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COMMENTS ON THE ELECTROWEAK PHASE TRANSITION*

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ABSTRACT

We report on an investigation of various problems related to the theory of the electroweak phase transition. This includes a determination of the

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nature of the phase transition, a discussion of the possible role of higher order radiative corrections and the theory of the formation and evolution of the bubbles of the new phase. We find in particular that no dangerous linear terms appear in the effective potential. However, the strength of the first order phase transition is 2/3 times less than what follows from the one-loop approximation. This rules out baryogenesis in the minimal version of the electroweak theory.

1. With the recognition that baryon number violation is unsuppressed at high temperature in the standard model has come the realization that the electroweak phase transition might be the origin of the observed asymmetry between matter and antimatter [1, 2, 3]. In order to have sufficient departure from equilibrium, it is necessary that this transition be rather strongly first order. As a result, there has been renewed interest in understanding under what circumstances the transition is first order, and how the transition proceeds. The minimal standard model almost certainly cannot produce the observed asymmetry: it has too little CP violation, and, as we will see, its phase transition is too weakly first order for a Higgs more massive than the present experimentalphalimit. Nevertheless, for considering the features of the phase transition, it is a useful prototype, because it is weakly coupled and comparatively simple.

Despite its apparent simplicity, understanding the phase transition in this theory has turned out to be surprisingly complicated, and the literature now contains contradictory claims on almost every point. In the first papers on this subject it was assumed that the phase transition is second order [4]. Later it was shown that if the Higgs mass is sufficiently small, the phase transition becomes first order [5]. The problem of bubble formation was considered in some detail; it was argued that at least initially the bubble walls typically are rather thick [6]. While bubble wall evolution was only touched upon in these early efforts [6], simple arguments suggested that, e.g. for a 50 GeV Higgs, the motion of the wall would be non-relativistic.

Recently, quite different views have been expressed about all of these issues. Brahm and Hsu, in an interesting paper [7], have argued that infrared effects spoil the one-loop analysis and claim to reliably establish that the transition is second order. Anderson and Hall [8] have thoroughly considered a number of aspects of the phase transition. Most of their results are in qualitative agreement with the earlier treatments [4, 5] and with our previous results [9], while providing some quantitative improvement. However, they argue that the initial bubbles are thin. Gleiser and Kolb [10], and Tetradis [12] argue that fluctuations are so large that the transition does not proceed through the formation of (critical) bubbles. There has also been controversy as to whether or not bubble wall motion is ultrarelativistic [13, 14].

In the present note, we will attempt to deal with these various issues. We will focus on the minimal standard model. As we have said, this model is not realistic, but we expect that the arguments and methods described here can be extended to more realistic situations. We focus on this case only because it is the simplest – and even here, we will frequently have to content ourselves with rather crude calculations. We will be able to give at least partial answers to each of the questions raised above. We will argue that, for sufficiently small coupling, the phase transition can be reliably shown to be first order. In particular, the linear term in the potential found in Ref. [7] is not present. On the other hand, we will see that the transition is more weakly first order than suggested by the one loop analysis. We will show that the bubbles are thick when they form, and the bubble nucleation rate cannot be reliably computed in the thin wall approximation. We will show that, for the range of Higgs masses considered here, the transition does in fact proceed by nucleation of critical bubbles. Finally, we will make some estimates of the bubble wall thickness and velocity. We will see that the early estimates of Ref. [6] are only reliable if particle mean free paths are very long. In practice, the relevant mean free paths are short, and the velocity of the bubble wall somewhat larger than these early estimates suggest. The problem of bubble wall propagation turns out to be surprisingly complicated, and we mention some of the issues which must be considered. We will see, however, that, for the first order transitions being considered here, the motion of the wall tends to be non-relativistic. In this paper, we will outline our treatment of each of these issues and describe the major results; details will be given in a subsequent publication [15].

