physical Higgs scalar. The Higgs mass is a free parameter, while its couplings to fermions and gauge bosons are predicted by the model. Within the ***minimal*** Supersymmetric SM (SSM), there are five physical scalars: two neutral CP-even scalars, one neutral CP-odd scalar and a charged pair of scalars. Their masses and couplings to fermions and gauge bosons depend on two free parameters. Most experimental searches are based on these two models.

The SM leaves many theoretical questions unanswered. It is commonly believed that answers to such questions lie with New-Physics (NP) that takes place at a high energy scale Λ_{NP}:

$$\begin{aligned} &\Lambda_{NP} \gg \Lambda_{EW}, \\ &\Lambda_{EW} = \frac{2M_W}{9} = 246\ GeV, \end{aligned} \tag{1.1}$$

(Λ_{EW} is the Electroweak breaking scale). Most theories that go beyond the Standard Model have many more fundamental scalar multiplets beyond the single doublet of the minimal SM. Most supersymmetric theories that go beyond the Supersymmetric Standard Model have many more scalar multiplets beyond the two doublets of the minimal SSM. In this work, ***we study general multi-scalar models, under the assumption that new physics exists at some high energy scale.*** Assuming that there is physics beyond the SM at a new energy scale $\Lambda_{NP} >> \Lambda_{EW}$, we may describe its effects on the Higgs sector in the following ***way: All dimensionful parameters in the Higgs potential are of $O(\Lambda_{NP}^2)$, as long as the EW symmetry (and SUSY in supersymmetric models) are maintained to low enough energies.*** It must be stressed that fine-tuning of parameters in the Higgs potential is required in order that $\Lambda_{EW} \ll \Lambda_{NP}$.† As a crucial aspect of our philosophy in this paper, we shall demand that ***only the minimal number of fine-tunings be performed in order that*** $\Lambda_{EW} \ll \Lambda_{NP}$.

This property is general enough to encompass a wide variety of theories beyond the SM. In particular, we have in mind all those models where the $SU(2)_L \otimes U(1)_Y$ symmetry is embedded in a larger symmetry group. The breaking of this larger symmetry occurs when a scalar $SU(2)_L \otimes U(1)_Y$-singlet N assumes a VEV: $\Lambda_{NP} \sim \langle N \rangle$. Scalar non-singlets Δ have couplings to N of the general form:

$$\eta N N \Delta^\dagger \Delta, \tag{1.2}$$

where η is a quartic coupling constant. When N assumes a VEV, this leads to a mass term for Δ:

$$m^2 \Delta^\dagger \Delta = \eta \langle N \rangle^2 \Delta^\dagger \Delta. \tag{1.3}$$

Thus, indeed, $m^2 = O(\Lambda^2_{NP})$. Other effects due to the existence of singlets with large VEVs will be discussed, but they are of lesser significance in low-energy phenomena.

In this paper we shall try to be as general as possible. We shall not refer to any specific model for the New Physics, nor shall we fix the high energy scale beyond the requirement of eq. (1.1). The only crucial ingredient is the property of the Higgs potential described above. In addition, we shall assume that all ***dimensionless*** scalar couplings are of $O(1)$. Our conclusions hold as long as the value of the couplings is not as small as the ratio $(\Lambda_{EW}/\Lambda_{NP})$, and not as large as to make a perturbative analysis meaningless.

Higgs singlets do not couple to any of the known fermions and gauge bosons. Their low-energy effects, if any, are small and model-dependent. Higgs non-singlets, on the other hand, couple to the known gauge bosons in a model-independent way. Higgs doublets are likely to couple to the known fermions. Our interest lies in

† In general, such fine-tuning is also required in supersymmetric models. However, supersymmetric models possess the theoretically pleasing feature that such fine-tuning need only be performed at tree-level, and is stable under radiative corrections. The problem of the naturalness of the gauge hierarchy is a well-known puzzle of particle physics, and we have nothing new to add in this area.

the non-singlet part of the Higgs sector, with a special emphasis on multi-doublet models. The breaking of the EW symmetry occurs when non-singlet scalars, usually ***doublets*** Φ, assume a VEV: $\Lambda_{EW} \sim \langle\Phi\rangle$. Doublets are needed to give the SM fermions their masses. The experimentally successful $\rho \equiv M_W^2/(M_Z^2 \cos^2\theta_W) = 1$ relation implies that, in general, scalar multiplets which are neither singlets nor doublets of the SM should carry only very small VEVs (compared to $\langle\Phi\rangle$). Most of the properties of such models are well represented by a simple two doublet model (four-doublet in the supersymmetric case). We explicitly study only these simple cases and explain how the results generalize for models with an arbitrary number of various multiplets.

The idea in the basis of this work is closely related to the extended survival hypothesis of ref. [3]: "Higgses acquire the maximum mass compatible with the pattern of symmetry breaking". In ref. [4] it was shown that the above hypothesis is realized in many explicit examples of GUTs. Here, we show how this is unavoidable in a large class of models, and specify the necessary conditions. The focus of ref. [3] and of subsequent works is-on the implications of this hypothesis on the symmetry breaking, on the renormalization group equations and on fermion masses. Our work concentrates on the implications on the scalar sector itself at low energies. The possibility that within multi-scalar models the low-energy effective theory is that of the minimal SM was pointed out in ref. [5]. In this paper, we emphasize the generality of this scenario and show how it ***necessarily*** emerges in "reasonable" theoretical frameworks. We concentrate on the modifications to the effective low-energy theory due to the extended scalar sector. Some of our results were derived within specific models by earlier authors. Here, we have attempted to give a general discussion where only a minimal set of assumptions is used. We include brief reviews of specific examples whenever these help clarify our arguments.

The structure of this paper is as follows. In sections 2 - 8 we study multi-scalar models with the $SU(2)_L \otimes U(1)_Y$ symmetry of the SM as the only low-energy symmetry. In section 2 we introduce our notation. In section 3 we study the effects of scalar singlets on the non-singlet sector. In particular, we describe the effects which are ***not*** accounted for by our method of effective potential for the scalar non-singlets with large dimensionful parameters. In sections 4 - 8 we study in detail the implications of such an effective potential for the scalar non-singlets.

In section 4 we study the Higgs potential of the two doublet model and derive the scalar spectrum. In section 5 we study the couplings to quarks and the implications for flavor changing neutral currents (FCNC). We identify the various sources for CP-violation and their dependence on Λ_{NP} in section 6. In the next two sections we study bounds on the light Higgs mass: the tree-level unitarity upper bound (section 7) and the Linde-Weinberg lower bound (section 8). A possible source of additional light scalars is a spontaneously broken global symmetry. We study this possibility in section 9. In section 10 we explain how our results generalize to the case of an arbitrary multi-scalar model. Another mechanism for protecting light scalar masses is supersymmetry. In sections 11 - 12 we give a detailed study of multi-scalar models with both SUSY and the EW symmetry of the SSM as the only low-energy symmetries. We study the Higgs potential and the scalar spectrum (section 11) and the scalar induced FCNC (section 12). Finally, we recapitulate our assumptions and present the main conclusions in section 13.

2. Notation

The scalar doublets in a general basis are denoted by Φ_i (where i runs from 1 to k = the number of scalar doublets):

$$\begin{aligned}\Phi_i =&\sqrt{\tfrac{1}{2}}\begin{pmatrix}\phi_i^+\\ \phi_i^0\end{pmatrix},\\ \phi_i^0 =&v_i + R_i + iI_i.\end{aligned} \tag{2.1}$$

The normalization of the VEVs v_i is such that

$$v^2 \equiv \sum_{i=1}^{k} v_i^2 = (246\ GeV)^2. \tag{2.2}$$

A scalar doublet $\widetilde{\Phi}_i$ has a hypercharge opposite to that of Φ_i:

$$\widetilde{\Phi}_i = i\sigma_2\Phi_i^* = \sqrt{\tfrac{1}{2}}\begin{pmatrix}\phi_i^{0*}\\ -\phi_i^-\end{pmatrix}. \tag{2.3}$$

When working in a two doublet scheme, we define an angle β by:

$$\tan\beta \equiv \frac{v_2}{v_1};\quad s_\beta \equiv \sin\beta;\quad c_\beta \equiv \cos\beta. \tag{2.4}$$

Many of our results are derived in a rotated basis, where only one of the scalar doublets has a VEV. We denote the field with a non-vanishing VEV by Φ and those fields with a vanishing VEV by Φ_i' [where i runs from 1 to $(k-1)$]:

$$
\begin{aligned}
\Phi &= \sqrt{\tfrac{1}{2}}\begin{pmatrix} \phi^+ \\ v + R + iI \end{pmatrix}, \\
\Phi_i' &= \sqrt{\tfrac{1}{2}}\begin{pmatrix} \phi_i'^+ \\ R_i' + iI_i' \end{pmatrix}.
\end{aligned}
\tag{2.5}
$$

A third useful basis is that of mass eigenstates. If CP is conserved, then the lightest neutral (CP-even) scalar is denoted by h, the heavier ones by H_i. CP-odd scalars (which are pseudoscalars in their diagonal couplings to fermions) are denoted by A_i and charged scalars by $H_i^\pm$. For later convenience, we introduce the unitary mixing matrix in the neutral CP-even sector between the $\{R, R'\}$ and $\{h, H_i\}$ bases:

$$
\begin{aligned}
h &= \mathcal{U}_{00} R + \sum_{j=1}^{k-1} \mathcal{U}_{0j} R_j', \\
H_i &= \mathcal{U}_{i0} R + \sum_{j=1}^{t-1} \mathcal{U}_{ij} R_j'.
\end{aligned}
\tag{2.6}
$$

Our convention for the ordering of masses is as follows:

$$
\begin{aligned}
&M^2(h) < M^2(H_1) < M^2(H_2) < \cdots < M^2(H_{k-1}), \\
&M^2(A_1) < M^2(A_2) < \cdots < M^2(A_{k-1}), \\
&M^2(H_1^\pm) < M^2(H_2^\pm) < \cdots < M^2(H_{k-1}^\pm).
\end{aligned}
\tag{2.7}
$$

When working in a two doublet model we suppress the sub-index 1. Note that the number of ***physical*** pseudoscalars and of ***physical*** pairs of charged scalars is $(k-1)$, because one of the pseudoscalars is the would-be Goldstone boson "eaten" by the Z boson and one pair of the charged scalars is the would-be Goldstone boson "eaten?' by the $W^\pm$.

Scalar singlets are denoted by N:

$$
N = \sqrt{\tfrac{1}{2}}(n + R_N + iI_N). \tag{2.8}
$$

In general, the singlets will serve as a specific realization for the existence of a high-energy scale with $\Lambda_{NP} = \mathbf{n}$.

3. Effects of Scalar Singlets

Our purpose in this section is to show that the effects of a scalar singlet with a VEV at a high energy scale Λ_{NP} on the scalar non-singlet sector can be divided into two categories:

a. The dimensionful parameters of the Higgs potential for the non-singlets are of the order of the high energy scale. The implications of this property are the subject of study in the following sections.

b. There is mixing between the singlet and non-singlets. We describe the consequences of this mixing in this section.

