arXiv:hep-th/9311017 02 Nov 1993

6392
September 1993
T/E

Quantization of Gauge Field Theories on the Front-Form without Gauge Constraints I : The Abelian Case*

Ovid C. Jacob†

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94309

ABSTRACT

Recently, we have proposed a new front-form quantization which treated both the x^+ and the x^- coordinates as front-form 'times.' This quantization was found to preserve parity explicitly. In this paper we extend this construction to local Abelian gauge fields . We quantize this theory using a method proposed originally by Faddeev and Jackiw . We emphasize here the feature that quantizing along both x^+ and x^- , gauge theories does not require extra constraints (also known as 'gauge conditions') to determine the solution uniquely.

Submitted to Phys. Rev. D

⋆ Work supported by the Department of Energy, contract DE–AC03–76SF00515.
† jacob@slacvm.slac.stanford.edu

1. Introduction

Front-form quantization is usually done by quantization along the front $x^+ = const$. Usually this is done by quantizing a system with constraints[1] [2] [3] [4] . In a previous papers[5] [6] [7] we introduced a quantization which treated x^+ and x^- on equal footing.[8] The main argument given was that this new approach was manifestly parity invariant. We also pointed out that this new approach had the same number of degrees of freedom as the equal-time approach.

We'd like to point out that in some work involving initial value problems in gravity using front-form coordinates[9] [10] [11] [12] [13] , the initial data for these coordinates is also specified along **both** $x^+ = const$ and $x^- = const$ surfaces as well as at $x^+ = x^- = 0$. R. Penrose [11] points out that in this approach there are *no* constraints . In this paper we study how this approach could bypass much of the difficulty coming from the presence of constraints in the presence of local Abelian ($U(1)$) gauge symmetry. (A future paper will look at the non-Abelian ($SU(N)$) case. The point is as follows: in usual gauge theory quantization, the gauge condition is a relation (constraint) between the quantizing degrees of freedom (the initial data). We show in this work that using the two null hyperplanes, we don't need any constraints between initial data.

There are two points which we should mention. First, the use of the reduced phase space quantization of Faddeev and Jackiw[14] (see also[15]) allows us to get the commutation relations easily. Second, as they point out, if the two-form (which goes in defining the equations of motion) is invertible, then there are **no constraints** . This fact coupled with Penrose's remark [11] seem to imply that using two null hyperplanes, we always have an invertible two-form , hence never any constraints. Obviously this greatly facilitates the quantization procedure.

2. Reduced Phase Space Quantization of QED

We follow the reduced phase space quantization of Faddeev and Jackiw [14] to study QED :

$$\mathcal{L} = -\frac{1}{2}F_{\mu\nu}F^{\mu\nu} + \frac{i}{2}\partial^{\nu}\overline{\psi}\gamma_{\nu}\psi - \overline{\psi}\frac{i}{2}\partial^{\nu}\gamma_{\nu}\psi - m\overline{\psi}\psi + \mathcal{L}_I \tag{2.1}$$

where $\mathcal{L}_I$ is the interaction part of the Lagrangean:

$$\mathcal{L}_I = -e\overline{\psi}\gamma^{\nu}\psi A_{\nu} \tag{2.2}$$

The Euler-Lagrange equations of motion are

$$\partial_{\mu}F^{\mu\nu} - e\overline{\psi}\gamma^{\nu}\psi = 0 \tag{2.3}$$

$$(i\gamma_{\mu}\partial^{\mu} - m - e\gamma_{\mu}A^{\mu})\psi = 0 \tag{2.4}$$

To obtain the Hamiltonians for evolution along x^+ and x^-, we follow the approach of previous papers [5] and [6] , so we write $\mathcal{L}$ out explicitly :

$$\mathcal{L}d^4x = \{ -\frac{1}{2}F_{ij}F^{ij} - F_{+-}F^{+-} - F_{+i}F^{+i} - F_{-i}F^{-i}$$

$$-2e\psi_+^{\dagger}A_+\psi_+ - 2e\psi_-^{\dagger}A_-\psi_- + e\psi_+^{\dagger}\gamma_0\gamma_i A^i\psi_- + e\psi_-^{\dagger}\gamma_0\gamma_i A^i\psi_+ + \psi_+^{\dagger}\frac{i\partial^-}{2}\psi_+ - \frac{i}{2}(\partial^-\psi_+^{\dagger})\psi_+$$

$$+\psi_-^{\dagger}\frac{i\partial^+}{2}\psi_- - \frac{i}{2}(\partial^+\psi_-^{\dagger})\psi_- + -\psi_+^{\dagger}\gamma_0\gamma_i\frac{i\partial_i}{2}\psi_- + \frac{i}{2}(\partial_i\psi_-^{\dagger})\gamma_0\gamma_i\psi_+$$

$$-\psi_-^{\dagger}\gamma_0\gamma_i\frac{i\partial_i}{2}\psi_+ + \frac{i}{2}(\partial_i\psi_+^{\dagger})\gamma_0\gamma_i\psi_- - m\psi_+^{\dagger}\frac{\gamma^-}{2}\psi_- - m\psi_-^{\dagger}\frac{\gamma^+}{2}\psi_+\}\frac{dx^-dx^+d^2x_{\perp}}{2} \tag{2.5}$$