2. We first consider the question of the order of the phase transition. The standard approach to this problem consists of computing the effective potential to one loop order. At high temperature, the one-loop expression for V_T is given, to a good approximation, by

$$V(\phi,T) = D(T^2 - T_o^2)\phi^2 - ET\phi^3 + \frac{\lambda_T}{4}\phi^4 \ . \tag{1}$$

Here $\lambda = m_H^2/2v_o^2$, and

$$D = \frac{1}{8v_o^2}(2m_W^2 + m_Z^2 + 2m_t^2) \ , \quad E = \frac{1}{4\pi v_o^3}(2m_W^3 + m_Z^3) \sim 10^{-2} \ , \tag{2}$$

$$T_o^2 = \frac{1}{4D}(m_H^2 - 8Bv_o^2) \ , \quad B = \frac{3}{64\pi^2}(2m_W^4 + m_Z^4 - 4m_t^4) \ , \tag{3}$$

$$\lambda_T = \lambda - \frac{3}{16\pi^2 v_o^4}\left(2m_W^4 \ln\frac{m_W^2}{a_B T^2} + m_Z^4 \ln\frac{m_Z^2}{a_B T^2} - 4m_t^4 \ln\frac{m_t^2}{a_F T^2}\right) \ . \tag{4}$$

Here $v_o = 246$ GeV is the value of the scalar field at the minimum of $V(\phi, 0)$ and $\ln a_B = 2\ln 4\pi - 2\gamma \simeq 3.51$, $\ln a_F = 2\ln\pi - 2\gamma \simeq 1.14$ [4, 9, 8].

The behavior of $V(\phi, T)$ is reviewed in Refs. [16, 17]. At very high temperatures the only minimum of $V(\phi, T)$ is at $\phi = 0$. A second minimum, ϕ_1, appears at $T = T_1$, where

$$T_1^2 = \frac{T_o^2}{1 - 9E^2/8\lambda_{T_1} D} \ , \quad \phi_1 = \frac{3ET}{2\lambda_T} \ . \tag{5}$$

The values of $V(\phi, T)$ in the two minima become equal to each other at the temperature T_c, where

$$T_c^2 = \frac{T_o^2}{1 - E^2/\lambda_{T_c} D} \ , \quad \phi_c = \frac{2ET}{\lambda_T} \ . \tag{6}$$

The minimum of $V(\phi, T)$ at $\phi = 0$ disappears at the temperature T_o, when the field ϕ in the second minimum is $\phi_o = 3ET/\lambda_T$.

3. The first order character of the transition is due to the ϕ^3 term in the potential. The appearance of such a term non-analytic in $|\phi|^2$ is the signal of an infrared problem, and raises concerns about the validity of the perturbation expansion. Indeed, these issues were raised in the early work on the subject [5, 18, 19]. The problem of infrared divergences at high temperatures in theories with light or massless particles has received extensive attention in the literature and is discussed in many textbooks [16, 20]. It is well known that these problems arise from the zero-frequency terms in the discrete frequency sums which appear in the computation of equilibrium quantities at finite temperature. The corresponding Feynman diagrams are those appropriate to a three dimensional field theory. For simplicity, we will consider here the contribution of W bosons. In the present case, the problem is most easily analyzed in Coulomb gauge, $\vec{\nabla} \cdot \vec{W} = 0$. The relevant propagators are those for the Coulomb lines, D_{00}, and transverse lines, D_{ij}. For

zero frequency,

$$D_{00} = \frac{1}{\vec{k}^2} \ , \quad D_{ij}(\vec{k}) = \frac{1}{\vec{k}^2 + m_W^2(\phi)} P_{ij}(\vec{k}) \ , \tag{7}$$

where $m_W = g\phi/2$, and $P_{ij} = \delta_{ij} - \frac{k_i k_j}{\vec{k}^2}$. The 'Goldstone' field and the physical Higgs scalar have standard scalar propagators with mass terms which are independent of the gauge coupling. The cubic term is readily extracted from the zero-frequency piece of the determinant. 2/3 of it arises from the transverse gauge bosons; the other 1/3 is obtained from the Coulomb line.