We explicitly study a model with a single real scalar singlet R_N, and a single scalar doublet Φ. The singlet assumes a VEV, $\langle R_N \rangle = n$, which represents the high energy scale: $\Lambda_{NP} = O(n)$. The doublet assumes a VEV, $\langle \phi^0 \rangle = v$, which determines the electroweak breaking scale: $\Lambda_{EW} = O(v)$. The Higgs potential for these scalars is:

$$V = m_1^2 \Phi^\dagger \Phi + \tfrac{1}{2} m_N^2 R_N^2 + \eta_1 (\Phi^\dagger \Phi)^2 + \tfrac{1}{4}\eta_2 R_N^4 + \tfrac{1}{2}\eta_3 (\Phi^\dagger \Phi) R_N^2. \tag{3.1}$$

For simplicity, we imposed a discrete symmetry $R_N \to -R_N$. This does not affect our results. The VEVs are determined by:

$$\begin{aligned} n^2 &= -\frac{m_N^2}{\eta_2} + O(v^2), \\ v^2 &= -\frac{m_1^2 + \frac{1}{2}\eta_3 n^2}{\eta_1}. \end{aligned} \tag{3.2}$$

Following our assumption that the dimensionless couplings, η_1, η_2 and η_3 are of O(1), it immediately follows that the *dimensionful* parameter m_N^2 is of order n^2, namely $O(\Lambda_{NP}^2)$. It also follows that $m_1^2 = O(n^2)$. However, in order that the EW symmetry will be maintained to low enough energies, the combination

$$m_{1(eff)}^2 \equiv m_1^2 + \tfrac{1}{2}\eta_3 n^2, \tag{3.3}$$

[which is, apriori, $O(n^2)$] has to be fine-tuned to $O(v^2)$. In a general model with k doublets, there are $\frac{1}{2}k(k+1)$ effective mass terms for the doublets. In section 4 we find that k combinations of them need to be fine-tuned in order that $\Lambda_{EW} \ll \Lambda_{NP}$.

The scalar masses can be easily derived from the above potential. There are two physical degrees of freedom, R and R_N. The 2×2 mass matrix for these two neutral scalars is:

$$M^2(R, R_N) = \begin{pmatrix} 2\eta_1 v^2 & \eta_3 vn \\ \eta_3 vn & 2\eta_2 n^2 \end{pmatrix} \tag{3.4}$$

The two mass eigenstates have masses [to $O(v^2)$]:

$$\begin{aligned} M^2(h) &= \left(2\eta_1 - \frac{\eta_3^2}{2\eta_2}\right) v^2, \\ M^2(H) &= 2\eta_2 n^2 + \frac{\eta_3^2}{2\eta_2} v^2. \end{aligned} \tag{3.5}$$

To first order in v/n, the mass eigenstates are given by:

$$\begin{aligned} h &= R - \left(\frac{\eta_3 v}{2\eta_2 n}\right) R_N, \\ H &= \left(\frac{\eta_3 v}{2\eta_2 n}\right) R + R_N. \end{aligned} \tag{3.6}$$

The heavy mass eigenstate H has its mass at the high energy scale Λ_{NP}. It is dominantly a singlet. It couples to the known gauge bosons and fermions only through its small doublet component. Thus, any effects it may have on the low-energy world are suppressed by, at least, a factor of order $\Lambda_{EW}^2/\Lambda_{NP}^2$ due to its large mass and another factor of order $\Lambda_{EW}^2/\Lambda_{NP}^2$ due to the small admixture of the doublet. We omit such contributions from further considerations, as they are negligible compared to the $O(\Lambda_{EW}^2/\Lambda_{NP}^2)$ effects of the New Physics which will be studied in subsequent sections.

One effect of the scalar singlet on the light sector is to shift the mass of the light mass eigenstate h by $-\frac{\eta_3^2}{2\eta_2}v^2$. Note that the shift is $O(\Lambda_{EW}^2)$ and it does not depend on the high energy scale Λ_{NP}. Does that mean that the effects of the singlet are not decoupled from the low-energy world even for arbitrary high scale Λ_{NP}? To answer that we have to check whether we can experimentally distinguish between a minimal SM Higgs and the $\Lambda_{NP} \to \infty$ limit of the case in study. Within the SM, the mass of the Higgs boson is an arbitrary parameter. However, once it is measured, the Higgs self-interactions are determined. The trilinear and the quartic self-couplings are given by $\frac{M^2(h)}{2v}$ and $\frac{M^2(h)}{16v^2}$, respectively. We calculated the self-couplings of the light mass eigenstate Within the singlet-doublet model and found that they again are given by $\frac{M^2(h)}{2v}$ and $\frac{M^2(h)}{16v^2}$, but $M^2(h)$ is now given by Eq. (3.5). In other words: in the limit $n \to \infty$, the η_1 parameter is effectively redefined:

$$\eta_{1(eff)} \equiv \eta_1 - \frac{\eta_3^2}{2\eta_2}. \tag{3.7}$$

If a measurement of the Higgs self-coupling were practical, ***experiments would find the same relation between the Higgs mass and its self interactions as in the minimal*** SM. The above result is expected, since if we integrate out the heavy degrees of freedom from the theory, we are left with precisely the minimal SM, with appropriately renormalized low-energy parameters.

A second effect is that the light mass eigenstate, which is dominantly a doublet, has a small component of a singlet of order $\Lambda_{EW}/\Lambda_{NP}$. The consequences are straightforward: all the Yukawa and gauge couplings of h are modified with respect to the minimal SM values by the factor

$$1 - n_h \left(\frac{\eta_3^4}{8\eta_2^2}\right)\left(\frac{v^2}{n^2}\right), \tag{3.8}$$

where n_h is the number of h scalars appearing at the vertex. Thus, the existence of scalar singlets with VEVs at a high energy scale implies a universal correction to the Yukawa and gauge couplings of scalar non-singlets, of order $\Lambda_{EW}^2/\Lambda_{NP}^2$. All other effects are accounted for by an effective potential for the scalar non-singlets, where all dimensionful parameters are of the order of the high energy scale. In what follows, we study the consequences of such a potential for the low-energy theory, without further invoking the existence of scalar singlets or any other specific realization of the New Physics.

4. The Higgs Potential and Scalar Masses in the Two Doublet Model

Our starting point is the Higgs potential for two scalar doublets. For simplicity, we impose CP symmetry and do not include trilinear couplings. Both assumptions will be relaxed when relevant.

$$\begin{aligned} V =& m_1^2\Phi_1^\dagger\Phi_1 + m_2^2\Phi_2^\dagger\Phi_2 - m_{12}^2(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1) \\ &+\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ &+\tfrac{1}{2}\lambda_5\left[(\Phi_1^\dagger\Phi_2)^2 + (\Phi_2^\dagger\Phi_1)^2\right] + \tfrac{1}{2}(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1)(\lambda_6\Phi_1^\dagger\Phi_1 + \lambda_7\Phi_2^\dagger\Phi_2). \end{aligned} \tag{4.1}$$

Our basic assumptions on the parameters of this potential are the following:

(a) All dimensionless couplings (λ_i) are 0(1).

(b) We perform the minimal number of fine-tunings of parameters such that $v^2 \simeq O(\Lambda_{EW}^2)$.

(c) Apart from the fine-tuning requirement of ***(b),*** all unconstrained dimensionful couplings are $O(\Lambda_{NP}^2)$, the ***high*** scale of new physics.

The conditions for an ***extremum*** are:*

$$\begin{aligned} m_1^2 =& m_{12}^2\frac{v_2}{v_1} - \lambda_1 v_1^2 - \tfrac{1}{2}(\lambda_3+\lambda_4+\lambda_5)v_2^2 - \tfrac{3}{4}\lambda_6 v_1 v_2 - \tfrac{1}{4}\lambda_7\frac{v_2^3}{v_1}, \\ m_2^2 =& m_{12}^2\frac{v_1}{v_2} - \lambda_2 v_2^2 - \tfrac{1}{2}(\lambda_3+\lambda_4+\lambda_5)v_1^2 - \tfrac{3}{4}\lambda_7 v_1 v_2 - \tfrac{1}{4}\lambda_6\frac{v_1^3}{v_2}. \end{aligned} \tag{4.2}$$

Only two of the three dimensionful parameters are constrained by the extremum conditions. We choose m_{12}^2 to be the free parameter. By our assumptions, the unconstrained parameter, $m_{12}^2 = O(\Lambda_{NP}^2)$. However, the combinations $(m_{12}^2 - \frac{v_1}{v_2}m_1^2)$ and $(m_{12}^2 - \frac{v_2}{v_1}m_2^2)$ are both $O(\Lambda_{EW}^2)$. This implies that two fine-tunings are needed in order that the $SU(2)_L \otimes U(1)_Y$ symmetry be broken at the appropriate scale.

$\star$ Here, we assume that v_1 and v_2 are nonzero. If one of the VEVs is zero, simply proceed to eq. (4.3).

We now rotate to a basis $\{\Phi, \Phi'\}$, where only Φ carries a VEV:

$$\begin{aligned} \Phi &= \frac{1}{v}\left(v_1\Phi_1 + v_2\Phi_2\right), \\ \Phi' &= \frac{1}{v}\left(-v_2\Phi_1 + v_1\Phi_2\right). \end{aligned} \tag{4.3}$$

The Higgs potential is

$$\begin{aligned} V =& \mu_0^2\Phi^\dagger\Phi + \mu_1^2\Phi'^\dagger\Phi' - \mu_{01}^2(\Phi^\dagger\Phi' + \Phi'^\dagger\Phi) \\ &+\delta_1(\Phi^\dagger\Phi)^2 + \delta_2(\Phi'^\dagger\Phi')^2 + \delta_3(\Phi^\dagger\Phi)(\Phi'^\dagger\Phi') + \delta_4(\Phi^\dagger\Phi')(\Phi'^\dagger\Phi) \\ &+\tfrac{1}{2}\delta_5\left[(\Phi^\dagger\Phi')^2 + (\Phi'^\dagger\Phi)^2\right] + \tfrac{1}{2}(\Phi^\dagger\Phi' + \Phi'^\dagger\Phi)(\delta_6\Phi^\dagger\Phi + \delta_7\Phi'^\dagger\Phi'). \end{aligned} \tag{4.4}$$

The extremum conditions are

$$\mu_0^2 = -\delta_1 v^2, \quad \mu_{01}^2 = \tfrac{1}{4}\delta_6 v^2. \tag{4.5}$$

These equations are precisely the two fine-tuning conditions required to implement EW symmetry-breaking at the correct energy scale. This result easily generalizes to models with N Higgs doublets: we obtain N conditions which are satisfied by $\mu_0^2, \quad \mu_{01}^2, \ldots, \mu_{0N-1}^2$.

> ***If all dimensionful couplings are of $O(\Lambda_{NP}^2)$, then N fine-tunings must be performed on an N scalar doublet model in order to break the EW symmetry at the low-energy scale.***

The value of μ_i^2 $(i = 1, \ldots, N-1)$ are not constrained by the extremum conditions. According to our above assumptions $\mu_i^2 = O(\Lambda_{NP}^2)$.