where $\psi_{\pm} = \Lambda_{\pm}\psi$ and $\Lambda_{\pm} = \frac{1}{2}\gamma^0\gamma^{\pm}$. Note also that $\partial_{\nu} = \frac{\partial}{\partial x^{\nu}}$ so that $\partial^- = 2\partial_+ = 2\frac{\partial}{\partial x^+}$ and $\partial^+ = 2\partial_- = 2\frac{\partial}{\partial x^-}$. The corresponding conjugate momenta for

x^+-derivatives are

$$\pi_A^i = \frac{\partial \mathcal{L}}{\partial(\partial^- A_i)} = -\frac{1}{2}F^{+i} \tag{2.6}$$

$$\pi_\psi(x) = \frac{\partial \mathcal{L}}{\partial(\partial^- \psi)} = \frac{i}{2}\psi_+^\dagger \tag{2.7}$$

$$\pi_{\psi^\dagger}(x) = \frac{\partial \mathcal{L}}{\partial(\partial^- \psi^\dagger)} = -\frac{i}{2}\psi_+ \tag{2.8}$$

For the momenta corresponding to x^--derivatives we get similar forms:

$$\rho_A^i = \frac{\partial \mathcal{L}}{\partial(\partial^+ A_i)} = -\frac{1}{2}F^{-i} \tag{2.9}$$

$$\rho_\psi(x) = \frac{\partial \mathcal{L}}{\partial(\partial^+ \psi)} = \frac{i}{2}\psi_-^\dagger \tag{2.10}$$

$$\rho_{\psi^\dagger}(x) = \frac{\partial \mathcal{L}}{\partial(\partial^+ \psi^\dagger)} = -\frac{i}{2}\psi_- \tag{2.11}$$

We rewrite $\mathcal{L}d^4x$ in the following way

$$\mathcal{L}d^4x = \frac{1}{2}\{\pi_A^i\partial^- A_i + \pi_A^i\partial^- A_i + \rho_A^i\partial^+ A_i + \rho_A^i\partial^+ A_i$$

$$+\pi_\psi\partial^-\psi_+ + \pi_\psi\partial^-\psi_+ + (\partial^-\psi_+^\dagger)\pi_{\psi^\dagger} + (\partial^-\psi_+^\dagger)\pi_{\psi^\dagger}$$

$$+\rho_\phi\partial^+\phi + \rho_\phi\partial^+\phi + \rho_\psi\partial^+\psi_- + \rho_\psi\partial^+\psi_- + (\partial^-\psi_-^\dagger)\rho_{\psi^\dagger} + (\partial^-\psi_-^\dagger)\rho_{\psi^\dagger}\}d^4x$$

$$-\mathcal{H}dx^+ - \mathcal{K}dx^- + \mathcal{M}d^4x \tag{2.12}$$

The meaning of these terms is as follows : the first bracket represents the p-q-dot terms which go into the definitions of the canonical commutation relations; the second and third term are the Hamiltonians which define the evolution of the system along x^+ , given by $\mathcal{H}$, and along x^- given by $\mathcal{K}$ finally, the last term

contains the remaining pieces which give the 'constraints', though are not 'true constraints' [14] as are consistent with the gauge field equations of motions. The Hamiltonians $\mathcal{H}$ and $\mathcal{K}$ are :

$$\mathcal{H} = \frac{dx^- dx_\perp^2}{2}\{\frac{1}{2}(B^{-2}) + 2e\psi_+^\dagger A_+\psi_+ + 2e\psi_-^\dagger A_-\psi_- + \psi_+^\dagger\gamma_0\gamma_i\frac{i\partial_i}{2}\psi_- + e\psi_+^\dagger\gamma_0\gamma_i A^i\psi_-$$

$$-\frac{i}{2}(\partial_i\psi_-^\dagger)\gamma_0\gamma_i\psi_- e\psi_-^\dagger\gamma_0\gamma_i A^i\psi_+ + m\psi_+^\dagger\frac{\gamma^-}{4}\psi_- + m\psi_-^\dagger\frac{\gamma^+}{4}\psi_+\} \tag{2.13}$$

$$\mathcal{K} = \frac{dx^+ dx_\perp^2}{2}\{\frac{1}{2}(B^{+2}) + 2e\psi_-^\dagger A_-\psi_- + 2e\psi_+^\dagger A_+\psi_+ + \psi_-^\dagger\gamma_0\gamma_i\frac{i\partial_i}{2}\psi_- e\psi_-^\dagger\gamma_0\gamma_i A^i\psi_+$$

$$-\frac{i}{2}(\partial_i\psi_+^\dagger)\gamma_0\gamma_i\psi_- + e\psi_+^\dagger\gamma_0\gamma_i A^i\psi_- + m\psi_-^\dagger\frac{\gamma^+}{4}\psi_+ + m\psi_+^\dagger\frac{\gamma^-}{4}\psi_-\} \tag{2.14}$$

where $B^- = B^+ = \frac{1}{\sqrt{2}}F^{12}$, and for the 'constraints' we get whatever is left over

$$\mathcal{M} = \{ -\partial_i A_+ F^{+i} - \partial_i A_- F^{-i} - F_{+-}F^{+-} + 2eA_+\psi_+^\dagger\psi_+ + 2eA_-\psi_-^\dagger\psi_-\} \tag{2.15}$$

Well, we can rewrite is as (up to total derivatives) :

$$\mathcal{M} = A_+ C_\pi + A_- C_\rho \tag{2.16}$$

and the 'constraints' C_π and C_ρ are

$$C_\pi = -\partial_- F^{-+} - \partial_i F^{i+} + 2e\psi_+^\dagger\psi_+ \tag{2.17}$$

$$C_\rho = -\partial_+ F^{+-} - \partial_i F^{i-} + 2e\psi_-^\dagger\psi_- \tag{2.18}$$

We see that $C_\pi = C_\rho = 0$ identically by the classical equation of motion for the gauge fields , as in equation [(2.3)] for $\nu = +, -$.