In this gauge it is rather easy to see how higher orders of perturbation theory behave. At one loop, it is well-known that the Coulomb line, even for $\phi = 0$, acquires a mass $m_D^2 = (N_g + N_c) g^2 T^2/3$ [21]. In order to obtain a sensible perturbation theory for small ϕ, it is necessary to partially resum the perturbation expansion, i.e. to use in each order of the loop expansion a Coulomb propagator with this effective mass. In the infra-red, this resummation corresponds to integrating out all heavy modes (to one loop order), leaving only the $\omega = 0$ modes, with a Debye mass. In practice, the tree level gauge boson mass at the minimum of the potential is small compared to m_D. As a result, repeating the one loop calculation with this effective mass, the contribution to the cubic term from the Coulomb line disappears. The same is not true, however, for the transverse bosons. For zero ϕ, gauge invariance forbids a one-loop mass for these fields; the transverse polarization tensor is in fact given by $\Pi_{ij} = \frac{15 g^2 T \sqrt{\vec{k}^2}}{32} P_{ij}$. As a result, the contribution to the cubic term from these fields survives, and the net effect is to reduce the coefficient of the cubic term in eq. (1) by 2/3: [1]

$$E = \frac{1}{6\pi v_o^3} (2 m_W^3 + m_Z^3) \ . \tag{8}$$

This reduction means that, for a given Higgs mass, the phase transition is significantly less first order than one expects from the one loop analysis.

[1] After we obtained this result, we received a very interesting paper by Carrington [22] where the modification of the cubic term by high order corrections was also considered. Even though the author did not claim that these corrections reduce the cubic term by a factor of 2/3, after some algebra one can check that his result is equivalent to ours.

It has been pointed out that a minimal requirement of the phase transition is that the sphaleron rate after the transition be sufficiently small that the baryon number not be washed out. Using the (unimproved) one loop result, this gives a limit of about $42 - 55$ GeV [23, 9], at best just barely consistent with the present experimental constraints [24]. Allowing for the correction obtained here, the limit on m_H is reduced by about 25%, clearly ruling out baryogenesis in the model.

More generally, however, we can ask about the behavior of the perturbation expansion, particularly at small ϕ. Before making general remarks, it is helpful to consider the two loop diagrams involving transverse gauge bosons and scalars (Fig. 1). The zero frequency pieces of these diagrams, separately, give contributions $\sim g^3|\phi|T^3$. If one combines these diagrams, however, being careful about combinatorics, they have the structure of an insertion of a polarization tensor on the transverse gauge boson line. Because, as mentioned above, this tensor vanishes at zero momentum, the sum of these diagrams is less singular at small ϕ, and simply gives a correction to the quadratic term. We have checked all other diagrams at two loop order and shown that there are no linear terms in the potential.

The authors of Ref. [7] found a linear contribution to the potential by simply substituting the scalar mass found at one loop back into the one loop calculation. Such a procedure is generally reliable when calculating Green's functions or tadpoles. Indeed, it is well known that the sum of the geometric progression, which appears after the insertion of an arbitrary number of polarization operators $\Pi(T)$ into the propagator $(k^2 + m^2)^{-1}$, simply gives $(k^2 + m^2 + \Pi(T))^{-1}$. However, this trick does not work for the closed loop diagram for the effective potential, which contains $\ln(k^2 + m^2)$. A naive substitution of the effective mass squared $m^2 + \Pi(T)$ instead of m^2 into $\ln(k^2 + m^2)$ corresponds to a wrong counting of higher order corrections. The simplest way to avoid the ambiguity is to calculate tadpole diagrams for $\frac{dV}{d\phi}$ instead of the vacuum loops, and then integrate the result with respect to ϕ. One can easily check by this method as well, that no linear terms appear in the expression for $V(\phi, T)$.