From the above potential for the two-doublet model, we can easily derive the masses for the various scalars. For the two neutral scalars {R, R'} we have a 2 x 2 mass matrix:

$$M^2(R,R') = \begin{pmatrix} 2\delta_1 v^2 & \frac{1}{2}\delta_6 v^2 \\ \frac{1}{2}\delta_6 v^2 & \mu_1^2 + \frac{1}{2}(\delta_3 + \delta_4 + \delta_5)v^2 \end{pmatrix}. \tag{4.6}$$

The mass eigenvalues are:

$$\begin{aligned} M^2(h) &\simeq 2\delta_1 v^2 - \tfrac{1}{4}\delta_6^2\frac{v^4}{\mu_1^2}, \\ M^2(H) &\simeq \mu_1^2 + \tfrac{1}{2}(\delta_3 + \delta_4 + \delta_5)v^2, \end{aligned} \tag{4.7}$$

where we have dropped higher order terms in v^2/μ_1^2. To zeroth order in v^2/μ_1^2 the basis {h, *H*} can be identified with the basis {R, R'}.

The physical pseudoscalar can be identified with I', and the physical charged scalar can be identified with $\phi'^{\pm}$ with masses:

$$
\begin{aligned}
M^2(A) =&\mu_1^2 + \tfrac{1}{2}(\delta_3 + \delta_4 - \delta_5)v^2,\\
M^2(H^{\pm}) =&\mu_1^2 + \tfrac{1}{2}\delta_3 v^2.
\end{aligned}
\tag{4.8}
$$

The above results are simple to understand on general grounds. As the members of any multiplet are degenerate in the EW symmetry limit, mass-squared splittings within a multiplet must be of $O(\Lambda_{EW}^2)$. As a result, the neutral scalar which is a member of the same multiplet as the massless would-be Goldstone bosons has mass of $O(\Lambda_{EW})$. Other multiplets are not protected by any symmetry and acquire masses of the $O(\Lambda_{NP})$. The members of each such multiplet are degenerate in mass to $O(\Lambda_{EW}^2/\Lambda_{NP}^2)$.

5. Yukawa Interactions and FCNC

Our purpose in this section is to understand the scalar-mediated. contributions to FCNC. We follow the notation and formalism of ref. [6]. The Yukawa interactions in the two doublet model are given by

$$
-\mathcal{L}_Y = \overline{\Psi^0_{Li}}\left(F_{ij}\tilde{\Phi}_1 + F'_{ij}\tilde{\Phi}_2\right)U^0_{Rj} + \overline{\Psi^0_{Li}}\left(G_{ij}\Phi_2 + G'_{ij}\Phi_1\right)D^0_{Rj} + h.c., \tag{5.1}
$$

where left-handed quark doublets are denoted by Ψ^0_{Li}. The sub-index i is a generation index ($i = 1,2,3$), and the superscript 0 denotes an interaction eigenstate. Right-handed quark singlets are denoted by D^0_{Ri} and U^0_{Ri}. For simplicity we impose CP symmetry so that the 3 x 3 Yukawa matrices (F, F', G, G') and the VEVs (v_1, v_2) are real. (This condition will be relaxed in the next section.) The quark mass matrices are given by:

$$
\begin{aligned}
M_u =&\sqrt{\tfrac{1}{2}}\left(F + \frac{v_2}{v_1}F'\right)v_1,\\
M_d =&\sqrt{\tfrac{1}{2}}\left(G + \frac{v_1}{v_2}G'\right)v_2.
\end{aligned}
\tag{5.2}
$$

We now rotate to the basis $\{\Phi, \Phi'\}$. The Yukawa interactions of the neutral scalars are given by:

$$
\begin{aligned}
-\mathcal{L}_Y^U = & \sum_{q=u,c,t} \left(\frac{m_q}{v}\right) \left[R\bar{q}q - \left(\frac{v_2}{v_1}\right) R'\bar{q}q - i\left(\frac{v_2}{v_1}\right) I'\bar{q}\gamma_5 q\right] \\
& + \left[\overline{U^0_{Li}} F'_{ij} \frac{v}{\sqrt{2}v_1}(R' + iI')U^0_{Rj} + h.c.\right]
\end{aligned}
\tag{5.3}
$$

$$
\begin{aligned}
-\mathcal{L}_Y^D = & \sum_{q=d,s,b} \left(\frac{m_q}{v}\right) \left[R\bar{q}q + \left(\frac{v_1}{v_2}\right) R'\bar{q}q + i\left(\frac{v_1}{v_2}\right) I'\bar{q}\gamma_5 q\right] \\
& - \left[\overline{D^0_{Li}} G'_{ij} \frac{v}{\sqrt{2}v_2}(R' + iI')D^0_{Rj} + h.c.\right]
\end{aligned}
\tag{5.4}
$$

where ***q*** denotes quark ***mass*** eigenstates. The scalar ***R*** couples diagonally in flavor space. Moreover, its couplings are the same as those of the single scalar of the minimal SM. Recall that the light scalar ***h*** is to be identified with the field ***R, to*** $O(v^2/\mu_1^2)$. We conclude that ***the mass and the Yukawa interactions of the light scalar h are identical to those of the single Higgs of the minimal SM, up to effects of order*** $\Lambda^2_{EW}/\Lambda^2_{NP}$.

The non-diagonal couplings of neutral scalars are contained in the last terms of $\mathcal{L}_Y^U$ and $\mathcal{L}_Y^D$. We are mainly interested in the scalar contributions to FCNC in the kaon system $[\Delta M(K^0)]$. We therefore study FCNC of the down sector in the two generation case. Generalization to the three generation case and application to the up sector are straightforward.

We work in the basis where M_u is diagonal. We define V_L and V_R to be the unitary matrices that diagonalize M_d:

$$
\widehat{M_d} = V_L M_d V_R^\dagger
\tag{5.5}
$$

where $\widehat{M_d}$ is diagonal. The Cabibbo matrix is $V_L^{\dagger}$. We further define

$$
G'' = \frac{v}{\sqrt{2}v_2} V_L G' V_R^\dagger .
\tag{5.6}
$$

The non-diagonal scalar couplings are given by:

$$
\begin{aligned}
& R'\left[\left(G''_{12} + G''_{21}\right)\left(\bar{d}s + \bar{s}d\right) + \left(G''_{12} - G''_{21}\right)\left(\bar{d}\gamma_5 s - \bar{s}\gamma_5 d\right]\right. \\
& + iI'\left[\left(G''_{12} - G''_{21}\right)\left(\bar{d}s - \bar{s}d\right) + \left(G''_{12} + G''_{21}\right)\left(\bar{d}\gamma_5 s + \bar{s}\gamma_5 d\right)\right].
\end{aligned}
\tag{5.7}
$$

The strength of the non-diagonal couplings $(G''_{12} \pm G''_{21})$ depends on the details of the model. Note, however, that R' couples to $CP = +1$ combinations while I' couples to CP = -1 combinations, as expected in the CP-symmetric case.

The most stringent limits on FCNC come from the mixing of neutral mesons. Both the light scalar h and the heavier neutral scalars H and A contribute to this mixing at tree-level. However, for AS = 2 processes, the light scalar contribution is suppressed by an additional factor of $O(\Lambda_{EW}^2/\Lambda_{NP}^2)$ over the heavy ones. The contribution from the neutral heavy scalars may be very large due to several enhancement factors:

a. The Higgs amplitude is second order in Yukawa interaction (tree diagram), while the W amplitude is fourth order in weak interaction (box diagram).

b. There is no GIM mechanism for Higgs-induced FCNC. This is most significant in $K - \bar{K}$ mixing, where the dominant contribution within the minimal SM comes from c quarks and thus suppressed by the factor m_c^2/M_W^2.

c. The relevant matrix element for $P - \bar{P}$ mixing, where P is a $Q\bar{q}$ meson, is $\langle P^0|\overline{q_L}Q_R\overline{q_R}Q_L|\bar{P}^0\rangle$ which gives additional enhancement:

$$\frac{\langle P^0|\overline{q_L}Q_R\overline{q_R}Q_L|\bar{P}^0\rangle}{\langle P^0|\overline{q_L}\gamma_\mu Q_L\overline{q_L}\gamma^\mu Q_L|\bar{P}^0\rangle} = \frac{3}{4}\left[\left(\frac{M_P}{m_Q+m_q}\right)^2+\frac{1}{6}\right] \tag{5.8}$$

in the vacuum saturation approximation. For the kaon system, this factor is about 7.6.

If, for simplicity, we assume $M^2(H) = M^2(A) \equiv M^2(\phi'^0)$ [which is true to leading order in $(\Lambda_{EW}^2/\Lambda_{NP}^2)$] and put $G''_{12} = G''_{21}$, then the ϕ'^0 contribution to M_{12} (the mixing term between K^0 and $\bar{K}^0$) is

$$M_{12}^{\phi'^0} = \frac{(G''_{12})^2}{M^2(\phi'^0)}\frac{\langle K^0|\,\overline{d_L}s_R\overline{d_R}s_L\,|\bar{K}^0\rangle}{M_K} \tag{5.9}$$

This is to be compared with the W contribution in the SM:

$$M_{12}^W = \frac{G_F}{2\sqrt{2}}\frac{\alpha}{4\pi\sin^2\theta_W}(\cos\theta_c\sin\theta_c)^2\left(\frac{m_c^2}{M_W^2}\right)\frac{\langle K^0|\,\overline{d_L}\gamma_\mu s_L\overline{d_L}\gamma^\mu s_L\,|\bar{K}^0\rangle}{M_K}. \tag{5.10}$$

We shall require that

$$M_{12}^{\phi'^0} \le M_{12}^W. \tag{5.11}$$

If we naively assume G''_{12} = O(1) we get a very stringent limit [7]:

$$G''_{12} = 0(\ 1) \Longrightarrow M(\phi') \geq 2000\ TeV. \qquad (5.12)$$

However, this result is probably misleading [8,9]. The known values of fermion masses, some of which are much smaller than M_W, hint that Yukawa couplings may be much smaller than gauge couplings.

The actual limit that we can derive from FCNC is model dependent, and in specific models it is, indeed, much less restrictive than the bound (5.12). The simplest and most illuminating example is the minimal Left-Right Symmetric (LRS) [10] model. The Higgs sector of this model consists of two doublets (and other multiplets which are of no relevance to our discussion here). The $SU(2)_L \otimes SU(2)_R$ symmetry imposes the following constraints on the Yukawa matrices defined in (5.1):

$$F_{ij} = G_{ij};\ F'_{ij} = G'_{ij}. \qquad (5.13)$$

This enables us to give the Yukawa matrices in terms of the quark mass matrices:

$$\begin{aligned} G &= \frac{\sqrt{2}}{v_1^2 - v_2^2}\left(v_1 M_u - v_2 M_d\right), \\ G' &= \frac{\sqrt{2}}{v_1^2 - v_2^2}\left(v_1 M_d - v_2 M_u\right). \end{aligned} \qquad (5.14)$$

We get the following value for the non diagonal couplings (G" is symmetric if both C and P hold):

$$G''_{12} = G''_{21} = \frac{v^2}{v_1^2 - v_2^2}\sin\theta_c\cos\theta_c\frac{m_c}{v}. \qquad (5.15)$$

Note that $G''_{12} = O(10^{-3}) \ll 1$.

