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Baryogenesis from Flat Directions of the Supersymmetric Standard Model

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Abstract

Baryogenesis from the coherent production of a scalar condensate along a flat direction of the supersymmetric extension of the standard model (Affleck-Dine mechanism) is investigated. Two important effects are emphasized. First, nonrenormalizable terms in the superpotential can lift standard model flat directions at large field values. Second, the finite energy density in the early universe induces soft potentials with curvature of order the Hubble constant. Both these have important implications for baryogenesis, which requires large squark or slepton expectation values to develop along flat directions. In particular, the induced mass squared must be negative. The resulting baryon to entropy ratio is very insensitive to the details of the couplings and initial conditions, but depends on the dimension of the nonrenormalizable operator in the superpotential which stabilizes the flat direction and the reheat temperature after inflation. Unlike the original scenario, an acceptable baryon asymmetry can result without subsequent entropy releases. In the simplest scenario the baryon asymmetry is generated along the LH_u flat direction, and is related to the mass of the lightest neutrino.

1 Introduction

One of the features which distinguishes supersymmetric field theories from ordinary ones is the existence of "flat directions" in field space on which the scalar potential vanishes. At the level of renormalizable terms, such flat directions are generic. Supersymmetry breaking lifts these directions and sets the scale for their potential. From a cosmological perspective, these flat directions can have profound consequences. The parameters which describe the flat directions can be thought of as expectation values of massless chiral fields (moduli). These expectation values can start out displaced from the true minimum. Oscillations about the minimum occur when the Hubble constant becomes comparable to the effective mass. These oscillations have an equation of state without pressure, and so amount to a coherent condensate of zero-momentum particles, redshifting like matter. The coherent production of scalar fields along flat directions emerges as a generic feature of any supersymmetric theory.

String theories contain a number of perturbative flat directions whose potential is generated in the presence of supersymmetry breaking. At early times the moduli can in general have Planck scale expectation values relative to the true minimum. Because these fields probably decay only through Planck suppressed interactions, they would dominate the energy density of the universe before decaying. This potential cosmological catastrophe is the string version [1] of the "Polonyi problem" [2].

The main focus of this paper will be on another class of flat directions, which are present in the minimal supersymmetric standard model (MSSM). In these directions combinations of fields carrying baryon or lepton number, such as squarks and sleptons, have non-zero expectation values. If baryon and lepton number are explicitly broken it is possible to excite a non-zero baryon or lepton number along such directions, as first suggested by Affleck and Dine [3]. We will refer to this as the Affleck-Dine (AD) mechanism of baryogenesis, and the associated fields as AD fields. Eventually, this condensate decays leaving the universe with a non-zero baryon or lepton number.

In this paper, we carefully re-examine the coherent production of scalar fields along flat directions. In order to make any quantitative predictions about the production of coherent condensates in the early universe, a number of important questions must be faced. First, how are the flat directions lifted in the early universe? Second, for the AD mechanism, how do the baryon

number violating terms in the potential arise? And third, what are the "initial conditions" for the flat directions? These are the questions which we wish to address.

We begin by observing that the finite energy density of the early universe, breaks supersymmetry by a "large" amount [4, 5]. Gravitational strength interactions between the background energy density and flat directions give soft supersymmetry breaking terms, with scales which are parametrically of order the Hubble constant. At very early times it is this breaking which is most important in lifting the flat directions. This is in contrast to the usual assumption that the potential for flat directions arises only from the supersymmetry breaking terms that determine the soft potential in the present universe. According to the standard picture (with Hubble scale masses neglected), fields are effectively frozen in the early universe (up to quantum deSitter fluctuations during inflation) and highly overdamped. However, the supersymmetry breaking due to the finite energy density gives rise to masses of order H. The scalar field is therefore parametrically close to critically damped at early times, and can efficiently evolve to an instantaneous minimum of the potential. This qualitatively changes the scenario for the coherent production of scalar fields as discussed below.

In the case of the moduli problem, the existence of a nontrivial potential at early times with a minimum which does not necessarily coincide with the minimum at late times just brings the cosmological problem into sharper focus, and calls into question some proposed solutions. It also suggests a possible solution if the minimum of the moduli potential at early and late times coincides. This can be technically natural if the minima lie at a point of enhanced symmetry. We will discuss briefly this solution and its principle limitation: the dilaton.

The finite density supersymmetry breaking potential also has an important impact on the AD mechanism. A flat direction can easily have a minimum far from the origin, giving rise to large expectation values for squark and/or slepton fields. On the other hand, because the curvature of the potential is of order the (instantaneous) Hubble constant, if the minimum of the potential is at the origin, quantum fluctuations of this field during inflation will not lead to a net baryon numbe (as is frequently assumed) since the correlation volume for the fluctuations is generally much smaller than the present Hubble volume.

There is another important issue which must be dealt with in the case of

the AD field: the flat directions are expected to be lifted by higher dimension operators. This has two effects. First, the expectation value along a flat direction at early times is determined by a balance between the induced soft potential and the higher order superpotential terms, and is typically small compared to the scale of the higher dimension operators. On the other hand, these operators themselves generally violate baryon and lepton number. As a result, the baryon number per particle in the condensate is order one. However, the fraction of the energy (and finally entropy) density carried by the condensate is typically rather small, and a sensitive function of the dimension of these operators. Because of this it is possible for the AD mechanism to produce the correct baryon to entropy ratio even without additional entropy releases, as opposed to the usual claims in the literature. In the end, we find that the mechanism is quite robust. There exist many flat directions which are broken only by operators of high dimension, and for which an acceptable baryon number is obtained. As for the decay of the AD field, we find that generally it evaporates by scattering with the thermal plasma, not by free decay as assumed in the early literature. It is also possible for the AD direction to be exactly flat in the supersymmetric limit, and we will discuss this possibility as well.

The finite density supersymmetry breaking also helps to answer the question of initial conditions along flat directions, which are usually assumed in some ad hoc way. In a cosmological scenario which includes inflation, the conditions relevant here are simply the values of the scalar fields along the flat directions at the end of inflation. During inflation the finite vacuum energy breaks supersymmetry and generates a soft potential. If the duration of inflation is sufficient to solve the flatness and horizon problems, the flat directions are efficiently driven toward an instantaneous minimum of the potential. This sets the "initial conditions" for the subsequent evolution.

The structure of the paper is as follows. In the following section the relevant properties of flat directions are reviewed. In section 3, supersymmetry breaking in the early universe is discussed, and the supergravity interactions responsible for transmitting this breaking to the flat directions presented. We show that within a cosmological scenario with an inflaton with sufficiently low reheat temperature to evade the gravitino problem, the finite energy supersymmetry breaking is important during inflation and the inflaton matter dominated era following inflation. In section 4 the impact on the AD mechanism is discussed. The classical evolution along a flat direction is studied

for the case of a negative soft mass squared. The possibility of realizing the AD mechanism with a small positive mass squared is also considered. The Polonyi problem is reconsidered in the light of the finite energy density supersymmetry breaking in section 5. The appendix contains a list of flat directions for the standard model.

Throughout we assume for simplicity a hidden sector model of supersymmetry breaking in which the zero density breaking is transmitted to the visible sector by gravitational scale interactions. The gravitino mass, $m_{3/2}$, then sets the weak scale, and is related to the intermediate scale of supersymmetry breaking by $m_{3/2} \sim M_{\rm INT}^2/M_p$.

2 Flat Directions

Supersymmetric theories commonly have directions with no classical potential. The space of all flat directions is usually referred to as the moduli space. In string theory moduli fields which parameterize an internal conformal field theory are common. In some cases the degeneracy for these moduli arises from a world sheet symmetry. In other cases it can be understood in terms of space time discrete R symmetries [6]. More generally flat directions are common in supersymmetric field theories, particularly ones with a large number of fields, such as the MSSM. In this case flat directions arise as accidental degeneracies along which both D and F components vanish. In this section we review some properties of flat directions which are important for the AD mechanism. We also discuss the effects which lift the flat directions, namely higher dimension operators and soft supersymmetry breaking terms.

The classical degeneracy along flat directions is protected from perturbative quantum corrections in the supersymmetric limit by the nonrenormalization theorem [7]. The degeneracy can be lifted by nonperturbative quantum corrections. For the flat directions relevant to the AD mechanism these effects are unimportant since no visible sector gauge couplings become strong in the early universe. We will therefore assume the potential vanishes on the moduli space in the supersymmetric and $M_p \to \infty$ limit. The potential then appears as a result of supersymmetry breaking and nonrenormalizable terms in the superpotential.

A flat direction is parameterized by a full chiral superfield, including scalar, fermionic, and auxiliary components. Here, however, the term "flat

direction" will usually refer only to the scalar component as we are interested in the coherent production of scalar fields. A single flat direction necessarily carries a global $U(1)$ quantum number. A condensate of the flat direction can therefore carry a net particle number for some $U(1)$, as required for the AD mechanism. As discussed in subsequent sections, condensates of standard model flat directions turn out to decay in the early universe through renormalizable couplings when the temperature is well above the weak scale. At such temperatures anomalous sphaleron processes which violate $B+L$ are in equilibrium [8]. The relevant quantum number the condensate must carry in order to give a nonvanishing B after sphaleron processing is therefore $B-L$. The minimal standard model contains a large number of directions which are flat with respect to the renormalizable interactions and carry $B-L$. The subspace on which the gauge potential arising from D terms vanishes is 37 dimensional. There are a large number of directions in this subspace for which all F components also vanish. A typical example of a renormalizable flat direction, carrying $B-L=-1$, is

$$Q_1^\alpha = \frac{1}{\sqrt{3}} \begin{pmatrix} \phi \\ 0 \end{pmatrix} \qquad L_1 = \frac{1}{\sqrt{3}} \begin{pmatrix} 0 \\ \phi \end{pmatrix} \qquad \bar{d}_2^\alpha = \frac{1}{\sqrt{3}} \phi$$

where superscripts are color indices, subscripts are for generation, and ϕ is the complex field parameterizing the flat direction (with canonical kinetic term). A list of standard model flat directions is given in the appendix.

It is often convenient to characterize a flat direction by a composite gauge invariant operator, X, formed from the product of m chiral superfields which make up the direction. For example, the direction given above may be parameterized by the invariant $X = Q_1 L_1 \bar{d}_2$ ($m=3$). The scalar component of the composite operator is related to the canonical field, ϕ, parameterizing the flat direction by a relation of the form $X = c\phi^m$. Fields can take on nonzero values along multiple flat directions simultaneously, although F flatness is then generally not maintained for other directions. For example, the renormalizable F terms vanish if both $\bar{u}_1 \bar{d}_1 \bar{d}_2$ and $Q_3 L_1 \bar{d}_1$ are nonzero, while $F^*_{H_d}$ does not vanish if $\bar{u}_1 \bar{d}_1 \bar{d}_2$ and $Q_2 L_1 \bar{d}_1$ are nonzero. For multiple flat directions the relation between the invariant composite operators and the ϕ fields with canonical normalization is in general highly nonlinear. However most of the relevant dynamics for the AD mechanism does not require the treatment of multiple directions. In fact, even when there are multiple flat directions, it is the lowest dimension operator generating a potential for one

of the flat directions which determines the ultimate baryon to entropy ratio. Unless stated otherwise we therefore consider the dynamics of a single flat direction in what follows.

