# A Finite Zitterbewegung Model for Relativistic Quantum Mechanics* 

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#### Abstract

Starting from steps of length $h / m c$ and time intervals $h / m c^{2}$, which imply a quasi-local Zitterbewegung with velocity steps $\pm c$, we employ discrimination between bit-strings of finite length to construct a necessarily $3+1$ dimensional eventspace for relativistic quantum mechanics. By using the combinatorial hierarchy to label the strings, we provide a successful start on constructing the coupling constants and mass ratios implied by the scheme. Agreement with experiment is surprisingly accurate.


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## 1. INTRODUCTION

Quantum mechanics, including relativistic quantum mechanics and second quantized field theory, is usually thought to require an infinite dimensional Hilbert space and complex amplitudes for its mathematical articulation. Since neither are obviously present in the laboratory in any easily recognizable form, realistically inclined physicists tend to distrust the formalism without having any generally acceptable way to replace it. Russell once described the relationship between an ordinary man and his philosophy: he is in the same situation as a man who is stuck with an old wife whom he has ceased to love and knows no acceptable way of getting rid of, yet will let no outsider cast aspersions on her. Many practicing physicists face a similar dilemma when confronted with the foundations of quantum mechanics.

Since quantum mechanics involves discretc and countable (hence in fact noninfinite) phenomena such as mass values, energy levels, conserved quantum numbers and the like, one attractive possibility would appear to be replacing the continuum structure of conventional quantum mechanics by a rigidly finite and discrete theory. As a consequence of a research program started by Bastin and Kilmister over thirty years ago, such a theory now exists ${ }^{[1]}$ and has found a mathematical basis in the ordering operator calculus drawn from computer science ${ }^{[2]}$.

A practical starting point for replacing the continuum assumptions of elementary particle physics by a finite and discrete basis is to assume that all changes in spacial "position" are quantized with the Lorentz-invariant step length $h / m c$ and that all changes in temporal ordering are quantized with a Lorentz-invariant time interval of $h / m c^{2}$. This model, pioneered by Stein ${ }^{[3-6]}$, clearly implies a quasi-local Zitterbewegung with velocity changes quantized to $\pm c$. This model is developed here, following lines laid out this year ${ }^{[7,8]}$.

There are two major questions which occur immediately to physicists with a continuum bent: what happens to Lorentz invariance and what happens to rotational invariance? Obviously both are "broken" if the standard of comparison is
the continuum theory. The question for a physicist is rather, are there observable consequences of this symmetry breaking?

Take Lorentz invariance first. Clearly there is no way to show that a velocity is exactly zero or exactly c by measurement. The best we can do is to say that in particular experimental circumstances the velocity parametcr $\beta=V / c$ lics in the range $\frac{1}{N_{\min }} \leq \beta \leq 1-\frac{1}{N_{\max }}$. So we need make Lorentz transformations only for rational fraction velocity parameters within these limits. As we have discussed elsewhere ${ }^{[1,2]}$, for the interval between two events at $(0,0)$ and $(z, t)$, defining $\beta=z / c t$, the interval $\tau^{2}=c^{2} t^{2}-z^{2}=(c t+z)(c t-z)$ is invariant under the transformation $\left(c t^{\prime}+z^{\prime}\right)=\rho(c t+z),\left(c t^{\prime}-z^{\prime}\right)=\rho^{-1}(c t-z)$, which is equivalent to the Lorentz transformation

$$
\begin{equation*}
z^{\prime}=\gamma_{\rho}\left(z+\beta_{\rho} c t\right) ; c t^{\prime}=\gamma_{\rho}\left(c t+\beta_{\rho} z\right) ; \gamma_{\rho}^{2}=\frac{1}{2}\left(\rho+\rho^{-1}\right) ; \beta_{\rho}^{2}=1-1 / \gamma_{\rho}^{2} \tag{1.1}
\end{equation*}
$$

Note that the square root implied by the definition of $\gamma_{\rho}$ must be represented by a rational fraction in our discrete theory; the appropriate value can be arrived at using the experimental limits already assumed. Consequently any model which generates rational fraction velocities in such a way that the empirical results cannot show the discreteness by integer counts in units of $h / m_{p} c$ and/or times in units of $h / m_{p} c^{2}$ will be indistinguishable in its observable consequences from a continuum theory, so far as the direct consequences of Lorentz invariance are concerned.

Rotational invariance is another matter. It is already broken in the conventional theory by angular momentum quantization. For a system with angular momentum $j \hbar$, where $j$ takes integer or half-integer values we can only define $2 j+1$ discrete angles. As Hans-Christian Pauli keeps reminding us, total angular momentum is not Lorentz invariant, so we will have no need to derive it. Once we have produced a model in which we get integer and half-integer components of angular momentum consistently with our step-length quantization, and can relate the implied discrete rotations to our discrete Lorentz transformations, we are through with the construction. We believe that it is up to the continuum theorists to justify rotational
invariance in the face of angular momentum quantization. It is their problem, not ours.

The theory we employ is based on ordered finite strings of the symbols " 0 " and " 1 " which combine under exclusive or (XOR), i.e. "bit-strings". They were originally introduced into the theory for quite different reasons than the current application. It was subsequently discovered that they have the requisite properties to model both the Zitterbewegung with which we start and the quantum numbers - of the standard model of quarks and leptons. This is less surprising than it might seem at first sight, since Bastin's philosophy all along was that "space", "time" and "particles" have to be constructed together, and bit-strings were introduced with that objective in mind.

Bit-strings neatly take care of both properties we have already seen that we need as modeling elements. A string of length $n$ will have $k$ " 1 "'s with $0 \leq k \leq n$. It can be used to define a velocity parameter on the interval $(-1,1)$ by taking $\beta=\frac{2 k}{n}-1$. This fact has a much deeper significance; it comes from the definition of attribute velocity in the ordering operator calculus ${ }^{[2]}$. If we have two distinct strings with $k_{1}+k_{2}<n$, when they combine by exclusive or to form a third string it will have a value of $k$ in the range $\left|k_{1}-k_{2}\right| \leq k \leq k_{1}+k_{2}$, precisely the correct limits for the addition of angular momenta. We can base our model on bit-strings with confidence that we can construct these critical aspects of relativistic quantum mechanics. All we need from the fundamental theory at this stage is the fact that it generates a sufficiently large universe of bit-strings which, for our initial purposes, are arbitrary both as to length and composition.

## 2. CONSTRUCTION OF COMPLEX WAVE FUNCTIONS FROM BIT-STRINGS

### 2.1. Why complex amplitudes?

Examination of the foundational ideas ${ }^{[2]}$ needed to construct a finite and discrete relativistic quantum mechanics from bit-strings ${ }^{[1]}$ led McGoveran to the conclusion that the non-classical statistics in quantum mechanics (eg. complex probability amplitudes rather than real probabilities) can be modeled in any system whose multiple paths between two "events" share indistinguishable elements. Since he discussed this at ANPA $11^{[9]}$ and also here at ANPA WEST 6 in the talk just prior to mine ${ }^{[10]}$ I refer you to his papers for details.

Consider the standard double slit experiment shown in figure 1. In both cases we make two calibration runs with one or the other slit blocked, and then open both. When we have insured that the particle goes through one slit or the other by the firing of a counter, the number of paths - which is proportional to the number of counts recorded - in the two calibration runs, $P_{1}$ or $P_{2}$, allow us to predict the outcome when both slits are open and the particles go through one at a time (i.e. $P_{12}=0$ ) to be simply the sum of the two individual cases, $P_{1}+P_{2}=P$. This prediction is verified experimentally. Neglecting corrections due to the structure of the counters which I have discussed elsewhere ${ }^{[11]}$, the two calibration runs in which the counters do not fire give a distribution which is the same as in the first case, i.e. $P_{\overline{1}} / P_{1}=P_{\overline{2}} / P_{2}$. However when both slits are open, $P_{\overline{1} \overline{2}} \neq P_{\overline{1}}+P_{\overline{2}}$ ! In fact experiments give the double slit interference pattern characteristic of wave motion, where the difference in path lengths can be computed using $\lambda=h / p$, with $p=\gamma \beta m c$.

We assume, as will be the case in our model, that the two paths are independently generated and hence define a joint probability space with $P_{\overline{1}} P_{\overline{2}}$ elements. A convenient way to parameterize the situation is to write $P_{\overline{1}}^{2}+P_{\overline{2}}^{2}=P^{2}-2 P_{\overline{1}} P_{\overline{2}}^{2} \equiv$ $R_{12}^{2}$, which is identically satisfied if the two paths simply add. If, due to indistinguishable paths which we do not know how to assign to either $P_{\overline{1}}$ or $P_{\overline{2}}$, we
have indeed made the two independent in the sense that the product $P_{\overline{1}} P_{\overline{2}}$ is no longer constrained other than by the inequality $2 P_{1} P_{\overline{2}}<P^{2}$, we can adopt $R_{12}^{2}$ as the measure of the square of the number of paths in this new space. Taking the product $2 P_{\overline{1}} P_{\overline{2}}=f^{2} P^{2}$ where $f$ is some rational fraction less than unity, we thus arrive at the general result

$$
\begin{equation*}
P_{\overline{1}}^{2}+P_{\overline{2}}^{2}=R_{12}^{2}=P^{2}(1-f)(1+f) \tag{2.1}
\end{equation*}
$$

which has been derived by McGoveran by considering case counts including indistinguishables. We can now define

$$
\begin{equation*}
\psi=P_{\overline{1}}+i P_{\overline{2}} \tag{2.2}
\end{equation*}
$$

with the normalization condition

$$
\begin{equation*}
\psi^{*} \psi=R^{2} \tag{2.3}
\end{equation*}
$$

Clearly we can divide $\psi$ by $R$ to get the normalization condition $\psi^{*} \psi=1$ when we are modeling the situation in which a single system engages in the two events at the two endpoints with certainty. Once we have this general result, it is simply a matter of mathematical convenience whether we use real or complex amplitudes to model this constraint, and norm it to unity when the probability of the system traversing the "space" between the two events is unity.