The contribution of H and A to $K - \bar{K}$ mixing is [11, 12]:

$$M_{12}^{\phi'^0} = \sqrt{2}G_F\left(\cos\theta_c \sin\theta_c\right)^2\left(\frac{m_c^2}{M_W^2}\right)\left(\frac{M_W^2}{M^2(\phi'^0)}\right)\left(\frac{v_1^2+v_2^2}{v_1^2-v_2^2}\right)^2\frac{\langle K^0|\,\overline{d_L}s_R\overline{d_R}s_L\,|\bar{K}^0\rangle}{M_K} \qquad (5.16)$$

which gives [13, 14]:

$$M(\phi'^0) \geq 10^2 M_W \approx 8\ TeV. \qquad (5.17)$$

Indeed, the limit is much weaker than in eq. (5.12). In the minimal LRS model, there is no significant scalar contribution to FCNC if the masses of $\boldsymbol{H}$ and $\boldsymbol{A}$ are above 8 TeV (other experimental constraints, e.g. from $B-\bar{B}$ mixing, are weaker). Calculations within other specific models give similar results [9].

6. *CP* Violation

The study of CP violation within multi-scalar models is complicated: there are many ways in which CP violation reveals itself. Furthermore, due to a plethora of unknown parameters, any quantitative study is necessarily model dependent. Our purpose is to identify the possible sources of CP violation and to understand their general dependence on the scale Λ_{NP}. As we do not attempt a quantitative discussion, we look for a way to introduce CP violation which is simple and economical in parameters.

Consider the case of spontaneous CP-violation [15, 16], when all Yukawa couplings and Higgs couplings are real. The source of CP violation is a relative phase ξ between the VEVs of the Φ_1 and Φ_2 doublets. Without loss of generality we choose v_1 to be real:

$$\langle\Phi_1\rangle = \sqrt{\tfrac{1}{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix},\ \langle\Phi_2\rangle = \sqrt{\tfrac{1}{2}}\begin{pmatrix} 0 \\ v_2 e^{i\xi} \end{pmatrix}. \qquad (6.1)$$

Unfortunately, in our specific framework and with two doublets, the vacuum is CP conserving: if we assume $\sin\xi \neq 0$, the extremum condition is

$$\cos\xi = \frac{m_{12}^2 - \frac{1}{4}(\lambda_6 v_1^2 + \lambda_7 v_2^2)}{\lambda_5 v_1 v_2} \qquad (6.3)$$

With $m_{12}^2 >> v^2$, there is no solution to this equation. Thus the minimum corresponds to $\sin\xi = 0$ and there is no spontaneous CP violation.

We could insist on spontaneous CP violation by extending the Higgs sector to, say, three doublets. The calculation becomes cumbersome and not very illuminating. Instead, we allow a complex (CP-violating) m_{12}^2 term:

$$V_{CPV} = -m_{12}^2\left(e^{-i\sigma}\Phi_1^\dagger\Phi_2 + e^{i\sigma}\Phi_2^\dagger\Phi_1\right). \qquad (6.3)$$

Although this is an explicit CP violating term, there are many cases in which such a term arises from a spontaneous breaking at a high scale. Suppose we have a singlet scalar N which has a coupling to the doublets of the following form:

$$\eta_{12}\left(N^2\Phi_1^\dagger\Phi_2 + N^{*2}\Phi_2^\dagger\Phi_1\right). \tag{6.4}$$

If this singlet assumes a complex VEV, $\langle N\rangle = ne^{-i\sigma/2}$, **a** term of the form (6.3) (with $m_{12}^2 = \eta_{12}n^2$) arises.

The complex term [eq. (6.3)] in the Higgs potential induces a relative phase between the two VEVs. For $v^2/m_{12}^2 << \cos\sigma, \sin\sigma$, the condition on ξ is

$$\sin\xi = \sin\sigma\left[1 + O\left(\frac{v^2}{m_{12}^2}\right)\right]. \tag{6.5}$$

The effects of CP violation in the quark sector are induced by the complex VEV. Thus, the study of CP violation in this sector is equivalent to its study in a model where CP breaking is genuinely spontaneous.

Spontaneous CP violation within a two doublet model with an approximate discrete symmetry was studied in detail and much clarity in ref. [6]. Although the starting point for the study of ref. [6] is-very different from ours, we find it convenient to follow their notation and line of reasoning. Moreover, since our work and that of ref. [6] have approximate flavor conserving neutral currents, many of the features that emerge are common to both approaches.

There are four sources of CP violation in our framework:

(i) Complex quartic couplings.
In the $\{\Phi,\Phi'\}$ basis, the potential is:

$$\begin{aligned}V =&\mu_0^2\Phi^\dagger\Phi + \mu_1^2\Phi'^\dagger\Phi' - \mu_{01}^2(e^{-i\rho}\Phi^\dagger\Phi' + e^{i\rho}\Phi'^\dagger\Phi)\\ &+\delta_1(\Phi^\dagger\Phi)^2 + \delta_2(\Phi'^\dagger\Phi')^2 + \delta_3(\Phi^\dagger\Phi)(\Phi'^\dagger\Phi') + \delta_4(\Phi^\dagger\Phi')(\Phi'^\dagger\Phi)\\ &+\tfrac{1}{2}\delta_5\left[e^{-i\chi}(\Phi^\dagger\Phi')^2 + e^{i\chi}(\Phi'^\dagger\Phi)^2\right]\\ &+\tfrac{1}{2}\left[\delta_6(e^{-i\rho}\Phi^\dagger\Phi' + e^{i\rho}\Phi'^\dagger\Phi)(\Phi^\dagger\Phi) + \delta_7(e^{-i\kappa}\Phi^\dagger\Phi' + e^{i\kappa}\Phi'^\dagger\Phi)(\Phi'^\dagger\Phi')\right].\end{aligned} \tag{6.6}$$

A complex quartic coupling leads to both CP-odd couplings (when the number of pseudoscalars in the vertex is odd) and CP-even couplings (when the number of pseudoscalars in the vertex is even). The relations among the couplings and phases in this basis and in the original basis are rather complicated. All phases vanish in the limit $\sigma \to 0$, although in general these phases need not be small. For example:

$$\tan\chi = s_\xi \frac{2\lambda_5 c_\xi (s_\beta^4 - c_\beta^4) + (\lambda_6 - \lambda_7) s_\beta c_\beta}{2(\lambda_1 + \lambda_2 - \lambda) s_\beta^2 c_\beta^2 + \lambda_5 c_{2\xi} + (\lambda_6 - \lambda_7) s_\beta c_\beta (s_\beta^2 - c_\beta^2) c_\xi}, \quad (6.7)$$

where $s_\xi \equiv \sin\xi$, $c_\xi \equiv$ cost, etc., and $\lambda \equiv \lambda_3 + \lambda_4 + \lambda_5 \cos 2\xi$.

(ii) Mixing of scalars and pseudoscalars.

The mass of the charged scalar remains unchanged [eq. (4.8)]. However, the masses of the neutral scalars are given now by a 3 x 3 matrix, as there is mixing between the scalars and the pseudoscalar. In the basis {R, R', I'} we have:

$$M^2 = \begin{pmatrix} 2\delta_1 v^2 & \frac{1}{2}\delta_6 c_\rho v^2 & \frac{1}{2}\delta_6 s_\rho v^2 \\ \frac{1}{2}\delta_6 c_\rho v^2 & \mu_1^2 + \frac{1}{2}(\delta_3 + \delta_4 + \delta_5 c_\chi) v^2 & \frac{1}{2}\delta_5 s_\chi v^2 \\ \frac{1}{2}\delta_6 s_\rho v^2 & \frac{1}{2}\delta_5 s_\chi v^2 & \mu_1^2 + \frac{1}{2}(\delta_3 + \delta_4 - \delta_5 c_\chi) v^2 \end{pmatrix}. \quad (6.8)$$

Again, to zeroth order in (v^2/μ_1^2), the light scalar h is to be identified with R. In addition, h has both R' and I' components of $O(v^2/\mu_1^2)$. The mixing θ between the R' and I' states is generally not small: tan 26 = tan χ where $\tan\chi$ is given in eq. (6.7). This source of CP violation was studied in refs. [17,18].

(iii) CP-odd Yukawa couplings.

The flavor-changing term of eq. (5.4) becomes now:

$$-\left[\overline{D^0_{Li}} G'_{ij} \frac{v e^{-i\xi}}{\sqrt{2} v_2} (R' + iI') D^0_{Rj} + h.c.\right]. \quad (6.9)$$

Instead of eq. (5.7) we get

$$R'[r_+(\bar{d}s + \bar{s}d) + is_+(\bar{d}s - \bar{s}d) + r_-(\bar{d}\gamma_5 s - \bar{s}\gamma_5 d) + is_-(\bar{d}\gamma_5 s + \bar{s}\gamma_5 d)]$$
$$+iI'[r_-(\bar{d}s - \bar{s}d) + is_-(\bar{d}s + \bar{s}d) + r_+(\bar{d}\gamma_5 s + \bar{s}\gamma_5 d) + is_+(\bar{d}\gamma_5 s - \bar{s}\gamma_5 d)] \quad (6.10)$$

where $r_\pm$ and $s_\pm$ are functions of the G''_{ij} matrix elements and the phase ξ. In the $s_\pm$ terms, the fields R' and I' couple to the "wrong" CP eigenstates, which gives yet another source of CP violation.

All CP violating effects described so far do not appear in the minimal SM. They involve either a heavy scalar or the small pseudoscalar component in ***h*.** In either case, they are suppressed by at least one power of $\Lambda^2_{EW}/\Lambda^2_{NP}$. The actual magnitude of these effects is model dependent. However, it is-obvious that by taking Λ_{NP} to be large, we can make these effects small enough as not to violate any experimental data. Conversely, it is possible to construct models where, for low enough Λ_{NP}, CP-violating effects from the Higgs sector are significant. A fourth source of CP violation is the following:

(iv) A Kobayashi-Maskawa (KM) phase [19].

The quark mass matrices are:

$$
\begin{aligned}
M_u &= \sqrt{\tfrac{1}{2}}\left(F + e^{-i\xi}\frac{v_2}{v_1}F'\right)v_1,\\
M_d &= \sqrt{\tfrac{1}{2}}\left(G + e^{-i\xi}\frac{v_1}{v_2}G'\right)v_2.
\end{aligned}
\tag{6.11}
$$

In the basis where M_u is diagonal, the CKM mixing matrix is $V_L^\dagger$ ***[see*** eq. (5.5)]. In general, M_d of eq. (6.11) is complex, and consequently so is V_L. ***A multi-Higgs model with spontaneous CP-breaking leads, in general, to a non-trivial KM* phase.** The low-energy effective theory contains one Higgs only, and we can choose its VEV to be real. The original relative phase between VEVs will show up in the complex Yukawa couplings of the light Higgs boson.

In summary, multi-scalar models possess many sources of ***CP*** violation; The strength of each CP-violating contribution is model dependent. However, in the limit that the scale of new physics is very large, the KM phase remains the only effective source of ***CP*** violation in the low-energy theory.

7. Unitarity Bounds

In a unitary gauge theory, each (suitably normalized) partial wave amplitude satisfies $|a_J| \leq 1$. For a weakly coupled theory this should be satisfied by the tree level amplitudes ("tree-level unitarity"). If not, the theory is strongly coupled and perturbation theory breaks down.