The absence of linear terms does not automatically mean that higher order corrections are completely under control. A general investigation of the infrared problem in the non-Abelian gauge theories at finite temperature suggests that the results which we obtained are reliable for $\phi \gtrsim \frac{g}{2} T \sim T/3$ [18, 19]. Thus, a more detailed investigation is needed to study behavior of the theories with $m_H \gtrsim 10^2$ GeV near the critical temperature, since the scalar field, which appears at the moment of the phase transition in these theories, is very small (see Fig. 2). However, we expect that our results are reliable for strongly first order phase transitions with $\phi \gtrsim T$, which is quite sufficient to study (or to rule out) baryogenesis in the electroweak theory.

4. We turn now to the problem of bubble formation. At high temperatures, this becomes a problem in classical thermodynamics. One looks for a stationary point of the free energy, with the property that $\phi \to 0$ as $r \to \infty$, i.e. a solution of the classical field equations with potential $V_0 + V_T$. Some care is required in solving this equation, however, since necessarily one is constructing a saddle point of the action (unstable modes corresponding to bubble growth or collapse). As a result, if one makes a poor approximation, one overestimates the probability of bubble formation. We believe this is the case of the analysis of Ref. [8], where formation of bubbles was studied in the thin wall approximation. This approximation works well if the height of the barrier between the two minima is much larger than the difference between the values of the effective potential in each of them. This is not the case for the phase transition with $m_H \lesssim 60$ GeV. Numerical solution of the equations yields an action typically a few times larger than that obtained from the thin wall approximation.

Even though the thin wall approximation fails, one can still study bubble formation analytically in a wide class of theories. Indeed, we have found that in the vicinity of the phase transition one can write, to a very good approximation,

$$\frac{S_3}{T} = \frac{38.8\, D^{3/2}}{E^2} \cdot \Big(\frac{\Delta T}{T}\Big)^{3/2} \times f\Big(\frac{2\,\lambda_o\, D\, \Delta T}{E^2\, T}\Big). \tag{9}$$

where

$$f(\alpha) = 1 + \frac{\alpha}{4}\Big[1 + \frac{2.4}{1-\alpha} + \frac{0.26}{(1-\alpha)^2}\Big]. \tag{10}$$

Parameter α changes from 0 to 1 in the range of temperatures for which the phase transition is possible. We have found that eq. (10) is correct with an accuracy about 1% in the most interesting range $0 \leq \alpha \leq 0.95$.

5. In the case that the transition is weakly first order, it is natural to ask whether the transition actually proceeds through formation of bubbles, or if other sorts of fluctuations might be more important. In most treatments, it is assumed that the transition occurs once the bubble nucleation rate is large enough that the universe can fill with bubbles. In practice, because of the extremely slow expansion rate at the time of the transition, this means that the barrier is still high enough that the naive calculation of the nucleation rate gives an extremely small result; the three dimensional action is of order $130-140$. Given that the rate of formation of critical bubbles is so small, one might expect that other types of fluctuations which might equilibrate the two phases would be extremely rare. In Refs. [10, 11, 12], however, it has been argued that this is not the case. Roughly speaking, these authors arrive at this conclusion by estimating the mean square fluctuation of the scalar field about the symmetric minimum, $\phi^2_{rms} = <\phi^2>$, and comparing this with the value of the field at the other minimum. A rough estimate leads them to the conclusion that the $<\phi^2> \sim mT \sim \phi_c^2$, so that it is not meaningful to consider the system as sitting in one vacuum or the other. Here m is the Higgs field mass near $\phi = 0$. Subcritical bubbles, they argue, equilibrate the two phases even before one reaches the temperature T_c.