### 2.2. Bit-Strings

We specify a bit-string

$$
\begin{equation*}
\mathbf{X}(S)=\left(\ldots, b_{i}^{x}, \ldots \ldots\right)_{S} \tag{2.4}
\end{equation*}
$$

by its $S$ ordered elements

$$
\begin{equation*}
b_{i}^{x} \in 0,1 ; \quad i \in 1,2, \ldots . S ; 0,1, \ldots, S \in \text { ordinal integers } \tag{2.5}
\end{equation*}
$$

and its norm by

$$
\begin{equation*}
|\mathbf{X}(S)|=\Sigma_{i=1}^{S} b_{i}^{x}=X \tag{2.6}
\end{equation*}
$$

Define the null string by $\mathbf{0}(S), b_{i}^{0}=0$ for all $i$ and the anti-null string by $\mathbf{1}(S)$, $b_{i}^{1}=1$ for all $i$. Define discrimination (XOR) by

$$
\begin{equation*}
\mathbf{X} \oplus \mathbf{Y}=\left(\ldots, b_{i}^{x y}, \ldots\right)_{S}=\mathbf{Y} \oplus \mathbf{X} ; b_{i}^{x y}=\left(b_{i}^{x}-b_{i}^{y}\right)^{2} \tag{2.7}
\end{equation*}
$$

from which it follows that

$$
\begin{equation*}
\mathbf{A} \oplus \mathbf{A}=\mathbf{0} ; \mathbf{A} \oplus \mathbf{0}=\mathbf{A} \tag{2.8}
\end{equation*}
$$

We will also find it useful to define

$$
\begin{equation*}
\overline{\mathbf{A}}=\mathbf{A} \oplus \mathbf{1} ; \text { hence } \mathbf{A} \oplus \overline{\mathbf{A}} \oplus \mathbf{1}=\mathbf{0} \tag{2.9}
\end{equation*}
$$

### 2.3. ONE DIMENSIONAL AMPLITUDES

- Consider two independently generated strings $\mathbf{A}(S), \mathbf{B}(S)$ restricted by $|\mathbf{A} \oplus \mathbf{B}|=n$ and $A-B=c$. We call these the boundary conditions. We now construct two substrings $\mathbf{a}(n), \mathbf{b}(n)$ by the following recursive algorithm starting from $i, j=0$ and ending at $i=S, j=n$.

$$
\begin{gathered}
i:=i+1 \\
\text { if } b_{i}^{A}=1 \text { and } b_{i}^{B}=0 \text { then } j:=j+1 \text { and } b_{j}^{a}:=1 \text { and } b_{j}^{b}:=0 \\
\text { if } b_{i}^{A}=0 \text { and } b_{i}^{B}=1 \text { then } j:=j+1 \text { and } b_{j}^{a}:=0 \text { and } b_{j}^{b}:=1 \\
\text { if }\left(b_{i}^{A}-b_{i}^{B}\right)^{2}=0 \text { then } j, b_{j}^{a} \text { and } b_{j}^{b} \text { do not change }
\end{gathered}
$$

Once we have made this construction,

$$
\begin{equation*}
\mathbf{a}(n) \oplus \mathbf{b}(n) \oplus \mathbf{1}(n)=\mathbf{0}(n) \tag{2.10}
\end{equation*}
$$

and we can interpret the string a as representing a "random walk" in which a " 1 " represents a step forward and a " 0 " represents a step backward, as in the Stein ${ }^{[3-6]}$
paradigm. Define

$$
\begin{equation*}
a_{j}=\Sigma_{k=1}^{j} b_{k}^{a} ; \quad b_{j}=\Sigma_{k=1}^{j} b_{k}^{b} \tag{2.11}
\end{equation*}
$$

We call the "points" $\left(a_{j}-b_{j}, j\right)$ connecting $(0,0)$ to $(c, n)$ a trajectory; the new ordering parameter $j$ then represents "causal" time order along the trajectory. Note that $a+b=n$ and $a-b=A-B=c$ for any trajectory because of our boundary conditions.

We can also define a $p a t h$ in the larger space $s_{i}, A_{i}, B_{i}$ where

$$
\begin{gather*}
s_{i}=\Sigma_{k=1}^{i} s_{k}=\Sigma_{k=1}^{i} b_{k}^{A} b_{k}^{B} \\
s_{i}^{\prime}=\Sigma_{k=1}^{i} s_{k}^{\prime}=\Sigma_{k=1}^{i}\left(1-b_{k}^{A}\right)\left(1-b_{k}^{B}\right)  \tag{2.12}\\
A_{i}=\Sigma_{k=1}^{i} b_{k}^{A}\left(b_{k}^{A}-b_{k}^{B}\right)^{2}+s_{k} ; B_{i}=\Sigma_{k=1}^{i} b_{k}^{B}\left(b_{k}^{A}-b_{k}^{B}\right)^{2}+s_{k}
\end{gather*}
$$

Note that by construction $A_{i}-B_{i}=a_{j}-b_{j}$ and hence $A_{i}, B_{i}$ is tied to the same trajectory in the $\left(a_{j}-b_{j}, j\right)$ plane; it acquires a third "orthogonal" coordinate due to those cases when both $A_{i}$ and $B_{i}$ are incremented by 1 . Note also that there is no way from our boundary conditions or from the trajectory to tell those cases from those where $i$ advances but neither $A_{i}$ nor $B_{i}$ nor $s_{i}$ is incremented. All we know is that $s_{A B}=\Sigma_{k=1}^{S} b_{k}^{A} b_{k}^{B}$, lies in the range $0 \leq s_{A B} \leq S-n$ and that $s_{A B}^{\prime}=\Sigma_{k=1}^{S}\left(1-b_{k}^{A}\right)\left(1-b_{k}^{B}\right)=S-n-s_{A B}$. It is these indistinguishable paths which create the interfering alternatives in our model.

We now ask how many paths characterized by some ordering parameter $s=$ $0,1,2, \ldots, S-n$ satisfy our boundary conditions. By construction each path is tied to the $n$ points which compose a trajectory, and can bc chosen in $n^{s}$ ways. Note that we have broken the causal connection between path and trajectory. Of the total number of ways of choosing a path characterized by $s$ from the $S!/(S-s)$ ! possibilities, only $S!/ s!(S-s)$ ! are distinct. Consequently, the probability of having
a path characterized by $s$ is

$$
\begin{equation*}
\frac{S!/ s!(S-s)!}{S!/(S-s)!}=\frac{1}{s!} \tag{2.13}
\end{equation*}
$$

Thus the total number of paths is

$$
\begin{equation*}
P(n ; S)=\Sigma_{s=0}^{S-n} \frac{n^{s}}{s!}=\Sigma_{s=0}^{S-n} p_{s}(n) \equiv \exp _{S-n}(n) \tag{2.14}
\end{equation*}
$$

where $\exp _{S_{-n}}(n)$ is the finite exponential. This is a general result for the transport operalor referring to attribute distance as has been proved by McGoveran in FDP, Theorems 36-40, pp 55-58.

Although Eq. 2.14 specifies the total number of paths, given $S$ and $n$, it conceals a four-fold ambiguity arising from the construction. However the sequence of paths is generated, the order adopted in the sum implies a recursive generation of the terms $p_{s}(n)=n^{s} / s!$ given by

$$
\begin{equation*}
p_{s+1}(n)=n p_{s}(n) /(s+1) ; p_{0}(n)=1 \tag{2.15}
\end{equation*}
$$

The first ambiguity is the fact that we do not know whether $S-n$ is even or odd outside of the uninteresting case $S=n$ when paths and trajectories coincide; hence we do not know whether the sum terminates in an even or an odd term. The second ambiguity arises because, however $s$ is ordered, we do not know how many cases arise because both $A_{i}$ and $B_{i}$ are incremented, or neilher; in terms of the notation introduced in Eq. 2.12, we do not know whether the value we are summing over should be called $s$ or $s^{\prime}$. To include this dichotomy we split the even and odd sequences themselves into two sequences corresponding to these alternatives which we call 11 and 00 , giving four recursion relations:

$$
p_{s+4}^{e, 11}(n)=\frac{n^{4}}{(s+4)(s+3)(s+2)(s+1)} p_{s}^{e, 11}(n) ; p_{0}^{e, 11}(n)=1
$$

- 

$$
\begin{gather*}
p_{s+4}^{o, 11}(n)=\frac{n^{4}}{(s+4)(s+3)(s+2)(s+1)} p_{s}^{o, 11}(n) ; p_{1}^{o, 11}(n)=n \\
p_{s+4}^{e, 00}(n)=\frac{n^{4}}{(s+4)(s+3)(s+2)(s+1)} p_{s}^{e, 00}(n) ; p_{2}^{e, 00}(n)=\frac{1}{2} n^{2} \\
p_{s+4}^{o, 00}(n)=\frac{n^{4}}{(s+4)(s+3)(s+2)(s+1)} p_{s}^{o, 00}(n) ; p_{3}^{o, 00}(n)=\frac{1}{6} n^{3} \tag{2.16}
\end{gather*}
$$

- At some point which depends on whether (a) $S-n$ is even or odd and/or $2 s_{A B}$ is greater or less than $S-n$, this four-fold ordering of the terms in the sum over $s$ has to stop, and may or may not leave some terms unaccounted for. Calling the contribution of these terms to the sum $\Delta P$, we find that our construction allows us to decompose the sum over paths as follows:

$$
\begin{equation*}
P(n ; S)=\Sigma_{s=0}^{S-n}\left[p_{s}^{e, 11}+p_{s}^{o, 11}+p_{s}^{e, 00}+p_{s}^{o, 00}\right]+\Delta P \tag{2.17}
\end{equation*}
$$

We are now in the general situation discussed at the start of this chapter, except that our construction has provided us with four types of path rather than two. Now that we have recognized that the amplitudes - whose square gives a quantity which can be normed to form a probability - can be complex, we have no conceptual barrier to forming real combinations which can be negative as well as positive. The obvious choice is to form those which lead to the finite sines and cosines, i.e. by subtracting the two components of the odd or even series from each other:

$$
\begin{gather*}
R \cos _{S-n}(n)=R \Sigma_{k=0}^{\frac{1}{2}(S-n)}(-1)^{k} \frac{n^{2 k}}{(2 k)!}=\Sigma_{s=0}^{(S-n)}\left[p_{s}^{e, 11}-p_{s}^{e, 00}\right]  \tag{2.18}\\
R \sin _{S-n}(n)=R \Sigma_{k=1}^{\frac{1}{2}(S-n)}(-1)^{k+1} \frac{n^{2 k-1}}{(2 k-1)!}=\Sigma_{s=1}^{(S-n)}\left[p_{s}^{o, 11}-p_{s}^{n, 00}\right] \tag{2.19}
\end{gather*}
$$

The two constructions can now be combined by taking the normalized wave func-
tion to be

$$
\begin{equation*}
\psi_{S-n}(n)=\exp _{S-n}(i n)=\Sigma_{s=0}^{\frac{1}{4}(S-n)} \frac{(i n)^{s}}{s!} \tag{2.20}
\end{equation*}
$$

Thus, by taking proper account of the interference between independently generated paths which share indistinguishable elements, we claim to have derived Feynman's prescription ${ }^{[12]}$. for calculating the quantum mechanical wave function as a "sum over paths" with imaginary finite and discrete steps.

### 2.4. Construction of Space-time Coordinates

$1+1$ dimensions
In any universe of bit strings of length $S$, all quadruples such that

$$
\begin{equation*}
\mathbf{A} \oplus \mathbf{B} \oplus \mathbf{C} \oplus \mathbf{D}=\mathbf{0} \tag{2.21}
\end{equation*}
$$

are called events. Note that this implies that

$$
\begin{equation*}
\mathbf{A} \oplus \mathbf{B}=\mathbf{C} \oplus \mathbf{D} ; \mathbf{A} \oplus \mathbf{C}=\mathbf{B} \oplus \mathbf{D} ; \mathbf{A} \oplus \mathbf{D}=\mathbf{B} \oplus \mathbf{C} \tag{2.22}
\end{equation*}
$$

$$
\begin{equation*}
\mathbf{A}=\mathbf{B} \oplus \mathbf{C} \oplus \mathbf{D} ; \mathbf{B}=\mathbf{C} \oplus \mathbf{D} \oplus \mathbf{A} ; \mathbf{C}=\mathbf{D} \oplus \mathbf{A} \oplus \mathbf{B} ; \mathbf{D}=\mathbf{A} \oplus \mathbf{B} \oplus \mathbf{C} \tag{2.23}
\end{equation*}
$$

Consider an event defined by four independently generated strings $\mathbf{F}, \mathbf{B}, \mathbf{R}, \mathbf{L}$ whose norms are $F, B, R, L$; all must be less than or equal to $|\mathbf{1}|=S$. For the moment we need only define a fifth integer $n$ by

$$
\begin{equation*}
|\mathbf{F} \oplus \mathbf{B}|=n=|\mathbf{R} \oplus \mathbf{L}| \tag{2.24}
\end{equation*}
$$

Our intent is to construct a discrete square coordinate mesh $\left(z_{i}, t_{j}\right)$ with $(2 n+$ $1)^{2}$ points within which we can model piecewise continuous ordered trajectories
$\left(z_{k}, t_{k}\right)$ which connect the "endpoint" $(0,0)$ to some "endpoint" $(z, t)$ lying on the boundary of the square

$$
\begin{equation*}
t= \pm n,-n \leq z \leq n ; z= \pm n,-n \leq t \leq n \tag{2.25}
\end{equation*}
$$

The order parameter $0 \leq k \leq n$ traverses any space-time point along the trajectory only once; in addition we require that

$$
\begin{equation*}
z_{k+1}-z_{k}= \pm 1 ; t_{k+1}-t_{k}= \pm 1 ;(\text { four choices }) \tag{2.26}
\end{equation*}
$$

The description is static in the sense that it can be read either from 0 to $n$ or from $n$ to 0 and still describe the same trajectory. Note that in contrast to previous discussions, (a) we consider space-like as well as time-like trajectories, and (b) that the length of the strings $S \geq n$ is not specified; it is some finite integer named in advance of the construction. Note further that since we specify both endpoints, we are describing a completed process. The "wave functions" we will eventually construct on this mesh will be "born collapsed". All our results will belong to the "fixed past"; whether we should or should not use our theory to predict the future, either in a deterministic or a statistically deterministic sense, is a separate issue we will not discuss in this paper. We have picked our boundary conditions $(0,0)---(z, t)$ in the process of specifying the problem.