Tree-level unitarity is most sensitively tested in the scattering of longitudinal gauge bosons [20]. Within the minimal SM, consider the limit $s >> M^2(h) \gg M_W^2$. The $J = 0$ amplitude for the elastic scattering of the state $(2W_L^+W_L^- + Z_LZ_L + hh)/\sqrt{8}$ is

$$|a_0| = \frac{3G_F}{8\sqrt{2}\pi}M^2(h). \tag{7.1}$$

If we demand that $|a_0| \leq 1$, we obtain an upper bound on the Higgs mass:*

$$M^2(h) \leq \frac{8\sqrt{2}\pi}{3G_F} = (1\ TeV)^2. \tag{7.2}$$

We now turn to the multi-scalar case [22]. The trilinear couplings relevant to the above process have the form:

$$\frac{i}{2}g_{\mu\nu}g^2\left(W^{\mu+}W^{\nu-} + \frac{1}{\cos^2\theta_W}Z^\mu Z^\nu\right)\sum v_iR_i. \tag{7.3}$$

As $\boldsymbol{R} = \frac{1}{v}\sum v_iR_i$, these interaction can be written as:

$$ig_{\mu\nu}g\left(M_W\,W^{\mu+}W^{\nu-} + \frac{M_Z}{\cos\theta_W}Z^\mu Z^\nu\right)R. \tag{7.4}$$

Thus, there are no couplings of this type to other scalar fields; only $\boldsymbol{R}$ contributes to the processes in question. Therefore, eq. (7.2) holds for the $\boldsymbol{RR}$ element of the scalar mass-squared matrix in the {R, R'}-basis. In terms of physical masses, the

---

⋆ One can obtain a slightly stronger bound by employing the unitarity inequality $|\text{Im}\ a_J| \geq |a_J|^2$, as noted in ref. [21], which is easily seen to imply that $|\text{Re}\ a_J| \leq \frac{1}{2}$. Since tree-level amplitudes are real, this will reduce the Higgs mass bounds quoted above by a factor of $\sqrt{2}$.

unitarity bound reads:

$$\mathcal{U}_{00}^2 M^2(h) + \sum_{i=1}^{k-1} \mathcal{U}_{i0}^2 M^2(H_i) \leq (1\ TeV)^2 \tag{7.5}$$

where the mixing matrix $\mathcal{U}$ was defined in eq. (2.6). This is consistent with our previous analysis, which found that $M^2(h) = O(\Lambda_{EW}^2)$, $M^2(H_i) = O(\Lambda_{NP}^2)$, $\mathcal{U}_{00} =$ O(1) and $\mathcal{U}_{i0} = O(\Lambda_{EW}^2/\Lambda_{NP}^2)$, i = 1,. . . , $k - 1$. This leads to the following conclusion: ***The unitarity bound on $M^2(h)$ is identical to that on the mass of the single Higgs in the minimal SM,*** up to corrections of order $\Lambda_{EW}^2/\Lambda_{NP}^2$. This is a well-known result, first obtained in ref. [22].

In general, no unitarity bounds of order Λ_{EW} are obtained for any other physical Higgs boson mass. For example, in the model discussed in section 4, in the limit of $\Lambda_{NP} \gg \Lambda_{EW}$, $M^2(h) \sim O(\delta_1 \Lambda_{EW}^2)$ [see eq. (4.6)], whereas all other $M^2(H_i) \sim \mu_i^2 = O(\Lambda_{NP}^2)$. The unitarity bound [eq. (7.2)] should be thought of as a bound on the light Higgs self-coupling (or, more precisely, the self-coupling of the scalar field R), namely $\delta_1 \leq 8\pi/3$. The bare mass terms $\mu_i^2 (i = 1, \ldots, N - 1)$ can take arbitrarily large values without violating unitarity. For a general discussion on the implications of unitarity on the magnitude of various Higgs self-couplings in arbitrary models, see ref. [22].

8. The Linde-Weinberg Bound

The Linde-Weinberg bound [23, 24] refers to a calculation of the minimal allowed value for the mass of the physical Higgs of the SM. This bound is obtained by requiring that radiative corrections preserve the tree-level $SU(2)_L \otimes U(1)_Y$ breaking vacuum as the global minimum [25]. In this section we work in the $\{\Phi, \Phi'\}$ basis. Without loss of generality, we employ the following tree-level Higgs potential:

$$V = \mu_0^2 \Phi^\dagger \Phi + \mu_1^2 \Phi'^\dagger \Phi' + \delta_1 (\Phi^\dagger \Phi)^2 + \delta_2 (\Phi'^\dagger \Phi')^2 + \delta (\Phi^\dagger \Phi)(\Phi'^\dagger \Phi'). \tag{8.1}$$

The contribution of the scalar doublet Φ' with a vanishing VEV, $\langle \Phi' \rangle = 0$, to the one-loop effective potential is:

$$V_{eff}(\phi) = \tfrac{1}{2}\mu_0^2\phi^2 + \tfrac{1}{4}\delta_1\phi^4 - \frac{i}{2}\sum\int\frac{d^4k}{(2\pi)^4}\ln\left[\frac{k^2-(\mu_1^2+\frac{1}{2}\delta\phi^2)+i\epsilon}{k^2-\mu_1^2+i\epsilon}\right], \quad (8.2)$$

where $\phi = v + R$ in the notation of eq. (2.5). The summation in eq. (8.2) is over the four scalar degrees of freedom in Φ'. Using the potential of eq. (8.1), these contributions are all equal and result in a factor of 4. (Had we used the Higgs potential of eq. (4.4), then we should replace δ by δ_3, $\delta_3+\delta_4+\delta_5$ and $\delta_3+\delta_4-\delta_5$ for the contributions of $\phi'^{\pm}$, R' and I' respectively.) In the calculation we employ dimensional regularization and use the physical scheme.† The final result is:

$$\begin{aligned} V_{eff} =& \tfrac{1}{2}m^2\phi^2 + \tfrac{1}{4}\lambda\phi^4 \\ &+\frac{1}{64\pi^2}\sum\left[\left(\mu_1^2+\tfrac{1}{2}\delta\phi^2\right)^2\ln\left(\frac{\mu_1^2+\frac{1}{2}\delta\phi^2}{\mu_1^2}\right)-\tfrac{1}{2}\delta\phi^2\left(\mu_1^2+\tfrac{3}{4}\delta\phi^2\right)\right]. \end{aligned} \quad (8.3)$$

To find the mass of R we calculate $M^2(R) = (d^2V/d\phi^2)\big|_{\phi=v}$. We find

$$M^2(R) = -2m^2 + \sum\frac{\delta^2v^2}{32\pi^2}\left[1-\frac{2\mu_1^2}{\delta v^2}\ln\left(\frac{\mu_1^2+\frac{1}{2}\delta v^2}{\mu_1^2}\right)\right]. \quad (8.4)$$

The above result in the case of $m^2 = 0$ was previously obtained in ref. [26]. By requiring that $V(v) < V(0) \equiv 0$, it follows that

$$m^2 < \sum\frac{\delta^2v^2}{128\pi^2}\left[1+\frac{4\mu_1^2}{\delta v^2}-\frac{8\mu_1^2}{\delta^2v^4}\left(\mu_1^2+\tfrac{1}{2}\delta v^2\right)\ln\left(\frac{\mu_1^2+\frac{1}{2}\delta v^2}{\mu_1^2}\right)\right]. \quad (8.5)$$

Consequently,

$$[M^2(R)]_{min} = \sum\frac{\delta^2v^2}{64\pi^2}\left\{1-\frac{4\mu_1^2}{\delta v^2}\left[1-\frac{2\mu_1^2}{\delta v^2}\ln\left(\frac{\mu_1^2+\frac{1}{2}\delta v^2}{\mu_1^2}\right)\right]\right\}. \quad (8.6)$$

This holds for any μ_1^2 value. It is interesting to study the two limits, $\mu_1^2 \to 0$ and $\mu_1^2 \to \infty$. Let us denote the masses of R', I' or $\phi'^{\pm}$ by M'^2.

† In the physical scheme, the renormalized mass and coupling constant are defined by $m^2 = (d^2V_{eff}/d\phi^2)\big|_{\phi=0}$ and $\lambda = 6(d^4V_{eff}/d\phi^4)\big|_{\phi=0}$.

(*i*) $\mu_1^2 = 0,\ M'^2 = \frac{1}{2}\delta v^2$.

In this case

$$[M^2(R)]_{min} = \sum \frac{\delta^2 v^2}{64\pi^2} = \frac{1}{16\pi^2 v^2} \sum M'^4. \tag{8.7}$$

This is a well known result for the contribution of scalars that acquire their masses from the spontaneous breaking of $SU(2)_L \otimes U(1)_Y$.

(ii) $\mu_1^2 \to \infty,\ M'^2 \to \mu_1^2$.

In this case

$$[M^2(R)]_{min} = \sum \frac{\delta^3 v^4}{192\pi^2 \mu_1^2} = \frac{v^4}{192\pi^2} \sum \frac{\delta^3}{M'^2}. \tag{8.8}$$

This is our new result in this section: it gives the contribution to the Linde-Weinberg bound from scalar doublets that do not carry a VEV, and have large masses which do not come from EW symmetry breaking.

To summarize, the contribution to the Linde-Weinberg bound from scalars which do not carry a VEV is:

$$[M^2(R)]_{LW} = \frac{1}{16\pi^2 v^2}\,\mathrm{Tr}\{M_l^4\} + \frac{v^4}{192\pi^2}\,\mathrm{Tr}\{\delta_h^3/M_h^2\}\,. \tag{8.9}$$

where Φ'_l are those scalars which acquire masses from spontaneous $SU(2)_L \otimes U(1)_Y$ symmetry breaking only, and Φ'_h are those scalars which have "bare" mass terms at a scale much larger than Λ_{EW}. In eq. (8.9), we use the $\mathrm{Tr}\{\cdots\}$ notation to indicate a sum over all scalar degrees of freedom ***(i.e.*** the quantities in braces should be viewed as diagonal matrices whose dimensions are equal to the number of contributing scalars). It is easy to obtain the complete expression for $[M^2(R)]_{LW}$ by including the gauge bosons, quarks and leptons. Since all these particles get their mass from the Higgs VEV, one must modify eq. (8.9) by simply replacing $\mathrm{Tr}\{M_l^4\}$ with $\mathrm{Str}\{M_l^4\}$, where

$$\mathrm{Str}\{\cdots\} \equiv \sum_i C_i(2J_i + 1)(-1)^{2J_i}\{\cdots\}, \tag{8.10}$$

and the sum is taken over all particles of spin J_i and C_i counts electric charge and color degrees of freedom.

We emphasize here that the Linde-Weinberg bound is a limit on the ***RR*** element of the scalar mass-squared matrix. In terms of physical masses, the Linde-Weinberg bound reads:

$$\mathcal{U}_{00}^2 M^2(h) + \sum_{i=1}^{k-1} \mathcal{U}_{i0}^2 M^2(H_i) \geq [M^2(R)]_{LW}. \tag{8.11}$$

Following arguments similar to those of the previous section, we arrive at the following conclusion: ***The Linde- Weinberg bound on $M^2(h)$ is identical to that on the mass of the single Higgs in the minimal SM,*** up to corrections of order $\Lambda_{EW}^2/\Lambda_{NP}^2$.

If the scalar mass is not much larger than 1 TeV, and if the quartic coupling δ is not much smaller than 1, then the contribution of a heavy scalar doublet to the Linde- Weinberg bound may be significant:

$$[M^2(R)]_{LW}^{\Phi'} = (2.8\ GeV)^2 \left(\frac{\delta}{1.0}\right)^3 \left[\frac{1\ TeV}{M(\Phi')}\right]^2. \tag{8.12}$$

Note that the sign of the shift is unknown, as it depends on the sign of δ.