Typically, several supermultiplets gain mass in a flat direction, $m_\perp = \lambda\langle\phi\rangle$. For example, in the $Q_1 L_1 \bar{d}_2$ direction given above, the Yukawa couplings in the superpotential, $\left(\lambda_u H_u \bar{u}_1 + \lambda_d H_d \bar{d}_1 + \lambda_e H_e \bar{e}_1 + \lambda_d H_d Q_2\right)\langle\phi\rangle$, lead to masses for quark, lepton and Higgs superfields. Gauge symmetries are also broken along flat directions, with the broken gauge supermultiplets gaining mass by the super Higgs mechanism, $m_g = g\langle\phi\rangle$. In the $Q_1 L_1 \bar{d}_2$ example, the standard model gauge group is broken to a $SU(2)_C \times U(1)$ subgroup. Along more general flat directions the gauge group is typically completely broken.

In the early universe the relevant scale for excitations in a radiation dominated era is of course the temperature. Analogously the scale for quantum deSitter fluctuations in an inflationary era is the Hubble constant. Far out along a flat direction the modes which gain mass from Yukawa and gauge couplings become heavier than these excitation scales, and therefore decouple. This is why the moduli space is the relevant subspace on which the dynamics takes place when the fields are large.

The directions referred to above as "flat" can be lifted by supersymmetry breaking and terms in the superpotential. The resulting potential is of central importance to the discussion of the evolution along flat directions in the early universe. For the AD mechanism the origin of the potential terms which violate the $U(1)$ carried by the direction is also crucial. First consider the potential arising from the superpotential. In general since a flat direction can be represented by an invariant operator, it can appear to some power in the superpotential. These superpotential terms could vanish accidentally and not lift the direction; the nonrenormalization theorem makes this technically natural. Certain symmetries can also forbid all such terms in the superpotential, as discussed at the end of this section. However to be as general as possible, unless stated otherwise, we will assume all superpotential terms consistent with gauge symmetry and R parity are present.

The nonrenormalizable terms in the superpotential which lift the flat directions are of two types. First, since the direction can be written as an invariant, X, it can appear to some power

$$W = \frac{\lambda}{n M^{n-3}} X^k = \frac{\lambda}{n M^{n-3}} \phi^n \tag{1}$$

where $X = \phi^m$, $n = mk$, and M is some large mass scale such as the GUT or Planck scale. Under our assumptions above, the lowest value of k is 1 or 2 depending on whether the direction is even or odd under R parity. The second type of term which lifts the flat direction contains a single field not in the flat direction and some number of fields which make up the direction,

$$W = \frac{\lambda}{M^{n-3}} \psi \phi^{n-1} \tag{2}$$

For terms of this form, F_ψ is nonzero along the flat direction. An example of this type is the direction $\bar{u}_1 \bar{u}_2 \bar{u}_3 \bar{e}_1 \bar{e}_2$ which is lifted by $W = (\lambda/M) \bar{u}_1 \bar{u}_2 \bar{d}_2 \bar{e}_1$, since $F^*_{\bar{d}_2} = (\lambda/M) \bar{u}_1 \bar{u}_2 \bar{e}_1$ is nonzero along the direction. In the flat space limit, with minimal kinetic terms, the lowest order contributions of either type of superpotential term, (1) or (2), give a potential

$$V(\phi) = \frac{|\lambda|^2}{M^{2n-6}} (\phi^* \phi)^{n-1}. \tag{3}$$

These terms always dominate the potential for sufficiently large field value. While the soft terms discussed below can in principle have either sign, these terms make a positive contribution to the potential (provided $\phi \ll M_p$). This has the consequence, as discussed in subsequent sections, of limiting the fields to be parameterically less than M_p. All the flat directions listed in the Appendix can be lifted by nonrenormalizable operators of the type discussed above with $n \leq 6$.

The superpotential contribution to the potential (3) has the interesting property that it conserves the $U(1)$ carried by the flat direction despite the fact that the superpotentials (1) and (2) violate the $U(1)$. This is because for a single term in the superpotential there is always an accidental R symmetry under which ϕ has charge $R = 2/n$. Higher order operators can violate this accidental symmetry in the potential through interference with the lowest order term. However, the coefficients of such terms are suppressed by additional powers of the heavy scale M, and so are subdominant. In principle, with multiple flat directions, such interference terms could arise at the same order as the $U(1)$ conserving terms. This would require two flat directions made out of the same number of fields, but with different $B - L$. Examination of the list of flat directions in the appendix, reveals that this occurs for directions made of five fields. In this case, $\phi^5 = L L \bar{d} \bar{d} \bar{d}$ carries $B - L = -3$, while all other directions made of five fields carry $B - L = 1$. However,

for this example, including all superpotential terms consistent with R parity gives a $B-L$ conserving potential which lifts the directions at lower order. The $B-L$ violating interference terms are then again subdominant. We therefore conclude that nonzero F terms arising from the nonrenormalizable superpotential give rise (predominantly) to a $U(1)$ conserving potential (3).

The other source of potential terms for "flat" directions is supersymmetry breaking. In the flat space limit these can be represented by soft breaking terms. The general form of the soft terms is fixed. The lowest order term is just a mass term

$$V(\phi) = m^2 \phi^* \phi \tag{4}$$

In addition if there are self couplings of the flat direction in the Kahler potential or superpotential (as discussed above), A type terms can arise. The lowest order A terms are of the form

$$V(\phi) = \frac{A}{M^{n-3}} \phi^n \tag{5}$$

where there are n fields in the flat direction. Assuming R-parity is unbroken in the early universe all A terms for standard model flat directions are nonrenormalizable. There are two important points to note about the soft terms. The first is the magnitude of these terms. In the present universe, assuming hidden sector supersymmetry breaking, both m and A are of order the weak scale, $m_{3/2}$. However, as we show in the next section, the finite energy density in the early universe necessarily breaks supersymmetry, inducing soft parameters of order the Hubble constant, $m \sim H$, and $A \sim H$ for $H > m_{3/2}$. This differs from the usual assumption that the soft parameters are order $m_{3/2}$ in the early universe, and has dramatic consequences for the evolution. The second important point is that the A term violates the $U(1)$ carried by the flat direction. This will be the source of $B-L$ violation necessary to generate a net $B-L$ in the evolution of the AD flat direction. In addition, the coefficient of the A term is in general complex. The relative phase between this and the "initial" phase of the flat direction is the source of CP violation in the AD mechanism. As discussed in section 4, all the potential terms (3), (4), and (5) turn out to be important in determining the evolution of the flat direction.

It is worth noting that because of the nonrenormalization theorem for the superpotential, even operators consistent with all symmetries need not appear in the superpotential. In certain instances the absence of the gauge

invariant operators which could lift the flat direction can be guaranteed by an R symmetry. Directions of this type are therefore exactly flat in the supersymmetric limit. Only soft terms contribute to the potential along such directions. In the absence of a superpotential the potential does not necessarily grow like a power for large fields. Field values of order M_p can therefore in principle develop, and higher order terms in the soft potential then become important. The general form of the $U(1)$ conserving soft potential is (again, specializing for simplicity to the case of a single field)

$$V(\phi) = m^2 M_p^2 \ f(\phi^* \phi / M_p^2) \tag{6}$$

where f is some function. Likewise the general $U(1)$ violating soft potential in this case is

$$V(\phi) = m^2 M_p^2 \ g(\phi^n / M_p^n) \tag{7}$$

Notice that the $U(1)$ violating terms start at order m^2. This is because A terms (with coefficient m) are proportional to W and so vanish if $W = 0$. As shown in the next section the scale for m again turns out to be the Hubble constant for $H > m_{3/2}$. The scenario for the evolution of the AD field with $W = 0$ turns out to quite different than in the case with a superpotential. This is in fact the picture that was originally adopted by Affleck and Dine [3]. We briefly comment on the cosmology of the AD mechanism for this case in section (4.6).

The tree level superpotential also vanishes exactly for string moduli. The potential for these directions therefore also arises from soft terms of the form (6), again with the soft parameters set by the Hubble constant for $H > m_{3/2}$. We comment on the modification and possible solution of the Polonyi problem in the presence of such terms in section (5).

3 Supersymmetry Breaking in the Early Universe

As discussed in the previous section, flat directions can be lifted by supersymmetry breaking and nonrenormalizable terms in the superpotential. In this section the potential along flat directions arising from supersymmetry breaking in the early universe is considered. Generally it has been assumed that in the early universe the relevant scale for the soft breaking parameters m

and A are of order $m_{3/2}$ (assuming hidden sector supersymmetry breaking). Our main observation is that the finite energy density in the early universe necessarily breaks supersymmetry. As discussed below, for $H > m_{3/2}$ this breaking is dominant over breaking from a hidden sector, and determines the soft potential along flat directions.

First consider how the finite energy density breaks supersymmetry. A nonzero expectation value for the energy density, and therefore the Hamiltonian, implies the supercharge does not annihilate the vacuum, thereby breaking supersymmetry. The specific form of the breaking depends on the cosmological epoch. During an inflationary epoch the vacuum energy is positive by definition. Nonzero vacuum energy necessarily requires finite F and/or D components for some matter fields, thereby signaling supersymmetry breaking. In a post inflationary epoch before reheating occurs, the energy density is dominated by the oscillations of the inflaton. Again the time averaged vacuum energy is nonzero, thereby breaking supersymmetry. Supersymmetry is also broken in a radiation dominated era. Here the boson and fermion thermal occupation numbers are distinct so the background is not supersymmetric. A similar quantum mechanical effect also exists during inflation; deSitter fluctations give bosons and fermions distinct nonzero occupation numbers. As discussed below, these thermal and quantum effects turn out to be less important at large field values than the classical supersymmetry breaking from the finite vacuum energy.

The finite energy supersymmetry breaking can be transmitted to flat directions by either renormalizable or nonrenormalizable interactions. The effect of renormalizable interactions is contained in the effective potential arising from integrating out the states which gain a mass along the direction, $m_\perp = g\langle\phi\rangle$, where g here represents a gauge or Yukawa coupling. In flat space at zero temperature supersymmetry guarantees that the bosonic and fermionic functional determinants in the effective potential cancel to all orders. At finite temperature or in deSitter space, the boson and fermion occupation numbers differ, and a nonvanishing effective potential does arise. However, at finite temperature for $\phi \gg T/g$, the states which gain a mass along the flat direction effectively decouple, $m_\perp \gg T$. The effective potential arising from integrating out these states is therefore exponentially suppressed in this region, and can be neglected. In deSitter space, where the Hubble constant sets the scale for quantum excitations, an analogous decoupling takes place for $\phi \gg H/g$. For large field values the induced potential from

renormalizable interactions is therefore unimportant [9].

Nonrenormalizable interactions can also transmit the supersymmetry breaking to flat directions. This contribution to the potential arises from integrating out fields which do not gain a mass along the flat direction. Unlike the case of renormalizable interactions, this effective potential can induce a mass for a flat direction which is roughly independent of the magnitude of the fields (so long as it is less than M_p). For large fields, nonrenormalizable interactions are therefore more important than renormalizable ones. To illustrate this effect consider the global limit with a term in the Kahler potential of the form

$$\delta K = \int d^4\theta \ \frac{1}{M_p^2} \chi^\dagger \chi \phi^\dagger \phi \tag{8}$$

where χ is a field which dominates the energy density of the universe, ϕ is a canonically normalized flat direction, and $M_p = m_p/\sqrt{8\pi}$ is the reduced Planck mass. No symmetry prevents such a term, which can be present already at the Planck scale. In fact, the existence of such operators is guaranteed in the presence of Yukawa couplings since they are necessary counterterms for operators generated by loop diagrams [10, 11]. If χ dominates the energy density, then $\rho \simeq \langle \int d^4\theta \chi^\dagger \chi \rangle$. In a thermal phase the expectation value is just the thermal mean value of the χ component kinetic terms, $\rho \simeq g_* T^4$. During inflation it is given by the inflaton F components, $\rho = F_\chi^* F_\chi = V(\chi)$. In the inflaton matter dominated era after inflation the expectation value is again the total energy density, $\rho = \dot{\chi}\dot{\chi}^* + V(\chi)$. The interaction (8) therefore gives an effective mass for ϕ of

$$\delta \mathcal{L} = (\rho/M_p^2)\phi^\dagger \phi \tag{9}$$

(note that a positive contribution in the Kahler potential gives a negative contribution to m^2). In a flat expanding background the energy density is related to the expansion rate, H, by Einstein's equations, $\rho = 3H^2 M_p^2$. This implies that the soft mass induced by the finite energy supersymmetry breaking is $m^2 \sim H^2$. This is a generic result, independent of what specifically dominates the energy density. For $H > m_{3/2}$, this source for the soft mass is more important than any hidden sector breaking.