Any space-time point $\left(z_{k}, t_{k}\right)$ not on the axes $\left(z_{k}, 0\right),\left(0, t_{k}\right)$ lies in one of the four quadrants $(+,+) \leftrightarrow R>L, F>B,(-,+) \leftrightarrow R<L, F>B,(+,-) \leftrightarrow$ $R>L, F<B,(-,-) \leftrightarrow R<L, F<B$. We define our bounding endpoints in terms of our basic parameters, and four new parameters $r, l, f, b$ by $|t|>z \leftrightarrow$ $z=R-L=r-l ; t=n=r+l,|t|<-z \leftrightarrow z=R-L=r-l ; t=-n$, $|z|<t \leftrightarrow z=n=f+b ; t=F-B=f-b,|z|<-t \leftrightarrow z=-n ; t=F-B=f-b$.

The advantage of introducing the new parameters $r, l, f, b$ is that they make it
easy to define what will become Lorentz invariants. Explicitly

$$
\begin{gather*}
t^{2}-z^{2}=\tau^{2}=4 r l=n^{2}\left(1-\beta^{2}\right) \text { with } \beta=\frac{2 r}{n}-1 \\
z^{2}-t^{2}=-\tau^{2}=4 f b=n^{2}\left(1-\omega^{2}\right) \text { with } \omega=\frac{2 f}{n}-1 \tag{2.27}
\end{gather*}
$$

As we have shown many times $[6,7]$ it is easy to give meaning to the concept of Lorentz invariance in our discretc contcxt. Defining $r^{\prime}=\rho r, l^{\prime}=\rho^{-1} l, r^{2}$ is obviously invariant, and if we define $\gamma_{\rho}=\frac{1}{2}\left(\rho+\rho^{-1}\right), \beta_{\rho}^{2}=1-\frac{1}{\gamma_{\rho}^{2}}$ we have immediately that

$$
\begin{equation*}
z^{\prime}=\gamma_{\rho}\left(z+\beta_{\rho} t\right) ; t^{\prime}=\gamma_{\rho}\left(t+\beta_{\rho} z\right) \tag{2.28}
\end{equation*}
$$

## $3+1$ dimensions

To distinguish space from time in the model, we include additional spacial dimensions which we require to be homogeneous and isotropic in the sense that none of the symmetry properties depend on the choice of the labels $x, y, z, \ldots$. One of the great conceptual advantages of our constructive approach is that McGoveran has proved that in our theory the extension from $1+1$ space-time to $2+1$ and $3+1$ has to stop there (FDP Section 3.4, pp 30-34). To see how this applies in our context, fix the F , B pair as defining the universal ordering parameter $j$ for causal spacetime events, and try to construct not only the $z$ coordinate from the $\mathrm{R}, \mathrm{L}$ pair as above but three additional independently generated pairs $W_{+}, W_{-} ; X_{+}, X_{-}, Y_{+}, Y_{-}$ to construct the coordinates $w=W_{+}-W_{-}, x=X_{+}-X_{-}, y=Y_{+}-Y_{-}$, and for consistency in the notation replace $\mathrm{L}, \mathrm{R}$ by $Z_{-}, Z_{+}$with $\mathrm{z}=Z_{+}-Z_{\ldots}$.

Following the same procedure as above, we generate four substrings $\mathbf{w}_{+}(n)$, $\mathbf{x}_{+}(n), \mathbf{y}_{+}(n), \mathbf{z}_{+}(n)$. Since these four strings are independent by hypothesis, they cannot discriminate to the null string, so we need a definition of event appropriate to this situation. We take this to be those values of $j$ for which all four strings
-
have accumulated the same number of "1"'s, i.e.

$$
\begin{equation*}
\Sigma_{k=1}^{j} b_{k}^{w_{+}}=\Sigma_{k=1}^{j} b_{k}^{x_{+}}=\Sigma_{k=1}^{j} b_{k}^{y_{+}}=\Sigma_{k=1}^{j} b_{k}^{z_{+}} \tag{2.29}
\end{equation*}
$$

The extension to $D$ rather than 4 spacial dimensions is obvious. This reduces the probability of events occurring after $j$ space-time steps in $D$ dimensions to the probability of obtaining the same number of " 1 "'s in $D$ independent Bernoulli sequences after $j$ trials ${ }^{[13]}$,

$$
\begin{equation*}
p(j)=\frac{1}{2^{j D}} \Sigma_{k=0}^{j}\binom{j}{k}^{D}<j^{-\frac{D-1}{2}} \tag{2.30}
\end{equation*}
$$

Clearly this definition of events defines a "homogeneous and isotropic" d-space, but the probability of being able to continue to find events for large values of $j$ vanishes for $D>3$. Consequently we need only consider three spacial dimensions. Thus, provided we have some clear way to label independent bit strings, we can extend our construction of $1+1$ space-time to $3+1$ space-time, but no further.

As we have discussed elsewhere ${ }^{[7,8]}$, it is now straightforward to derive wave functions are solutions of the Schroedinger, Kline-Gordon and Dirac equations when there are enough paths so that we cannot tell our exact combinatorial results from the appropriate sine, cosine, exponential and Bessel functions.

## 3. LABEL SPACE

So far we have been discussing what in the general theory would be called content strings ${ }^{[1]}$. In order to apply our theory to elementary particle physics, we need to identify the first part of the string with the four levels of the combinatorial hierarchy. The novelties here compared to the basic paper ${ }^{[1]}$ have to do with the generation problem and quark confinement. The scheme I have been using uses strings of length 16 , with 2 slots for the neutrinos, 4 for the electrons and gammas, and 8 for the quarks and gluons, leaving two unaccounted for. Unfortunately I
didn't realize until after the measurement of the $Z_{0}$ width that these could be used for the three generations; so I lost the chance of making a prediction before the experiment that (as turns out to be the case) our theory can only accommodate three generations.

Interpretation of the experiment is as clear for us as for the conventional particle physicists since all it requires is the Fermi constant and the number of generations of massless neutrinos. Now we have crossed the generation gap, we can see why the muon should be about $3 \cdot 7 \cdot 10$ electron masses, and we should be able to tie up the $\pi, \mu, \nu_{\mu}, \nu_{e}$ system into a neat little package, computing lifetimes as well as widths. The same reasoning suggests that the $\tau$-lepton should be less than $3 \cdot 7$ muon masses. Empirically it is closer to 17 than 21 muon masses, presumably because it contains some electrons not well represented by being bound into $\mu$ 's. Getting this correction right will give us a new handle on how to bound state calculations. It will also give us another leg up on conventional theories since they have hardly any idea as to how to get any mass relations across the generation gap.

The other generation problem has to do with quarks, which is tougher because they are confined. From our point of view, confinement is a direct consequence of McGoveran's theorem, since once we have identified the three asymptotically conserved quantum numbers as charge, lepton number and baryon number, we know that we cannot see the colored quarks as free particles. Since the theorem is quite explicit about how the probability of finding events at large relative distances falls off, we should be able to go from that to some sort of asymptotic behavior for the "confining potential". Putting that together with the " handy-dandy formula" for bound states and resonances ${ }^{[7]}$ we should be able to get a handle on effective quark masses even though they do not appear as free particles. In particular, it might be possible to put seven colored gluons together as a closed "glueball" structure in label space, in which the valence quarks are confined. String models of something like this structure, taking the string tension from parameter fits, seem to do pretty well. We should be able to calculate something related to the parameters they take from experiment.

There is some urgency to doing all this, since the top quark is the only particle which has not been observed, but which most theorists believe has to be there. It might be found in the next year or so, but if the current Fermilab experiment directed to that end fails because it is too heavy, we might have five years rather than one in which to make our critical prediction. Unfortunately we will have to understand "running coupling constants" in our own terms, and get the five other quark masses right, before there is any reason to believe our calculations. That will involve a lot of work, and I doubt that I can carry it through without a younger collaborator who rcally understands current high energy physics.

In conclusion, I sketch how, once the wave function construction given above is nailed down (I failed to explain David's combinatorial counting adequately to an interested but skeptical audience at SLAC on February 9) we might attempt to present the mass calculations in a paper aimed at the Physical Review.

1) Point out that when the label space closes off at $2^{127}+136$ labels, we can identify this as the formation of a "particle" which has the Planck mass, and is unstable against particle emission by the Noyes-Dyson argument. Since at this point all slots are indifferently interpretable, all constituent masses are the same, and the particle emitted which starts the construction of space-time has to be the stable proton. Finding a language which will convince particle physicists that this is a reasonable way to calculate the basic mass of the proton as gravitational energy will be the major task here.
2) Once we have the stable proton in space-time, we can back down from level 4 to levels $1,2,3$, identify the electromagnetic coupling, and use the Parker-Rhodes calculation to generate the electron mass as a finite electromagnetic self-energy. I was already able to present this as a self-energy calculation in PITCH ${ }^{[14]}$; I think this can now be made much more convincing to conventional theorists.
3) We already have the quantum numbers for weak-electromagnetic unification, but no firm argument as to why the coupling constants connect in the way needed to predict the $W$ and $Z_{0}$ masses at the "tree level". Once the electron mass
calculation of $m_{p} / m_{e}$ is accepted as due to intermediate nucleon-antinucleon pairs due to electromagnetic coupling, essentially the same calculation give $M_{W} / m_{e}$ as due weak interactions with intermediate $W-\nu$ states. Since the mass is given once, not twice, the two results must agree. I have worked this out and get the standard weak-electromagnetic unification formula connecting $\alpha, G_{F}, M_{W}, M_{Z}$ and $\sin ^{2} \theta_{W}$ with all the factors right. This now give rigorous justification for the prediction of $\sin ^{2} \theta_{W}$ and the McGoveran correction to it, that we had argued for earlier on less firm grounds.
4) With the electron stabilized electromagnetically, the Noyes-Dyson argument for the pion as a collection of electron-positron pairs now becomes rigorous, and as indicated above, now we can identify the $\mu$ and the $\tau$-lepton within our scheme, should allow us to do a good job on low energy leptonic decays.
5) Thanks to the handy-dandy formula we can think of the pion as a bound state of a nucleon-antinucleon pair, and since we have good values for the masses, we have a good value for the pion-nucleon coupling constant, and can do low energy nuclear physics in a reasonable way. More importantly from a fundamental point of view, this calculation should also give us a handle for thinking about the pion as a quark-antiquark pair rather than as a nucleon-antinucleon pair. I am sure that it is no coincidence that the coupling constant is about $2 \times 7$, since 7 is the proper strong coupling constant for quarks and gluons. Now we could try to go on to strange quarks, kaons, etc. and begin to understand how to get effective masses and running coupling constants for the light (up, down, strange) quarks. This would really crack the generation problem, and lead through charm and beauty to the true, sometimes called the top, quark.