9. Spontaneously Broken Global Symmetries

Spontaneously broken global continuous symmetries imply the existence of massless Goldstone bosons. They may also lead to light scalars in addition to the Goldstone bosons, even in the framework of models with New Physics at a high energy scale. In what follows, we study this possibility.

Our interest is in global symmetries which are broken by VEVs of Higgs non-singlets. Again, we explicitly study a two doublet model. In section 10 we generalize to higher multiplets.

We take the model discussed in section 4, but impose an additional global $U(1)_G$ symmetry. The doublets Φ_1 and Φ_2 carry different $U(1)_G$ charges. This is just the original axion model of Peccei and Quinn [27]. The Higgs potential is then:

$$\begin{aligned} V =& m_1^2\Phi_1^\dagger\Phi_1 + m_2^2\Phi_2^\dagger\Phi_2 \\ &+\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1). \end{aligned} \tag{9.1}$$

We have only two dimensionful parameters, and both are constrained by the minimum conditions which yield:

$$
\begin{aligned}
m_1^2 &= -\lambda_1 v_1^2 - \tfrac{1}{2}(\lambda_3 + \lambda_4)v_2^2,\\
m_2^2 &= -\lambda_2 v_2^2 - \tfrac{1}{2}(\lambda_3 + \lambda_4)v_1^2.
\end{aligned}
\tag{9.2}
$$

The masses of the various scalars are easily computed. As expected, the two pseudoscalars are massless: one is the would-be Goldstone boson "eaten" by the Z, I_Z. The other, I_G, is a true Goldstone boson (the axion), due to the spontaneous breaking of $U(1)_G$.

Of the two charged pairs, one is the would-be Goldstone boson "eaten" by the W, $H_W^{\pm}$. The other has a mass at the EW breaking scale:

$$
M^2(H^{\pm}) = -\tfrac{1}{2}\lambda_4 v^2, \tag{9.3}
$$

where $v^2 = v_1^2 + v_2^2$ is the sum of the squares of the scalar VEVs. Note that H^* would be massless if $\lambda_4 = 0$; in this case the global symmetry is $SU(2)_G$, as the Lagrangian is invariant under independent $SU(2)$ rotations of Φ_1 and Φ_2. In this case, the massless $H^{\pm}$ is the charged Goldstone boson of the spontaneously broken $SU(2)_G$.

For the two neutral scalars, we find the following mass matrix (in the basis $\{R, R'\}$):

$$
M^2 = \frac{1}{v^2}\begin{pmatrix} 2(\lambda_1 v_1^4 + \lambda_2 v_2^4 + \lambda_+ v_1^2 v_2^2) & (2\lambda_2 - \lambda_+)v_2^3 v_1 - (2\lambda_1 - \lambda_+)v_1^3 v_2 \\ (2\lambda_2 - \lambda_+)v_2^3 v_1 - (2\lambda_1 - \lambda_+)v_1^3 v_2 & 2(\lambda_1 + \lambda_2 - \lambda_+)v_1^2 v_2^2 \end{pmatrix} \tag{9.4}
$$

where $\lambda_+ \equiv \lambda_3$ +X4. The two neutral scalars have their masses at the EW breaking scale. In general, there is a substantial mixing between R and R'. This may lead to significant Higgs induced FCNC, as they are not suppressed by any small parameter. In such a case, it may be necessary to invoke the natural flavor conservation (NFC) mechanism [28]. As Φ_1 and Φ_2 have different $U(1)_G$ charges, it may well be that the quark assignments under $U(1)_G$ are such that there is automatic NFC. ***The same global symmetry that led to the additional light scalars may lead to NFC.***

Note that if we take the two VEVs to significantly differ from each other, say $v_2^2 << v_1^2$, then one of the neutral scalars, which we denote by H_G, is much lighter than Λ_{EW}:

$$M^2(H_G) \simeq \left[2\lambda_2 - \frac{\lambda_+^2}{2\lambda_1}\right] v_2^2. \tag{9.5}$$

This is because v_2 now signifies the $U(1)_G$ breaking scale, Λ_G. As in the $U(1)_G$ symmetry limit the scalar H_G and the pseudoscalar I_G are degenerate, mass splitting between them cannot be larger than $O(\Lambda_G^2)$.

To summarize:

a. When the breaking of a continuous global symmetry is due to non-singlets, the whole multiplet to which the Goldstone boson belongs is light, namely at or below Λ_{EW}.

b. There may be "dangerous" FCNC. However, the additional symmetry may naturally lead to NFC.

c. If the scale of the symmetry breaking is much below the EW breaking scale, then there is at least one very light neutral scalar, in addition to the Goldstone boson.

The existence of the light scalar doublet is, in this model, a signature of an additional symmetry [$U(1)_G$] which is maintained to a low energy scale. However, a weaker condition will suffice to guarantee the lightness of this additional doublet. Consider the ***discrete*** symmetry D:

$$D: \Phi_2 \rightarrow -\Phi_2 \tag{9.6}$$

(or equivalently $\Phi_1 \rightarrow -\Phi_1$). This symmetry requires $m_{12}^2 = \lambda_6 = \lambda_7 = 0$ in the potential of eq. (4.1). A λ_5-term, which explicitly breaks the $U(1)_G$ symmetry, is allowed. Note that the symmetry D is spontaneously broken at the electroweak scale. If we compute the scalar spectrum, we discover that all five Higgs degrees of freedom are light (of order Λ_{EW}). The neutral pseudoscalar is no longer a massless Goldstone boson; its mass is $M^2(A) = -\frac{1}{2}\lambda_5 v^2$. Otherwise, the features discussed above remain intact. For example, the quark assignments under D may be such that there is automatic NFC.

In section 4 we mentioned that two fine-tunings are required to maintain the EW symmetry to low energy. Equivalently, one may say that these two fine-tunings are required to make one Higgs doublet light. In order to make the *two* scalar doublets light, one needs to fine-tune all three dimensionful parameters in the potential of eq. (4.1). Any additional symmetry which gives $m_{12}^2 = 0$ will do just that. As long as we allow only ***minimal*** fine-tuning, a second light doublet is a signature of a spontaneously broken symmetry which contains the symmetry D of eq. (9.6).

In all cases discussed in this section, the global symmetries (whether continuous or discrete) imply that there are more than one scalar with mass-squared of the form $\lambda\Lambda_{EW}^2$ (even when $\Lambda_{NP} >> \Lambda_{EW}$). Thus, in contrast to the scenario described in section 7, when any of these scalars has a mass much larger than Λ_{EW}, tree-level unitarity is violated. (For a particular example, see ref. [29].)

10. Generalization to Higher Multiplets

How do our results generalize to the case of an arbitrary number of various multiplets? To answer this question, we need to introduce an additional ingredient into our analysis: we assume that ***all scalars which are neither singlets nor doublets carry VEVs which are either very small (compared to Λ_{EW}) or zero.*** This assumption is strongly supported by the experimental determination of the ρ parameter [30]: $\rho = 0.998 \pm 0.0086$. The proximity of ρ to 1 generally implies that the VEVs of various multiplets other than the usual doublets are, at least, an order of magnitude smaller than v. There could be some complicated scenarios where various multiplets have VEVs of $O(v)$ and still give $\rho = 1$ [31] (due to a so-called "custodial" $SU(2)$symmetry), but we assume that none of these exotic scenarios holds.

The generalization of our results to scalar multiplets $\Delta(T, Y)$ (T is the weak isospin and Y is the hypercharge) with vanishing VEVs is simple:

a. The minimum conditions do not constrain the dimensionful parameter m_Δ^2 of the quadratic term $m_\Delta^2\Delta^\dagger\Delta$. By our assumption $m_\Delta^2 = O(\Lambda_{NP}^2)$. Consequently, the masses of these scalars are heavy.

b. In the EW symmetry limit all members of such a multiplet are degenerate. Consequently, mass differences within a multiplet are $O(\Lambda_{EW}^2)$.

c. With the one exception of a triplet with hypercharge Y = -1 which may couple to neutrinos, all higher multiplets do not couple to the known fermions.

d. The unitarity bound on the Higgs mass remains unaffected, since there is no mixing of the higher multiplet with the doublet Higgs boson.

e. The Linde-Weinberg bound gets contributions of a form similar to those from additional doublets: 'one may repeat the analysis of section 8, with A replacing Φ'. This result applies also to gauge-singlet scalars.

A more complicated case is that of higher multiplets with small but non vanishing VEVs. In this case, trilinear couplings play an essential role. To sketch the general argument, we study a model with a doublet $\Phi(1/2, 1/2)$ and a triplet A(l,O). We use the following simplified Higgs potential for the neutral fields ϕ and δ:

$$v = \tfrac{1}{2}m_1^2\phi^*\phi + \tfrac{1}{2}m_2^2\delta^2 - \tfrac{1}{2\sqrt{2}}m_{12}\phi^*\phi\delta + \tfrac{1}{4}\lambda_1(\phi^*\phi)^2 + \tfrac{1}{4}\lambda_2\delta^4. \quad \textbf{(10.1)}$$

We denote $l \equiv$ (6). The minimum conditions are:

$$\begin{aligned} m_1^2 &= \tfrac{1}{\sqrt{2}}m_{12}l - \lambda_1 v^2, \\ m_2^2 &= \tfrac{1}{2\sqrt{2}}m_{12}\frac{v^2}{l} - \lambda_2 l^2. \end{aligned} \quad \textbf{(10.2)}$$

Of the three dimensionful parameters, only m_1^2 has to be fine-tuned to satisfy the EW symmetry below Λ_{NP}. Taking $m_2^2 = O(\Lambda_{NP}^2)$ and $m_{12} = O(\Lambda_{NP})$ we get:

$$l \approx \frac{m_{12}v^2}{2\sqrt{2}m_2^2} = O(\Lambda_{EW}^2/\Lambda_{NP}) \ll v. \quad \textbf{(10.3)}$$

The p-parameter constraint is automatically satisfied for large enough Λ_{NP}.

The mass matrix for the neutral scalars gives, to leading order,

$$M^2(\delta) = m_2^2 = O(\Lambda_{NP}^2) \quad \textbf{(10.4)}$$

and mixing between the doublet and the triplet at $O(\Lambda_{EW}^2/\Lambda_{NP}^2)$. The triplet is indeed heavy and has only small mixing with the doublet. This mixing affects fermionic interactions: the Yukawa couplings of scalars which are dominantly doublets may have a universal correction of $O(l^2/v^2)$, where ***1*** is the VEV of a Higgs non-doublet. The unitarity bound and the Linde-Weinberg bound on $M^2($ ***h)*** are still identical to those on the mass of the single Higgs in the minimal SM, up to corrections of 0(Λ_{EW}^2 $/\Lambda_{NP}^2)$.