In order to be concrete about the evolution along flat directions, we will work under the assumption that there is an inflationary phase sufficient to solve the horizon and flatness problems. This requires $N > 60$ e-foldings of

the scale factor during inflation. We also assume this inflation gives rise to the density and temperature fluctuations in the present universe. In most models this occurs for $H \sim 10^{13-14}$ GeV during inflation. We also make the standard assumption that after inflation, the universe enters a matter dominated epoch in which the energy density is dominated by coherent oscillations of the inflaton. When the inflaton decays, the universe enters an era in which the energy density is dominated by the thermalized decay products of the inflaton. We assume that the associated "reheat" temperature, T_R, is sufficiently low to avoid the "gravitino problem" [12]. The precise bound on T_R depends on the gravitino mass and branching ratios, but cannot be much larger than 10^9 GeV [12]. The Hubble constant at reheating, H_R, and T_R are related by $g_* T_R^4 \sim 3 H_R^2 M_p^2$. Avoiding the gravitino problem therefore requires $H_R \ll m_{3/2}$. With this restriction the induced potential from finite density supersymmetry breaking is only important (ignoring any pre-inflationary evolution) during inflation and in the pre-reheating era dominated by inflaton oscillations. Since the inflaton dominates the energy density in both phases, we only need to consider the soft potential induced by couplings of the inflaton to the flat directions.

Since the important couplings between the inflaton and flat directions arise from Planck scale operators, supergravity interactions should be included. The supergravity scalar potential is

$$V = e^{K/M_p^2} \left(D_i W K^{i\bar{j}} D_{\bar{j}} W^* - \frac{3}{M_p^2} |W|^2 \right) + \frac{1}{8} f_{ab}^{-1} D^a D^b \qquad (10)$$

where $D_i W \equiv W_i + K_i W / M_p^2$ is the Kahler derivative, $W_i \equiv \partial W / \partial \varphi_i$, $K^{i\bar{j}} \equiv (K_{i\bar{j}})^{-1}$, f_{ab} is the gauge kinetic function. $W(\varphi)$ and $K(\varphi^\dagger, \varphi)$ are the superpotential and Kahler potential, $D^a \equiv K_\varphi T^a \varphi$, where φ includes in general the flat directions, inflaton(s), and hidden sector. By assumption the inflaton dominates the energy density during inflation, and prior to reheating. The largest piece of (10) is then for the inflaton. It is perhaps unlikely that D terms in the inflaton sector give a significant contribution to the inflaton potential [13]. The potential is not sufficiently flat along D non-flat directions to give a reasonable number of e-foldings [14, 15, 16]. If the inflaton potential arises from F terms in some sector,

$$V(I) \simeq e^{K(I^\dagger, I)/M_p^2} \left(F_I^* F^I - \frac{3}{M_p^2} |W(I)|^2 \right) \qquad (11)$$

where $F_I^* F^I \equiv D_I W(I) K^{\bar{I}I} D_{\bar{I}} W^*(I^*)$. The term in parenthesis necessarily has positive expectation value and a nontrivial potential along flat directions is obtained as described below. Even if D terms dominate the inflaton potential, a nontrivial potential along flat directions can result. In this case Kahler potential couplings (such as (8)) give a nontrivial potential along flat directions from $K_I T^a I$, where I is the inflaton.

Under the assumptions spelled out above, the induced soft potential for the flat directions arises from couplings to the inflaton F components. However, the specific form of the induced soft potential can depend on the scalar value of the inflaton. During inflation $I \sim M_p$ is possible. In fact in "natural" models I changes by order M_p per e-folding during inflation [14, 15, 16]. In this case since the inflaton vacuum energy is necessarily positive, $W(I)/M_p$ can be at most the same order as $D_I W(I)$. Using the relation between the energy density and Hubble constant, $\rho = 3H^2 M_p^2$, gives the scales $D_I W(I) \sim H M_p$ and $W(I) \sim H M_p^2$. The pure supergravity corrections to the potential are considerably simplified if $I \ll M_p$. In this limit $K_I \ll M_p$, $D_I W \to W_I$, and $|W|/M_p \ll W_I$. The inflaton potential then reduces to $V(I) \simeq W_I^* W^I$ (up to corrections of $\mathcal{O}(I/M_p)^2$). I may be of order M_p during inflation, but after inflation $I \ll M_p$ if, as we assume, the energy density after inflation is dominated by the coherent oscillations of the inflaton field. In this era we assume that the relevant potential for the inflaton is just that of a harmonic oscillator, $V(I) \simeq m_I^2 (I - I_0)^* (I - I_0)$. If the inflaton potential is at all natural, it is unlikely that the mass of the inflaton, m_I, is much smaller than H_I, the Hubble constant during inflation. Therefore well after inflation $\langle I^* I \rangle \sim (H^2/m_I^2) M_p^2 \sim (H^2/H_I^2) M_p^2 \ll M_p^2$ and the supergravity corrections are small. This distinction will only be important for the induced A and μ terms discussed below.

The general form for the induced potential from (10) along flat directions depends on whether or not the flat direction is lifted by nonrenormalizable terms in the superpotential. First consider terms which are independent of the superpotential along the flat direction. In this case the potential for a flat direction arising from supersymmetry breaking terms in the supergravity potential comes from the following sources:

1) The e^{K/M_p^2} prefactor

$$e^{K(\phi^\dagger, \phi)/M_p^2} V(I)$$

2) Cross terms in the Kahler derivative between the flat direction Kahler

potential and inflaton superpotential

$$K_\phi K^{\phi\bar{\phi}} K_{\bar{\phi}} \frac{|W(I)|^2}{M_p^4}$$

3) Kahler potential couplings between the inflaton and flat direction

$$K_\phi K^{\phi\bar{I}} D_{\bar{I}} W^*(I) \frac{W(I)}{M_p^2} + h.c.$$

With the scales for the inflaton potential potential terms given above, all these give the general form

$$V(\phi) = H^2 M_p^2 \; f(\phi/M_p) \tag{12}$$

where f is some function. Notice that the overall scale of the potential is set by the Hubble constant, $V'' \sim H^2$, and the scale for variations in the potential is M_p. This is the form of the induced potential for string moduli or standard model directions which are exactly flat in the supersymmetric limit. For string moduli the minimum of the induced potential (12) is in general displaced by order M_p from the true minimum arising from hidden sector supersymmetry breaking. However if there is a point of enhanced symmetry on the moduli space, the potential (no matter what the source) is necessarily an extremum about this point (for the potential induced by the finite density breaking this follows since the Kahler potential is a minimum about a symmetry point). For standard model fields, the origin is always an enhanced symmetry point, so the potential is always an extremum at the origin.

An important special case of the general form (12) results for a minimal Kahler potential for the flat direction, $K(\phi^\dagger, \phi) = \phi^\dagger \phi$. Assuming F terms dominate the inflaton energy density as in (11), and using the relation between the energy density and expansion rate, $V = 3H^2 M_p^2$, the resulting induced potential for $\phi \ll M_p$ is then just a mass term $m_\phi^2 \phi^* \phi$, with

$$m_\phi^2 = \left(2 + \frac{F_I^* F_I}{V(I)}\right) H^2 \tag{13}$$

For $I \ll M_p$, $V(I) \simeq F_I^* F_I$ as discussed above. (These expressions should be corrected by $V_F(I)/(V_F(I) + V_D(I))$ if inflaton D components contribute to

the energy density). The important feature of this contribution is that it is *positive* with coefficient of order one. This will have important implications for the AD mechanism discussed in the next section. With minimal Kahler terms only, $\phi = 0$ is stable and the large expectation values required for baryogenesis do not result. With general Kahler terms though, $|m_\phi^2| \sim H^2$, with either sign possible. In this paper we will assume that this is the case. However, it is possible to choose special forms for the Kahler potential couplings between the flat direction and inflaton which partially cancel the minimal supergravity induced mass (13). It has been suggested that no-scale like forms of Kahler potentials (which often arise at tree level in string theory) might accomplish this [17, 18]. It is also important to recognize that symmetries can protect a compact subspace of a flat direction from receiving a soft potential. This is the case for a Goldstone boson of a spontaneously broken symmetry.

There are additional contributions to the potential induced from the finite density supersymmetry breaking if a flat direction is lifted by nonrenormalizable terms in the superpotential. These arise from:
1) cross terms in the Kahler derivative between the derivative of the flat direction superpotential and inflaton superpotential,

$$W_\phi K^{\phi\bar{\phi}} K_{\bar{\phi}} \frac{W^*(I)}{M_p^2} \ + h.c.$$

2) cross terms between the flat direction and inflaton superpotential

$$\left(\frac{1}{M_p^2} K_I K^{I\bar{I}} K_I - 3\right)\left(\frac{W(\phi)^* W(I)}{M_p^2} \ + h.c.\right)$$

3) Kahler potential couplings between the flat direction and inflaton

$$W_\phi K^{\phi\bar{I}} D_{\bar{I}} W^*(I) \ + h.c.$$

With the nonrenormalizable superpotential terms (1) and (2) and the inflaton scales given above, all these have the form of a generalized A term

$$V(\phi) = H M_p^3 g(\phi^n / M_p^n) \tag{14}$$

where g is some function. The induced A terms have the important effect of defining the intial phase of the AD field, as discussed in the next section. The

possible A terms considerably simplify if $I \ll M_p$ as is the case after inflation. Using the inflaton scales given above for $I \ll M_p$, only the $W_\phi K^{\phi \bar{I}} W^*_{\bar{I}} + h.c.$ term can contribute significantly. Since all the fields are $\ll M_p$ in this case the Kahler potential couplings can be expanded in powers of M_p. The only term of the required form which gives rise to an A term is then

$$\frac{1}{M_p} \int d^4\theta \; I \phi^\dagger \phi \tag{15}$$

If I is a composite field rather than an elementary singlet, then only terms bilinear in the canonically normalized inflaton field can appear in the Kahler potential and such a term does not exist. It is therefore possible in some models for the induced A terms to vanish after inflation. The same conclusions hold for other dimension 3 soft terms and the induced μ term as discussed in section 4.4.

In sum, for $H > m_{3/2}$, the soft potential for the flat direction (away from the origin) is set by the supersymmetry breaking due the finite energy density, with soft breaking scale given by the Hubble constant. This is our most important result. Previously it had been (implicitly) assumed that the hidden sector supersymmetry breaking set the scale for the soft potential. Self couplings from nonrenormalizable superpotential terms have also not been consistently included in discussions of the evolution of flat directions.