Let us write the $\mathcal{L}dx^4$ with the explicit momenta dependence (up to total derivatives which we can discard [14],[16]), so as to make the resulting commutation relation clear :

$$\mathcal{L}d^4x = \frac{1}{2}2\{\pi^i_A dA_i - A_i d\pi^i_A + \pi_\psi d\psi_+ - d\pi_\psi \psi_+ + d\psi^\dagger_+ \pi_{\psi^\dagger} - \psi^\dagger_+ d\pi_{\psi^\dagger}\}\frac{dx^- dx_\perp}{2}$$

$$+\frac{1}{2}2\{\rho^i_A dA_i - A_i d\rho^i_A + \rho_\psi d\psi_- - d\rho_\psi \psi_- + d\psi^\dagger_- \rho_{\psi^\dagger} - \psi^\dagger_- d\rho_{\psi^\dagger}\}\frac{dx^+ dx_\perp}{2}$$

$$-\mathcal{H}dx^+ - \mathcal{K}dx^- + A_+ C_\pi d^4x + A_- C_\rho d^4x \tag{2.19}$$

We see now that we have two types of evolutions, one along x^+, for which the first term in equation (2.19) gives the commutation relations along surfaces $x^+ = y^+$ according to the form:

$$[\xi^a, \xi^b] = \Gamma^{-1}_{ab} \quad a, b = 1, ..8 \tag{2.20}$$

with

$$\xi^1 = \pi^1_A, \xi^2 = \pi^2_A, \xi^3 = \pi_\psi, \xi^4 = \pi_{\psi^\dagger}, \xi^5 = A^1, \xi^6 = A^2, \xi^7 = \psi_+, \xi^8 = \psi^\dagger_+ \tag{2.21}$$

and

$$\Gamma_{15} = \Gamma_{26} = \Gamma_{37} = \Gamma_{48} = 2 = -\Gamma_{48} = -\Gamma_{57} = -\Gamma_{62} = -\Gamma_{51} \tag{2.22}$$

and all the other Γ's are 0 . The second term in equation (2.19) gives the commutation relations along surfaces $x^- = y^-$ according to the form :

$$[\eta^a, \eta^b] = \Delta^{-1}_{ab} \quad a, b = 1, ..8 \tag{2.23}$$

with

$$\eta^1 = \rho^1_A, \eta^2 = \rho^2_A, \eta^3 = \pi_\psi, \eta^4 = \pi_{\psi^\dagger}, \eta^5 = A^1, \eta^6 = A^2, \eta^7 = \psi_-, \eta^8 = \psi^\dagger_- \tag{2.24}$$

and

$$\Delta_{15} = \Delta_{26} = \Delta_{37} = \Delta_{48} = 2 = -\Delta_{48} = -\Delta_{57} = -\Delta_{62} = -\Delta_{51} \tag{2.25}$$

and all the other Δ's are 0 . Going now to the quantum commutators, we get the

following relations for fields at equal $x^+ = y^+$, the usual front-form 'time' :

$$[A^i(x^+, x^-, x_\perp), \pi_A^j(y^+, y^-, y_\perp)]_{x^+=y^+} = \frac{i}{2}\delta(x^- - y^-)\delta^2(x_\perp - y_\perp)\delta^{ij} \tag{2.26}$$

$$\left\{\psi_+(x^+, x^-, x_\perp), \pi_\psi(y^+, y^-, y_\perp)\right\}_{x^+=y^+} = +\frac{i}{2}\Lambda_+\delta(x^- - y^-)\delta^2(x_\perp - y_\perp) \tag{2.27}$$

$$\left\{\psi_+^\dagger(x^+, x^-, x_\perp), \pi_{\psi^\dagger}(y^+, y^-, y_\perp)\right\}_{x^+=y^+} = -\frac{i}{2}\Lambda_+\delta(x^- - y^-)\delta^2(x_\perp - y_\perp) \tag{2.28}$$

Thus, the physical (quantized) degrees of freedom on $x^+ = 0$ are A^i , ψ_+ and $\psi_+^\dagger$.