I close with an update of the tabulation of our current results.

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## FIGURE CAPTIONS

1) a) the situation when a counter guarantees that the particle goes through one slit or the other. b) the situation when we do not know which slit the particle went through.
a)


$P_{1} \quad P_{2}$

$$
\begin{gathered}
P_{12}=0 \\
P=P_{1}+P_{2}
\end{gathered}
$$

A


$$
\frac{P_{\overline{1}}}{P_{1}}=\frac{P_{\overline{2}}}{P_{2}}
$$

$$
\begin{gathered}
P_{\overline{12}} \neq 0 \\
P \neq P_{\bar{T}}+P_{\overline{2}}
\end{gathered}
$$

- $3+1$ asymptotic space-time
- combinatorial free particle Dirac wave functions
- supraluminal synchronization and correlation without supraluminal signaling
- discrete Lorentz transformations for event-based coordinates
- relativistic Bohr-Sommerfeld quantization
- non-commutativity between position and velocity
- conservation laws for Yukawa vertices and 4- events
- crossing symmetry, CPT, spin and statistics


## Gravitation and Cosmology

- the equivalence principle
- electromagnetic and gravitational unification
- the three traditional tests of general relativity
- event horizon
- zero-velocity frame for the cosmic background radiation
- mass of the visible universe: $\left(2^{127}\right)^{2} m_{p}=4.84 \times 10^{52} \mathrm{gm}$
- fireball time: $\left(2^{127}\right)^{2} \hbar / m_{p} c^{2}=3.5$ million years
- critical density: of $\Omega_{V i s}=\rho / \rho_{c}=0.01175\left[0.005 \leq \Omega_{V i s} \leq 0.02\right]$
- dark matter $=12.7$ times visible matter [10??]
- baryons per photon $=1 / 256^{4}=2.328 \ldots \times 10^{-10}\left[2 \times 10^{-10} ?\right]$


## Unified theory of elementary particles

- quantum numbers of the standard model for quarks and leptons with confined quarks and exactly 3 weakly coupled generations
- gravitation: $\hbar c / G m_{p}^{2}=2^{127}+136=1.70147 \ldots\left[1-\frac{1}{3.7 \cdot 10}\right] \times 10^{38}$ $=1.6934 \ldots \times 10^{38}\left[1.6937(10) \times 10^{38}\right]$
- weak-electromagnetic unification:

$$
\begin{aligned}
& G_{F} m_{p}^{2} / \hbar c=\left(1-\frac{1}{3.7}\right) / 256^{2} \sqrt{2}=1.02758 \ldots \times 10^{-5}\left[1.02684(2) \times 10^{-5}\right] \\
& \sin ^{2} \theta_{W e a k}=0.25\left(1-\frac{1}{3.7}\right)^{2}=0.2267 \ldots[0.229(4)] \\
& M_{W}^{2}=\pi \alpha / \sqrt{2} G_{F} \sin ^{2} \theta_{W}=\left(37.3 G e v / c^{2} \sin \theta_{W}\right)^{2} ; M_{Z} \cos \theta_{W}=M_{W}
\end{aligned}
$$

- the hydrogen atom: $\left(E / \mu c^{2}\right)^{2}\left[1+\left(1 / 137 N_{B}\right)^{2}\right]=1$
- the Sommerfeld formula: $\left(E / \mu c^{2}\right)^{2}\left[1+a^{2} /\left(n+\sqrt{j^{2}-a^{2}}\right)^{2}\right]=1$
- the fine structure constant: $\frac{1}{\alpha}=\frac{137}{1-\frac{1}{30 \times 127}}=137.0359674 \ldots[137.0359895(61)]$
- $m_{p} / m_{e}=\frac{137 \pi}{\frac{3}{14}\left(1+\frac{2}{7}+\frac{4}{49}\right) \frac{4}{5}}=1836.151497 \ldots[1836.152701(37)]$
- $m_{\pi}^{ \pm} / m_{e}=275\left[1-\frac{2}{2 \cdot 3 \cdot 7 \cdot 7}\right]=273.1292 \ldots[273.1263(76)]$
- $m_{\pi^{0}} / m_{e}=274\left[1-\frac{3}{2 \cdot 3 \cdot 7 \cdot 2}\right]=264.21428 . .[264.1160(76)]$
- $\left(G_{\pi N}^{2} m_{\pi^{0}}\right)^{2}=\left(2 m_{p}\right)^{2}-m_{\pi^{0}}^{2}=\left(13.86811 m_{\pi^{0}}\right)^{2}$
[ ( )] = empirical value (error) or range


## APPENDIX

To illustrate the rapid pace of developments since ANPA 10, I collect here (a) the homework problems I proposed for ANPA 10, (b) contributions to the first four issues of the ANPA WEST Journal and (c) recent letters to Wheeler and Dyson.

## HOMEWORK FOR ANPA 10

## Queries

We take $\hbar, c$, and $G$ as measured by current scale invariant techniques, and define our dimensional units of mass $[\mathrm{M}]$, length $[\mathrm{L}]$ and time $[\mathrm{T}]$ by

$$
[M] \equiv(\hbar c / G)^{\frac{1}{2}} ;[L] \equiv \hbar /[M] c ;[T] \equiv[L] / c
$$

It is taken as understood in our work that a fundamental theory such as ours must computc cverything else as pure numbers in terms of ratios to these units and provide rules of correspondence, consistent with the current practice of physics, that will enable us to say how successful we have been in making such calculations.

Query 1. To what extend do you agree or disagree with this statement? What arguments would you advance in support of it? What experimental or logical evidence would convince you that this is a bad starting point for a fundamental theory?

It is often thought, by people who have followed the ANPA programme, that we have by now predicted, up to a factor of $[1 \pm O(1 / 137)]$, the following physical consequences, where the symbols have their usual significance:

$$
[M]=\left(2^{127}+136\right)^{\frac{1}{1}} m_{p} ; \hbar c / e^{2}=137=2^{2}-1+2^{3}-1+2^{7}-1
$$

Qu. 2. What arguments would you advance to support this conclusion? What experimental or logical evidence would convince you that these results are wrong or misleading?
Qu. 3. Can you explain why you believe in, or do not believe in, the Parker-Rhodes formula for the proton electron mass ratio

$$
m_{p} / m_{e}=137 \pi /[(3 / 14)(1+2 / 7+4 / 49)(4 / 5)]
$$

Qu. 4. Using the recent results establishing momentum conservation, can you (a) calculate the "center of mass" correction to the Bohr formula $\left[m_{e} \rightarrow m_{e} /(1+\right.$
$m_{e} / m_{p}$ ) and (b) see if a consistent discrete calculation provides a new route to the Parker-Rhodes formula?

Qu. 5. Can you, by using the relativistic discrete theory, including angular momentum and "elliptical orbits", obtain the Sommerfeld fine structure splitting for the hydrogen spectrum, and by using instead the spin degree of freedom show that this is consistent with the Dirac calculation of the same quantity?
Qu. 6. By treating the $(1)_{N_{L}}$ label (i.e. the unique label in the full 4-level $2^{127}+136$ bit string representation of the hierarchy which interacts with everything) as the Newtonian "quantum" in the same way that the coulomb "quantum" is treated in - the previous exercises, can you solve the Kepler problem?

Qu. 7. Can you show that our theory predicts the gravitational red shift for light emitted from any massive object.
Qu. 8. Can you show that Newtonian gravitation in our theory predicts only half the observed deflection of apparent stellar positions by the sun? Can you extend the gravitational theory to provide spin 2 gravitons in addition to the Newtonian term, and show that one can then get the experimental result?
Qu. 9. By using spin 2 gravitons in the Kepler problem (Qu. 6) in analogy to the Dirac version of the Sommerfeld problem (Qu. 5), can you calculate the precession of the perihelion of Mercury?
Qu. 10. Can you show that the mass of the neutral pion is 274 times the electron mass to an accuracy of better than one electron mass?
Qu. 11. Is the identification of $\left(2^{127}+136\right)^{2}$ as an estimate of the baryon number (and charged lepton number) of the universe, which seems natural in the context of program universe, a necessary consequence of theories of the type we are constructing?
Qu. 12. Is the fact that particles currently known can only be identified with reasonable assurance at level 3 , that all such particles are "visible" (intcract clectromagnetically either directly or indirectly), and that from the statistical point of view labels that close on the first two levels will be 127/10 times more prevalent an indication that there should be roughly 10 times as much "dark" as "visible" matter in the universe? Realize that although these labels are not identified, they, like any label in the scheme, must interact gravitationally.
Qu. 13. Does the success of the Noyes-Dyson argument for the mass of the neutral pion (Qu. 10) take us far enough to calculate the two gamma decay lifetime of this particle ( $0.87 \times 10^{-16}$ seconds)?
Qu. 14. How do we calculate the mass of the $W$ and the $Z_{0}$ ? If we can do this the $\pi^{ \pm}-\pi^{0}$ and neutron-proton mass splitings should follow.

Qu. 15. Can we calculate some approximation to the "gluon condensate" which allows Namyslowski to get "running masses" for quarks and gluons? If so, most of strong interact physics should follow in due course.
Qu. 16. Are there quantum geons?

From ANPA WEST Journal, 1, No. 1.
A Conversation with Pierre Noyes about ANPA History

Pierre Noyes is the American champion of "bit string" physics, and one of its chief architects. He received his doctorate in theoretical physics from Berkeley where his mentors were Chew, Serber and Wick, and afterwards spent a year with Peierls. He worked on nuclear forces at Rochester and then at Livermore, where he also worked on nuclear weapons; in 1969 he cancelled all his security clearances in protest against the Vietnam war. Since 1962 he has been a professor at the Stanford Linear Accelerator Center. He has published close to a hundred papers in elementary particle physics focused on strong interactions and the quantum mechanical three body problem. Since 1979 much of his effort has gone into trying to turn the combinatorial hierarchy into a comprehensive theory of the physical world.
$A N P A$ West. In simple language, what is the combinatorial hierarchy?
Noyes. It's a mathematical procedure generating two sequences of numbers. Onc of these sequences specifies the scale constants of the physical universe, while the other forces the construction to terminate after four steps, showing that there are no other basic scale constants.

ANPA West. What are these scale constants?
Noyes. You might say that they fix the place of humanity in the universe. The human scale is a few feet, 100 or so pounds, and at least a second to make a decision. The mass of the universe sending light to us is 75 orders of magnitude larger than 100 pounds. The mass of the smallest particle we know to have mass is 32 orders of magnitude less than 100 pounds. The size of the smallest system we can measure is around 22 orders of magnitude smaller than a few feet. The age
of the visible universe is around 15 billion years. The shortest times we can now infer are around 23 orders of magnitude less than a second. What the hierarchy is about is a way to compute these numbers which works as physics.
$A N P A$ West. This linking of physics to the human place in the universe is very interesting, but I'm a little confused. What you just described are rough magnitudes which would seem to depend on the accident of human size, and yet what I think of as the scale constants are exact dimensionless numbers which belong, or should belong, to physical theory; could you explain a little more how these things are related?

Noyes. We are finite beings that can only spend part of our time counting such things. Most of our time we must spend filling our bellies, producing and taking care of our progeny and trying to help others to do the same. How far we can count in the time available relates our evolved structure to the rest of these ongoing enterprises.

ANPA West. When and how was the hierarchy discovered?
Noyes. It was discovered by Parker-Rhodes in 1961. The story as I recall hearing it a decade after the facts is that Ted Bastin posed the challenge to Fredrick ParkerRhodes of how to generate a sequence with one or two small numbers, something of the order of a hundred, some very large number, and stop. Frederick did indeed generate the sequence $3,10,137,2^{127}$ in suspiciously accurate agreement with the scale constants. This was a genuine discovery. The termination is at least as significant! There's a relatively simple rule for the sequence - the real problem is to find some "stop rule" that terminates the construction.