4 Evolution of the AD Scalar

The evolution of the fields parameterizing a flat direction is governed by the classical equations of motion. For the AD mechanism, assuming the baryon number does not average to zero over the current horizon size, only the zero mode of the field is relevant. The equation of motion for the zero mode is just that of a damped oscillator

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \tag{16}$$

where the damping term, proportional to the Hubble constant, arises because of the expanding background. The behavior of the solutions of (16) are well know. For $H^2 \gg V''(\phi)$ the field is overdamped and the friction term dominates the evolution. For $H^2 \ll V''(\phi)$ the field is underdamped and the

inertial term dominates the evolution. Previously, the implicit assumption has been that the potential for ϕ arose from hidden sector supersymmetry breaking, $V''(\phi) \sim m_{3/2}^2$. If this were the case, at early times when $H \gg m_{3/2}$ the field would be highly overdamped, and effectively frozen at some "initial" value. When $H \sim m_{3/2}$ the field would begin to oscillate about a local minimum. However, as discussed in the previous section the scale for the soft potential arising from the finite density supersymmetry breaking is the Hubble constant. At early times the fields are parameterically near critically damped. During inflation when H is roughly constant, the fields can therefore very effectively evolve to an instantaneous minimum. This has very important consequences for the AD mechanism of baryogenesis, which requires large field values to develop.

Crucial in assessing the possibility of baryogenesis is the sign of the induced mass squared at $\phi = 0$. As discussed in the previous section, with minimal Kahler terms the m^2 is of order H with positive coefficient. In this case as long as the field is within the basin of attraction (which is very likely for directions which are lifted by nonrenormalizable terms) the average value of the field evolves to $\phi = 0$ exponentially in time. This is in contrast to the usual statement that "scalars are not damped during inflation." Within supergravity, scalars can be very effectively damped away during inflation. After inflation no coherent production results, and the AD mechanism does not occur. Quantum deSitter fluctuations do excite the field with $\langle \delta\phi^2 \rangle \sim H^2$ for $m \sim H$, but with a correlation length of $l \sim \mathcal{O}(H^{-1})$. Any resulting baryon number then averages to zero over the present universe. In addition, the relative magnitude of the B violating A term in the potential is small for $H \ll M$. One possibility might be that $m^2 > 0$ but $m^2 \ll H^2$. The correlation length for deSitter fluctuations in this limit is $l \simeq H^{-1} e^{3H^2/2m^2}$ [19]. This is only large compared to the horizon size if $(m/H)^2 < \frac{1}{40}$. Although baryogenesis may be possible in this case, as discussed in section 3, $m^2 \ll H^2$ requires arranging couplings in the Kahler potential partially cancel the minimal supergravity contribution (13) to m^2 over a wide range of values for the inflaton field (including after inflation). Although this is possible in toy models, it seems likely to require fine tuning in any realistic example.

However, if the sign of the induced mass squared is negative a large expectation value for a flat direction can develop. With nonminimal Kahler terms this is perhaps just as likely as a positive mass squared. The magnitude of

the field is then set by a balance with nonrenormalizable terms in the superpotential which lift the flat direction. The post inflationary evolution turns out to be remarkably simple and independent of the details of the potential. We first summarize the salient features of the evolution in the negative mass squared scenario and then explain each point in more detail in subsequent subsections.

- During inflation, the AD field evolves exponentially to the minimum of the potential, determined by the induced negative mass squared and nonrenormalizable term in the superpotential. This process may be thought of as establishing "initial conditions" for the subsequent evolution of the field. The B violating A terms play an important role in determining the initial phase of the field.

- Subsequent to inflation, the minimum of the potential is time dependent (as it is tied to the instantaneous value of the Hubble parameter). The AD field oscillates near this time dependent minimum with decreasing amplitude

- When $H \sim m_{3/2}$ the soft potential arising from hidden sector supersymmetry breaking becomes important and the sign of the mass squared becomes positive. At this time, the B-violating A term arising from the hidden sector is of comparable importance to the mass term, thereby imparting a substantial baryon number to the condensate. The fractional baryon number carried by the condensate is near maximal, more or less independent of the details of the flat direction. Subsequent to this time, the baryon number violating operators are negligible so the baryon number (in a comoving volume) is constant.

- The inflaton decays when $H < m_{3/2}$ (consistent with the gravitino bound on the reheat temperature). The baryon to entropy ratio subsequent to this is

$$\frac{n_b}{s} \approx \frac{n_b}{n_\phi}\frac{T_R}{m_\phi}\frac{\rho_\phi}{\rho_I} \tag{17}$$

  where n_b and n_ϕ are baryon and AD field number densities, T_R is the reheat temperature, $m_\phi \sim m_{3/2}$ is the low energy mass for the AD field, and ρ_ϕ and ρ_I are the AD field and inflaton mass densities (both at the time of inflaton decay).

The final baryon density depends principally on the reheat temperature and the dimension of the operator which stabilizes the flat direction (in the supersymmetric limit), through the factor ρ_ϕ/ρ_I. The net baryon number density is a robust prediction which depends only weakly on the other variables in the problem, such as the numerical values of the coupling constants (and their phases) appearing in the superpotential.

In the following four subsections we discuss details of the evolution and resulting baryon to entropy ratio in the negative mass squared scenario outlined above, assuming the flat direction is lifted by nonrenormalizable operators. In section 4.5 the evolution for positive (small) mass squared is considered. In section 4.6 we consider the evolution in the case that the direction is exactly flat in the supersymmetric limit.

4.1 The Inflationary Epoch

The large Hubble scale mass is clearly important to the evolution of the field, as the field is parameterically near critically damped. We now consider in detail the evolution of the AD field during inflation. The flat direction is assumed to be stabilized, even in the absence of supersymmetry breaking, by a high dimension operator in the superpotential of the form (1) or (2). During inflation the Hubble parameter is roughly constant. Given the discussions of sections 2 and 3 the relevant potential during inflation then takes the form

$$V(\phi) = -cH_I^2|\phi|^2 + \left(\frac{a\lambda H_I \phi^n}{nM^{n-3}} + \ h.c.\right) + |\lambda|^2\frac{|\phi|^{2n-2}}{M^{2n-6}} \qquad (18)$$

where c and a are constants of $\mathcal{O}(1)$, and M is some large mass scale such as the GUT or Planck scale. For $H_I \gg m_{3/2}$ soft terms arising from the hidden sector are of negligible importance. For $c > 0$ the potential (18) has an unstable extremum at the origin. As discussed at the beginning of section 3, there is a contribution to the potential for $\phi \sim H_I$ coming from deSitter fluctuations of the fields coupled to the flat direction by renormalizable couplings. This gives a positive mass squared contribution to the free energy of $\delta m^2 \sim g^2 H_I^2$, where g is a gauge or Yukawa coupling. However, for $c \sim \mathcal{O}(1)$ the origin remains unstable. Even if the origin is a local minimum from this effect (which might happen if a large number of fields become massless at $\phi = 0$) the global minimum at large ϕ is unaffected. For very large ϕ, (18)

grows as $|\phi|^{2n-2}$. For $H_I \ll M_p$ this limits $\phi \ll M_p$ just on energetic grounds. The only soft terms which are important are therefore the lowest order ones, namely the mass and A terms.

The minimum of the potential (18), is given by

$$|\phi_0| = \left(\frac{\beta H_I M^{n-3}}{\lambda}\right)^{\frac{1}{n-2}} \tag{19}$$

where β is a numerical constant which depends on a, c, and n. Notice that ϕ_0 is parameterically between H_I and M. For example, with $H_I \sim 10^{13}$ GeV, $M/\lambda \sim M_p$, and $n = 4$, $\phi_0 \sim 10^3 H_I$. The minimum is larger for greater n. The A term in (18) violates the $U(1)$ carried by ϕ and gives n discrete minima for the phase of ϕ. The potential in the angular direction goes like $\cos(\theta_a + \theta_\lambda + n\theta)$ where $\phi = |\phi|e^{i\theta}$, etc. During inflation if c is not too small, the field quickly settles into one of the minima.

In order to see just how fast the field evolves to the minima, it is useful to consider explicitly the evolution of the magnitude, ignoring for the moment the A term. At the beginning of inflation ϕ might be arbitrary. However a simple constraint arises by requiring that the energy density of the inflaton, $3H_I^2 M_p^2$, be greater than that in ϕ (otherwise inflation could not take place). For $M/\lambda \sim M_p$ this gives $\phi/\phi_0 < (M_p/H_I)^{1/(n-1)(n-2)}$. For $n = 4$ this gives $\phi < 10\phi_0$. For larger n the maximum value is even smaller. As a worst case, suppose ϕ did start near this maximal value. Then we expect that the field oscillates rapidly with a period much less than H_I^{-1}. The amplitude of the oscillations, ϕ_m, is expected to decrease with a characteristic time H_I^{-1}. The time rate of change of the energy in ϕ can be found from the equations of motion,

$$\frac{dE}{dt} = \dot{\phi}\frac{d}{d\phi}(T+V) = -3H_I\dot{\phi}^2 = -6H_I(E-V) \tag{20}$$

where T is the kinetic energy. Using the expression for $V(\phi)$ for large ϕ and averaging over a period gives $\dot{\phi}_m \simeq -6H_I/(2n-1)\phi_m$. We therefore conclude that in the large ϕ regime, ϕ decreases exponentially towards smaller values,

$$\phi_m \simeq e^{-6H_I t/(2n-1)}\phi_i \tag{21}$$

where ϕ_i is the initial value of the field with espect to the origin. Thus after just a few e-foldings ϕ is near a minima.

Once near a minimaum, the field evolves like a damped harmonic oscillator. For the region in which the potential is approximately harmonic, the equation of motion, neglecting the a term, is

$$\ddot{\phi}' + 2H\dot{\phi}' - 2(n-2)cH_I^2\phi' = 0 \tag{22}$$

where $\phi' = \phi - \phi_0$. So long as c is not too small, the system will quickly settle into a minimum. The field undergoes deSitter fluctuations about the minimum, with amplitude $\langle \delta\phi^2 \rangle \sim H_I^2$. This is a small perturbation in the radial mode since $\delta|\phi|/\phi_0 \sim H_I/\phi_0 \ll 1$, and has a very small correlation length $l \sim H_I^{-1}$. If a is not too small, the angular mode also has a mass of order H_I. The fluctuations are then also a small perturbation in this mode, $\delta\theta \sim H_I/\phi_0 \ll 1$, again with a small correlation length. At the end of inflation, over regions large compared to the current horizon size, ϕ is left with essentially a constant "initial" phase. As discussed in section 3 it is possible in principle that the finite density A term is very small during inflation (if the inflaton is composite and $I \ll M_p$). In this case there is no potential for the phase of ϕ. The phase then undergoes a random walk from deSitter fluctuations. But by the end of inflation, the correlation length for $\delta\theta$ is necessarily larger than the current horizon. So again the present universe is left with an essentially constant (random) "initial" phase [20].

We conclude that at the end of inflation, the average value of the field is at one of its minima, with a large expectation value (19). In addition the field has a definite value for its phase, which is constant over scales large compared to the present horizon. This amounts to the "initial" conditions for the subsequent evolution.

4.2 Post-Inflation: Inflaton Matter Dominated Era

After inflation the universe enters a matter era dominated by the coherent oscillations of the inflaton. During a matter era the Hubble constant is related to the expansion time by $H = \frac{2}{3}t^{-1}$. The equation of motion for ϕ is then

$$\ddot{\phi} + \frac{2}{t}\dot{\phi} + V'(\phi) = 0 \tag{23}$$

where $V(\phi)$ is still given by (18), though the dimensionless constants c and a may be different, and H is now time dependent. To simplify the analysis, we will neglect a during this phase of the evolution. (If $a \neq 0$ and different from

that during inflation then the phase simply evolves to a different value.) The most important feature of (23) is that the minimum of the potential, ϕ_0, now decreases with time. Since the potential grows like a power law for large ϕ, one might guess that if the field starts out not too far from the minimum at early times, it will closely track the minimum.

