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TOP TEN MODELS CONSTRAINED BY $b \to s\gamma$

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ABSTRACT

The radiative decay $b \to s\gamma$ is examined in the Standard Model and in nine classes of models which contain physics beyond the Standard Model. The constraints which may be placed on these models from the recent results of the CLEO Collaboration on both inclusive and exclusive radiative B decays is summarized. Reasonable bounds are found for the parameters in some cases.

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1 Introduction

The Standard Model (SM) of electroweak interactions is in complete agreement with present experimental data.[1] Nonetheless, it is believed to leave many questions unanswered, and this belief has resulted in numerous attempts to discover a more fundamental underlying theory. The search for new physics is conducted via a three-prong attack: (i) direct production of new particles at high energy colliders, (ii) deviations from SM predictions in precision measurements, and (iii) indirect observation of new physics in rare or forbidden processes. The first approach relies on a discovery via the direct production of exotic particles or observation of new reactions. The second and third techniques offer a complementary strategy by searching for indirect effects of new physics in higher order processes. In particular, the probing of loop induced couplings can provide a means of testing the detailed structure of the SM at the level of radiative corrections where the Glashow-Iliopoulos-Maiani[2] (GIM) cancellations are important. This talk will focus on the latter option, and will examine the radiative decay $b \to s\gamma$.

Radiative B decays are one of the best testing grounds of the SM due to recent progress on both theoretical and experimental fronts. The CLEO Collaboration has observed[3] the exclusive decay $B \to K^*\gamma$ with a branching fraction of $B(B \to K^*\gamma) = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$ and has also placed an upper limit on the underlying quark-level process of $B(b \to s\gamma) < 5.4 \times 10^{-4}$ at the 95% C.L. Using a conservative value of the ratio of exclusive to inclusive decay rates based on lattice calculations,[4] the observation of the exclusive process also implies the lower bound $B(b \to s\gamma) > 0.65 \times 10^{-4}$ at 95% C.L. On the theoretical side, the reliability of the calculation of the quark-level process $b \to s\gamma$ is improving as partial calculations of the next-to-leading logarithmic QCD corrections to the effective Hamiltonian now exist.[5] These new results have inspired a large number of investigations of this decay in various classes of models, which can be summarized by the following list:

- "Top Ten" Models Constrained by $b \to s\gamma$

1. Standard Model	6. Supersymmetry
2. Anomalous Top-Quark Couplings	7. Three-Higgs-Doublet Model
3. Anomalous Trilinear Gauge Couplings	8. Extended Technicolor
4. Fourth Generation	9. Leptoquarks
5. Two-Higgs-Doublet Models	10. Left-Right Symmetric Models

In what follows, I will summarize the contributions that $b \to s\gamma$ receives in each of these models and the constraints placed on the model parameters by the CLEO data.

2 Models

2.1 Standard Model

In the SM, the quark-level transition $b \to s\gamma$ is mediated by W-boson and t-quark exchange in an electromagnetic penguin diagram. The matrix element for this process at the electroweak scale is governed by the $\sigma_{\mu\nu}q^\nu(1+\gamma_5)$ dipole operator. The QCD corrections to this process are calculated[6] via an operator product expansion based on the effective Hamiltonian

$$H_{eff} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} c_i(\mu) O_i(\mu) \,, \tag{1}$$

which is then evolved from the electroweak scale down to $\mu = m_b$ by the Renormalization Group Equations. Here, V_{ij} represents the relevant Cabibbo-Kobayashi-Maskawa (CKM) factors. The O_i are a complete set of renormalized dimension six operators involving light fields which govern $b \to s$ transitions. They consist of six four-quark operators, O_{1-6}, the electromagnetic dipole operator, O_7, and the chromo-magnetic dipole operator, O_8. The Wilson coefficients, c_i, of the $b \to s$ operators are evaluated perturbatively at the W scale where the matching conditions are imposed and are evolved down to the renormalization scale μ. The explicit expressions for $c_{7,8}(M_W) = G_{7,8}(m_t^2/M_W^2)$ can be found in the literature.[7] The partial decay width is given by

$$\Gamma(b \to s\gamma) = \frac{\alpha G_F^2 m_b^5}{128\pi^4} |V_{ts}^* V_{tb} c_7(m_b)|^2 \,. \tag{2}$$

To obtain the branching fraction, the inclusive rate is scaled to that of the semi-leptonic decay $b \to X\ell\nu$. This procedure removes uncertainties in the calculation due to an overall factor of m_b^5 which appears in both expressions, and reduces the ambiguities involved with the imprecisely determined CKM factors. The result is then rescaled by the experimental value[8] of $B(b \to X\ell\nu) = 0.108$. The semi-leptonic rate is calculated incorporating both charm and non-charm modes, and includes both phase space and QCD corrections.[9] The calculation of $c_7(m_b)$

employs the partial next-to-leading log evolution equations from Ref. 5 for the coefficients of the $b \to s$ transition operators in the effective Hamiltonian, the $\mathcal{O}(\alpha_s)$ corrections due to gluon bremsstrahlung,[10] corrections[11] for $m_t > M_W$, a running α_{QED} evaluated at m_b, and the 3-loop evolution of the running α_s which is fitted to the global value[1] at the Z mass scale. The ratio of CKM elements in the scaled decay rate, $|V_{tb}V_{ts}/V_{cb}|$, is taken to be unity.