For fields at equal $x^- = y^-$, a new front-form 'time', we get:

$$[A^i(x^+, x^-, x_\perp), \pi_A^j(y^+, y^-, y_\perp)]_{x^-=y^-} = \frac{i}{2}\delta(x^- - y^-)\delta^2(x_\perp - y_\perp)\delta^{ij} \tag{2.29}$$

$$\left\{\psi_-(x^+, x^-, x_\perp), \rho_\psi(y^+, y^-, y_\perp)\right\}_{x^-=y^-} = +\frac{i}{2}\Lambda_-\delta(x^+ - y^+)\delta^2(x_\perp - y_\perp) \tag{2.30}$$

$$\left\{\psi_-^\dagger(x^+, x^-, x_\perp), \rho_{\psi^\dagger}(y^+, y^-, y_\perp)\right\}_{x^-=y^-} = -\frac{i}{2}\Lambda_-\delta(x^+ - y^+)\delta^2(x_\perp - y_\perp) \tag{2.31}$$

Here, the physical (quantized) degrees of freedom on $x^- = 0$ are A^i , ψ_- and $\psi_-^\dagger$. Note that A^+ and A^- do not enter in the list of physical (quantized) degrees of freedom.

The equations of motions are now like in Faddeev and Jackiw [14]

$$\Gamma_{ab}\partial^-\xi^b = \frac{\partial\mathcal{H}}{\partial\xi^a} \tag{2.32}$$

for the x^+ variation, and

$$\Delta_{ab}\partial^-\eta^b = \frac{\partial\mathcal{K}}{\partial\eta^a} \tag{2.33}$$

for the x^- variation . For $a = 5$ and $b = 1$, equation (2.32) gives

$$\partial_+F^{+1} = +2e\psi_+^\dagger\gamma_0\gamma_i\psi_- + 2e\psi_-^\dagger\gamma_0\gamma_i\psi_+ \tag{2.34}$$

which is just the equation of motion [(2.3)] for $\nu = 1$. For $a = 7$ and $b = 3$ we

recover the equation of motion for $\psi_+^\dagger$

$$i\partial^-\psi_+^\dagger = i\frac{\partial_i\psi_-^\dagger}{2}\gamma_0\gamma_i - m\psi_-^\dagger\frac{\gamma_0}{2} + 2e\psi_+^\dagger A_+ \tag{2.35}$$

We get similar results from (2.33).

But what is the meaning of the fields A^+ and A^- ? They obey the following coupled set of differential equations ,according to C_π and C_ρ :

$$\frac{1}{2}\partial^+\partial^- A^+ - \frac{1}{2}(\partial^+)^2 A^- - (\partial^i)^2 A^+ = -\partial^i\partial^+ A^i + 2e\psi_+^\dagger\psi_+ \tag{2.36}$$

$$\frac{1}{2}\partial^-\partial^+ A^- - \frac{1}{2}(\partial^-)^2 A^+ - (\partial^i)^2 A^- = -\partial^i\partial^- A^i + 2e\psi_-^\dagger\psi_- \tag{2.37}$$

We've arranged the equations so that all the known fields, the independent fields are on the right-hand side, and the 'new' fields are on the left-hand side. The point is that these are **not** constraint equations since they are not relations between the initial data , since neither A^+ nor A^- get initialized on either hyperplane! We introduce these new fields so that we preserve Lorentz covariance and so that we have the same equations of motion in the Euler-Lagrange case and the Hamiltonian case.

Inverting these equations, we got the following equations for A^+ and A^- :

$$A^+ = ((\partial^i)^2)^{-1}\partial^i\partial^+ A^i - ((\partial^i)^2)^{-1}e\psi_+^\dagger\psi_+ + (\partial^+\partial^- - (\partial^i)^2)^{-1}e\psi_+^\dagger\psi_+$$

$$-((\partial^i)^2)^{-1}(\partial^+\partial^- - (\partial^i)^2)^{-1}(\partial^+)^2 e\psi_+^\dagger\psi_+ \tag{2.38}$$

$$A^- = ((\partial^i)^2)^{-1}\partial^i\partial^+ A^i - ((\partial^i)^2)^{-1}e\psi_-^\dagger\psi_- + (\partial^-\partial^+ - (\partial^i)^2)^{-1}e\psi_-^\dagger\psi_-$$

$$-((\partial^i)^2)^{-1}(\partial^-\partial^+ - (\partial^i)^2)^{-1}(\partial^-)^2 e\psi_-^\dagger\psi_- \tag{2.39}$$

To fully define these fields, we need to define the operators $((\partial^i)^2)^{-1}$ and $(\partial^+\partial^- - (\partial^i)^2)^{-1}$. Then we'll have A^+ and A^- completely determined in terms of known fields.

This is quite straight-forward. For the definition of $(\partial^+)^{-1}$, we use the idea of Zhang and Harindranath [16] of taking anti-periodic boundary conditions for all the fields. This determines then the definition for this operator we are considering. It is

$$\frac{1}{\partial^+}f(x^-) = \frac{1}{2}\int \frac{dk^+}{2\pi}e^{-ik^+x^-}\{\frac{1}{k^+ + i\epsilon} + \frac{1}{k^+ - i\epsilon}\}f(k^+) \tag{2.40}$$

which leads to the following form for its square

$$(\frac{1}{\partial^+})^2 f(x^-) = \frac{1}{2}\int \frac{dk^+}{2\pi}e^{-ik^+x^-}\{\frac{1}{k^+ + i\epsilon} + \frac{1}{k^+ - i\epsilon}\}^2 f(k^+) \tag{2.41}$$

In position space, the operator $(\partial^+)^{-1}$, is just the convoluted epsilon distribution [16], while the operator $(\partial^+)^{-2}$, becomes

$$\frac{1}{2}\int_{-\lambda}^{\lambda} dx^-\epsilon(x^- - x'^-)\epsilon(x^- - x"^-) = -|x'^- - x"^-| + \lambda \tag{2.42}$$

As Zhang and Harindranath point out, it is crucial that we take this definition in getting a consistent specification of the front-form singularity $k^+ = 0$.