Greater insight into the solutions of (23), can be gained by making changes of variables. Since the minimum decreases as a power law in t it is useful to rescale time as

$$z = \log t$$

and define the dimensionless field χ with respect to the instantaneous minimum

$$\phi = \chi\phi_0(t) = \chi\left(\frac{\beta}{\lambda}M^{n-3}e^{-z}\right)^{\frac{1}{n-2}}$$

where $\beta = \sqrt{c'/(n-1)}$ for $a = 0$, and $c' = \frac{4}{9}c$. The equation of motion in these rescaled variables is then

$$\ddot{\chi} + \left(\frac{n-4}{n-2}\right)\dot{\chi} - \left[c' + \frac{n-3}{(n-2)^2}\right]\chi + c'\chi^{2n-3} = 0. \tag{24}$$

The rescaled problem is so simple because the effective mass term, Hubble damping term, and acceleration term are all homogeneous in z. The equation of motion (24) has two important properties. First, there is a fixed point at

$$\bar{\chi} = \left(1 + \frac{n-3}{c'(n-2)^2}\right)^{\frac{1}{2n-4}} \tag{25}$$

For reasonable values of the parameters this is just slightly larger than the position of the instantaneous minimum. So if ϕ starts at this fixed point it remains there, i.e. $\phi(t) = \bar{\chi}\phi_0(t)$, and ϕ tracks just behind the decreasing minimum. Second, the damping in the rescaled problem depends on n. For $n > 4$ the effective damping in the rescaled problem is positive. In this case the fixed point is attracting. For general initial conditions, the field oscillates about $\bar{\chi}$ with decreasing amplitude. For $n = 4$ there is no damping of χ. The field ϕ therefore oscillates about the attracting point with an envelope which decreases in time in proportion to the instantaneous minimum. For $n < 4$ the damping is negative, but this corresponds to a direction which is not even

flat at the renormalizable level. We conclude that for $n \geq 4$ the magnitude of ϕ decreases with the instantaneous minimum.

The oscillatory motion about the fixed point in the rescaled problem is physically reasonable. If ϕ starts at a large value it is underdamped ($V'' \gg H^2$) and gets driven to smaller values by the acceleration term. Eventually it reaches small values of ϕ where it is overdamped ($V'' \ll H^2$) and slows due to the friction term. Still later, the instantaneous minimum catches up and overtakes ϕ, again leaving it in an underdamped regime, and so forth. As the instantaneous minimum decreases, the field therefore naturally oscillates about a point at which $V''(\phi) \sim H^2$, which is necessarily close to $\phi_0(t)$. Numerical evolution of (24) supports this picture, and the n dependence explained above.

It is possible that the field does not start near the minimum when the AD field potential during and subsequent to inflation are very different. This would only occur when I during inflation is of order M_p, so that there can be terms which change the qualitative form of the potential during inflation but are negligible afterwards. In this case, one can proceed using the adiabatic approximation of the previous section. Here, because the Hubble constant is time dependent, the field is only damped with a power law dependence, and decreases at a rate $\phi \propto t^{-1/(n-2)}$.

4.3 Late Stage of Evolution: $H \sim m_{3/2}$

The most interesting behavior of the fields is for $H \sim m_{3/2}$. Until this time, the quadratic term, and A terms are of comparable importance (unless $a \ll 1$), and there is no sense in which baryon number is conserved. Once $H \ll m_{3/2}$, the baryon number per comoving volume is frozen. The potential, including now the low energy soft terms arising from hidden sector supersymmetry breaking is

$$V(\phi) = m_\phi^2 |\phi|^2 - \frac{c'}{t^2}|\phi|^2 + \left(\frac{(A m_{3/2} + a H)\lambda \phi^n}{n M^{n-3}} + h.c. \right) + |\lambda|^2 \frac{|\phi|^{2n-2}}{M^{2n-6}} \quad (26)$$

where $m_\phi \sim m_{3/2}$. At early times the field tracks near the time dependent minimum as discussed in the last section. Therefore when $H \sim m_{3/2}$ all the terms in (26) have comparable magnitudes. Since the soft terms have magnitudes fixed by $m_{3/2}$ the field is no longer near critically damped, but

becomes underdamped as H decreases beyond $m_{3/2}$. In addition, the $m_\phi^2|\phi|^2$ term comes to dominate the $-cH^2\phi^2$ term as H decreases. The field therefore begins to oscillate freely about $\phi = 0$ when $H \sim m_{3/2}$, with "initial" condition given by $\phi_0(t)$ (eq. (19)) with $t \sim m_{3/2}^{-1}$. The oscillation of the field is the coherent condensate, $n_\phi \simeq m_\phi|\phi|^2$.

Crucial for the generation of a baryon asymmetry are the B violating A terms in (26). However, as discussed above when $H \sim m_{3/2}$ all the terms have comparable magnitude, including the A terms. Since $V_B \sim V_{\not B}$ when the field begins to oscillate freely a large fractional baryon number is generated in the "initial" motion of the field when m^2 becomes positive. Notice that in this negative mass squared scenario n_b/n_ϕ is roughly *independent* of λ/M. This is because the value of the field is determined precisely by a balance of (negative) soft mass squared term and nonrenormalizable supersymmetric term. That the B violating A term also has the same magnitude follows from supersymmetry since its magnitude is the root mean square of the soft mass term and nonrenormalizable supersymmetric term. In this scenario there is no need for ad hoc assumptions about the initial value of the field when it begins to oscillate freely. The expectation that $n_b/n_\phi \sim \mathcal{O}(1)$ falls out naturally.

The important role of CP violation is also dictated by the A terms. As discussed in section 4.1 at early times the potential for the phase of ϕ goes like $\cos(\theta_a + \theta_\lambda + n\theta)$. As H decreases below $m_{3/2}$ the low energy A term becomes more important and the angular potential goes like $\cos(\theta_A + \theta_\lambda + n\theta)$. When the field begins to oscillate freely a nonzero $\dot{\theta}$ is therefore generated if $\theta_a \neq \theta_A$. This is of course required in order to generate a nonzero baryon number since $n_b = 2|\phi|^2\dot{\theta}$. The resulting baryon number therefore depends on the CP violating phase $\theta_a - \theta_A$, i.e. on a relative phase between the inflaton and hidden sectors. Alternately, as discussed in section (4.1) it is possible in principle for a to vanish during and after inflation. The initial phase is then random (but constant over scales large compared to the present horizon). The CP violation from the initial phase is then effectively spontaneous. It is interesting to note that if this is the case, the net baryon number averaged over all inflationary domains vanishes.

Let us now consider in detail the numerical evolution of the field equation in this late stage of evolution. In the discussion which follows, we will assume $a = 0$ after inflation, and take the initial phase, θ_i, of ϕ as an input. We

have also done the analysis with $a \neq 0$ and find no qualitative difference. It is useful to once again work with rescaled variables. The field is rescaled as

$$\phi \to \left(\frac{m_{3/2} M^{n-3}}{\lambda} \right)^{\frac{1}{n-2}} \phi$$

From the arguments above and (19), up to a numerical constant of order unity, this is just the value of the field when $H \sim m_{3/2}$. All other mass scales and time are rescaled with respect to $m_{3/2}$. The equation of motion (26) with $a = 0$ and $\theta_A + \theta_\lambda = 0$ is then

$$\ddot{\phi} + \frac{2}{t}\dot{\phi} + \left(m_\phi^2 - \frac{c'}{t^2} \right) \phi + A \left(\phi^* \right)^{n-1} + (n-1) \left(\phi^* \phi \right)^{n-2} \phi = 0 \qquad (27)$$

The equation of motion for the real and imaginary parts (appropriate for numerical integration) are

$$\begin{aligned} \ddot{\phi}_R + \frac{2}{t}\dot{\phi}_R + \left(m_\phi^2 - \frac{c'}{t^2} \right) \phi_R + A|\phi|^{n-1} \cos\left((n-1)\theta\right) + (n-1)|\phi|^{2n-4}\phi_R = 0 \\ \ddot{\phi}_I + \frac{2}{t}\dot{\phi}_I + \left(m_\phi^2 - \frac{c'}{t^2} \right) \phi_I - A|\phi|^{n-1} \sin\left((n-1)\theta\right) + (n-1)|\phi|^{2n-4}\phi_I = 0 \end{aligned} \qquad (28)$$

where $\phi = \phi_R + i\phi_I$, and $\theta = \text{Arg } \phi$. The initial ϕ and $\dot{\phi}$ for $t \ll 1$ were chosen such that the field tracks the fixed point (ignoring the $m_{3/2}^2$ mass term). The equations of motion (28) were then integrated forward in time to $t \gg 1$. In this regime n_b/n_ϕ asymptotes to a constant value. A typical trajectory in the ϕ plane is shown in fig. 1 for $m_\phi = c' = -a = 1$, $n = 4$, and $n\theta_i = \frac{9}{10}\pi$. The field tracks near the fixed point until the $m_{3/2}$ mass and A terms become important. When $t \sim 1$ ($H \sim m_{3/2}$) the field feels a "torque" from the A term, and spirals inward in the harmonic potential. The nonzero $\dot{\theta}$ in the trajectory gives rise to the baryon number. For the trajectory in fig. 1 $n_b/n_\phi \simeq .8$, as can be estimated by eye from the eccentricity of the ellipse. The fractional baryon number carried by the condensate is shown in fig. 2. as a function of $n\theta_i$ for $m_\phi = c' = -a = 1$, and $n = 4$. Notice that n_b/n_ϕ is 0 for $\theta_i = 0$ which corresponds to a minimum of $V(\theta)$, and changes sign at $n\theta_i = \pi$, which is a maximum of $V(\theta)$. For this choice of parameters,

before spiraling in the harmonic part of the potential, the field goes through one angular oscillation about the minimum of $V(\theta)$ while the A term is still important. This is the origin of the zero in n_b/n_ϕ for $n\theta_i \simeq 0.65\pi$. For this initial value the field receives an equal and opposite integrated torque as it oscillates through $V(\theta)$ while the A term is important. Integrating over θ_i, the rms n_b/n_ϕ is 0.3. This is typical of the result for other n and reasonable soft parameters.

At very late stages of the evolution when $H \ll m_{3/2}$, the only potential term which is relevant in (26) is the soft mass term $m_\phi^2|\phi|^2$ which is of course B conserving. The baryon number created during the epoch $H \sim m_{3/2}$ is therefore conserved by the classical evolution of ϕ for $H \ll m_{3/2}$.

4.4 Baryon to Entropy Ratio

As discussed in the previous subsection, the fractional baryon number stored in the condensate is likely to be near maximal, independent of the order at which the flat direction is lifted. The relevant physical quantity of interest however is the baryon to entropy ratio, which is $n_b/s \sim 10^{-10}$ in the present universe. In this section we show that n_b/s depends in an essential way only on the reheat temperature after inflation, T_R, and the magnitude of the nonrenormalizable operator which lifts the flat direction. These in turn determine the fractional energy density stored in the AD field. The reheat temperature depends on details of the inflationary model, and introduces some uncertainty in the final answer. The dependence on some nonrenormalizable B or L violating operator could in principle relate n_b/s to some B or L violating process observable in the laboratory. Unfortunately this is generally not the case because the B or L violation occur through higher dimension operators, the effects of which are negligible at small field value. However, the success of the preferred scenario is related to the lightest neutrino mass, as discussed below.