ANPA West. And Parker-Rhodes did that too??
Noyes. Yes, that's where the second sequence comes in; it measures the "raw
$-$
material" for the first, and after four steps this runs out, so you can't keep going. By the way, he had only recently joined Bastin, Kilmister, Amson and Pask, who had started this work in the 50 's.

ANPA West. When did you first become involved?
Noyes. I first heard of the hierarchy when Ted Bastin gave seminars on it at Stanford in 1971 and 1972. As an empiricist, my first reaction was that any apriori scheme of this sort must be mystical nonsense. However, I went to the second seminar and realized that 137 was given by an old argument due to Dyson, interpreted as counting the maximum number of electron-positron pairs that could exist within their own Compton wavelength. Having reduced the argument to counting, I realized that the same argument could be applied to gravity, and then I was hooked!

ANPA West. What were the major steps in the development of the theory since then?

Noyes. Stein's random walk connection between relativity and quantum mechanics. Kilmister's scheme for generating bit strings. Gefwert's constructive philosophy. Manthey's program universe. McGoveran's ordering operator calculus, which unifies the limiting velocity of relativity with the commutation relations of quantum mechanics in what has to be a "space" of 3 dimensions. This revolutionary unification will be discussed at ANPA 10.

ANPA West. How do you see this work applying outside of physics?
Noyes. That even in a "hard science" it can be more important to understand how we think and how we communicate with each other about it than what we are thinking "about".

ANPA West. How and when did ANPA take shape as an organization?
Noyes. I had the idea of forming ANPA in 1979 when I learned from an investment counsel that many corporations with money to give away didn't know where to put it. I thought we could offer something of interest to them and of use to the world. I got Kilmister, Bastin, Parker-Rhodes and Amson to join me in making a framework. Following our first international meeting at Kilmister's "Red Tiles Cottage" in Sussex, we have held 8 annual international meetings in Cambridge and will have our tenth in August, 1988. Our initial hopes for outside funding did not materialize, but we believe we have shown that what we do is worth supporting.

ANPA West. What is the role of ANPA today?
Noyes. To pursue as best we can alternatives to establishment views about science and society that can lead to quantitative and testable predictions about the uncertain future. To find a route to a better future that can grow out of our fixed past.

## From ANPA WEST Journal, 1, No. 1. WHY DISCRETE PHYSICS?

At the beginning of this century physicists startcd grappling with two revolutionary ideas: quantized action and relativity. Nearly a century later, there is still no consensus as to how (or even whether) they can work together to describe gravitation in a satisfactory way. Technical success in describing the physical universe has been achieved at the cost of large experimental programs and much sophisticated mathematics, but basic conceptual clarity is, for many of us, still lacking.

One of the contentions made by those who practice discrete, combinatorial physics is that the difficulties of the conventional theory stem, in large part, from the attempt to embed what are basically discrete and finite quantum particles in a continuous space-time background which is postulated rather than constructed. In
contrast we use a fully constructive and necessarily finitely computable approach in which the interconnections between events bring us those aspects, and so far as we can see at present, only those aspects of "particles", "space" and "time" that are needed to explain contemporary cosmological observations and contemporary experiments in high energy particle physics.

In particular we necessarily have an event horizon, a reasonable estimate of the contemporary universal matter and radiation density - which extrapolates backward correctly to the "time" when the radiation broke away from the matter in the cosmic fireball- and a simple explanation of why there is at least ten times more "dark" than electromagnetically interacting "matter". One of the early successes of combinatorial physics (due to Amson, Bastin, Kilmister and Parker-Rhodes) was the calculation of an excellent approximation to the dimensionless strength of the electromagnetic interaction and its ratio the gravitational interaction in hydrogen. We also necessarily have a limiting velocity, quantized action, the correct electronproton mass ratio, the quantum numbers of the standard model for quarks and leptons, and a start on quantitative calculations of problems in elementary particle physics. All of this is achieved by deep philosophical analysis, guided of course by contemporary experience in physics but without the use of sophisticated continuum mathematics. Instead our subtleties come from the novel and rapidly expanding group of core concepts underlying contemporary computer science.

Gefwert pointed out to us that any constructive physics must be computable. I went to Manthey and together we constructcd program universe as the simplest way we could think of implementing Kilmister's ideas on generation and discrimination. The initial elaborations I insisted on to reach familiar physics turned out to be unnecessary; the current "stripped down" algorithm seems to give all the structure needed for modeling contemporary physics! This effort became much more systematic when we started listening to and really hearing about McGoveran's modeling methodology. He gave us immediately a general understanding of the limiting velocity, the reason for distantsupraluminal correlations without signaling (Einstein-Podolsky-Rosen), and a proof that for large numbers of events we only
need a common "3-space". Etter picked up on these ideas and derived much of the Lorentz structure. Discussions between Stein, Karmanov and myself eventually have led to a simple and fully discrete version of the Lorentz transformations of special relativity. Meanwhile, McGoveran had proved that any theory such as ours will have non-commutativity, and enabled us to identify the quantum of action. Earlier work in the quantum numbers of the standard model of quarks and leptons and a discrete scattering theory then fell into place.

The world view which emerges has something in common with the Democritean slogan "Atoms and the void suffice!" as modified by Epicurus to include the possibility of free will and exclude simplistic reductionism. We have a multiply connected sequence of synchronizable distinct events with no "space in between", yet satisfy the requirements of special (and perhaps general) relativity and our understanding of Bell's theorem. The stabilization of "particles" and more complex systems of connected events against a "background" of arbitrary change gives us a reasonable way of talking about the "age of the universe", our solar system and planet, paleontology... let alone more recent "history". We might say that eventually, by chance, events and the void suffice. Yet just because we have a fixed past and uncertain future, we have no way to escape moral responsibility for our actions, - or our decisions not to take action. For me, it is this dimension of our alternative natural philosophy that has the deepest significance.

From ANPA WEST Journal, 1, No. 2.

## ON TO QED

The time has come to make a frontal assault on the best protected fortress of the physics establishment - quantum electrodynamics (QED). In 1974 my advice ${ }^{[15]}$, was that
"We should learn from our comrades in Southeast Asia that we must 'know our enemy' and attack where he is weak, not where he is strong. The strongest point in the defense of local field theory [my then current and continuing enemy,
among others] is obviously QED [Quantum Electrodynamics], so we should leave this [attack] to the last and try to outflank it by finding weaker points."

By 1989 we are in a much more advantageous strategic and tactical situation.
I tabulate below the major victories already achieved ${ }^{[16]}[, 1]$ - none of which can be reached by standard methods. Conventional theories take as brute facts the general structural results which we have established. Our gravitational theory and cosmology are in accord with observation, and we find both more plausible than the conventional pictures. The way we view elementary particle structure has a simpler and more self-coherent origin than the received wisdom allows. All of our quantitative results are for numbers that standard theories have to take from experiment, and often do not allow to be calculated. This solid body of firm conclusions gives us a very strong strategic position. What is lacking is some decisive calculation that goes beyond what conventional theory has achieved in a region where it assumes novel theoretical or experimental predictions are possible.

The results now in hand open up a number of possible exciting physical applications of and improvements in our theory. I will discuss several of these in my paper ${ }^{[17]}$ for ANPA WEST 5. Among these, the breakthrough achieved by McGoveran last year in calculating the fine structure constant ${ }^{[18]} \alpha$ offers a unique tactical opportunity for us to make calculations in quantum electrodynamics that are outside the grasp of conventional physics.

The fine structure constant $\alpha=e^{2} / h c \simeq 1 / 137$ encapsulates much of nincteenth and twentieth century physics and chemistry. The symbol $e^{2}$ represents the laws of electrochemistry and chemical valence, as discovered by Faraday, and the square of the electric charge on the particulate electron as discovered by J.J.'Ihompson. The limiting velocity $c$ (the velocity of light) refers back to Maxwell and Einstein. Similarly Planck's constant $h=2 \pi \hbar$ was the start of quantum mechanics. In 1966 Amson, Bastin, Kilmister and Parker-Rhodes computed the first approximation $1 / \alpha=137$. Taken together with the 1978 Parker-Rhodes calculation of the proton-electron mass ratio this now opens up most of the physics of
here and now to attack by our theory. Discrete and combinatorial physics (our theory) is ahead of conventional methods because establishment physicists have to take " $\alpha$ " from experiment; the highest ambition of particle theorists is to calculate both the weak ( $\beta$ - decay) and strong (quark) interactions and the particle mass ratios using only this number $\alpha$.

Bohr showed that the electron mass $m_{e}$ taken together with $c, h$ and $\alpha$ are enough to explain the visible and ultraviolet light (line spectrum) emitted and absorbed by hydrogen. But these spectral lines have a doublet "fine structure" measured by $\alpha^{2}$ - hence the name. This fine structure was computed by Sommerfeld in 1916, and in an apparently different way by Dirac in 1929. The next correction is called the "Lamb shift" and involves $\alpha^{3}$, but by the time one tries to compute $\alpha^{4}$ effects both the strong and the weak interactions have to be taken into account. At this point one needs to calculate millions of terms, which means that even the algebra has to be done on super computers. Hence in our view QED is defended by four rings of fortifications - the effects proportional to $\alpha, \alpha^{2}, \alpha^{3}, \alpha^{4}$. Fach class of effects is about a hundred times smaller than the last, and usually much more than a hundred times harder to calculate.

Conventional calculations have succeeded in achieving agreement with experiment for many effects of order $\alpha^{3}$ and some of order $\alpha^{4}$. Models of both the weak and the strong interactions generalized from QED have had some striking successes - thanks to a generous input of empirical data. The success was bought by considerable technical complexity. The fine structure constant measures the probability of emission and absorption of radiation; yet when the same particle emits and absorbs this radiation, the effect is infinite. Such effects can be made finite by adding additional infinite terms to the theory crafted to cancel the calculated infinities; this process is called "renormalization". Sophisticated "non-Abelian gauge theories" have recently bounded this confusion at the cost of predicting a "vacuum" energy density $10^{120}$ times too large to meet the cosmological requirements. Herculean efforts are needed to keep the (model) universes from shutting themselves down before they can gasp. We are plagued by none of these difficulties.

Assuming that the conventional theorist has successfully found his way through the mine field described in the last paragraph, he still has difficulty properly connecting the basically non-relativistic (low velocity) model of the hydrogen atom (Bohr or Dirac) to these very high (virtual) energy effects. A current problem for him is "positronium". Positronium is an atom made up of the familiar negatively charged electron and its positively charged "anti-particle", the positron. Together they annihilate, and "all is gamma rays" (like when the Teller and the anti-Teller meet), but before this happens, they emit light (spectral lines) which Bohr could compute; a first approximation to the fine structure can be obtained by following Sommerfeld or Dirac. But this is not enough. One way the bound state problem shows up is that $\alpha^{3}$ terms in the calculation of the decay lifetime of positronium have not yet been articulated. They would have to be a hundred times larger than expected in order to explain the experimental results. This fact in itself shows that the conventional method of calculation is breaking down: even the $\alpha^{2}$ term is suspiciously large.

Trouble now exists close to the heart of quantum field theory. This fact became manifest at an auspicious time for us. Thanks to McGoveran ${ }^{[34]}$, we have already breached the second $\left(\alpha^{2}\right)$ line of defense surrounding QED. Some mopping up operations are still needed; a lot of technical development will have to be carried out before we can tackle positronium directly. The significant fact is that we now know how to make relativistic bound state calculations in a simple way. Apparently all that is needed is a lot of hard work. I now raise the cry: On to QED! Seize the time!