The prediction for the $b \to s\gamma$ branching fraction as a function of the top-quark mass in the SM is shown in Fig. 1a, taking $\mu = m_b = 5\,\mathrm{GeV}$. The solid curve represents the rate with the inclusion of the partial next-to-leading log evolution of the operator coefficients, while the dashed curve corresponds to the leading log case. The effect of the known next-to-leading order terms is to decrease the QCD enhancements of the rate by $\sim 15\%$. Figure 1b displays the dependency of the branching fraction (for $m_t = 165\,\mathrm{GeV}$) on the choice of the renormalization scale for the Wilson coefficients. The uncertainty introduced by the choice of the value of m_c/m_b in calculating $B(b \to X\ell\nu)$ is also shown in this figure, where the region between the curves corresponds to $m_c/m_b = 0.316 \pm 0.013$. We see that the $b \to s\gamma$ branching fraction increases by $\sim 20\%$ as the renormalization scale μ is varied from m_b to $m_b/2$. The overall variation in the SM prediction for $B(b \to s\gamma)$ due to the combined freedom of choice in μ and m_c/m_b can be as large as $30 - 40\%$! Once the full next-to-leading order corrections have been computed, this large dependence on the renormalization scale will diminish. For now, this dependence represents an additional theoretical uncertainty.[12] When determining constraints on new physics from this decay, we choose values for these parameters which yields the most conservative SM rate; for most of the models discussed here μ is taken to be 5.0 GeV. Most of the parameter constraints presented here are not very sensitive to the remaining uncertainties in the calculation of the branching fraction arising from higher order QCD corrections, as $B(b \to s\gamma)$ is a steep function of the parameters in these cases.

2.2 Anomalous Top-Quark Couplings

The possibility of anomalous couplings between the top-quark and the gauge boson sector has been examined in the literature.[13] Future colliders such as the LHC and NLC can probe these effective couplings down to the level of $10^{-18} - 10^{-19}$ e-cm, but they rely on direct production of top-quark pairs, whereas $b \to s\gamma$ provides the opportunity to probe the properties of the top-quark before it is produced directly. If the t-quark has large anomalous couplings to on-shell photons and gluons, the resulting prediction[14] for the $b \to s\gamma$ rate would conflict with experiment. The most general form of the Lagrangian which describes the interaction between top-quarks and on-shell photons (assuming operators of dimension-five or less, only) is

$$\mathcal{L}_{t\bar{t}\gamma} = e\bar{t}\left[Q_t\gamma_\mu + \frac{1}{2m_t}\sigma_{\mu\nu}(\kappa_\gamma + i\tilde{\kappa}_\gamma\gamma_5)q^\nu\right]tA^\mu + h.c.\,, \tag{3}$$

where Q_t is the electric charge of the t-quark, and $\kappa_\gamma(\tilde{\kappa}_\gamma)$ represents the anomalous magnetic (electric) dipole moment. A similar expression is obtained for $\mathcal{L}_{t\bar{t}g}$. Note that a non-vanishing value for $\tilde{\kappa}_\gamma$ would signal the presence of a CP-violating amplitude. In practice, only the coefficients of the magnetic dipole and chromo-magnetic dipole $b \to s$ transition operators, O_7 and O_8 respectively, are modified by the presence of these couplings. The coefficients of these operators at the W scale can be written as

$$\begin{aligned} c_7(M_W) &= G_7^{SM}(m_t^2/M_W^2) + \kappa_\gamma G_1(m_t^2/M_W^2) + i\tilde{\kappa}_\gamma G_2(m_t^2/M_W^2)\,, \\ c_8(M_W) &= G_8^{SM}(m_t^2/M_W^2) + \kappa_g G_1(m_t^2/M_W^2) + i\tilde{\kappa}_g G_2(m_t^2/M_W^2)\,. \end{aligned} \tag{4}$$

The functions G_i are obtained by inserting the above couplings into the Feymann diagrams in which the photon is emitted from the top-quark line, and extracting the pure dipole-like terms after performing the loop integrations and are given in Ref. 14. All other Lorentz structures vanish due to electromagnetic gauge invariance and the fact that the photon is on-shell. When the resulting branching fraction and the CLEO data are combined, the constraints shown in Fig. 2 are obtained. In Fig. 2a, the 95% C.L. allowed region of the anomalous magnetic dipole operator as a function of m_t lies between the curves for the cases $\kappa_g = 0$ (solid curves) and $\kappa_g = \kappa_\gamma$ (dashed curves). In Fig. 2b, the 95% C.L. allowed region for the anomalous electric dipole moment lies beneath the curves. The bounds on the chromo-dipole moments are found to be weak, since they only enter the decay rate via operator mixing. For $m_t = 150\,\mathrm{GeV}$, κ_γ is constrained to lie in the range $(-2.6$ to $3.4) \times 10^{-16}$ e-cm, and $\tilde{\kappa}_\gamma < 5.1 \times 10^{-16}$ e-cm.

The chiral structure of the top-bottom charged current is also probed by $b \to s\gamma$. It has been determined[15] that consistency with the CLEO results restricts the potential deviation from the $v - a$ structure of the tbW coupling to be less than a few percent.