While we believe that for some range of parameters subcritical bubbles may be important, we do not believe that this is the case for the Higgs masses under consideration here. In estimating ϕ_{rms}, one should be careful to consider only long wavelength modes. Short wavelength modes will be associated with configurations with large gradient terms, which will collapse in a microscopic time. Also, one must be very careful with factors of π and 2. A more detailed investigation based on the stochastic approach to tunneling gives the estimate for the amplitude of relevant fluctuations $<\phi^2> \sim \frac{mT}{\pi^2}$ [25]. Combining this estimate with our results for the mass m, i.e. for the curvature of the effective potential near $\phi = 0$ in the relevant temperature interval, $T_o < T < T_1$, one obtains an estimate $\phi_{rms} \sim .1\, T$.

From this analysis, we see that the typical amplitude of the relevant scalar field fluctuations is substantially less than the separation of the two minima, unless the phase transition is very weakly first order. Even for $m_H \sim 60$ GeV, the distance between the two minima remains five times greater than ϕ_{rms}. Including fluctuations with $k \gg k_{max}$ will give larger ϕ_{rms}, but these will collapse in a microscopic time, and will not serve to equilibrate the two phases.

6. Understanding the motion of the wall is important mainly in the context of baryogenesis. We have already seen that no baryons will be generated in the single Higgs theory, unless the Higgs mass is well below the present experimental limit. We expect, however, that, for $m_H \sim 35$ GeV (for which $\phi/T > 1$), the bubble wall motion in this model will have many features in common with more realistic theories of baryogenesis, which require the phase transition to be strongly first order. Even in this simple model, determining the wall velocity and shape is a difficult problem, and we will content ourselves with rather crude estimates. In our analysis, we will assume that the wall achieves a steady state after some time. In particular, there is a frame, which we refer to as the 'wall frame', in which the scalar field and the particle distributions are independent of time. We will assume that the principle source of damping of the walls motion is elastic scattering of particles from the wall; if there are additional sources of damping, they can only slow the wall even further. We will treat the velocity of the wall as a small parameter. We will have to check its validity *a posteriori*. With these assumptions, it is helpful to consider two limiting cases, depending on whether the size of the wall, δ, is large or small compared to the relevant mean free paths for elastic scattering, ζ. As an estimate of these mean free paths, we follow Ref. [3] and take the longitudinal and transverse gluon propagators to include a mass proportional to m_D. For top quarks, this yields $\sigma_t = 16\,\pi\alpha_s^2/3m_{el}^2$. Multiplying by the flux one obtains $\zeta \sim 4\,T^{-1}$ for quarks. For W's and Z's the result is about three times larger. These numbers are consistent with results which have been obtained for the stopping power [26]. To get some feeling for the wall size, consider the system at temperature T_c. At this temperature, the two phases can coexist, separated by a static domain wall. The ϕ field in this domain wall is readily obtained by quadrature; for a 35 GeV Higgs, one finds $\delta \sim 40\,T^{-1}$.

This suggests that the thin wall limit may not be a good one, but it is instructive to consider it in any case. In this limit, a typical particle passes through the wall, or is reflected from it, without scattering. An estimate in this limit was given in Ref. [6]. If the problem is treated semiclassically, it is straightforward to calculate the extra, velocity-dependent force on the wall, assuming that all particles approaching the wall from either side are described by an equilibrium distribution at some temperature. (Note that particles moving away from the wall are not described by an equilibrium distribution in this case; these particles are assumed to be equilibrated far from the wall, at some possibly different temperature and velocity). In this model it is straightforward to calculate the force on the wall to linear order in v. For bosons, the leading term is of order m^3T, while for fermions, it is of order m^4. Equating this force to the pressure difference on the two sides of the wall gives the velocity. Defining

$$\epsilon = \frac{T_c - T}{T_c - T_o} \ , \tag{11}$$

we can obtain an approximate equation for the velocity, valid for small ϵ:

$$v \sim \frac{\pi}{6} \frac{\epsilon}{(1+\epsilon)} \, \mathcal{C}(m_t, \lambda) \ , \tag{12}$$

where the correction $\mathcal{C}$ comes predominantly from top quarks and is roughly of order one. For $m_t = 120$ GeV and $m_H = 35$ GeV (for which $\epsilon \sim 1/4$), we find $v \sim 0.05$.