We treat the sibling operator $(\partial^-)^{-1}$, like we did $(\partial^+)^{-1}$. We have

$$\frac{1}{\partial^-}f(x^+) = \frac{1}{2}\int \frac{dk^-}{2\pi}e^{-ik^-x^+}\{\frac{1}{k^- + i\epsilon} + \frac{1}{k^- - i\epsilon}\}f(k^-) \tag{2.43}$$

This leads to the following form for its square

$$(\frac{1}{\partial^-})^2 f(x^+) = \frac{1}{2}\int \frac{dk^-}{2\pi}e^{-ik^-x^+}\{\frac{1}{k^- + i\epsilon} + \frac{1}{k^- - i\epsilon}\}^2 f(k^-) \tag{2.44}$$

Just like above, the operator $(\partial^-)^{-1}$, is just the convoluted epsilon distribution

[16], while the operator $(\partial^-)^{-2}$, becomes

$$\frac{1}{2}\int_{-\lambda}^{\lambda} dx^- \epsilon(x^+ - x'^+)\epsilon(x^+ - x"^+) = -|x'^+ - x"^+| + \lambda \tag{2.45}$$

The other two operators are simpler to define. $((\partial^i)^2)^{-1}$ becomes

$$((\partial^i)^2)^{-1} f(x_\perp) = \int \frac{d^2k_\perp}{(2\pi)^2} \frac{e^{+ik_\perp x_\perp}}{-k_\perp^2 + i\epsilon} f(k_\perp) \tag{2.46}$$

while $(\partial^+\partial^- - (\partial^i)^2)^{-1}$ becomes

$$(\partial^+\partial^- - (\partial^i)^2)^{-1} = \int \frac{d^2k_\perp}{(2\pi)^2}\frac{dk^+dk^-}{(2\pi)^2}\frac{e^{-ikx}}{4k^+k^- - k_\perp^2 + i\epsilon} f(k^+, k^-, k_\perp) \tag{2.47}$$

.

3. Quantization of the Fields

Now that we have the commutation relations, we are ready to define the fields A^i and ψ. According to [11], using two null hyperplanes, the initial data must be specified on each of the hyperplanes as well as on their intersection . In this case, we will have initialization on the two surfaces $x^+ = 0$ and $x^- = 0$. We will require, though, that on the intersection of these surfaces, at $x^+ = x^- = 0$ these fields satisfy certain consistency conditions. This works out as follows.

On $x^+ = 0$ we have then :

$$A_i(x^+ = 0, x^-, x_\perp) = \int \frac{d^2k_\perp}{(2\pi)^3}\frac{dk^+}{2k^+}\{\epsilon_i(k^+, k_\perp)a(k^+, k_\perp)e^{-ik.x} + \epsilon_i^*(k^+, k_\perp)a^\dagger(k^+, k_\perp)e^{+ik.x}\} \tag{3.1}$$

$$\psi_+(x^+ = 0, x^-, x_\perp) = \int \frac{d^2k_\perp}{(2\pi)^3}\frac{dk^+}{2k^+}\sum_\lambda \{b(k^+, k_\perp)u_+(k^+, k_\perp, \lambda)e^{-ik.x}$$

$$+ d^\dagger(k^+, k_\perp)v_+(k^+, k_\perp, \lambda)e^{+ik.x}\} \tag{3.2}$$

In this case, $ik.x = ik^+x^- - ik_\perp.x_\perp$ and the polarization vector is $\epsilon_i(k^+, k_\perp)$.

On the other hyperplane, $x^- = 0$ we get similar forms:

$$A_i(x^- = 0, x^+, x_\perp) = \int \frac{d^2k_\perp}{(2\pi)^3}\frac{dk^-}{2k^-}\{\hat{\epsilon}_i(k^-, k_\perp)\hat{a}(k^-, k_\perp)e^{-i\hat{k}.x} + \hat{\epsilon}_i^*(k^-, k_\perp)\hat{a}^\dagger(k^-, k_\perp)e^{+i\hat{k}.x}\} \tag{3.3}$$

$$\psi_-(x^- = 0, x^+, x_\perp) = \int \frac{d^2k_\perp}{(2\pi)^3}\frac{dk^-}{2k^-}\sum_\mu \{\hat{b}(k^-, k_\perp)u_-(k^-, k_\perp, \mu)e^{-i\hat{k}.x}$$

$$+\hat{d}^\dagger(k^-, k_\perp)v_-(k^-, k_\perp, \mu)e^{+i\hat{k}.x}\} \tag{3.4}$$

Here , $i\hat{k}.x = ik^-x^+ - ik_\perp.x_\perp$.