2.3 Anomalous Trilinear Gauge Couplings

The trilinear gauge coupling of the photon to W^+W^- can also be tested by the $b \to s\gamma$ process. Anomalous γWW vertices can be probed by looking for deviations from the SM in tree-level processes such as $e^+e^- \to W^+W^-$ and $p\bar{p} \to W\gamma$, or by their influence on loop order processes, for example the $g-2$ of the muon. In the latter case, cutoffs must be used in order to regulate the divergent loop integrals and can introduce errors by attributing a physical significance to the cutoff.[16] However, some loop processes, such as $b \to s\gamma$, avoid this problem due to cancellations provided by the GIM mechanism and hence yield cutoff independent bounds on anomalous couplings. The CP-conserving interaction Lagrangian for $WW\gamma$ interactions is

$$\mathcal{L}_{WW\gamma} = i\left(W^\dagger_{\mu\nu}W^\mu A^\nu - W^\dagger_\mu A_\nu W^{\mu\nu}\right) + i\kappa_\gamma W^\dagger_\mu W_\nu A^{\mu\nu} + i\frac{\lambda_\gamma}{M_W^2}W^\dagger_{\lambda\mu}W^\mu_\nu A^{\nu\lambda} + h.c.\,, \tag{5}$$

where $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$, and the two parameters $\kappa_\gamma = 1 + \Delta\kappa_\gamma$ and λ_γ take on the values $\Delta\kappa_\gamma, \lambda_\gamma = 0$ in the SM. In this case, only the coefficient of the magnetic dipole $b \to s$ transition operator O_7 is modified by the presence of these additional terms and can be written as

$$c_7(M_W) = G_7^{SM}(m_t^2/M_W^2) + \Delta\kappa_\gamma A_1(m_t^2/M_W^2) + \lambda_\gamma A_2(m_t^2/M_W^2)\,. \tag{6}$$

The functions $A_{1,2}$ are obtained in the same manner as described above for the anomalous top-quark couplings and are given explicitly in Ref. 17. As both of these parameters are varied, either large enhancements or suppressions over the SM prediction for the $b \to s\gamma$ branching fraction can be obtained. When one demands consistency with both the upper and lower CLEO bounds, a large region of the $\Delta\kappa_\gamma - \lambda_\gamma$ parameter plane is excluded; this is displayed in Fig. 3 from Rizzo[17] for $m_t = 150\,\mathrm{GeV}$. Here, the 95% C.L. bounds obtained from the lower limit on $B(b \to s\gamma)$ correspond to the dashed curves, where the region between the curves is excluded, while the constraints placed from the upper CLEO limit correspond to the diagonal solid lines, with the allowed region lying in between the lines. The allowed region in this parameter plane as determined from UA2 data[18] from the reaction $pp \to W\gamma$ is also displayed in this figure and corresponds to the region between the two almost horizontal lines. Combining these constraints, an overall allowed region is obtained and is represented by the two shaded areas in this figure. We see that a sizable area of the parameter space is ruled out! Note that the SM point in the $\Delta\kappa_\gamma - \lambda_\gamma$ plane (labeled by 'S') lies in the center of one of the allowed regions.

2.4 Fourth Generation

The implications of a fourth generation of quarks on the process $b \to s\gamma$ have been previously[19] examined. The possibility of a fourth family of fermions was a popular[20] potential extension to the SM before LEP/SLC data[1] precluded the existence of a light fourth neutrino. However, one should keep in mind that a fourth generation is consistent with the LEP/SLC data as long as the fourth neutrino is heavy, *i.e.*, $m_{\nu_4} \gtrsim M_Z/2$, and that such a heavy fourth neutrino could mediate[21] a see-saw type mechanism thus generating a small mass for $\nu_{e,\mu,\tau}$.

In the case of four families there is an additional contribution to $b \to s\gamma$ from the virtual exchange of the fourth generation up quark t'. The Wilson coefficients of the dipole operators are given by

$$c_{7,8}(M_W) = G_{7,8}(m_t^2/M_W^2) + \frac{V^*_{t's}V_{t'b}}{V^*_{ts}V_{tb}} G_{7,8}(m_{t'}^2/M_W^2)\,, \tag{7}$$

in the limit of vanishing up and charm quark masses. V_{ij} represents the 4x4 CKM matrix which now contains 9 parameters; 6 angles and 3 phases. We recall here that the CKM coefficient corresponding to the t-quark contribution, *i.e.*, $V^*_{ts}V_{tb}$, is factorized in the effective Hamiltonian as shown in Eqn. (1). In order to determine the allowed ranges of the nine parameters in the full 4x4 CKM matrix we demand consistency[22] with (i) unitarity and the determination of the CKM matrix elements extracted from charged current measurements, (ii) the ratio $|V_{ub}|/|V_{cb}|$, (iii) ϵ, (iv) $B^0 - \bar{B}^0$ mixing. 10^8 sets of the nine CKM mixing parameters are generated via Monte Carlo and subjected to the constraints (i)-(iv) for $m_t = 130-200\,\mathrm{GeV}$ and $m_{t'} = 200 - 400\,\mathrm{GeV}$. The surviving sets of CKM parameters are then used to calculate the range of $B(b \to s\gamma)$ in the four generation standard model. This branching fraction is displayed in Fig. 4 as a function of m_t where the vertical lines represent the allowed fourth generation range as $m_{t'}$ is varied in the above region, and the solid curve corresponds to the three generation value. We see that once the restrictions (i)-(iv) above are applied, the four generation $b \to s\gamma$ branching fraction is essentially (except for smaller values of m_t) within the range allowed by CLEO.

2.5 Two-Higgs-Doublet Models

Next we turn to two-Higgs-doublet models (2HDM), where we examine two distinct models which naturally avoid tree-level flavor changing neutral currents. In Model I, one doublet (ϕ_2) generates masses for all fermions and the other doublet (ϕ_1) decouples from the fermion sector. In the second model (Model II) ϕ_2 gives mass to the up-type quarks, while the down-type quarks and charged leptons receive their mass from ϕ_1. Each doublet obtains a vacuum expectation value (vev) v_i, subject to the constraint that $v_1^2 + v_2^2 = v^2$, where v is the usual vev present in the SM. The charged Higgs boson interactions with the quark sector are governed by the Lagrangian

$$\mathcal{L} = \frac{g}{2\sqrt{2}M_W} H^{\pm} \left[V_{ij} m_{u_i} A_u \bar{u}_i (1-\gamma_5) d_j + V_{ij} m_{d_j} A_d \bar{u}_i (1+\gamma_5) d_j \right] + h.c. \,, \qquad (8)$$