Although n_b/n_ϕ is not small, the total density in the condensate $\rho_\phi \sim m_{3/2}^2\phi^2$, is much smaller than the total density for $\phi_0 \ll M_p$. For $H \sim m_{3/2}$ the coherent oscillations of the inflaton still dominate the energy density as discussed previously, $\rho_I \sim 3H^2M_p^2$. Using the estimate (19) for ϕ_0 at

$H \sim m_{3/2}$, the fractional energy in the AD condensate at this time is

$$\frac{\rho_\phi}{\rho_I} \approx \left(\frac{m_{3/2} M^{n-3}}{\lambda M_p^{n-2}} \right)^{2/(n-2)} . \tag{29}$$

For $n = 4$, $\rho_\phi/\rho_I \sim 10^{-16}(M/\lambda M_p)$, while for $n = 6$, $\rho_\phi/\rho_I \sim 10^{-8}(M^3/\lambda M_p^3)^{1/2}$, Notice for smaller (λ/M^{n-3}) the direction is effectively flatter, and ϕ_0 and ρ_ϕ are larger. A greater total energy is therefore stored in the oscillating condensate for smaller λ or larger n. As discussed in section 3 the inflaton decays when $H < m_{3/2}$. Until the inflaton decays ρ_ϕ/ρ_I stays roughly constant as both the AD condensate and inflaton redshift like matter. The number density in the AD condensate is $n_\phi = \rho_\phi/m_\phi$. After the inflaton decays the baryon to entropy ratio is therefore

$$\frac{n_b}{s} \approx \frac{n_b}{n_\phi} \frac{T_R}{m_\phi} \frac{\rho_\phi}{\rho_I}. \tag{30}$$

where $s \approx \rho_I/T_R$. This formula only applies if $n_\phi < \rho_I/T_R$ at the time of decay. This is well satisfied for $n = 4$ or 6. If this inequality is not satisfied, the entropy is actually dominated by the AD decay, and n_b/s is order unity in this case. For T_R above the weak scale anomalous sphaleron processes are in equilibrium. So only directions with nonzero $B - L$ give a significant baryon number in this scenario. As long as the AD condensate decays through $B-L$ conserving decays after the inflaton, the estimate (30) is insensitive to the details of the decay. Once the amplitude of the field becomes small enough for degrees of freedom coupled to the flat direction by renormalizable couplings to be excited by the thermal plasma (i.e. $m_\perp = g\langle\phi\rangle < T$ where g is a gauge or Yukawa coupling), the condensate can decay by $B - L$ conserving thermal scatterings. The rate for this scattering is set by T rather than $m_{3/2}$ (as would be the case for a free decay [3]). For typical parameters the condensate evaporates by thermal scattering some time after the inflaton decays.

In order for the estimate given above to apply, the field must begin to oscillate freely when $H \sim m_{3/2}$, and evaporate by thermal scatterings with the plasma at a later epoch. Even though most of the inflaton energy is not converted to radiation until $H \sim H_R < m_{3/2}$, subsequent to inflation but before reheating there is still a dilute plasma with temperature $T \sim (T_R^2 H M_p)^{1/4}$ arising from the inflaton decay products [21]. Scatterings with

this ambient plasma must therefore be unimportant when $H \sim m_{3/2}$. This is the case if $g\langle\phi\rangle > T$. If this were not the case, the condensate would be highly damped by thermal scatterings and potentially evaporate *before* the epoch at which the baryon asymmetry is established. Using the scaling for the temperature given above,

$$\left.\frac{\langle\phi\rangle}{T}\right|_{H\sim m_{3/2}} \sim \frac{1}{\sqrt{T_R M_I}}\left(\frac{H M^{n-3}}{\lambda}\right)^{1/(n-2)} \tag{31}$$

For $n = 4$, $\phi/T \sim (M_I/T_R)^{1/2}(M/\lambda M_p)^{1/2}$, where $M_I = \sqrt{m_{3/2}M_p}$, while for $n = 6$, $\phi/T \sim (M_p/T_R)^{1/2}(M^3/\lambda M_p^3)^{1/4}$. It is clear that for the $n = 6$ directions and $T_R < 10^9$ GeV, the condensate survives intact when $H \sim m_{3/2}$, allowing for the successful creation of baryon number. However, the $n = 4$ case is somewhat borderline, depending on the value of Yukawa couplings along the direction, and the scale of the nonrenormalizable operator which sets the expectation value. For example, if $M/\lambda \sim 10^3 M_p$, thermal up quarks, with Yukawa coupling of order 10^{-4}, could scatter with the $L_i H_u$ directions (for any i) unless $T_R < 10^6$ GeV.

The total density in the AD condensate, and therefore n_b/s, is very sensitive to n, the order at which the flat direction is lifted. For $n > 4$ with $M \sim M_p$ and a reasonable T_R, n_b/s is generally too large, without additional entropy releases. For example, $n = 6$ naturally gives the correct n_b/s only when T_R is of order the weak scale. Such low reheat temperatures can in fact arise for composite flat directions which act as inflatons [15]. However, for $n = 4$

$$\frac{n_b}{s} \sim 10^{-10}\left(\frac{T_R}{10^6\ \mathrm{GeV}}\right)\left(\frac{10^{-3}M}{\lambda M_p}\right) \tag{32}$$

The parameters in (32) represent ones which satisfy the constraint on ϕ/T to avoid the thermal scatterings discussed above. This is quite a reasonable range for T_R to be consistent with the bounds from thermal gravitino production. The estimate (32) is in contrast the standard scenario [3] in which n_b/s is generally quite large (as discussed in section 4.6). The only directions which carry $B - L$ and can be lifted at $n = 4$ in the standard model are the LH_u directions. The nonrenormalizable operator is then

$$W = \frac{\lambda}{M}(LH_u)^2 \tag{33}$$

This operator may be present directly at the Planck scale, or could be generated, as in $SO(10)$ GUTs, by integrating out heavy standard model singlets, N, with couplings gLH_uN, and Dirac masses M_NNN, giving $\lambda/M = g^2/m_N$, where g is a Yukawa coupling which could be less than $\mathcal{O}(1)$ [22]. At low energies this is the operator which gives rise to neutrino masses. For baryogenesis along the LH_u direction in this scenario, n_b/s can therefore be related to the *lightest* neutrino mass since the field moves out furthest along the eigenvector of L_iL_j corresponding to the smallest eigenvalue of the neutrino mass matrix.

$$\frac{n_b}{s} \sim 10^{-10} \left(\frac{T_R}{10^6 \text{ GeV}}\right) \left(\frac{10^{-8} \text{ eV}}{m_\nu}\right) \tag{34}$$

Note that n_b/s is inversely proportional to the neutrino mass. A *smaller* nonrenormalizable term leads to a *larger* baryon number [23]. Assuming $T_R < 10^9$ GeV in order to satisfy the gravitino bound [12], then requires that at least one neutrino be lighter than roughly 10^{-5} eV. Including the considerations about thermal scatterings with the condensate at $H \sim m_{3/2}$ discussed above, would reduce the allowed T_R to roughly 10^6 Gev, and therefore the upper limit on the lightest neutrino to 10^{-8} eV.

It is intriguing that the LH_u direction is so successful. There are a few things worth noting about this particular direction. We haven't addressed at all the likelihood that any particular flat direction is favored. LH_u is special in that it contains a Higgs field. So for one of the LH_u nonzero all the directions listed in the appendix are lifted at the renormalizable level except $LL\bar{e}$ and $LL\bar{d}\bar{d}\bar{d}$. Including nonrenormalizable operators, it is not hard to see that these remaining directions will not have amplitudes larger than that along LH_u. So the estimate for n_b/s is robust for this direction.

The H_u content is also special for another reason. Even with minimal Kahler terms at the high scale, the induced mass squared can become negative after including quantum corrections (this is the origin of electroweak symmetry breaking in the radiative breaking scenario). The leading log correction just comes from renormalization group evolution from the high scale down to the particle mass which is being integrated out. For a flat direction the running comes from integrating out particles which couple through renormalizable couplings. So this amounts to running down to the scale $Q \simeq \phi$, since the modes coupled to the flat direction have a mass $m_\perp = g\phi$. For the LH_u direction the H_u component receives the largest modification because

of the large top quark Yukawa coupling. With minimal particle content the one loop beta function for $m_{H_u}^2$ is

$$\frac{d}{dq}m_{H_u}^2 \simeq 3g_2^2 m_2^2 + g_1^2 m_1^2 - 3\lambda_t^2 \left(m_t^2 + m_{\bar{t}}^2 + m_{H_u}^2 + A_t^2\right) \tag{35}$$

where $q = -(4\pi)^{-2}\ln(Q^2)$, m_1 and m_2 are the hypercharge and weak gaugino masses, and here $A_t \equiv Am_{3/2} + aH$. In principle all the mass parameters in (35) could be of order H at early times. The top Yukawa gives a negative contribution to the mass squared. Now the physical mass squared for the flat direction is $m_{LH_u}^2 = m_{H_u}^2 + m_L^2 + |\mu|^2$, where the last contribution comes from the effective $\mu H_u H_d$ term in the superpotential. For $H > m_{3/2}$ it is possible that the μ term induced by the finite density is much smaller than H. As discussed in section 3, after inflation $I \ll M_p$. The only source for an induced $\mu \sim H$ term in this era (ignoring any superpotential couplings) is therefore

$$\frac{1}{M_p}\int d^4\theta \; I H_u H_d \tag{36}$$

If the inflaton transforms nontrivially under some symmetry then such linear terms don't arise, and $\mu \ll H$ (likewise, A terms and gaugino masses are not induced with scales or order H after inflation in the absence of terms linear in the inflaton). If in fact $\mu \ll H$ after inflation, then it is possible that $m_{LH_u}^2$ turns negative at a high scale during and/or after inflation, allowing large a large expectation to develop. This is especially true in GUT models where the larger representations can give even larger negative contributions to the beta function between the Planck and GUT scales. When $H \sim m_{3/2}$, $m_{LH_u}^2$ can become positive from the positive $\mu^2 \sim m_{3/2}^2$ contribution. Even though $m_{H_u}^2 + \mu^2$ must turn negative from running to the weak scale in order to drive electroweak symmetry breaking in the present universe, $m_{LH_u}^2$ can remain positive at late times because of the m_L^2 contribution. Whether this scenario for inducing a negative mass squared actually is realized depends crucially on the beta function and therefore particle content at very high scales, and on the nature and couplings of the inflaton. This scenario does in fact work for certain values of standard model parameters.[1] However, it might be that with minimal kinetic terms, the LH_u direction is the most likely of all the directions which carry $B - L \neq 0$ to have a negative mass squared at early times.

[1] We thank Diego Castano for verifying this scenario numerically.

4.5 Evolution for Positive Mass Squared

It is possible for the induced mass squared along a flat direction to be either positive or negative. As mentioned at the beginning of section 4, baryogenesis along a flat direction with $m^2 \ll H^2$ but $m^2 > 0$ may be possible if the induced mass term is small. If $m^2 \ll H^2$, large deSitter fluctuations can result during inflation. The correlation length for these fluctuations is $l \simeq e^{3H^2/2m^2}$ [19]. The magnitude and phase of the field is correlated over this scale. If the baryon asymmetry is to be constant over the current horizon scale, $(m/H)^2 < \frac{1}{40}$. Ignoring any higher order terms in the potential, the fluctuations reach an equilibrium distribution after $N > (H/m)^2$ e-foldings with $\langle \delta\phi^2 \rangle \simeq 3H^4/8\pi^2 m^2$ [19]. Including higher order terms, the distribution of fluctuations saturates at $V(\phi) \sim H^4$. However, large correlation lengths only arise in regions where ϕ is highly overdamped, i.e. $V''(\phi) \ll H^2$. These deSitter fluctuations have been suggested as a possible mechanism to obtain large "initial" field values for baryogenesis [24]. It is important to note though, that the induced mass squared must be tuned to be small over the entire range of the inflaton during inflation.