The existence of the trilinear coupling was crucial to the above argument. If we had put $m_{12} = 0$, then m_2^2 would have been constrained by the minimum conditions, implying:

$$M^2(\delta) = O(l^2), \tag{10.5}$$

which is much lighter than the EW breaking scale. This result is easy to understand: when $m_{12} = 0$ there is an additional U(1) symmetry, $A \rightarrow e^{i\alpha}\Delta$. This symmetry is spontaneously broken by the VEV of A. The mass splitting between the-neutral scalar and the Goldstone boson is $O(l^2)$, which is just the result in eq. (10.5).*

This demonstrates the generalization of our discussion of spontaneously broken global symmetries to higher multiplets: as long as there is no symmetry which protects the scalar masses, they will be of the order of the highest scale in the model. The existence of light scalars is associated with spontaneously broken global symmetries. If the breaking is due to a non-singlet then, (in addition to a massless Goldstone boson in the continuous case) there exists a light neutral scalar. Its mass is of the order of the global symmetry breaking scale. Other members of the multiplet have their masses at the EW breaking scale.

The spontaneous breaking of a global continuous symmetry by a non-singlet may have important phenomenological implications [32]. The Z-boson will decay into the Goldstone boson and the neutral scalar, if the latter is light enough. If the scalar multiplet is not a doublet, then the contribution is to the ***invisible*** width of the Z. The recent measurement of this quantity in the SLC and LEP experiments [33] excludes such a global symmetry breaking by multiplets of hypercharge ≥ 1 [32], e.g. the triplet Majoron model [34].†

$\star$ Similarly to the case discussed in section 9, a discrete symmetry which forbids the m_{12} term, namely $\delta \rightarrow -\delta$, is sufficient to guarantee the lightness of the scalar δ.

† When $SU(2)_L \otimes U(1)_Y$ is broken by a VEV of a doublet and an additional $U(1)_G$ symmetry is broken by a VEV of a single multiplet $\Delta(T \neq 0, Y = 0)$, charge conservation is necessarily violated. If the breaking of $U(1)_G$ is due to a scalar $\Delta(T, Y \neq 0)$ with $T \neq |Y|$, $T(T+1) \neq 3Y^2$, charge conservation is maintained only with fine-tuning of dimensionless scalar couplings. Together with the results of ref. [32], this implies that, modulo some exotic scenarios, one needs either singlets or doublets to spontaneously break a global symmetry. We thank E. Carlson and H. Quinn for discussions on this point.

11. The Higgs Potential and Scalar Masses in a Four Doublet SSM

Another well-known mechanism for preserving light masses for scalars is supersymmetry. Thus, for the remainder of this paper we shall concentrate on "low-energy" supersymmetric models [35]. We determine to what extent the observations of previous sections still apply and examine new features which arise in the supersymmetric case. In this section we study a supersymmetric model with four Higgs doublets and one Higgs singlet (see also ref. [36]). SUSY breaking is introduced explicitly through soft terms [37]. This relatively simple case represents well the class of models in which we are interested, namely supersymmetric models with the following properties:

a. An extended Higgs sector with an arbitrary number of various multiplets. As anomaly cancellation implies that the Higgs doublet superfields come in pairs of opposite hypercharge [38], the simplest extension (with non-singlets) of the minimal SSM is to a model with four Higgs doublet superfields.

b. Three energy scales:

(i) The EW breaking scale (Λ_{EW}), set by the VEVs of the Higgs doublets.

(ii) The SUSY breaking scale (Λ_{SB}) set by the explicit soft SUSY breaking terms.

(iii) A high energy scale of NP (Λ_{NP}), set by the VEV of the Higgs singlet.

We assume:

$$\Lambda_{NP} \gg \Lambda_{SB} \geq \Lambda_{EW}. \tag{11.1}$$

We shall examine two cases: $\Lambda_{SB} \gg \Lambda_{EW}$ and $\Lambda_{SB} \sim \Lambda_{EW}$.

We use the following notations for the superfields: Doublet superfields are denoted by $\hat{\Phi}^i_{\pm j}$. The upper index ($i = 1,2$) denotes the component of the doublet. The first sub-index (+ or $-$) denotes the hypercharge ($+\frac{1}{2}$ or $-\frac{1}{2}$). The second sub index j runs from 1 to k where k is the number of doublets of a given hypercharge ($k_+ = k_-$). We explicitly study the $k = 2$ case. The singlet superfield is denoted by $\hat{N}$. The notations for scalar fields are as in section 2, with indices matching those of the superfields. We find it useful to introduce the following definitions:

$$v_+^2 \equiv \sum_{i=1}^{k}(v_{+i})^2; \qquad v_-^2 \equiv \sum_{i=1}^{k}(v_{-i})^2. \tag{11.2}$$

Our starting point is the superpotential for the four-doublet model:

$$\begin{aligned} W = \epsilon_{ij} &\left(\lambda_{11}\hat{\Phi}^i_{-1}\hat{\Phi}^j_{+1} + \lambda_{12}\hat{\Phi}^i_{-1}\hat{\Phi}^j_{+2} + \lambda_{21}\hat{\Phi}^i_{-2}\hat{\Phi}^j_{+1} + \lambda_{22}\hat{\Phi}^i_{-2}\hat{\Phi}^j_{+1}\right)\hat{N} \\ &+\tfrac{1}{3}\lambda_N\hat{N}^3 - \mu_N^2\hat{N} \end{aligned} \tag{11.3}$$

(An $\hat{N}^2$ term can be eliminated by redefining $\hat{N}$.) The scalar potential is separated into three parts, $V = V_F + V_D + V_{SB}$. The F-terms are derived from the superpotential and the D-terms from the gauge properties of the fields in the usual way. Soft SUSY breaking is introduced by

$$\begin{aligned} V_{SB} =& m^2_{-1}\Phi^{i*}_{-1}\Phi^i_{-1} + m^2_{-2}\Phi^{i*}_{-2}\Phi^i_{-2} + m^2_{+1}\Phi^{i*}_{+1}\Phi^i_{+1} + m^2_{+2}\Phi^{i*}_{+2}\Phi^i_{+2} \\ &-\lambda_{11}A_{11}\left[\epsilon_{ij}\Phi^i_{-1}\Phi^j_{+1}N + h.c.\right] - \lambda_{12}A_{12}\left[\epsilon_{ij}\Phi^i_{-1}\Phi^j_{+2}N + h.c.\right] \\ &-\lambda_{21}A_{21}\left[\epsilon_{ij}\Phi^i_{-2}\Phi^j_{+1}N + h.c.\right] - \lambda_{22}A_{22}\left[\epsilon_{ij}\Phi^i_{-2}\Phi^j_{+2}N + h.c.\right]. \end{aligned} \tag{11.4}$$

The high energy scales are:

$$\begin{aligned} &\Lambda_{NP} \sim n, \\ &\Lambda_{SB} \sim max\{m_{\pm i}, (A_{ij}n)^{1/2}\}. \end{aligned} \tag{11.5}$$

Note that the dimensionful parameters A_{ij} are not $O(\Lambda_{SB})$ but rather $A_{ij} = O(\Lambda^2_{SB}/\Lambda_{NP})$. If we take the explicit SUSY breaking parameters A_{ij}, $m^2_{\pm i} \to 0$, there is still a supersymmetric minimum:

$$V_{min}(A_{ij} = 0,\ m^2_{\pm i} = 0,\ v_{\pm i} = 0,\ n \neq 0) = 0 \tag{11.6}$$

for $\mu_N^2 = \frac{1}{2}\lambda_N n^2$. This enables us to study the case where SUSY is broken at an ***intermediate*** scale. The calculation of the conditions at the minimum and the mass matrices for the scalars is straightforward but cumbersome. We give only the final results to leading order in $v^2_{\pm i}/n^2$.

There are five neutral scalars. Three of them have their masses at the high energy scale $n = O(\Lambda_{NP})$:

$$
\begin{aligned}
M^2(H_4) &= 2\lambda_N^2 n^2, \\
M^2(H_3) &= -\tfrac{1}{2}(\lambda_{11}\lambda_{21} + \lambda_{12}\lambda_{22})\left(\frac{v_-^2}{v_{-1}v_{-2}}\right) n^2, \\
M^2(H_2) &= -\tfrac{1}{2}(\lambda_{11}\lambda_{12} + \lambda_{21}\lambda_{22})\left(\frac{v_+^2}{v_{+1}v_{+2}}\right) n^2.
\end{aligned}
\tag{11.7}
$$

In the basis $\{R_{-1}, R_{-2}, R_{+1}, R_{+2}, R_N\}$, the respective eigenvectors are:

$$
H_4 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}; \quad H_3 = \frac{1}{v_-}\begin{pmatrix} v_{-2} \\ -v_{-1} \\ 0 \\ 0 \\ 0 \end{pmatrix}; \quad H_2 = \frac{1}{v_+}\begin{pmatrix} 0 \\ 0 \\ v_{+2} \\ -v_{+1} \\ 0 \end{pmatrix}. \tag{11.8}
$$

The minimum conditions show that

$$
M^2(H_2) - M^2(H_3) = O(\Lambda_{SB}^2). \tag{11.9}
$$

One neutral scalar has its mass in the intermediate scale Λ_{SB}:

$$
H_1 = \frac{1}{V}\begin{pmatrix} -\frac{v_+}{v_-}v_{-1} \\ -\frac{v_+}{v_-}v_{-2} \\ \frac{v_-}{v_+}v_{+1} \\ \frac{v_-}{v_+}v_{+2} \\ 0 \end{pmatrix}; \quad M^2(H_1) = \frac{1}{\sqrt{2}}\left(\frac{v^2}{v_+^2 v_-^2}\right)\sum_{i,j}\lambda_{ij}v_{-i}v_{+j}A_{ij}n. \tag{11.10}
$$

One neutral scalar remains massless (in the limit $v_{\pm i} \to 0$):

$$
h = \frac{1}{v}\begin{pmatrix} v_{-1} \\ v_{-2} \\ v_{+1} \\ v_{+2} \\ 0 \end{pmatrix}; \quad M^2(h) = 0. \tag{11.11}
$$

When we "switch on" the various $v_{\pm i}$, the above vectors are no longer the mass eigenstates {h, H_i} but rather the {R, R_i'} states. In particular, The lightest scalar

h has a mass of $O(v^2)$. Its dominant component is the R-field, but it has components of R_1' at order $\Lambda_{EW}^2/\Lambda_{SB}^2$ and of the heavy scalars at order $\Lambda_{EW}^2/\Lambda_{NP}^2$. In the case $\Lambda_{SB} \sim \Lambda_{EW}$ there may be a substantial mixing between R and R_1'.