where g is the usual SU(2) coupling constant and V_{ij} represents the appropriate CKM element. In model I, $A_u = \cot\beta$ and $A_d = -\cot\beta$, while in model II, $A_u = \cot\beta$ and $A_d = \tan\beta$, where $\tan\beta \equiv v_2/v_1$ is the ratio of vevs. In both models, the $H^{\pm}$ contributes to $b \to s\gamma$ via virtual exchange together with the top-quark, and the dipole $b \to s$ operators $O_{7,8}$ receive contributions from this exchange. At the W scale the coefficients of these operators take the generic form

$$c_{7,8}(M_W) = G_{7,8}(m_t^2/M_W^2) + \frac{1}{3\tan^2\beta} G_{7,8}(m_t^2/m_{H^{\pm}}^2) + \lambda F_{7,8}(m_t^2/m_{H^{\pm}}^2) \,, \qquad (9)$$

where $\lambda = -1/\tan\beta$, $+1$ in Model I and II, respectively. The analytic form of the functions $F_{7,8}$ can be found in Ref. 23. Since the $H^{\pm}$ contributions all scale as $\cot^2\beta$ in Model I, enhancements to the SM decay rate only occurs for small values of $\tan\beta$. The relative minus sign between the two $H^{\pm}$ contributions in this model also gives a destructive interference for some values of the parameters. Consistency with the CLEO lower and upper limits excludes[24] the shaded regions in the $m_{H^{\pm}} - \tan\beta$ parameter plane presented in Fig. 5a, assuming $m_t = 150\,\mathrm{GeV}$. Here, the shaded region on the left results from the CLEO upper bound and the shaded slice in the middle is from the lower limit. In Model II, large enhancements also appear for small values of $\tan\beta$, but more importantly, $B(b \to s\gamma)$ is always larger than that of the SM, independent of the value of $\tan\beta$. This is due to the $+\tan\beta$ scaling of the $F_{7,8}$ term in Eq. (9). In this case the CLEO upper bound excludes[24,25] the region to the left and beneath the curves shown in Fig. 5b for the various values of m_t as indicated. In this case, the bounds are quite sensitive[12] to the uncertainties arising from the higher order QCD corrections. We note that the $H^{\pm}$ couplings present in Model II are of the type present in Supersymmetry. However, the limits obtained in supersymmetric theories also depend on the size of the other super-particle contributions to $b \to s\gamma$ and are generally much more complex[26] as discussed below.

2.6 Supersymmetry

In the supersymmetric standard model flavor mixing is also present in the squark sector and hence flavor changing neutral current processes are sensitive to the masses and mixings of the super-partners. For example, $K^0 - \bar{K}^0$ mixing has been shown to place[27] stringent constraints on the level of degeneracy for the first two generations of squarks (if one assumes CKM-like mixing). One should also be reminded, of course, that magnetic moment transition operators, including $b \to s\gamma$, vanish in the exact supersymmetric limit.[28]

There are five classes of contributions to $b \to s\gamma$ in supersymmetric theories; the virtual exchange of (i) the up-type quarks and the W boson in the SM, (ii) the up-type quarks and the $H^{\pm}$ of Model II above, (iii) the up-type squarks and charginos, $\tilde{\chi}_i^{\pm}$, (iv) the down-type squarks and neutralinos, $\tilde{\chi}_i^0$, and (v) the down-type squarks and gluinos, $\tilde{g}$. As discussed above the contributions from (i) and (ii) are large and interfere constructively. It has been shown[26,29] that contributions (iv) and (v) are usually small in the minimal supersymmetric model and are not competitive with those induced by W boson and $H^{\pm}$ exchange. However, the chargino contributions (iii) can be large, and for some range of the parameter space can cancel the $H^{\pm}$ contributions to give a value of $B(b \to s\gamma)$ at or even below the SM prediction.

Several recent analyses of the chargino contributions have appeared in the literature.[28,29] The size and relative sign of these contributions depend on the parameters present in the chargino mass matrix and on those responsible for the masses and mixings of the squark sector. Assuming unification at a high energy scale, we take these parameters to be the common soft-breaking gaugino mass m_λ, the universal scalar mass m_0, the supersymmetric higgsino mass parameter μ, the universal trilinear soft-breaking scalar term in the superpotential A, $\tan\beta$, as well as m_t. Here we will consider the case where the up and charm squark masses are degenerate, and will examine the effects of the possibly large stop-squark mass splitting due to the potentially sizeable off-diagonal terms in the stop mass matrix. The chargino-squark contributions to the Wilson coefficients for the $b \to s$ transition dipole operators are given by[28,29]

$$
\begin{aligned}
c_{7,8}^{\tilde{\chi}^\pm}(M_W) \simeq & \sum_{j=1}^{2} \left\{ \frac{M_W^2}{\tilde{m}^2_{\chi_j^\pm}} |V_{j1}|^2 G_{7,8}\left(\frac{\tilde{m}^2}{\tilde{m}^2_{\chi_j^\pm}}\right) - \frac{M_W U_{j2} V_{j1}}{\tilde{m}_{\chi_j^\pm}\sqrt{2}\cos\beta} H_{7,8}\left(\frac{\tilde{m}^2}{\tilde{m}^2_{\chi_j^\pm}}\right) \right. \\
& + \sum_{k=1}^{2} \left[-\frac{M_W^2}{\tilde{m}^2_{\chi_j^\pm}} \left| V_{j1} T_{k1} - \frac{m_t V_{j2} T_{k2}}{M_W \sqrt{2}\sin\beta} \right|^2 G_{7,8}\left(\frac{\tilde{m}^2_{t_k}}{\tilde{m}^2_{\chi_j^\pm}}\right) \right. \\
& \left.\left. + \frac{M_W U_{j2} T_{k1}}{\tilde{m}_{\chi_j^\pm}\sqrt{2}\cos\beta} \left(V_{j1} T_{k1} - \frac{m_t V_{j2} T_{k2}}{M_W \sqrt{2}\sin\beta} \right) H_{7,8}\left(\frac{\tilde{m}^2}{\tilde{m}^2_{\chi_j^\pm}}\right) \right] \right\},
\end{aligned} \tag{10}
$$