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## From ANPA WEST Journal 1, No.3, (Spring, 1989) ADVANCES ON TWO FRONTS

Until recently our theory* could calculate numerical values for a few elementary particle parameters, but could only give heuristic or at best semi-quantitative estimates of the range for the next order corrections; for cosmology, the arguments were much vaguer. This situation changed dramatically when McGoveran made the fine structure breakthrough last year. McGoveran recently noted that, following the same line of reasoning, the correction to the "weak" (Fermi theory of $-\beta$-decay) $G_{F}$ should be $\left(1-\frac{1}{3 \times 7}\right.$ ), which works to four significant digits. Once one recognizes that $\sin \theta_{W_{e a k}}$ is also a coupling constant factor, and has lowest order value $\frac{1}{2}$ because of the missing "right-handed" neutrinos, the same factor of $\left(1-\frac{1}{3 \times 7}\right)$, also corrects the sine of the weak angle and brings our calculation within the currently accepted experimental range.

For our model, in the absence of further information, one would expect baryon number zero, i.e. equal numbers of 0 's and 1 's in the bit-strings. However, we have to start with 1 rather than 0 to get off the ground. By the time we have formed 256 labels, and hence have the fourth power of that number of particles which have engaged in $(2,2)$ scatterings (our scatterings conserve baryon number) and have closed off the first generation of quarks and leptons, we will still have an unavoidable bias of one part in $256^{4}$; hence $n_{B} / n_{\gamma}=1 / 256^{4}=2.328 \times 10^{-10}$. Since the higher generations repeat the same construction, this will also be the bias for the whole scheme when the $2^{127}+136$ labels have been constructed. The current observational value of the number of baryons per photon is $n_{B} / n_{\gamma}=2.8 \times 10^{-8} \Omega h_{o}^{2}$ where $\Omega$ is the ratio of the relevant matter density to the critical density, and $h_{o}$ the ratio of the hubble constant to the currently accepted value $\left[\frac{1}{2}<h_{o}<1\right]$. Observationally $\Omega h_{o}^{2}=0.007$, and hence $n_{B} / n_{\gamma}=1.96 \times 10^{-10}$. It begins to look
like we can write a reasonable version of Genesis. Whether our Deuteronomy comes out as it should remains to be seen.

## FINITE AND DISCRETE RELATIVISTIC QUANTUM MECHANICS ${ }^{\dagger}$ H.Pierre Noyes, SLAC, Stanford, CA 94305

Conventional theories take the structure of relativistic quantum mechanics as given. The two empirical constants $c$ and $\hbar$ are connected to the arbitrary historical standards of mass, length and time by various, hopefully self-consistent, means. A third fundamental constant such as $e^{2}, m_{e}, m_{p}, M_{\text {Planck }}$ has to be taken from experiment before theoretical "predictions" can be attempted. Even then there is no consensus on how to calculate the ratios between them, - a clear requirement for any fundamental physical theory that allows only empirical standards for mass, length and time, or some equivalent like $c, \hbar$ and $m_{p}$ to dictate the common units for the inter-comparison of experiments between laboratories.

Our theory [H.P.Noyes and D.O.McGoveran, Physics Essays 2, 76 (1989)] does not take relativistic quantum mechanics for granted. We accept the principles of finiteness, discreteness, finite computability, absolute non-uniqueness, and require the formalism to be strictly constructive. We construct (rather than postulate) the limiting velocity and discrete events, and then derive the Lorentz transformations and the non-commutativity of position and velocity. Our basic algorithm uses the combinatorial hierarchy to calculate scattering probabilities and hence coupling constants and mass ratios such as (in first approximation):

$$
\begin{aligned}
e^{2} / \hbar c & \simeq 1 / 137 ; \quad G m_{p}^{2} / \hbar c=\left[m_{p} / M_{P l a n c k}\right]^{2} \simeq 1 / 1.7 \times 10^{38} \\
G_{F} m_{p}^{2} / \hbar c & \simeq 1 / 256^{2} \sqrt{2}=1.07896 \ldots \times 10^{-5} ; \sin ^{2} \theta_{W e a k} \simeq \frac{1}{4} ; m_{u \simeq d}(0) \simeq \frac{1}{3} m_{p} \\
m_{p} / m_{e} & \simeq \frac{137 \pi}{\frac{3}{14}\left(1+\frac{2}{7}+\frac{4}{49}\right) \frac{4}{5}}=1836.151497 \ldots ; \quad m_{\pi} / m_{e} \simeq 274 m_{e}
\end{aligned}
$$

Since we have already identified the role of $\hbar$ and $c$ in the theory we can take a third dimensional parameter such as $e^{2}, m_{e}, m_{p}, M_{\text {Planck }}$ from experiment and calculate a first approximation for the other three. From then on our iterative improvement of the theory is, in principle, much the same as for any other fundamental theory. For instance, the high dimension Kaluza-Klein theories coupled to a large number of Yang-Mills fields, compactified, in effect take the Planck mass $[\hbar c / G]^{\frac{1}{2}}$ as the third dimensional parameter. Weinberg [Phys. Lett. B 125, 265 (1983); P.Candelas and S. Weinberg, Nucl.Phys. B 237, 393 (1984).] calculates a first approximation to the coupling constants in this way. Our first approximations are much better than those achieved by more conventional methods. Of course one has to pay the price of learning new mathematical concepts based on

[^1]our constructive principles, and become familiar with the ordering operator calculus created by McGoveran [D.O.McGoveran, in DISCRETE AND COMBINATORIAL PHYSICS: Proc. ANPA 9, H.P.Noyes, ed., ANPA WEST, 25 Buena Vista Way, Mill Valley, CA 94141, 1988, pp 37-104].

## From ANPA WEST Journal 1, No.4(in press). <br> ABOLISH INFRARED SLAVERY

The current paradigm on which conventional particle physics and cosmology rests - second quantized relativistic field theory - has an Achilles heel. Despite its manifold quantitative successes, and the enormously creative role it has played in guiding high energy particle research, all its quantitative techniques rest, ultimately, on manipulating the theory into a form in which the interaction energy is small compared to some solved problem with a well defined "vacuum state". Then the interactions are seen as "perturbing" the calm of the vacuum by a small amount (eg. one part in 137). Two such interactions should then give an effect proportional to one part in the square of 137 , i.e. one part in 18,769 , and so on. Although this sequence of terms can rarely be added up to give a finite algebraic formula for the result, it seems reasonable to drop corrections that are smaller than current experimental error in the measurement of the quantity which is being calculated. This is called "perturbation theory".

These clever manipulations take their most sophisticated form in the theory of strong interactions - quantum chromodynamics or QCD. When strongly interacting particles are close together, the uncertainty principle forces them to have high momenta, - high enough to create virtual particle-antiparticle pairs, or new particles allowed by the discrete conservation laws. Wick understood this clearly enough in 1938 when he presented a simple but profound analysis of the physics behind Yukawa's 1935 meson theory; I have often called this the "Wick-Yukawa mechanism" for producing short-range interactions. Quantum chromodynamics is peculiar in that the coupling "constant" between quarks (the particles) and gluons (the quanta, or mesons) - and also for the self-coupling between gluons which distinguishes QCD from quantum electrodynamics (QED) - decreases as the energy increases. Consequently at high energy and short distance the effective coupling constant becomes small enough so that perturbation theory works. This is called "asymptotic freedom". But at low energy or long distance the colored quarks become so strongly interacting that they can never get away from each other. This is called color confinement. Since high energy corresponds to high ("ultraviolet") frequencies, this low energy corresponds to "infrared" frequencies, and color confinement is sometimes called "infrared slavery".

Since QCD (in the "standard model" form which so far has no experimental counter-indications) is supposed to be a well defined mathematical theory, one should be able to solve the equations directly without resorting to perturbation theory. However, the non-linear mathematics involved is not well enough understood to allow formal solutions that can be evaluated numerically. Instead, the continuum space-time of the theory is replaced by a finite mesh of discrete points and the differential operators in the field equations by finite difference equations. The resulting equations are so complicated that, although suggestive results have been obtained, they are nowhere good enough to calculate, for instance, the binding of a proton (two up quarks and a down quark) to a neutron (two down quarks and an up quark) to form a deuteron - the simplest complex nucleus, that of heavy hydrogen. By ganging several super-computers together, some people hope to get there in a decade or so, while others are studying how to construct specialized super-computers just for the task of solving "QCD on a lattice", which is the jargon for this class of problems.

Thank's to McGoveran's successful calculation of the binding energy of the hydrogen atom - the Sommerfeld formula, and the correction to the leading value of $1 / 137$ for the dimensionless electromagnetic interaction strength - I have realized that the same approach can be extended to strong interactions. This could be the first step toward abolishing infrared slavery!

Any system of two masses $m_{1}, m_{2}$ which binds to form a less massive system has three mass-energies associated with it. Since we wish our description to be Lorentz invariant, we use the square of the invariant four-momentum $s=E^{2}-p^{2}$, where E is the energy and p the momentum, rather than the masses in formulating the connection. One of the three terms is obviously $\left(m_{1}+m_{2}\right)^{2}$, and the second the square of the mass of the bound system, which we call $s_{0}$. The third is the interaction energy, which will be some fraction, which we call $f^{2}$, of some reference mass $m$. This interaction energy must be supplied in order to separate the system into its constituents $m_{1}$ and $m_{2}$, so the relativistically invariant expression connecting these three quantities is

$$
\left(f^{2} m\right)^{2}=\left(m_{1}+m_{2}\right)^{2}-s_{0}
$$

This expression is more general than the equation I presented at ANPA 11, and consequently more useful, as we will see shortly. I call it the HANDY-DANDY FORMULA.

If we rewrite the formula as $\left(f^{2} m\right)^{2}+s_{0}=\left(m_{1}+m_{2}\right)^{2}$, we have a "metric formula" similar to that of Pythagoras in which the interaction energy and the rest energy of the bound system are added in quadrature to produce the freeparticle measure. But in a discrete theory, we cannot always extract the implied
"square root" to obtain a rational answer. Then the product of the two "roots", i.e. $\left(m_{1}+m_{2}+s_{0}\right)\left(m_{1}+m_{2}-s_{0}\right)$ may prove to be more useful for physical interpretation. For a more careful treatment see McGoveran's contribution to Proceedings of ANPA 11, available from Faruq Abdullah at City University, London early next year.

If we take a dynamical rather than a static point of view, the measured quantity is the energy needed to separate the bound system into its two constituent masses $m_{1}$ and $m_{2}$ both at rest, and is called the "binding energy" $\epsilon$; the relativistic definition connecting it to the notation given above is $s_{0}=\left(m_{1}+m_{2}-\epsilon\right)^{2}$. In the application to the hydrogen atom, the mass of the proton $m_{p}$ and the electron $m_{e}$ are assumed known, and unconnected to the binding energy. To take account of 3 -momentum conservation we refer the calculation to the "reduced mass" of the system $m_{e p} \equiv m_{e} m_{p} /\left(m_{p}+m_{e}\right) \simeq m_{e}$ and obtain the result first achieved by Bohr in 1915

$$
\left(m_{e p}-\epsilon\right)^{2}\left[1+\alpha^{2}\right]=m_{e p}^{2} .
$$

Here the coupling constant $f^{2}=\alpha=e^{2} / \hbar c \simeq 1 / 137$ is called the "fine structure constant". But this is still only a relativistic correction to a basically nonrelativistic treatment. The strong interaction case is a better test of our basic ideas.

Back in 1949 Fermi and Yang found the work going on in elementary particle physics to be too hidebound and conservative. To shake things up a bit they noted that the recently discovered Yukawa particle (the pion) could be modeled as a bound state of a nucleon and an antinucleon with spin zero; all the (discrete) charge, spin, parity and isospin quantum numbers work out right. Their model makes it easy to understand how a proton can cmit a positive pion and change into a neutron, or a proton and an anti-neutron can fuse to produce a positive pion. Then we could drop the complicated apparatus of second quantized relativistic field theory for strong interactions. They challenged theorists to produce such a model; they didn't have a clue as to how to do it themselves.