We require now that the fields be consistent at $x^+ = x^- = 0$. This means that we have

$$A_i(x^+ = 0, x^- = 0, x_\perp) = A_i(x^- = 0, x^+ = 0, x_\perp) \tag{3.5}$$

This implies

$$\int \frac{d^2k_\perp}{(2\pi)^3}\frac{dk^+}{2k^+}\{\epsilon_i(k^+, k_\perp)a(k^+, k_\perp)e^{+ik_\perp.x_\perp} + \epsilon_i(k^+, k_\perp)^* a^\dagger(k^+, k_\perp)e^{-ik_\perp.x_\perp}\}$$

$$= \int \frac{d^2k_\perp}{(2\pi)^3}\frac{dk^-}{2k^-}\{\hat{\epsilon}_i(k^-, k_\perp)\hat{a}(k^-, k_\perp)e^{+ik_\perp.x_\perp} + \hat{\epsilon}_i(k^-, k_\perp)^*\hat{a}^\dagger(k^-, k_\perp)e^{-ik_\perp.x_\perp}\} \tag{3.6}$$

As k^+ and k^- are just dummy variables here, we get that

$$a(k^+, k_\perp) = \hat{a}(k^+, k_\perp), \quad a^\dagger(k^+, k_\perp) = \hat{a}^\dagger(k^+, k_\perp) \tag{3.7}$$

as well as

$$\epsilon_i(k^+, k_\perp) = \hat{\epsilon}_i(k^+, k_\perp) \tag{3.8}$$

and we need to point out that the variables are the **same** for both creation operators. So this means that

$$a(k^+, k_\perp) \neq \hat{a}(k^-, k_\perp), \quad \epsilon_i(k^+, k_\perp) \neq \hat{\epsilon}_i(k^-, k_\perp) \tag{3.9}$$

hence the field A_i has different effects on the two surfaces. On $x^+ = 0$, $A_i(x^+ = 0, x^-, x_\perp)$ creates or destroys vector quanta with momentum $k = (k^+, k_\perp)$ and

polarization $\epsilon_i(k^+, k_\perp)$. On $x^- = 0$, $A_i(x^- = 0, x^+, x_\perp)$ creates or destroys quanta with momentum $\hat{k} = (k^-, k_\perp)$ and polarization $\epsilon_i(k^-, k_\perp)$.[17]

The analysis for the fermion fields goes through just like in the previous paper [6] .

What about the fields A^+ and A^- ? As mentioned in the previous section, as these fields are not initialized on any of the surfaces, they do not constitute constraints. We have solved equations [(2.36)] and [(2.37)] in terms of the independent degrees of freedom A^i and ψ_+ , ψ_- in equations [(2.38)] and [(2.39)]. We get the following :

$$A^+(x^+, x^-, x_\perp) = ((\partial^i)^2)^{-1}\partial^i\partial^+ A^i(x^+ = 0, x^-, x_\perp)$$

$$-((\partial^i)^2)^{-1} e\psi_+^\dagger\psi_+(x^+ = 0, x^-, x_\perp) + (\partial^+\partial^- - (\partial^i)^2)^{-1} e\psi_+^\dagger\psi_+(x^+ = 0, x^-, x_\perp)$$

$$-((\partial^i)^2)^{-1}(\partial^+\partial^- - (\partial^i)^2)^{-1}(\partial^+)^2 e\psi_+^\dagger\psi_+(x^+ = 0, x^-, x_\perp) \qquad (3.10)$$

$$A^-(x^-, x^+, x_\perp) = ((\partial^i)^2)^{-1}\partial^i\partial^+ A^i(x^- = 0, x^+, x_\perp)$$

$$-(\partial^i)^2)^{-1} e\psi_-^\dagger\psi_-(x^- = 0, x^+, x_\perp) + (\partial^-\partial^+ - (\partial^i)^2)^{-1} e\psi_-^\dagger\psi_-(x^- = 0, x^+, x_\perp)$$

$$-((\partial^i)^2)^{-1}(\partial^-\partial^+ - (\partial^i)^2)^{-1}(\partial^-)^2 e\psi_-^\dagger\psi_-(x^- = 0, x^+, x_\perp) \qquad (3.11)$$

where we use the definitions [(2.46)], [(2.47)], [(2.41)] and [(2.44)] .

So all the fields coming in the definition of QED are defined and A^+ or A^- do not represent new modes or new quanta. It is important to point out here that our gauge field A has only **two** physical degrees of freedom , $A^i, i = 1, 2$. The fields A^+ and A^- are needed to guarantee Lorentz covariance, but are not gotten from some constraint equations.

Let us point out that these equations are different in nature than similar equations one gets in the case of constraint quantization. In the constrained case, one

needs to solve the constraint equation **before** quantization. This is often hard and sometimes impossible analytically. Here, we have already quantized our theory and are computing new fields, so we are past the quantization stage. The quantization procedure seems easier in this approach than in the constrained approaches [1], [2], [3], [4] .