In this discussion we have made a variety of oversimplifications. One which is potentially important is our assumption of equilibrium densities on both sides of the wall. Some fraction of incoming particles, however, are reflected from the wall, and this will tend to lead to an enhancement of the particle density in front of the wall. For definiteness, consider top quarks. As a crude estimate, suppose the fraction of reflected particles is f (of order m/T), and suppose that the mean free path for processes which can change top quark number is τ, with $\zeta \ll \tau$. Then the density in front of the wall is enhanced by an amount of order $fvn\sqrt{\frac{\tau}{\zeta}}$. For the thin wall case, this is likely to reduce the velocity of the wall somewhat. In the thicker wall cases considered below, however, this is likely to be more important.

Consider now the case $\delta \gg \zeta$. In this case, to model the deviations from equilibrium due to the finite velocity of the wall, we can break the wall into segments of length ζ, and repeat the thin wall analysis for each of these segments. In particular, we assume that the distributions on either side of each segment are at equilibrium. We also assume that the temperature and velocity of the particle distributions are constant across the wall. This should not be a bad approximation when there are many light species of particles in the system. One obtains for the v-dependent force on the wall, a result roughly suppressed by a factor of order $\sqrt{\zeta/\delta}$. For a Higgs mass of 35 GeV we obtain a velocity ~ 0.2.

In this thick wall case, the density enhancement described earlier could be extremely important. Here one expects the extra particles to be distributed more or less uniformly over the wall. This leads to an extra force $\Delta F \sim f n v \tau \ \Delta\rho \ /\delta$ where $\Delta\rho$ denotes the internal energy difference on the two sides of the wall. As an estimate of f, we can take the ratio of the equilibrium densities on the two sides of the wall; for top quarks, this gives a number of order 5%. This effect seems to be comparable to that of the preceding paragraph.

7. In this paper we have outlined a program for treating the phase transition in weakly coupled theories with scalar fields. From the standpoint of electroweak baryogenesis, it is important to find models consistent with current experimental bounds in which the phase transition is strongly first order. It would be interesting to find models where the bubble wall is thin; in such theories [3] electroweak baryogenesis can be very efficient. However, our work suggests that in many models, the wall will be slow and thick, and adiabatic analyses of the type of Ref. [2] will be relevant. In these cases, one may just barely be able to produce the observed asymmetry. Further improvements in both the theory of the phase transition and that of electroweak baryogenesis will be necessary to completely settle these questions.

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References

[1] V.A. Kuzmin, V.A. Rubakov and M.E.Shaposhnikov, Phys. Lett. 155B (1985) 36; M.E. Shaposhnikov, JETP Lett. **44** (1986) 465; Nucl. Phys. **B287** (1987) 757; Nucl. Phys. **B299** (1988) 797; A.I. Bochkarev, S.Yu. Khlebnikov and M.E. Shaposhnikov, Nucl. Phys. **B329** (1990) 490; L. McLerran, Phys. Rev. Lett. **62** (1989) 1075; N. Turok and P. Zadrozny, Phys. Rev. Lett. **65** (1990) 2331; Nucl. Phys. **B358** (1991) 471.

[2] L. McLerran, M. Shaposhnikov, N. Turok and M. Voloshin, Phys. Lett. **256B** (1991) 451; M. Dine, P. Huet, R. Singleton and L. Susskind, Phys.Lett. **257B** (1991) 351.

[3] A. Cohen, D.B. Kaplan and A.E. Nelson, Nucl. Phys. **B349** (1991) 727; Phys.Lett. **263B** (1991) 86; University of California, San Diego, preprint UCSD-PTH-91-20 (1991).