where $\tilde{m}_{\chi_j^\pm}$ represents the chargino masses, $\tilde{m}$ the up and charm squark masses, $\tilde{m}_{t_k}$ the stop-squark masses, U_{ij} and V_{ij} are the unitary matrices which diagonalize the chargino mass matrix, and T_{kl} diagonalizes the stop-squark mass matrix. These all are calculable in terms of the supersymmetry parameters listed above. The functions $H_{7,8}$ are given in Refs. 26,28,29. Contours of $B(b \to s\gamma)$, including the SM, $H^\pm$, and $\tilde{\chi}^\pm$ contributions, are displayed in Fig. 6 from Garisto and Ng[29] in the $m_\lambda - \mu$ parameter plane for four values of $A = \pm 1, \pm 2$ and taking $m_0 = 100\,\text{GeV}$, $m_t = 140\,\text{GeV}$, and $\tan\beta = 10$. It is immediately clear from the figure, that regions of parameter space do exist where $B(b \to s\gamma)_{\text{SUSY}}$ is at or below the SM value and is consistent with the CLEO bounds. It is found that the stop-squark and chargino contributions have a large destructive interfere with the SM and $H^\pm$ contributions when $\tilde{t}_1$ is light (*i.e.*, when there is a large stop mass splitting), $\tan\beta$ is large, and $A\mu < 0$. However, if all the up-type squarks are degenerate, the chargino contributions exactly cancel due to a SUSY-GIM mechanism. In this case, the $H^\pm$ mass is constrained to be large as shown in the previous section.

2.7 Three-Higgs-Doublet Models

New CP violating phases are present in models with three or more scalar doublets. These phases appear in charged scalar exchange and can influence CP asymmetries in neutral B decays, even if the Yukawa couplings obey natural flavor conservation.[30] For example, in a three-Higgs-Doublet model (3HDM) one can avoid tree-level flavor changing neutral currents by requiring that a different doublet generate a mass for the up-type quarks, the down-type quarks, and the charged leptons, respectively. In this case, the interaction Lagrangian between the quark sector and the two physical charged Higgs bosons is written as[31]

$$\mathcal{L} = \frac{g}{2M_W} \sum_{i=1,2} H_i^+ \bar{U} \left[Y_i M_u V_{CKM}(1-\gamma_5) + X_i M_d V_{CKM}(1+\gamma_5) \right] D + h.c. \,, \quad (11)$$

where X and Y are complex coupling constants that arise from the diagonalization of the charged scalar mixing matrix and obey the relation

$$\sum_{i=1,2} X_i Y_i^* = 1 \,. \quad (12)$$

Both $H_1^\pm$ and $H_2^\pm$ contribute to $b \to s\gamma$ and the Wilson coefficients $c_{7,8}$ at the matching scale M_W now become

$$c_{7,8}(M_W) = G_{7,8}(m_t^2/M_W^2) + \sum_{i=1,2} \left[\frac{|Y_i|^2}{3} G_{7,8}(m_t^2/m^2_{H_i^\pm}) + X_i Y_i^* F_{7,8}(m_t^2/m^2_{H_i^\pm}) \right] , \quad (13)$$

with the analytic expressions for the functions $F_{7,8}$ being the same as in the two-Higgs-Doublet case.[23] The $X_i Y_i^*$ term signals the existence of a relative phase in the $b \to s\gamma$ amplitude. When evolved down to the b-quark scale, the contributions proportional to $\mathcal{I}m\,(X_i Y_i^*)$ do not interfere with the remaining terms in $c_{7,8}(M_W)$ and do not mix with the 4-quark operators. Hence these terms only appear quadratically in the expression for the $b \to s\gamma$ rate. A conservative upper limit can be placed on the value of $|\mathcal{I}m\,(XY^*)|$ (where $\mathcal{I}m\,(XY^*) = \mathcal{I}m\,(X_1 Y_1^*) = -\mathcal{I}m\,(X_2 Y_2^*)$ as given in Eqn. (12) above) by letting the imaginary contribution alone saturate the CLEO upper bound. These constraints are displayed in Fig. 7 as a function of the lightest charged Higgs mass $m_{H_1^\pm}$ for various values of the heavier charged Higgs mass $m_{H_2^\pm}$, subject to the restraint $m_{H_1^\pm} < m_{H_2^\pm}$. The bottom solid curve corresponds to the case where the contribution of the second charged Higgs $H_2^\pm$ is neglected. We see that the constraints depend very strongly on the value of $m_{H_2^\pm}$ and that the bounds disappear when $m_{H_1^\pm} \simeq m_{H_2^\pm}$ due to an exact cancellation between the two $H_i^\pm$ contributions.