I now believe that the "handy-dandy formula" can be the starting point for meeting their challenge. Let the two nucleons have a mass $2 m_{N} \simeq 2 m_{p}$ and the pion a mass $m_{\pi} \simeq 274 m_{e} \simeq(274 / 1836.15 ..) m_{p} \simeq(1 / 7) m_{p}$, values we have already calculated in our program ${ }^{\star}$. As our reference energy, $m$, we take the smallest mass in the system, which is $m_{\pi}$ - the mass of the pion. Then most of the energy needed to liberate the nucleon and the anti-nucleon from this bound state will go into making the mass of the nucleon-antinucleon pair, and the coupling constant,

[^2]conventionally symbolized by $G^{2}$, will have to be greater than unity. Invoking the handy-dandy formula
$$
\left(G^{2} m_{\pi}\right)^{2}=2 m_{N}^{2}-m_{\pi}^{2}=\left(2 m_{N}\right)^{2}\left[1-\frac{m_{\pi}}{2 m_{N}}\right] \simeq\left(14 m_{\pi}\right)^{2}
$$

In this way we clain to have calculated $G^{2}=14$, which is close enough to the accepted value for this first attempt.

We now have three ways of deriving the "handy-dandy formula", due respectively to $\mathrm{Bohr}^{\dagger}$ and Sommerfeld ${ }^{\ddagger}$, Dirac ${ }^{\S}$ and McGoveran ${ }^{\text {§ }}$. Biedenharn ${ }^{*}$ has shown that the first two derivations rest on the same symmetry principles; we suspect that this is also true for our derivation. Using my relativistic finite particle number scattering theory, I recently found yet another way of getting the handydandy formula. The connection between the masses of the constituents, the mass of the resulting bound system, and the "coupling constant" $f^{2}$ turns out to be simply the constraint which says that there are precisely two particles in the system, in the approximation in which the amount of time they spend "outside the range of forces" is large compared to the time inside. This may sound a little peculiar for coulomb forces, which are usually described as having "infinite range", but from a modern point of view, this is the region of "asymptotic freedom". The short-range region is where one starts to encounter particle-antiparticle pairs at a distance of half a Compton wavelength or less. Indeed this is just the point where relativistic effects come in and where McGoveran and I have shown that the value of $1 / 137$ for the fine structure constant has to be modified because of these additional degrees of freedom.

The important point in all this is that nothing in either McGoveran's or my derivation of the handy-dandy formula requires the coupling constant $f^{2}$ to be small. As we showed here in the theory of pions and nucleons, the coupling constant $G^{2}=14$. Perturbation theory would then require one to neglect 196 compared to 14 even in the next approximation. This obvious nonsense is why conventional methods have yet to produce an adequate fundamental theory for nuclear physics. But for us this large coupling constant simply means that in such a system the two particles interact 14 times as often as they fail to interact. The formula still holds. This indeed is a start on abolishing infrared slavery!

[^3]Let's construct a quantum geon as the other end of the scale to your classical geon. Take as our dimensional standards $M_{P}=\left[\frac{\hbar c}{G}\right]^{\frac{1}{2}}, L=\frac{\hbar}{M_{P C}}, T=\frac{\hbar}{M_{P} c^{2}}$. Assume that we understand $c$ and $\hbar$ as units in an algebraic scheme, that $M_{P}$ is the largest elementary mass (and hence that the smallest elementary length is $L)$, and ask for the smallest elementary mass, which we call $M_{n}$. Clearly this can be constructed by assuming there is some mass $m$ (or equivalently some length $\ell=\frac{\hbar}{m c}$ ), which defines the geometric mean between the two ( $m^{2}=M_{P} M_{n} \Leftrightarrow$ $\ell^{2}=L L_{n}=\frac{\hbar^{2}}{M_{P} M_{n} c^{2}}$. This will be the mass characterizing the most complex possibilities, and empirically is within about $1 \%$ of the proton mass.

Note that the relativistic Bohr formula for the Coulomb binding encrgy cof two masses $m_{1}, m_{2}$ can be written as $\left(\frac{m_{1}+m_{2}-\epsilon}{m_{1}+m_{2}}\right)^{2}\left[1+\left(\frac{\alpha}{n}\right)^{2}\right]=1$ with $\alpha=\frac{e^{2}}{\hbar c}$, and that the Sommerfeld (or Dirac or McGoveran) modification can be obtained by $n \rightarrow n+\sqrt{j^{2}-\alpha^{2}} ; j \neq 0$. Quantum mechanically, this formula is simply the requirement that a bound state pole at $s=s_{0}=\left(m_{1}+m_{2}-\epsilon\right)^{2}$ with residue proportional to $\alpha$ have the unique value required by postulating that it contains exactly two structureless particles, or for scattering states the on-shell unitarity constraint (conservation of probability). If we go to the limit of two massless particles bound gravitationally (note that the binding energy can then only be defined relative to the mass $m$ which occurs in the coupling constant $\lambda=\frac{G m^{2}}{\hbar c}$ ), the formula becomes $\left(\frac{\epsilon}{m}\right)^{2}\left[1+\left(\frac{G m^{2}}{\hbar c}\right)^{2}\right]=1$. We predict that any two massless particles (neutrinos, photons, gravitons) will have a lowest bound state (if allowed by other symmetries) whose rest mass is $1.3 \times 10^{19}$ smaller than the proton mass. We call this a "quantum geon". The maximum number of these we can find within their own Compton wavelength is $1.7 \times 10^{38}$, for the same reason that, following Dyson, the maximum number of charged particle pairs we can find within their own Compton wavelength is $137 \approx \frac{2 m c^{2}}{\left[e^{2} / \frac{\hbar}{2 m c}\right]}$.

The Sommerfeld-Dirac-McGoveran formula, and the fact that we can construct neutrinos, photons, and gravitons without introducing electromagnetic interactions in our scheme now allows us to extend the discussion to "dark matter". Since there are only two chiral massless neutrinos in our context (i.e assuming that the distinctions which separate $\mu$ - and $\tau$ - neutrinos from $e$ - neutrinos do not generate mass), the simplcst geon is a $\nu \bar{\nu}$ pair in a $0^{-}$state. This should be called the ground state of "neutrinonium", but since that would inevitably be shortened, I christen it the ground state of neutrinium. Note that is lies below the "vacuum state" containing only zero energy neutrinos, photons and (non-interacting) gravitons an appropriately vacuous concept. We assume that the triplet state has a bound
state at zero energy - easy to arrange in our finite particle number relativistic scattering theory - which we identify as the photon. Similarly, we can require that a neutrino bound to the $0^{-}$geon be a massless state indistinguishable from the bare neutrino, and from the neutrino bound to the photon to give spin $1 / 2$. Then the exclusion principle will guarantee that the corresponding spin $3 / 2$ state will lie above threshold as a resonance in $\nu+\gamma$ scattering. Two photons bound to form spin zero will be the $0+$ quantum geon whose binding energy must be calculated to determine whether it is a bound, zero mass, or resonant state (we suspect the latter). Finally, two photons bound to form spin two with zero mass are a graviton. Although the "travelling" quanta (neutrinos, photons, gravitons) have only six helicity states, our spinless geons can aggregate gravitationally to form massive objects (ultimately approaching classical geons) and define coordinate systems with respect to which all five spin 2 states of the gravitons can be defined. The "confined" Newtonian quantum makes a sixth to which we add the two photon and the two neutrino states. Thesc are the 10 states of dark matter already postulated (ANPA WEST Journal 1, No.3, spring, 1989) which will be generated 12.7 times as often as the matter which carries electromagnetic coupling.

I hope you will like this extension of the geon idea to the quantum domain. A long time ago you advocated a "radical conservatism" based on gravitation, electromagnetism and neutrinos. This might be a way to do it! What do you think?

In previous centuries people used to deposit a sealed letter with the secretary of some scientific society in order to establish priority for a discovery not yet ready for formal publication. I'm sending this letter to you and to Freeman in a effort to accomplish the same purpose, but I'm not asking you to keep it secret!

Letter to Freeman Dyson, December 22, 1989

Thank you for your prompt response to my letter, and copy of your paper.
[Dyson's paper, to appear in American Journal of Physics, entitled"Feynman's Proof of the Maxwell Equations", starts from
Newton's equation

$$
\begin{equation*}
m \ddot{x}_{j}=F_{j}(x, \dot{x}, t) \tag{1}
\end{equation*}
$$

with commutation relations

$$
\begin{gather*}
{\left[x_{j}, x_{k}\right]=0,}  \tag{2}\\
m\left[x_{j}, \dot{x}_{k}\right]=i \hbar \delta_{j k}, \tag{3}
\end{gather*}
$$

and derives the Maxwell equations.]
I am happy to see that we are not the only ones who wanted to see Feynman's derivation. You have done all of us a big favor in making it available.

You remark that you see no particular connection between this derivation and the derivation of the $1+1$ Dirac equation which I mentioned. I enclose the current version of our paper, which McGoveran has approved. Since neither Stein or Karmanov have had a chance to read it as yet, please don't circulate this draft. However, feel free to show it to others at the Institute and discuss it with them if you feel so inclined. I agree that there is no obvious connection between our thinking and the paradoxes posed by the Feynman derivation, other than that we are using a Zitterbewegung model which was in his thoughts at the same time. I will return below to what I believe is a fruitful interaction between the derivation you have provided for us and our own program.

One of the first things that struck me in your reconstruction of Fcynman's proof is that only the two dimensional constants $\hbar$ and $m$ occur; the limiting velocity $c$ is missing both from the postulates (1),(2),(3) and from the results (4),(5),(6). This already goes a long way toward resolving the paradox of electromagnetic ("Lorentz invariant") field equations derived from a Galilean invariant definition of mass. So long as the system of equations plus interpretation has no way to specify $c$ as a dimensional constant, the theory is scale invariant. Although Eq's 5 and 6 imply a limiting velocity ( $c=1$ ), within this context one can always pick units so that to some unknowable accuracy it will be impossible to tell Galilean from Lorentz invariance. McGoveran has reached the same conclusion independently.

I am sure you are familiar with at least part of this story, and I imagine Feynman was as well. Free field QED depends only on $\hbar$ and $c$; it is scale invariant. In their classic paper on the measurability of electromagnetic fields, Bohr and Rosenfeld make use of the scale invariance of QED to justify their use of macroscopic apparatus and the non-relativistic uncertainty relations when they derive the commutation relations for $\vec{E}$ and $\vec{H}$. But they point out at the end of the paper that once a unit of mass is introduced, the derivation brcaks down. It was for this reason that they questioned the validity of second quantization for the matter field. Since Oppenheimer had me study this paper when I was a graduate student, I acquired this suspicion early, and never lost it. Nonrelativistic quantum mechanics is also scale invariant, depending as it does only on $\hbar$ and $m$. In my paper on the double slit experiment ${ }^{[11]}$ I point out that for this reason it can in principle be tested with macroscopic apparatus using cannon balls shot through holes in armor plate detected by the deflection of individual grains of bird shot scattered from the cannon balls - provided the whole experiment is carried out in the dark! Scale invariance in this case is broken once a limiting velocity is specified, just as

Feynman's derivation breaks down once this is done.
I hope this analysis removes the paradox for you. If so I would like your opinion as to whether a note to the American Journal of Physics along these lines would be in order. The moral I draw from this is that Feynman's program failed to turn up new physics because he did not confront the incompatibility of quantum mechanics with relativity at a fundamental level. He was still wedded to the continuum, so had to go instead to "renormalization" and the juggling of infinities. I was never happy with this, which is why I spent so many years on phenomenology, dispersion theory, and the non-relativistic three body problem. For these problems the initial theory is well defined, and the danger signals are easy to spot because of the Wick-Yukawa mechanism for producing new particulate degrees of freedom at short distance due to particle creation. But I was eventually led back to foundations, as you know. I believe that the Feynman derivation gives me a convenient starting point from which I can explain why we believe we can get new physics out of our discrete framework.