The induced mass must also be tuned to be numerically small after inflation. In a quadratic potential, the field evolves in the inflaton matter dominated era following inflation with $m^2 = c'/t^2$, as

$$\phi(t) \simeq \begin{cases} \phi_i t^{-\frac{1}{2}} \cos\left(\frac{1}{2}\sqrt{4c'-1}\ln(t/t_i)\right) & c' \geq \frac{1}{4} \\ \phi_i t^{-c'} & c' \ll 1 \end{cases} \tag{37}$$

where $\phi(t_i) = \phi_i$ If the induced mass is not also small after inflation the envelope of the field decreases rapidly as a power law to small values. For $c' \geq \frac{1}{4}$, the envelope of the field scales as $|\phi| \sim |\phi_i|\sqrt{H/H_{inf}}$. However, if $c' \ll 1$ after inflation, ignoring for the moment higher order terms in the potential, the field is highly overdamped in regions where the correlation length is large, $V(\phi)'' \ll H^2$, and therefore roughly consant after inflation. Eventually, as the induced mass contribution in the potential becomes small as H decreases, the higher order terms in the potential become more important. Once the higher order terms become important, $V(\phi)'' \sim H^2$, and the field becomes near critically damped and begins to move. This evolution can be analyzed with rescalings similar to those given in section 4.2. For $c' \ll 1$ there is again

an attracting point at

$$\bar{\phi}(t) \simeq \left(\frac{\gamma M^{n-3}}{\lambda t} \right)^{\frac{1}{n-2}} \tag{38}$$

where $\gamma = \sqrt{(n-3)/(n-1)}/(n-2)$. As the Hubble constant decreases the ϕ field therefore oscillates about this value, which is very similar to the attracting point in the negative mass squared scenario. When the field begins to oscillate freely at $H \sim m_{3/2}$ all the terms in the potential have roughly the same magnitude, and near maximal fractional baryon number can result. So the entirely (small) positive mass squared scenario is parametrically the same as the negative mass squared case, and gives a similar result for the baryon asymmetry.

4.6 Evolution for W=0

As discussed in section 2, it is possible that the superpotential vanishes along a flat direction. This can be enforced by a discrete R symmetry. This case may be thought of as the $n \to \infty$ or $\lambda \to 0$ limit of the preceding discussions. For exactly flat directions the potential arises solely from Kahler potential couplings, and is of the form (6) and (7). The typical scale for variations in the potential is therefore M_p. If $\phi = 0$ is unstable during and after inflation, a minimum can occur for $\phi \sim M_p$ at early times. When $H < m_{3/2}$ the minimum at large ϕ from the induced soft potential must disappear. There must be a minimum in the soft potential at $\phi = 0$ from hidden sector supersymmetry breaking since for MSSM fields $\phi \ll M_p$ in the present universe. The initial value for the field when $H \sim m_{3/2}$ is therefore of order M_p. The B or L violating soft potential (7) (arising from the Kahler potential couplings or hidden sector supersymmetry breaking) is then roughly equal in magnitude to the conserving potential when the field begins to oscillate freely. A large fractional baryon number is stored in the condensate as in the previous scenario, and $n_b/n_\phi \sim \mathcal{O}(1)$. Now however, for $\phi \sim M_p$, the energy density stored in the Affleck-Dine condensate is of order the energy density of the universe. So it is no longer true that ϕ represents a small fraction of the energy. This is the situation originally considered by Affleck and Dine [3].

Once the inflaton decays, there is a thermal background which can in principle scatter off the condensate as in the previous scenario. However, before inflaton decay the AD and inflaton fields have roughly equal energy

density. The value of the AD field at the era of inflaton decay is therefore $\phi \sim (\sqrt{g_*} T_R / m_{3/2}) T_R \gg T_R$. So any fields which have renormalizable couplings to ϕ (and therefore gain a mass of $m_\perp = g\phi$) are too heavy to be excited by the thermal plasma. As a result, the condensate remains after inflaton decay, and comes to dominates the energy density since the plasma energy density redshifts away. Once $m_\perp < m_\phi$, decays through renormalizable couplings become kinematically accessible. For $g > 10^{-5}$ the condensate decays essentially as soon as such decays are allowed, with an effective reheat temperature $T_{R,\phi} \sim m_\phi / \sqrt{g}$ [14]. Since the condensate decay is itself the dominant source of entropy, n_b/s is much different than in the previous scenario. For $T_{R,\phi} \gg m_\phi$ the decays give a relatively large number of low energy particles. In order to thermalize, these particles would have to gain energy through multibody scatterings to a smaller number of higher energy particles. However, baryon number conservation prevents this, thereby limiting the actual reheat temperature to $T_R \sim m_\phi$. Once the decay products do thermalize, the plasma can carry at most roughly one unit of baryon number per degree of freedom, giving $n_b/s \sim \mathcal{O}(1)$ [25]. In this large entropy case, the production of such a degenerate plasma can also be couched in terms of a chemical potential for baryon number [26]. For an acceptable baryon to entropy ratio to result the B violating parameters of the soft potential must be suppressed, or there must be an additional source of entropy at or below the electroweak phase transition.

As given in the appendix, the directions $QQQL$ and $\bar{u}\bar{u}\bar{d}\bar{e}$ carry $B-L=0$ but have nonzero $B+L$. If baryogenesis takes place along one of these directions with T_R greater than T_c, the temperature of the electroweak phase transition, anomalous sphaleron processes destroy the generated $B+L$ [8]. It has been suggested that the condensate can survive to a temperature below T_c, thereby suppressing the sphalerons if $\phi \sim T_c$ at T_c, and allowing baryogenesis along directions which carry $B-L=0$ [26]. However, as discussed above, the condensate decays at or above this scale. So it may be marginally possible for baryogenesis to take place along $B-L=0$ directions in the $W=0$ scenario. Baryogenesis can certainly take place along exact flat directions with $B-L \neq 0$ [3]. The main drawback here is the requirement for additional entropy releases below the electroweak phase transition.

5 Cosmological Evolution of String Moduli

String moduli are exactly flat (perturbatively) in the supersymmetric limit, and couple to standard model fields only through Planck scale suppressed interactions. The coherent production of string moduli leads to the string version [1, 14] of the Polonyi problem [2]. Such a condensate decays at a very low temperature, $T \sim 5$ keV, and leads to a number of cosmological problems. These include modification of the light element abundances [2], the requirement for baryogenesis at such a low temperature, and overproduction of LSPs [14]. In discussions of the cosmological evolution of string moduli, it is usually assumed that $V'' \sim m_{3/2}^2$ at very early times. If this were the case, the moduli would effectively be frozen for $H \gg m_{3/2}$, and begin to oscillate when $H \sim m_{3/2}$. If the initial displacement were $\mathcal{O}(M_p)$ the moduli dominate the energy density essentially as soon as oscillations begin, leading to the cosmological disasters mentioned above.

We have seen, however, that at early times because of the finite density induced soft potential, $V'' \sim H^2$, so that the fields are parameterically close to critically damped. During inflation the fields are therefore driven to a local minimum within a few e-foldings (unless the induced mass happens to be numerically much less than H). The induced potential of course remains after inflation. However, in general the minima of the induced potential do not necessarily coincide with the minima of the low energy potential. In fact since the scale for variations in the soft potential is $\mathcal{O}(M_p)$, one expects the minima to differ by this amount. Once $H \sim M_p$, the moduli start to oscillate freely about a minimum of the low energy potential with initial amplitudes of $\mathcal{O}(M_p)$.

As an example of the moduli evolution consider the following toy model

$$V = (m_{3/2}^2 + a^2 H^2)|\mathcal{M}|^2 + \frac{1}{2M_p^2}(m_{3/2}^2 + b^2 H^2)|\mathcal{M}|^4. \tag{39}$$

For $H \gg m_{3/2}$, the minimum lies at $(a/b)M_p$. For $H \ll m_{3/2}$, the minimum lies at M_p. For suitable a and b, the system sits near the first minimum until $H \sim m_{3/2}$. At this point, the field begins to oscillate about the second minimum, with "initial" amplitude of $\mathcal{M} \sim (1 - a/b)M_p$. This is, of course, just a statement of the original Polonyi problem. Our observation that the curvature of the potential is of order the Hubble constant at early times,

rather than ameliorating the problem, just gives a concrete realization of the initial conditions.

The present discussion suggests a solution of the moduli problem [4]. If the minima coincide at early and late times the moduli are driven to a minimum during inflation (up to quantum deSitter fluctuations). This is technically natural if there is a point of enhanced symmetry on moduli space. Very roughly, the moduli transform under some symmetry near such points. The lowest order invariants are therefore bilinears, and the potential (no matter what the source) is necessarily an extremum at such points. So it is possible that the potential is a minimum at a symmetry point at both early and late times. More precisely the moduli are composite fields near points of enhanced symmetry. At the symmetry point, the fields making up the moduli become massless (ignoring any nonperturbative effects). In string theory, there often exist points of enhanced gauge symmetry on moduli space. The moduli act as Higgs fields near the symmetry points. The most famous example of this is self dual point of $R \to \frac{2}{R}$ duality in toroidal compactification. At such points the Kaluza Klein $U(1)$ for each S^1 gets enlarged to $SU(2)$. Analogous points seem to be a generic feature of many compactifications.

As an example of enhanced symmetry, consider first moduli other than the dilaton. For these, it is possible in many instances to find points where all the moduli transform under a discrete symmetry. An example is provided by the Z_3 orbifold [27]. This orbifold can be constructed as a product of three two-dimensional tori, each exhibiting a Z_3 symmetry. (One of these Z_3's is modded out; the other two survive). The enhanced symmetry at this point is $SU(3) \times Z_3 \times Z_3$. All of the moduli in the twisted sectors are charged under $SU(3)$. Of the untwisted moduli, all but 3 transform under the Z_3's; these correspond to breathing modes for the three tori. However, for particular values of the radii and of the antisymmetric tensor fields (torsion), there are further enhanced symmetries. In particular, one can go to what would be the $SU(3)$ points of conventional 2-d toroidal compactifications, for each of the three tori. After modding out by the Z_3, six $U(1)$'s remain, under which all of the remaining moduli transform.

Clearly the Z_3 orbifold does not describe the real world. On the other hand, this example illustrates the possibility that all of the moduli (except the dilaton) can transform under some enhanced symmetry. For this scenario to be realized in a realistic example our vacuum must be at or near a point of enhanced symmetry. Any enhanced continuous gauge symmetries must

either be identified with part or all of the standard model gauge group or be spontaneously broken. In the latter case there would be extra light gauge bosons at the weak scale. Upon supersymmetry breaking, it is perfectly possible that some of the continuous gauge symmetries are broken by $\mathcal{O}(m_{3/2})$ vev's.

The main problem with this idea is the dilaton. One might hope that S duality could be realized in the effective potential. However, at the dilaton self dual point the four dimensional gauge coupling is likely to be very large. So if symmetries are the solution of the moduli problem, the dilaton is probably on different footing than the other moduli. The dilaton mass might arise from dynamics not directly associated with supersymmetry breaking [1, 14, 16]. In no-scale type theories it may also be possible in some circumstances to avoid the dilaton problem if the dilaton dominates the energy density at very early times [28]. Alternately a period of late inflation can in principle dilute the dilaton [14, 29].

6 Conclusions

Exact and approximate flat directions are a generic feature of supersymmetric theories. If low energy supersymmetry has anything to do with nature, these flat directions are likely to play an important role in cosmology. The coherent production of scalar fields along flat directions emerges as a generic feature of supersymmetric theories. In this paper, we have explored certain aspects of the cosmology of flat directions. Perhaps our most important, albeit quite simple, observation is that the scale for the induced soft potential at early times is of order H. This has dramatic consequences for the AD mechanism of baryogenesis, and the evolution of string moduli.