There are four neutral pseudoscalars. Three of them have thkir masses at the high energy scale Λ_{NP} and one at the intermediate scale Λ_{SB}. In the $v_{\pm i} \to 0$ limit:

$$M^2(A_i) = M^2(H_i) \; for \; i = 1,2,3,4. \tag{11.12}$$

The A_2, A_3 and A_4 fields are given in the $\{I_{-1}, I_{-2}, I_{+1}, I_{+2}, I_N\}$ basis by expressions similar to eq. (11.8). The A_2 and A_3 fields are degenerate to $O(\Lambda_{SB}^2/\Lambda_{NP}^2)$. The A_1 field is given by:

$$A_1 = \frac{1}{v}\begin{pmatrix} \frac{v_+}{v_-}v_{-1} \\ \frac{v_+}{v_-}v_{-2} \\ \frac{v_-}{v_+}v_{+1} \\ \frac{v_-}{v_+}v_{+2} \\ 0 \end{pmatrix}. \tag{11.13}$$

There are three pairs of physical charged--scalars. Two of them are heavy and" degenerate to 0($\Lambda_{SB}^2/\Lambda_{NP}^2$). 0 ne pair has mass at the intermediate scale. In the limit $v^2 \to 0$:

$$M^2(H_i^{\pm}) = M^2(H_i) \; for \; i = 1,2,3. \tag{11.14}$$

The corresponding eigenvectors are (in the basis $\{\phi_{-1}^+, \phi_{-2}^+, \phi_{+1}^+, \phi_{+2}^+\}$):

$$H_3^+ = \frac{1}{v_-}\begin{pmatrix} v_{-2} \\ -v_{-1} \\ 0 \\ 0 \end{pmatrix}; \quad H_2^+ = \frac{1}{v_+}\begin{pmatrix} 0 \\ 0 \\ v_{+2} \\ -v_{+1} \end{pmatrix}; \quad H_1^+ = \frac{1}{v}\begin{pmatrix} \frac{v_+}{v_-}v_{-1} \\ \frac{v_+}{v_-}v_{-2} \\ \frac{v_-}{v_+}v_{+1} \\ \frac{v_-}{v_+}v_{+2} \end{pmatrix}. \tag{11.15}$$

To conclude, the four doublets in our model give thirteen physical scalars. One neutral scalar is light $[O(\Lambda_{EW}^2)]$. The other scalars are divided to groups of four: each consists of a neutral scalar, a neutral pseudoscalar and a charged pair. One such group has its mass at Λ_{SB} . Its members are degenerate to $O(\Lambda_{EW}^2/\Lambda_{SB}^2)$. The other groups are heavy, namely with masses at Λ_{NP}. The members of each group are degenerate to $0(\Lambda^2_{EW}/\Lambda^2_{NP})$. The groups divide into pairs, each pair of groups is degenerate to $0(\Lambda^2_{SB}/\Lambda^2_{NP})$.

In SUSY models with a high energy scale, the full content of ***two*** Higgs doublets is "protected" from becoming heavy. ***The low energy spectrum** (below **or at the scale of** $SUSY$ **breaking) is just the spectrum of the minimal SSM.*** On the other hand, if $\Lambda_{SB} = O(\Lambda_{NP}) \gg \Lambda_{EW}$, then we recover the minimal non-supersymmetric Higgs sector, in accordance with the results of section 4.

12. Yukawa Interactions in the SSM

Our purpose in this section is to explain how scalar-mediated FCNC are suppressed in a multi-scalar SUSY framework. We study the four doublet model of the previous section, along the lines of our study of the SM in section 5.

The Yukawa interactions in the four doublet SUSY model are given by

$$-L_Y = \overline{\Psi^0_{Li}}\left(F_{ij}\Phi_{-1} + F'_{ij}\Phi_{-2}\right)U^0_{Rj} + \overline{\Psi^0_{Li}}\left(G_{ij}\Phi_{+1} + G'_{ij}\Phi_{+2}\right)D^0_{Rj} + h.c. \quad (12.1)$$

As before, we simplify the analysis by assuming CP symmetry, in which case the F and G matrices are real. The quark mass matrices are given by:

$$\begin{aligned} M_u &= \sqrt{\tfrac{1}{2}}\left(Fv_{-1} + F'v_{-2}\right), \\ M_d &= \sqrt{\tfrac{1}{2}}\left(Gv_{+1} + G'v_{+2}\right). \end{aligned} \quad (12.2)$$

We now rotate to the basis $\{\Phi, \Phi'_i\}$:

$$\begin{pmatrix} \tilde{\Phi}_{-1} \\ \tilde{\Phi}_{-2} \\ \Phi_{+1} \\ \Phi_{+2} \end{pmatrix} = \begin{pmatrix} \frac{v_{-1}}{v} & -\frac{v_+ v_{-1}}{v_-} & 0 & \frac{v_{-2}}{v_-} \\ \frac{v_{-2}}{v} & -\frac{v_+ v_{-2}}{v_-} & 0 & -\frac{v_{-1}}{v_-} \\ \frac{v_{+1}}{v} & \frac{v_- v_{+1}}{v_+} & \frac{v_{+2}}{v_+} & 0 \\ \frac{v_{+2}}{v} & \frac{v_- v_{+2}}{v_+} & -\frac{v_{+1}}{v_+} & 0 \end{pmatrix} \begin{pmatrix} \Phi \\ \Phi'_1 \\ \Phi'_2 \\ \Phi'_3 \end{pmatrix}. \quad (12.3)$$

The Yukawa interactions of the neutral scalars are given by:

$$\begin{aligned} \mathcal{L}^U_Y = \sum_{q=u,c,t} \left(\frac{m_q}{v}\right) & \left[R\bar{q}q - \left(\frac{v_+}{v_-}\right) R'_1\bar{q}q - i\left(\frac{v_+}{v_-}\right) I'_1\bar{q}\gamma_5 q\right] \\ & + \left[\overline{U^0_{Li}}\left(F_{ij}\frac{v_{-2}}{\sqrt{2}v_-} - F'_{ij}\frac{v_{-1}}{\sqrt{2}v_-}\right)(R'_3 + iI'_3)U^0_{Rj} + h.c.\right] \end{aligned} \quad (12.4)$$

$$
\begin{aligned}
-\mathcal{L}_Y^D = \sum_{q=d,s,b} \left(\frac{m_q}{v}\right) & \left[R\bar{q}q + \left(\frac{v_-}{v_+}\right) R_1'\bar{q}q + i\left(\frac{v_-}{v_+}\right) I_1'\bar{q}\gamma_5 q\right] \\
& + \left[\overline{D_{Li}^0}\left(G_{ij}\frac{v_{+2}}{\sqrt{2}v_+} - G_{ij}'\frac{v_{+1}}{\sqrt{2}v_+}\right)(R_2' + iI_2')D_{Rj}^0 + h.c.\right]
\end{aligned}
\tag{12.5}
$$

where q are quark ***muss*** eigenstates. The scalars $\boldsymbol{R}$ and R_1' couple diagonally in flavor space. Moreover, their couplings are the same as those of the two neutral scalars in the minimal SSM. They mix with each other (to form the light scalar $\boldsymbol{h}$ and the intermediate scalar H_1) at 0($\Lambda_{EW}^2/\Lambda_{SB}^2$). However, any combination of $\boldsymbol{R}$ and R_1' couples diagonally in flavor space. The fields $\boldsymbol{h}$ and H_1 have components of R_2' and R_3' (and thereafter non-diagonal couplings) only at 0($\Lambda_{EW}^2/\Lambda_{NP}^2$). The pseudoscalar I_1' also couples diagonally in flavor space, with identical couplings to those of the single pseudoscalar in the minimal SSM. The mass eigenstate A_1 has components of I_2' and I_3' only at 0(Λ_{EW}^2/ A",,).

We conclude that ***the masses-and the Yukawa interactions of the light scalar h and* of *the intermediate neutral scalar H_1 and pseudoscalar A_1 are identical to those of the three neutral scalars of the minimal SSM, up to effects* of *order*** $(\Lambda_{EW}^2/\Lambda_{NP}^2)$. In particular, this result holds in the case where $\Lambda_{SB} \sim \Lambda_{EW}$. All.FCNC effects. induced by scalar exchange are suppressed by inverse powers of Λ_{NP}, in accordance with our previous results (see section 5).

We caution the reader that in supersymmetric theories, other FCNC effects can arise which must be controlled in order to have a phenomenologically viable model. For example, off-diagonal squark-quark-gaugino interactions can lead to observable FCNC phenomena (see, e.g. ref. [39] and references quoted therein). However, such effects are not directly related to the multiplicity of the Higgs superfields and are beyond the scope of this paper.

13. Conclusions

In this work, we have studied models with an extended Higgs sector, allowing for an arbitrary number of various multiplets. We were interested in the low-energy effects of these additional multiplets. Our basic assumption was:

a. The Standard Model is a low-energy effective theory. There exists some New Physics at an energy scale Λ_{NP} which is much larger than the Electroweak breaking scale Λ_{EW}.

In addition, we assumed the following:

b. All unknown dimensionless couplings are O(1). More specifically, we do not allow the Higgs potential parameters to be as small as $(\Lambda_{EW}/\Lambda_{NP})$, so as not to obscure the difference between the two scales.

c. The main contribution to the W and Z masses comes from VEVs of Higgs doublets. Higher multiplets- carry much smaller (or vanishing) VEVs.

Our main conclusion is the following: if the only low-energy symmetry is the EW symmetry of the SM, then there is only one light scalar. Its properties are identical to those of the single Higgs of the minimal SM, up to small corrections of order $\Lambda^2_{EW}/\Lambda^2_{NP}$. That is, effects of the additional scalar particles are suppressed by inverse powers of the high energy scale. They may induce flavor changing neutral currents which become significant if the scale is a few TeV or lower. They may provide us with several new sources of CP violation. The only source for CP violation which does not depend in magnitude on the high energy scale is the usual KM phase in the quark mixing matrix. Additional scalars contribute to the Linde-Weinberg bound. The modification of the bound can be significant only if the scale of new physics is a few TeV or less.

The existence of additional light scalars is associated with additional symmetries at low energies. The best known example is that of Supersymmetry. If the only low-energy symmetry is the supersymmetric electroweak symmetry of the SUSY SM, then there are two light neutral scalars, one light neutral pseudoscalar and one light pair of charged scalars. The properties of these scalars are identical to those of the scalars in the ***minimal*** SUSY SM, up to small corrections of order $\Lambda^2_{EW}/\Lambda^2_{NP}$. If the scale of SUSY breaking is higher than that of EW breaking, then there will be one light scalar at the EW breaking scale, with properties identical to those of the SM, as before. The other above-mentioned light scalars have masses at the SUSY breaking scale. If the two scales are similar, then none of the two light neutral scalars is equivalent to the single Higgs of the minimal SM. However, both couple diagonally in flavor space.

Other cases in which we have additional light scalars are theories which possess spontaneously broken global symmetries. If this symmetry is continuous, then there is at least one massless Goldstone boson. If the symmetry (either continuous or discrete) is broken by a VEV for a Higgs non-singlet, there will be additiona. light scalars. If the additional Higgs multiplet is not a doublet, then there is a neutral scalar which is much lighter than the EW breaking scale. However, such a. possibility is highly unlikely in light of the recent SLC/LEP measurements of the number of light neutrino flavors.

In the case of no spontaneously broken global symmetries, all FCNC effects due to additional scalars are suppressed by the high energy scale of new physics. In these cases, there is no need to impose Natural Flavor Conservation (NFC) in the low-energy theory. If there are several light doublets as a result of a spontaneouslp. broken global symmetry, then these doublets necessarily do not all carry the same quantum numbers under the additional symmetry. Consequently, it is likely that the same symmetry that protects the scalars from becoming heavy, leads to NFC. In other words, in models where the EW symmetry is embedded in larger symmetry group, NFC is likely to be either unnecessary or automatic.

Most experimental Higgs searches are based on either the Minimal SM or the Minimal SUSY SM. We conclude that these two models are not just toy models where calculations are simple and free of ambiguities; they indeed represent a much larger class of models, including LRS and GUT frameworks. If experiments find a light scalar sector which is richer or different from either of these minimal models, it is an important clue to the existence of New Physics and additional symmetries at low energies.

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