4. Parity in Front-Form Quantization

We are ready now to study how the fields A^i , ψ_+ and ψ_- transform under parity. For this we use (Bjorken and Drell for instance[18]) :

$$\mathcal{P}A^i(x^+, x^-, x_\perp)\mathcal{P}^{-1} = -A^i(x^-, x^+, -x_\perp) \tag{4.1}$$

since under parity $(x^+, x^-, x_\perp) \to (x^-, x^+, -x_\perp)$ and the vector field has negative intrinsic parity. For the vector field we get

$$\mathcal{P}A^i(x^+ = 0, x^-, x_\perp)\mathcal{P}^{-1} = \mathcal{P}\int \frac{d^2k_\perp}{(2\pi)^3}\frac{dk^+}{2k^+}\{\epsilon_i(k^+, k_\perp)a(k^+, k_\perp)e^{-ik.x}$$

$$+\epsilon_i^*(k^+, k_\perp)a^\dagger(k^+, k_\perp)e^{+ik.x}\}\mathcal{P}^{-1} \tag{4.2}$$

This becomes

$$\mathcal{P}A^i(x^+ = 0, x^-, x_\perp)\mathcal{P}^{-1} = \int \frac{d^2(-k_\perp)}{(2\pi)^3}\frac{dk^-}{2k^-}\{ -\epsilon_i(k^-, -k_\perp)a(k^-, -k_\perp)e^{-ik'.x'}$$

$$-\epsilon_i^*(k^-, -k_\perp)a^\dagger(k^-, -k_\perp)e^{+ik'.x'}\} \tag{4.3}$$

if

$$\mathcal{P}a(k^+, k_\perp)\mathcal{P}^{-1} = a(k^-, -k_\perp), \quad \mathcal{P}a^\dagger(k^+, k_\perp)\mathcal{P}^{-1} = a^\dagger(k^-, -k_\perp)$$

and

$$\mathcal{P}\epsilon_i(k^+, k_\perp)\mathcal{P}^{-1} = -\epsilon_i(k^-, -k_\perp) \tag{4.4}$$

and $ik'.x' = ik^-x^+ - ik_\perp x_\perp$. Redefining variables $(k^-, -k_\perp) \to (l^-, l_\perp)$, we get

the result

$$\mathcal{P}A^i(x^+ = 0, x^-, x_\perp)\mathcal{P}^{-1} = -A^i(x^- = 0, x^+, -x_\perp) \tag{4.5}$$

Let us consider the fermion fields now. In this case we have the same result of the previous paper [6]

$$\mathcal{P}\psi(x^+, x^-, x_\perp)\mathcal{P}^{-1} = \gamma_0\psi(x^-, x^+, -x_\perp) \tag{4.6}$$

and we expect that fields defined on x^+ will be mapped into fields defined on x^- by parity. Indeed, that is what we find for ψ_+ .

We derive now these relations for arbitrary x^+ and x^-. Note that for the x^+ evolution we have

$$A^i(x^+, x^-, x_\perp) = e^{-iP^-x^+}A^i(x^+ = 0, x^-, -x_\perp) \tag{4.7}$$

or

$$\psi_-(x^+, x^-, x_\perp) = e^{-iP^-x^+}\psi_-(x^+ = 0, x^-, -x_\perp) \tag{4.8}$$

so that the parity-transformed field is

$$\mathcal{P}A^i(x^+, x^-, x_\perp)\mathcal{P}^{-1} = \mathcal{P}e^{-iP^-x^+}\mathcal{P}^{-1}\mathcal{P}A^i(x^+ = 0, x^-, -x_\perp)\mathcal{P}^{-1} \tag{4.9}$$

which becomes

$$\mathcal{P}A^i(x^+, x^-, x_\perp)\mathcal{P}^{-1} = e^{-iP^+x^-}A^i(x^- = 0, x^+, -x_\perp) \tag{4.10}$$

since

$$\mathcal{P}P^-\mathcal{P}^{-1} = \mathcal{P}\int \mathcal{H}\mathcal{P}^{-1} = \int \mathcal{K} = P^+ \tag{4.11}$$

by use of the equations (2.13) and (2.14). A similar result holds for the fermion case.

We also get the generator of x^- evolutions to transform properly as well since

$$\mathcal{P}P^+\mathcal{P}^{-1} = \mathcal{P}\int \mathcal{K}\mathcal{P}^{-1} = \int \mathcal{H} = P^- \tag{4.12}$$

again, by use of equations (2.14) and (2.13).

Since now the generators of evolution along x^+ and x^- ($\mathcal{H}$ and $\mathcal{K}$ respectively), transform properly under parity , we can evolve the parity relations obtained at $x^+ = 0$ and $x^- = 0$ to relations for arbitrary x^+ and x^-. For the vector case we get

$$\mathcal{P}A^i(x^+, x^-, x_\perp)\mathcal{P}^{-1} = -A^i(x^-, x^+, -x_\perp) \tag{4.13}$$

as expected from previous work [5] .

For the fermion case, we get [7]

$$\mathcal{P}\psi_+(x^+, x^-, x_\perp)\mathcal{P}^{-1} = \gamma_0\psi_-(x^-, x^+, -x_\perp) \tag{4.14}$$

which show very clearly that parity maps independent fields on $x^+ = 0$ $[\psi_+(x^+ = 0, x^-, x_\perp)]$, to independent fields on $x^- = 0$ $[\psi_-(x^- = 0, x^+, x_\perp)]$, demonstrating the it is crucial that we take **both** $x^+ = 0$ and $x^- = 0$ as quantizing surfaces if we desire to have fields with parity as an explicit symmetry as already noted [6].