[4] D.A. Kirzhnits, JETP Lett. **15** (1972) 529; D.A. Kirzhnits and A.D. Linde, Phys. Lett. **72B** (1972) 471; S. Weinberg, Phys. Rev. **D9** (1974) 3357; L. Dolan and R. Jackiw, Phys. Rev. **D9** (1974) 3320; D.A. Kirzhnits and A.D. Linde, JETP **40** (1974) 628.

[5] D.A. Kirzhnits and A.D. Linde, Ann. Phys. **101** (1976) 195.

[6] A.D. Linde, Phys.Lett. **70B** (1977) 306; **100B** (1981) 37; Nucl. Phys. **B216** (1983) 421.

[7] D. Brahm and S. Hsu, Caltech preprints CALT-68-1705 and CALT-68-1762 (1991).

[8] G. Anderson and L. Hall, LBL preprint LBL-31169 (1991).

[9] M. Dine, P. Huet and R. Singleton, University of California, Santa Cruz, preprint SCIPP 91/08 (1991); A.D. Linde and D.A. Linde, unpublished.

[10] M. Gleiser and E. Kolb, FNAL preprint FERMILAB-Pub-91/305-A (1991).

[11] M. Gleiser, E. Kolb and R. Watkins, Nucl. Phys. **B364** (1991) 411.

[12] N. Tetradis, preprint DESY 91-151.

[13] N. Turok, Princeton preprint PUPT-91-1273.

[14] L. McLerran and N. Turok, private communication.

[15] M.Dine, R.G. Leigh, P. Huet, A.D. Linde and D.A. Linde, preprint SCIPP-92-07, SLAC-PUB-5741, SU-ITP-92-7 (1992).

[16] A.D. Linde, *Particle Physics and Inflationary Cosmology* (Harwood, New York, 1990).

[17] M. Sher, Phys. Rep. **179** (1989) 273.

[18] A.D. Linde, Rep. Prog. Phys. **42** (1979) 389.

[19] A.D. Linde, Phys. Lett. **93B** (1980) 327;
D.J. Gross, R.D. Pisarski and L.G. Yaffe, Rev. Mod. Phys. **53** (1981) 1.

[20] J.I. Kapusta, *Finite Temperature Field Theory*, Cambridge University Press, 1989.

[21] O.K. Kalashnikov and V.V. Klimov, Sov. J. Nucl. Phys. **31** (1980) 699; H.A. Weldon, Phys. Rev. **D26** (1982) 1384, 2789; E. Braaten and R. Pisarski, Phys. Rev. **D42** (1990) 2156.

[22] M.E. Carrington, University of Minnesota preprint TPI-MINN-91/48-T (1991).

[23] A. Bochkarev, S. Kuzmin and M. Shaposhnikov, Phys. Lett. **244B** (1990) 27.

[24] ALEPH, DELPHI, L3 and OPAL Collaborations, as presented by M. Davier, Proceedings of the International Lepton-Photon Symposium and Europhysics Conference on High Energy Physics, eds. S. Hegerty, K. Potter and E. Quercigh (Geneva, 1991), to appear.

[25] A.D. Linde, Stanford University preprint SU-ITP-900 (1991), to be published in Nucl. Phys.

[26] E. Braaten and M.H. Thoma, LBL preprint LBL-30998 (1991).