The key to the success of our finite theory in modeling quantum mechanics with discrete step lengths in space and time, comes from our use of McGoveran's definition of attribute distance in the context of a conservation law applying to multiple computational paths between two endpoints. I use this first to derive the time dependence of the Schroedinger equation for an isolated system (constant Hamiltonian with eigenvalue $m c^{2}$ ) from our model. The connection to Feynman is that we allow steps both forward and backward in time with the backward steps representing an anti-system and the conservation law being that the number of systems minus the number of antisystems is the same at the two ends of the path. Suppose the number of steps forward is $F$ and the number of steps backward is $B$. We define the "time" $T$ (in units of our step length) from the beginning to the end of the path as $T=F-B$, and ask how many paths satisfy this constraint. We distinguish these paths from the "trajectory", which consists of the $T$ "points" $1,2, \ldots, \mathrm{~T}$ which may be traversed a large number of times before the boundary condition is met. We characterize the paths by the number of reVersals $V$. The portions of the paths forward and backward for a reversal which occurs outside the interval cancel out, so we can count only those which occur inside. Each of these can occur at any point, so the number of paths contributed by $V$ reversals will contain the factor $T^{V}$; that is, we are sampling with replacement. To calculate the number of paths, this must be multiplied by the probability that a path satisfying the boundary conditions with $V$ reversals will occur. Assume that it takes $S$ executions of a computational procedure to generate the sequence defining a particular path, of which $V$ call for a reversal. Then the total population of paths is simply the number of permutations which is $S!/(S-V)!$. Of these only the combinations $S!/ V!(S-V)$ ! are distinguishable, so the probability of generating
a distinguishable path is this number divided by the number of permutations, or $1 / V$ !. Consequently the number of distinguishable paths for $V$ reversals is $T^{V} / V$ !. This is a general result for the "transport operator" referring to attribute distance, as was shown by McGoveran some time ago [FDP, Theorems 36-40, pp 55-58].

What goes beyond this general result is the Feynman model that if the last step in the sequence is forward, a system arrives, while if the last step is backward, an anti-system arrives. Consequently if we have two paths that differ in this way we must subtract one from the other to get the net amplitude, positive if systems predominate and negative in the opposite case. Take as our boundary condition that prior to the start of the motion and after the end we have a system rather than an anti-system. Suppose $F>B$ so the motion is "forward in time" and that the first step is forward so starts with a system. Then for an even number of reversals it will end as a system, but for an odd number it will end as an anti-system, so the amplitude for this case is

$$
A_{+}=\Sigma_{V=0, \text { even }} \frac{(-T)^{V}}{V!} \rightarrow \cos T
$$

However, if the first step is backward, we must make at least one reversal for the end result to be a system, so

$$
A_{-}=\Sigma_{V=1, o d d} \frac{(-T)^{V}}{V!} \rightarrow-\sin T
$$

It is characteristic of the ordering operator calculus that when there are paths with interfering alternatives due to the sharing of indistinguishable possibilities which are generated by two independent arbitrary sequences ordered by a global ordering operator that we obtain at least ${ }^{\star}$ two different results; further, we do not have sufficient information to choose between them. However, as in this case, we do have the requirement that whichever paths we traversed, we arrived at the end result with unit probability and hence that the results must be added in quadrature and normalized [cf. Ref. 10 above]. Thus we have shown that with the Feynman conservation law connecting motion "backward in time" to anti-systems, a system that takes T discrete steps will arrive with probability $A_{+}^{2}+A_{-}^{2}$ which turns out to already be normalized to one system in our derivation.

To go from this derivation to the time dependence of the Schroedinger equation is now trivial. It is a matter of mathematical convenience to combine the two real sequences into one complex sequence $\psi=A_{+}+i A_{-}$with the rule that the probability is proportional to $|\psi|^{2}$. The unit step length is set by the usual quantization

[^4]condition $m c^{2}=E=h \nu=\hbar \omega$, and hence for finite step length but a number of reversals large compared to $h / m c^{2}$, we have found the solution of the continuum equation
$$
i \hbar \partial \psi / \partial t=E \psi
$$
and also derived the rule relating $|\psi|^{2}$ to probabilities rather than having to postulate it separately.

We can now go on to conncct up with your reconstruction of Feynman's derivation of the Maxwell equations. To begin with, we derive [Ref. 1 above] our discrete version of your Eq. 3, rather than postulating it. Breaking into our development at a point appropriate to this discussion, we can postulate that any mass $m$ executes a discrete Zitterbewegung - whose origin I return to below -at $\pm c$ with space steps $h / m c$ and time steps $h / m c^{2}$. We break scale invariance from the start, and require the theory to be quantum mechanical and relativistic at the same time. Concentrating for the moment on $1+1$ dimensions, and a particle which starts at $a$ at time $t_{a}$ and ends at $b$ at time $t_{b}$ which takes $R$ steps to the right and $L$ steps to the left to get there, we see that for this finite motion $z=(b-a)=c(R-L)(h / m c)$, $c t=c\left(t_{b}-t_{a}\right)=(R+L)(h / m c)$ and that the invariant interval $\tau$ is given by $\tau^{2}=c^{2} t^{2}-z^{2}=4 R L(h / m c)^{2}$ independent of the number of time steps $N=R+L$. This simple fact is one key to how we can retain our finite step length and still approximate our results by a continuum thcory for times large compared to $h / m c^{2}$.

The theory is Lorentz invariant in the following sense. If we go to a coordinate system with $R^{\prime}=\rho R, L^{\prime}=\rho^{-1} L, \tau$ is invariant, and we recover the usual Lorentz transformation with $\gamma^{2}=1 /\left(1-\beta^{2}\right)=\frac{1}{2}\left(\rho+\rho^{-1}\right)$, provided only $h / m c$ is invariant. This means that the same model allows us to define momentum as $p=\gamma \beta m c$ and energy as $E=\gamma m c^{2}$ with the usual invariance relation $E^{2}-p^{2} c^{2}=m^{2} c^{4}$ in terms of the finite velocity between the two endpoints $\beta=(R-L) /(R+L)=\frac{2 R}{N}-1$.

We can now go through the same type of derivation of the sum over paths for the left-right motion in the $z$ direction (assumed independently generated from the time motion, except for the same global ordering operator and the boundary conditions specified above) assuming that $p_{z}$ is conserved between the endpoints and arrive at the space wave function $e^{i p_{z} z}$, and by multiplying it by the independent time evolution get $\psi(z, t)$ as the solution of the Klein-Gordon equation

$$
\partial^{2} \psi / \partial z^{2}-\partial^{2} \psi / c^{2} \partial t^{2}=m^{2} c^{4} \psi
$$

where we have used $E^{2}=p_{z}^{2} c^{2}+m^{2} c^{4}$. Equivalently we could derive the equation in the rest system where $c t=\tau$ and then make a discrete Lorentz transformation to an arbitrary system. Now that we have derived the way the imaginary unit $i$ enters our
theory, we can carry through the same derivations using the Feynman relativistic prescription ${ }^{[12,]}{ }^{[19]}$ that we should sum over paths weighted by $(i \epsilon h / m c)^{V}$, except that we can take $\epsilon=1$ and still go to the continuum limit for sufficiently large space and time intervals. This is the route we follow in the enclosed paper ${ }^{[20]}$ deriving the $1+1$ Dirac equation, and giving an exact combinatorial result without taking the continuum limit.

I trust that by now I have given sufficient justification for the claim that in our model we have proved the commutation relation (3) as arising from $\dot{x}= \pm c$ with a discrete step length $\hbar / m c$, thus in this equation canceling out the velocity of light and removing the paradox. Our theory is Lorentz invariant in the discrete sense defined above. So far as (2) goes, the independence of the space directions, we have a more powerful result known as McGoveran's theorem [FDP, Theorem 13 , pp 30-34]

Theorem 13: The upper bound on the global $d$-dimensionality of a $d$-space of cardinality $N$ with a discrete, finite and homogeneous distance function is 3 for sufficiently large $N$.
In our context this a simple application of a theorem proved by Feller about independent Bernoulli sequences 35 years ago.

- When it comes to (1), we have to do a little more work in that so far we have not defined mass. In the fundamental theory this is obtained from our derivation of the ratio of the Planck mass to the proton mass, which incidentally makes the equivalence of gravitational and inertial mass in our theory a proved result rather than a postulate. But we still need some operational rule connecting mass to laboratory experiment. Following Mach, we use the Third Law (momentum conservation) to define mass ratios. Since $m_{p}$ is structurally defined in our theory and easy to connect to the usual laboratory definitions, this closes that loop, but then requires us to derive the Third Law. This is easy, since our definition of events requires us to have the step lengths in a system with two different masses be in some rational ratio, which then is conserved. Details will be presented elsewhere[cf. this paper, above]. Incidentally, a direct proof of the First Law (persistence of velocity as defined by the counter paradigm) has already been given ${ }^{[21]}$. Consequently we have already defined mass and, following Mach, (1) must be viewed as a definition of force. Of course we must replace this by using instead the time rate of change of momentum. Since we have defined momentum above, we also have the momentumposition commutation relation (22), relativistic Bohr-Sommerfeld quantization, etc. We have now derived, rather than postulated, your (1), (2), (3) and can accept Feynman's derivation as leading to the Maxwell equations, since in our theory the velocity which occurs in them is already required to be $c$ and the Lorentz invariance made compatible with our discrete versions of (1) and (3). I would very much
appreciate any comments you have on this and any suggestions you might have as to where and with how much detail we should attempt to publish it.

This still may leave you unsatisfied as to how the Maxwell equations emerge in an intuitive sense. But if you will refer to our basic paper (Ref. 4) you will find that we already had the spin quantum number for fermions even before the enclosed derivation of the Dirac equations, and knew how to use them as both sources and sinks of spin 1 photons and the coulomb interaction. We are forced by our theory to adopt the Wheeler-Feynman point of view that "photons" have to be defined in terms of their sources and sinks, but are better off than conventional QED because we need only a finite number of "soft photons" to define our metric, and have no ultraviolet or infrared divergences. Which brings me to the question of what causcs the Zittcrbcwegung in the first place.

My tentative conclusion is that what we have done is to model what in a second quantized field theory would be called the "vacuum fluctuations of the fields" or the "zero point energy" of the harmonic oscillators in those models in a finite and convergent manner. At this point we do not need to know the strength of the interactions, since the fluctuations only depend on $\hbar c$ if we know our mass in that unit. A good test of this will be whether we can derive the Casimir effect. I am already sure that it will be attractive for two plates and repulsive for a sphere, but whether we can get the coefficient right remains to be seen. Since we get both the Fermi constant and the electromagnetic $\alpha$ in close agreement to experiment in our first approximation, and the Dirac-Sommerfeld formula for Hydrogen at the same time as we get $\alpha$ to six or seven significant figures ${ }^{[22]}$ it looks like the Lamb shift should not be too difficult. If, following Parker-Rhodes, we think of the mass of the electron as due to the interaction with the electromagnetic field using the proton compton radius as the unit of length, we not only get the right result but can consistently think of our Zitterbewegung as a way of making a finite mass renormalization.

I could go on to other new results, but at this point I am primarily concerned with how you think our theory does, or does not, relate to the Feynman calculation. I hope to hear from you soon.

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[^0]:    * Work supported by the Department of Energy, contract DE-AC03-76SF00515.

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