Thus far we have looked at transformation properties of independent fields on $x^+ = 0$. It is quite straightforward to show that we get similar results for the fields which are initialized on $x^- = 0$:

$$\mathcal{P}A^i(x^-, x^+, x_\perp)\mathcal{P}^{-1} = -A^i(x^+, x^-, -x_\perp) \tag{4.15}$$

for the vector field and

$$\mathcal{P}\psi_-(x^-, x^+, x_\perp)\mathcal{P}^{-1} = \gamma_0\psi_+(x^+, x^-, -x_\perp) \tag{4.16}$$

for the fermion field [7] .

Let us examine the parity transformation properties of the fields A^+ and A^-. It is a straightforward exercise to check, using equations [(3.10)] and [(3.11)] that we get

$$\mathcal{P}A^+(x^+ = 0, x^-, x_\perp)\mathcal{P}^{-1} = A^-(x^- = 0, x^+, -x_\perp) \tag{4.17}$$

due to the transformation properties of the fields A^i and ψ_+. We likewise get

$$\mathcal{P}A^+(x^+, x^-, x_\perp)\mathcal{P}^{-1} = A^-(x^-, x^+, -x_\perp) \tag{4.18}$$

for arbitrary x^+.

For the other field A^-, results come out as expected as well

$$\mathcal{P}A^-(x^- = 0, x^+, x_\perp)\mathcal{P}^{-1} = A^+(x^+ = 0, x^-, -x_\perp) \tag{4.19}$$

due to the transformation properties of the fields A^i and ψ_-. We likewise get

$$\mathcal{P}A^-(x^-, x^+, x_\perp)\mathcal{P}^{-1} = A^+(x^+, x^-, -x_\perp) \tag{4.20}$$

for arbitrary x^-. This completes our demonstration that fields defined on $x^+ = 0$ and $x^- = 0$ transform properly under parity, and define QED consistently.

5. Acknowledgements

I would like to thank Dharam Ahluwalia and Prof. Stanley Brodsky for discussions and to thank Prof. Richard Blankenbecler for his continuing support.

REFERENCES

1. Kurt Sundermeyer, 'Constrained Dynamics', Springer-Verlag, New York, 1982

2. D. M. Gitman and I. V. Tyutin, 'Quantization of Fields with Constraints', Springer-Verlag, New York, 1990

3. Jan Govaerts , 'Hamiltonian Quantization and Constrained Dynamics', Leuven University Press, Leuven, Belgium, 1991

4. Marc Henneaux and Claudio Teitelboim, 'Quantization of Gauge Systems', Princeton University Press, Princeton, NJ, 1992

5. Ovid C. Jacob, 'Parity Conserving Light-Cone Quantization', SLAC-PUB-6188,May 1993, hep-th/9305076, Submitted to Mod. Phys. Lett A.

6. Ovid C. Jacob 'Parity and Front-Form quantization of Field Theories,' SLAC-PUB August 1993

7. see also Dharam V. Ahluwalia and Mikolaj Sawicki, 'Spinors on the Front-Form,' LANL preprint, October 1993

8. Couple of recent preprints also address the issue of consistency of field equations on light-cone (T. Heinzl and E. Werner, Regensburg preprint TPR-93-3) or for null-plane field theory (Norbert E. Ligterink and B.L.G. Bakker, Vrije Universiteit, Amsterdam preprint, 1993) . In both of these papers, the authors stay in the usual approach of taking only **one** light-cone (null-plane) 'time.' The Regensburg paper actually makes some rather strong claims (with scant support) regarding lack of need for a second light-like hyperplane, but they do it in the context of a mixed initial-boundary value problem, whereas here we consider an initial value problem.

9. R. Gambini, A. Restuccia, Phys. Rev. D17, 3150, (1978)

10. R. K. Sachs, J. Math. Phys. 3, 908, (1962)

11. R. Penrose, Gen. Relat. and Grav. 12, 225, (1980); Roger Penrose and Wolfgang Rindler, 'Spinors and Space-Time,' vol. I, Chapter 5 ,Cambridge University Press, New York, 1984

12. H. Müller zum Hagen and H-J Seifert, Gen. Relat. and Grav. 8, 259, (1977)

13. H. Bondi, M. G. J. van der Burg and A. W. K. Metzner, Proc. R. Soc. London, A269, 21, (1962)

14. L. D. Faddeev and R. Jackiw, Phys. Rev. Lett. 60, 1692,(1988)

15. R. Jackiw, '(Constrained) Quantization without Tears,' CTP-#2215, hep-th/9306075

16. Wei-Min Zhang and Avaroth Harindranath ,'Light-Front QCD: Role of Longitudinal Boundary Integrals', hepth@xxx/9302119; Wei-Min Zhang and Avaroth Harindranath ,'Residual Gauge Fixing in Light-Front QCD', hepth@xxx/9302107

17. Note that the Fourier decomposition goes through eventhough we have two 'times.' The point is that on each constant surface, be it $x^+ = 0$ or $x^- = 0$, there is only **one** front-form time, so that there is a consistent definition of a Fourier decomposition. On each of these surfaces, space-time looks like $2+1$ Minkowski space-time.

18. James D. Bjorken and Sidney D. Drell, 'Relativistic Quantum Fields,' Chapter 15 , McGraw-Hill, San Francisco, 1965