2.8 Extended Technicolor

The decay $b \to s\gamma$ has been investigated within the framework of various classes of Extended Technicolor (ETC) models in Ref. 32. These contributions were found to be either comparable or suppressed relative to those of the SM, since gauge invariance implies that the photon vertex is corrected only at higher order in these models. We note that the Z-boson couplings are modified at leading order in these theories and that large rates for the decays $B \to \mu\mu$ and $b \to s\mu\mu$ can be obtained.[32,33] The effective Lagrangian for ETC gauge boson exchange in these scenarios can be written as

$$\mathcal{L} = \frac{1}{f^2}(\bar{\psi}_L^i \gamma_\mu T_L)(\bar{U}_R \gamma^\mu u_R^j) Y_u^{ij} + \frac{1}{f^2}(\bar{\psi}_L^i \gamma_\mu T_L)(\bar{D}_R \gamma^\mu d_R^j) Y_d^{ij} + h.c. \,, \tag{14}$$

where T_L is a techni-doublet with the right-handed techni-partners U_R and D_R, ψ_L represents the left-handed quark doublets with u_R and d_R being the right-handed partners, the matrices $Y_{u,d}^{ij}$ parameterize the symmetry breaking, and i, j are generation indices.

The first class of models considered in Ref. 32 is that of "traditional" ETC, which contains the minimal set of interactions necessary to generate the third generation quark masses. In this case the ETC gauge boson spectrum is highly non-degenerate and the quark mass matrices are approximately given by

$$M_{u,d} \sim \frac{4\pi v^3}{f^2} Y_{u,d} \,. \tag{15}$$

Working in the basis where Y_u^{33} is normalized to unity, gives the relation $f^2 \sim 4\pi v^3/m_t$. The dominant contribution to $b \to s\gamma$ occurs when the ETC gauge boson is exchanged between purely left-handed doublets and when the photon is emitted from the technifermion line. This results in the magnetic moment operator

$$\frac{m_t}{4\pi v} \frac{m_b V_{ts}}{(4\pi v)^2} \bar{b}_R \sigma^{\mu\nu} s_L \frac{e}{2} f_{\mu\nu} \,. \tag{16}$$

Comparing this to the corresponding quantity in the SM (*i.e.*, $c_7(M_W) O_7$) shows[32] that the ETC contribution is suppressed with respect to the SM by a factor of $m_t/[4\pi v G_7(m_t^2/M_W^2)]$.

The second class of models considered in this reference are those which incorporate a techni-GIM mechanism which provide a GIM-like suppression of flavor changing neutral currents due to a restricted form of flavor symmetry breaking. The ETC scale becomes $f^2 = m_{ETC}^2/g_{ETC}^2$, where $m_{ETC}(g_{ETC})$ represents the mass (coupling) of the nearly degenerate ETC gauge bosons. Here, the dominant contribution to $b \to s\gamma$ results when the photon is attached to the ETC gauge boson line. Assuming $Y_u^{23} \sim V_{ts}$, the effective magnetic moment operator is estimated[32] to be

$$\frac{\xi_4 m_b V_{ts}}{4m_{ETC}^2}\bar{b}_R\sigma_{\mu\nu}s_L\frac{e}{6}F^{\mu\nu}\,, \tag{17}$$

with ξ_4 being a model dependent parameter. This contribution is expected[32] to yield a rate for $b \to s\gamma$ which is within 10% of that in the SM.

2.9 Leptoquarks

Leptoquarks are color triplet particles which couple to a lepton-quark pair and are naturally present in many theories beyond the SM which relate leptons and quarks at a more fundamental level. They appear in technicolor theories, models with quark-lepton substructure, horizontal symmetries, and grand unified theories based on the gauge groups SU(5), SO(10), and E_6. In all these scenarios leptoquarks carry both baryon and lepton number but their other quantum numbers, *i.e.*, spin, weak isospin, and electric charge, vary between the different models. The scalar and vector leptoquark interaction Lagrangians[34,35] which are renormalizable, baryon and lepton number conserving, and consistent with the symmetries of $SU(3)_C \times SU(2)_L \times U(1)_Y$ are given by

$$\begin{aligned}
\mathcal{L}_S &= (\lambda_{LS_0}\bar{q}_L^c i\tau_2\ell_L + \lambda_{RS_0}\bar{u}_R^c e_R)S_0^\dagger + \lambda_{R\tilde{S}_0}\bar{d}_R^c e_R\tilde{S}_0^\dagger + (\lambda_{LS_{1/2}}\bar{u}_R\ell_L \\
&\quad +\lambda_{RS_{1/2}}\bar{q}_L i\tau_2 e_R)S_{1/2}^\dagger + \lambda_{L\tilde{S}_{1/2}}\bar{d}_R\ell_L\tilde{S}_{1/2}^\dagger + \lambda_{LS_1}\bar{q}_L^c i\tau_2\vec{\tau}\ell_L\cdot\vec{S}_1^\dagger + h.c.\,, \\
\mathcal{L}_V &= (\lambda_{LV_0}\bar{q}_L\gamma_\mu\ell_L + \lambda_{RV_0}\bar{d}_R\gamma_\mu e_R)V_0^{\mu\dagger} + \lambda_{R\tilde{V}_0}\bar{u}_R\gamma_\mu e_R\tilde{V}_0^{\mu\dagger} + (\lambda_{LV_{1/2}}\bar{d}_R^c\gamma_\mu\ell_L \\
&\quad +\lambda_{RV_{1/2}}\bar{q}_L^c\gamma_\mu e_R)V_{1/2}^{\mu\dagger} + \lambda_{L\tilde{V}_{1/2}}\bar{u}_R^c\gamma_\mu\ell_L\tilde{V}_{1/2}^{\mu\dagger} + \lambda_{LV_1}\bar{q}_L\gamma_\mu\vec{\tau}\ell_L\cdot\vec{V}_1^{\mu\dagger} + h.c.\,.
\end{aligned} \tag{18}$$

Here the subscripts 0, 1/2, and 1 represent the SU(2) singlet, doublet, and triplet leptoquarks, respectively, the λ's are *a priori* unknown Yukawa coupling constants, the $L(R)$ index on the coupling reflects the chirality of the lepton, and the generation indices have been suppressed.