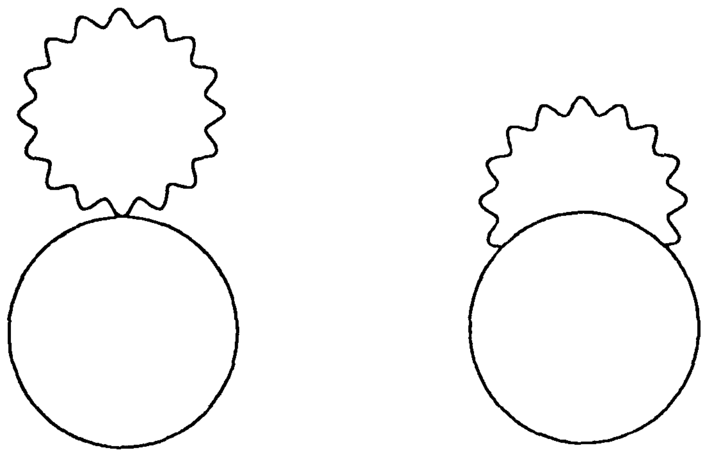

Fig.1: Two loop graphs that are singular in the infrared.

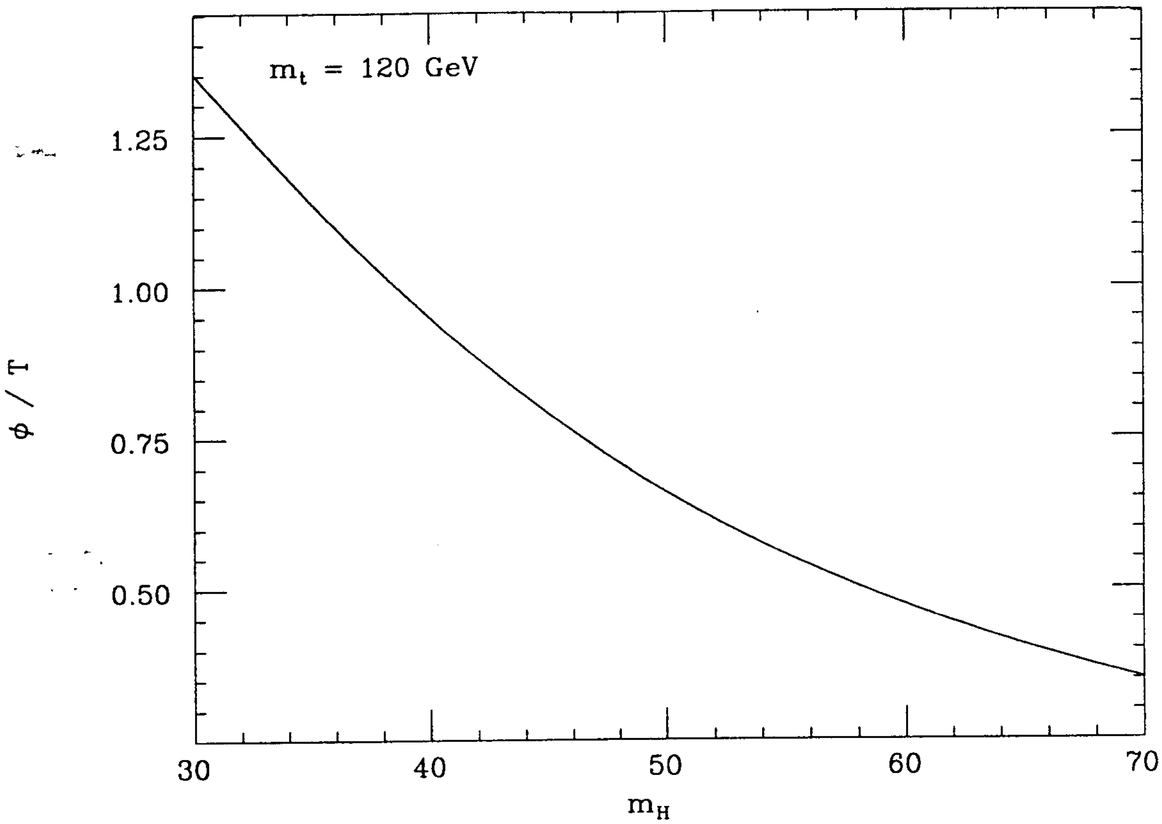

Fig.2: The value of ϕ/T at the point of tunneling, as a function of Higgs mass.