Leptoquarks can contribute to $b \to s\gamma$ by the virtual exchange of a charged lepton and a leptoquark in a penguin diagram. These diagrams have been calculated in Ref. 35, where the leptoquark contributions to $c_8(M_W)$ have been neglected. Using the approximation that the leptoquark contributions to the $b \to s\gamma$ amplitude must be smaller than that for the SM, Davidson *et al.*[35] derive the following bounds on the relevant combinations of the Yukawa coupling constants for scalar leptoquarks,

$$\lambda_L^{\ell b}\lambda_L^{\ell s}, \lambda_R^{\ell b}\lambda_R^{\ell s} < \frac{3\times 10^{-2}}{Q_\ell + \frac{1}{2}Q_{LQ}} \left(\frac{m_{LQ}}{100\,\mathrm{GeV}}\right)^2 , \tag{19}$$

where ℓ is a charged lepton of any generation, and $Q_\ell (Q_{LQ})$ are the electric charges of the exchanged lepton (leptoquark). Similarly, for non-gauge vector leptoquarks,

$$\lambda_L^{\ell b}\lambda_L^{\ell s}, \lambda_R^{\ell b}\lambda_R^{\ell s} < \frac{2\times 10^{-2}}{2Q_\ell + \frac{5}{2}Q_{LQ}} \left(\frac{m_{LQ}}{100\,\mathrm{GeV}}\right)^2 . \tag{20}$$

We note that other B decays, such as $B \to \ell^- \ell^{(\prime)+}$, can provide stronger constraints[35] on these leptoquark couplings.

2.10 Left-Right Symmetric Models

The last scenario of new physics that we will consider is the Left-Right Symmetric Model (LRM)[36] which is based on the extended gauge group $SU(2)_L \times SU(2)_R \times U(1)$. Such theories have been popular for many years, as both a possible generalization of the SM and in the context of grand unified theories such as $SO(10)$ and E_6. One prediction of these models is the existence of a heavy, charged, right-handed gauge boson $W_R^\pm$, which in principal mixes with the SM $W_L^\pm$ via a mixing angle ϕ to form the mass eigenstates $W_{1,2}^\pm$. This mixing angle is constrained[37] by data in polarized μ decay (in the case of light right-handed neutrinos) and from universality requirements to be $|\phi| \lesssim 0.05$. The exchange of a $W_R^\pm$ within a penguin diagram, in analogy with the SM $W_L^\pm$ exchange, can lead to significant deviations from the SM prediction for the rate in $b \to s\gamma$ which are sensitive to the sign and magnitude of the angle ϕ.

The first class of LRM we will discuss is that where the right-handed and left-handed CKM mixing matrices are assumed to be equal, *i.e.*, $V_R = V_L$. In this case, $W_R^\pm$ searches at the Tevatron collider together with the value of the $K_L - K_S$ mass difference constrain[38] the mass of $W_R^\pm$ to be at least $m_{W_R} > 1.6\kappa\,\mathrm{TeV}$, where $\kappa \equiv g_R/g_L$ is the ratio of right-handed to left-handed $SU(2)$ coupling constants. In the LRM the complete operator basis governing $b \to s$ transitions is expanded to include 20 operators. Two new four-quark operators $O_{9,10}$ which have different chirality structure are also present,[39,40] and left-right symmetry dictates the existence of a set of operators which have a flipped chirality structure compared to the standard set. The latter are obtained by the substitution $P_L \leftrightarrow P_R$ in the definition of O_{1-10}, where $P_{L,R} = (1 \pm \gamma_5)/2$. We denote the standard set of operators as "left-handed", *i.e.*, O_{1L-10L}, and the chirality flipped operators as "right-handed", O_{1R-10R}. These two sets of operators do not mix under the QCD evolution and thus can be treated independently. The expression for the partial decay width now becomes

$$\Gamma(b \to s\gamma) = \frac{\alpha(m_b) G_F m_b^5}{128\pi^4} |V_L^{ts*} V_L^{tb}|^2 \left(|c_{7L}(m_b)|^2 + |c_{7R}(m_b)|^2 \right) , \qquad (21)$$

where $c_{7L,R}(M_{W_1})$ are defined via the low-energy effective Hamiltonian

$$\mathcal{H}_{eff} = -\frac{G_F e m_b}{4\sqrt{2}\pi^2} \bar{s}\sigma_{\mu\nu} \left(c_{7L} P_R + c_{7R} P_L \right) b F^{\mu\nu} . \qquad (22)$$

The expressions for the coefficients c_{iL} and c_{iR} are evaluated via the one-loop matching conditions at the scale M_{W_1} and are given in Ref. 39. The branching fraction is obtained by scaling to the semi-leptonic decay rate as usual, except that possible $W_R^\pm$ contributions[41] to $b \to c\ell\nu$ must also be included. The resulting values for $B(b \to s\gamma)$ from the work of Rizzo[39] are displayed in Fig. 8a-b as a function of the tangent of the $W_L - W_R$ mixing angle, $\tan\phi$, for $M_{W_R} = 1.6\,\text{TeV}$. Figure 8a examines the branching fraction for various values of the top-quark mass assuming $\kappa = 1$, while Fig. 8b fixes $m_t = 160\,\text{GeV}$ and varies κ between 0.6 and 2. In both cases the solid horizontal line represents the CLEO bound. We see from the figures that for $\kappa = 1$, $\tan\phi$ is constrained to lie in the range $-0.02 \lesssim \tan\phi \lesssim 0.005$ and that these bounds strengthen with increasing values of κ. We also note that $b \to s\gamma$ was found not to be sensitive to the exact value of the W_R mass.

The assumption that $V_R = V_L$ is simple and attractive, but one should keep in mind that realistic and phenomenologically viable models can be constructed where V_R is unrelated to V_L, and hence the above constraints on the model parameters can be avoided. One such example is the model given in Gronau and Wakaizumi,[42] where B decays proceed only through right-handed currents. Using the form of V_L and V_R given in this reference, consistency with the B lifetime provides the bound $M_{W_R} \leq 416.2\kappa[|V_R^{cb}|/\sqrt{2}]^{1/2} \simeq 415\kappa\,\text{GeV}$. Collider bounds from the Tevatron can be satisfied[38] in this model if $\kappa \geq 1.5$ and $M_R \geq 600\,\text{GeV}$, assuming that the $W_R^\pm$ decays only into the known SM fermions as well as the right-handed neutrino. The value of $B(b \to s\gamma)$ as a function of $\tan\phi$ is presented in Fig. 8c from Ref. 39. Here, $\kappa = 1.5$ and $M_{W_R} = 600\,\text{GeV}$ is assumed and the outer(inner)-most solid line corresponds to $m_t = 120(200)\,\text{GeV}$. It is immediately clear that the allowed range of $\tan\phi$ is much more restricted than in the $V_L = V_R$ case and that both the upper and lower CLEO bounds will play a role. For $M_{W_R} = 600(800)\,\mathrm{GeV}$, the allowed ranges of $\tan\phi$ are found to be $-0.43 \times 10^{-3} < \tan\phi < 0$, and $0.40 \times 10^{-3} < \tan\phi < 0.81 \times 10^{-3}$ ($-0.32 \times 10^{-3} < \tan\phi < 0$, and $0.29 \times 10^{-3} < \tan\phi < 0.60 \times 10^{-3}$). These ranges are highly constrained and a more precise determination of the $b \to s\gamma$ branching fraction would finely-tune the values of the parameters in this model.

3 Conclusion

In summary, we have seen that the process $b \to s\gamma$ provides powerful constraints for a variety of models containing physics beyond the SM. In most cases, these constraints either complement or are stronger than those from other low-energy processes and from direct collider searches. We look forward to an exciting future in B physics!

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Fig. 1. The branching fraction for $b \to s\gamma$ in the Standard Model (a) as a function of the top-quark mass including QCD corrections to the leading log (dashed) and next-to-leading log order (solid). (b) Dependency of the branching fraction on the choice of renormalization scale μ for various values of the b-quark mass as indicated with $m_t = 150\,\mathrm{GeV}$.

Fig. 2. The allowed range of (a) κ_γ and (b) $\tilde{\kappa}_\gamma$ assuming $\tilde{\kappa}_g = 0$ (solid curve) or $\tilde{\kappa}_g = \tilde{\kappa}_\gamma$ (dashed curve).

Fig. 3. Allowed (shaded) region of the $\Delta\kappa_\gamma - \lambda_\gamma$ parameter plane from the CLEO upper and lower bounds on $b \to s\gamma$, assuming $m_t = 150\,\mathrm{GeV}$, and the UA2 event rate for $pp \to W\gamma$ as discussed in the text. The point in this plane representing the SM is labeled by S.

Fig. 4. Branching fraction for $b \to s\gamma$ as a function of m_t. The solid curve represents the three generation SM value and the vertical lines are the allowed ranges of $B(b \to s\gamma)$ in the four generation model.

Fig. 5. The excluded regions in the $m_{H^\pm} - \tan\beta$ plane resulting from the present CLEO bounds in (a) Model I (shaded area is excluded) for $m_t = 150\,\mathrm{GeV}$ and (b) Model II for various values of m_t as indicated, where the excluded regions lie to the left and below each curve.

Fig. 6. Contours of $B(b \to s\gamma)$ in units of 10^{-4} in the $m_\lambda - \mu$ parameter plane with $\tan\beta = 10$, $m_t = 140\,\mathrm{GeV}$, $m_0 = 100\,\mathrm{GeV}$, taking $A = +1, -1+2, -2$ in (a), (b), (c), (d), respectively. All masses in GeV.

Fig. 7. Constraints on $|\mathcal{I}m\,(XY^*)|$ as a function of the mass of the lightest charged Higgs boson, $m_{H_1^\pm}$ with $m_{H_2^\pm} = 100, 250, 500, 750,$ and $1000\,\mathrm{GeV}$ corresponding (from left to right) to the dashed, dash-dotted, solid, dotted, and dashed curves. The bottom solid curve represents the case where the $H_2^\pm$ contributions have been neglected. The allowed region lies beneath the curves.

Fig. 8. $B(b \to s\gamma)$ in the LRM assuming $V_R = V_L$ as a function of the tangent of the $W_L - W_R$ mixing angle, t_ϕ. (a) $\kappa = 1$ and $M_{W_R} = 1.6\,\mathrm{TeV}$, with $m_t = 120, 140, 160, 180,$ and 200 GeV corresponding to the dotted, dashed, dash-dotted, solid, and square-dotted curves, respectively. (b) $m_t = 160\,\mathrm{GeV}$ and $M_{W_R} = 1.6\,\mathrm{TeV}$ with κ varying between 0.6 (left-most dotted curve) and 2.0 (inner-most dash-dotted curve). (c) Gronau-Wakaizumi version of the LRM with $\kappa = 1.5$, $M_{W_R} = 600\,\mathrm{GeV}$ with the outer-most solid curve corresponding to $m_t = 120\,\mathrm{GeV}$, and is increased in each case by steps of 20 GeV.