The AD mechanism of baryogenesis is not generally obtained with a minimal Kahler potential for the standard model fields. In this case the induced potential has a minimum at the origin, and the fields are driven to small values during inflation. With nonminimal Kahler couplings the origin can be unstable and large expectation values along flat directions can result. Such nonminimal couplings can in fact be generated radiatively in the presence of Yukawa couplings. For directions lifted by nonrenormalizable terms in the superpotential, the fields begin to oscillate freely in the low energy potential at $H \sim m_{3/2}$ with an "initial" condition which is determined by a balance

between the induced soft mass term with and nonrenormalizable term. This guarantees that the B conserving and violating terms in the potential are the same order. The resulting baryon number per condensate particle is near maximal. However, since the "initial" field value is parameterically less than M_p, the baryon to entropy ratio can be quite small. Within this scenario the mechanism is quite robust. For the LH_u direction an acceptable baryon number results with a reasonable value for the reheat temperature after inflation. In this case the baryon asymmetry is related to the lightest neutrino mass.

For the moduli problem, the induced potential generally gives a concrete realization of the initial conditions, which are usually just assumed in an ad hoc way. However, it suggests a solution if there is an enhanced symmetry point on moduli space. The minimum of the potential can then in principle coincide at early and late times. However, it is not clear how the dilaton can fit into such a picture. If symmetries are the solution of the moduli problem, our vacuum is quite close to an enhanced symmetry point, which might have interesting phenomenological consequences.

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8 Appendix

In the global limit the scalar potential is a sum of gauge and superpotential contributions, $V = \frac{1}{2}g^2 D^a D^a + F_\varphi^* F_\varphi$, where $D^a = \varphi^* T^a \varphi$, and $F_\varphi^* = \partial W/\partial\varphi$. There are many flat directions in the standard model field space on which the potential vanishes with respect to the renormalizable superpotential. D flat directions can be parameterized through gauge invariant operators. In order to form such invariants it is useful to first construct potentially D, and F flat combinations of fields. A set of such operators which are F flat with

respect to the standard model Yukawa couplings

$$W = \lambda_u Q H_u \bar{u} + \lambda_d Q H_d \bar{d} + \lambda_e L H_d \bar{e}$$

are given in table 1. Throughout, unbarred fields are $SU(2)_L$ doublets while barred fields are $SU(2)_L$ singlets, and generation indices are suppressed. The Higgs fields appear in only a limited number of flat directions. The field H_d does not appear in table 1 contracted with Q or L since the Yukawa couplings give nonzero $F^*_{\bar{d}}$ and $F^*_{\bar{e}}$ respectively in this case. Likewise H_u does not appear contracted with Q as $F^*_{\bar{u}}$ would be nonzero. It can appear contracted with H_d or L though. The only flat directions involving Higgs fields are therefore $H_u H_d$ and $H_u L$. The superpotential μ term,

$$W = \mu H_u H_d$$

generates a nonzero F component if either Higgs is nonzero, $F^*_{H_u} = \mu H_d$ and $F^*_{H_d} = \mu H_u$. However μ can not be much larger than the weak scale. So this contribution to the potential is the same order as that from the zero density soft supersymmetry breaking masses. Directions which are lifted only by the μ term in the supersymmetric limit are therefore included in the list of renormalizable "flat" directions.

Flat directions can be constructed by tensoring together products of the combinations of fields that appear in table 1. A complete list of standard model flat directions made out of up to 7 fields is given in table 2. Gauge indices are contracted in an obvious manner. In general many possible directions exist for each invariant by permuting flavor indices. For example, the invariant $LL\bar{e}$ contains the independent F flat invariants $L_1 L_2 \bar{e}_3$, $L_1 L_3 \bar{e}_2$, and $L_2 L_3 \bar{e}_1$. For each invariant there is a single Goldstone boson for the spontaneously broken $U(1)$ global quantum number carried by the invariant, and its supersymmetric partner. The scalar components of the other directions in the invariant are Goldstone bosons for the spontaneously broken flavor symmetries.

Fields can take values along multiple directions simultaneously, and remain F flat, although this is not guaranteed. For example, the directions $Q_1 L_1 \bar{d}_2$, $\bar{u}_2 \bar{d}_2 \bar{d}_3$, and $L_1 L_2 \bar{e}_3$ can be nonzero and preserve the D and F flat conditions. Typically most directions not related by flavor are lifted when fields take on nonzero value along some direction. For example, when LH_u is nonzero, all the directions in table 2 are lifted except $LL\bar{e}$ and $LL\bar{d}\bar{d}\bar{d}$.

All the flat directions in table 2 made of given number of fields have the same $B-L$, with the exception of H_uH_d and $LL\bar{d}\bar{d}\bar{d}$. This may seem surprising, but follows from the following simple combinatorics. As listed in table 1 the number of fields minus $B-L$ equals 4 for units which do not involve Higgs fields, and is zero for $\bar{e}$. This has the effect that all directions made out of a given number of F flat combinations from table 1, any number of $\bar{e}$ fields, and no Higgs fields, have the same $B-L$. All the directions listed in table 2 without Higgs fields are in fact constructed in this way, with the exception of $LL\bar{d}\bar{d}\bar{d}$. This direction is made of two operators from table 1 while the other directions with 5 fields are made from one operator and some number of $\bar{e}$ fields.

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[9] The only exception is a Yukawa coupling of two fields in the flat direction directly to a field in the inflaton sector with small expectation value but large F component. The F component would then induce a soft mass for the flat direction of $m \sim g\sqrt{H M_p}$.

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[13] Nonzero D terms can arise during inflation in the presence of a linear Fayet-Iliopoulos D term. For example, consider an $SU(2) \times U(1)$ model with gauge couplings g and e, and field content $\varphi = (2, +1)$ and $\bar{\varphi} = (2, -1)$. With a linear D term for the $U(1)$, $\xi \int d^4\theta V$, there is flat direction $\varphi^*\varphi - \bar{\varphi}^*\bar{\varphi} = -\frac{1}{2}(e^2/(e^2+g^2))\xi$, along which the gauge potential does not vanish, $V_D = \frac{1}{8}(e^2g^2/(e^2+g^2))\xi^2$. With a nonminimal Kahler potential, inflation can take place along this direction.

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[18] M. K. Gaillard, H. Murayama, and K. Olive, preprint UMN-TH-1334-95, hep-ph 9504307.

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[20] If no potential for the phase of ϕ exists during inflation the baryons which are eventually produced have a (roughly) scale invariant spectrum

of density fluctuations, $\delta n_b/n_b \sim H_I/|\phi_0|$, where $|\phi_0|$ is the minimum during inflation. These fluctuations can not be too large in order that structure does not form too early after matter domination, i.e. $\delta\rho_b/\rho_{\rm tot} < 10^{-5}$. This gives a bound of $\phi_0/H_I < 10^4$ during inflation.

[21] E. Kolb and M. Turner, The Early Universe (Addison Wesley, New York, 1990).

[22] If this operator is generated by integrating out a heavy singlet with $m_N < H_I$, the singlet can be excited during inflation with amplitude $\langle \delta N^2 \rangle \sim H_I^2$. This gives an effective mass squared to the LH_u direction of $m^2 \sim g^2 H_I^2$. But to obtain an acceptable baryon number in this case $g \ll 1$. So the negative mass squared scenario can still be realized even if the singlet is excited during inflation.

[23] Of course for very small m_ν the baryon number must vanish as $m_\nu \to 0$. For $m_\nu < 10^{-20}$ eV the $\frac{\lambda}{M}(LH_u)^2$ term in the superpotential is less important in determining the value of the LH_u direction when $H \sim m_{3/2}$ than higher order terms in the soft potential. For m_ν less than this value the baryon number therefore does decrease (ignoring any $U(1)_L$ violation in the Kahler potential).

[24] J. Ellis, K. Enqvist, D. Nanopoulos, and K. Olive, Phys. Lett. B **191** (1987) 343; K. Enqvist, K. W. Ng, and K. Olive, Phys. Rev. D **37** (1988) 2111; H. Murayama and T. Yanagida, Phys. Lett. B **322** (1994) 349.

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Figure Captions

Figure 1. Trajectory in ϕ plane for $n\theta_i = \frac{9}{10}\pi$, with $m_\phi = c' = -a = 1$, and $n = 4$.

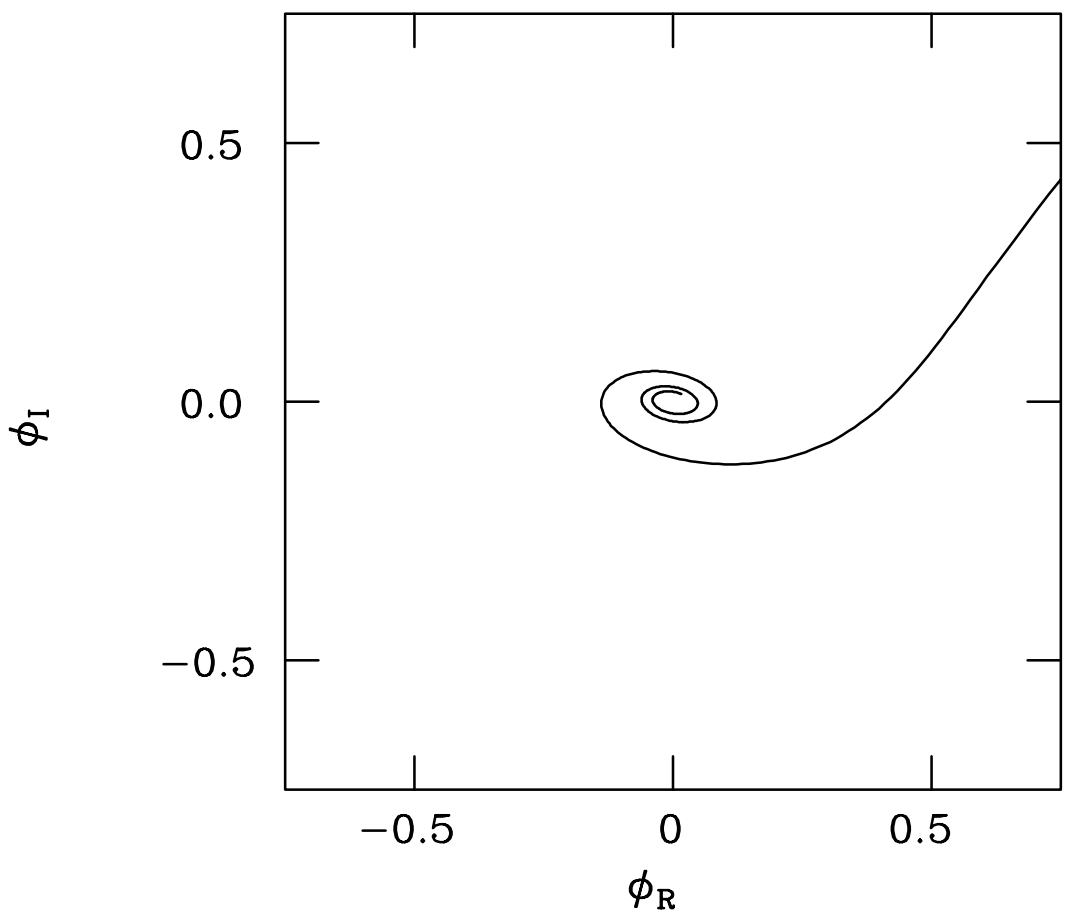
Figure 2. Fractional baryon number carried by condensate as a function of $n\theta_i$ for $m_\phi = c' = -a = 1$, and $n = 4$.

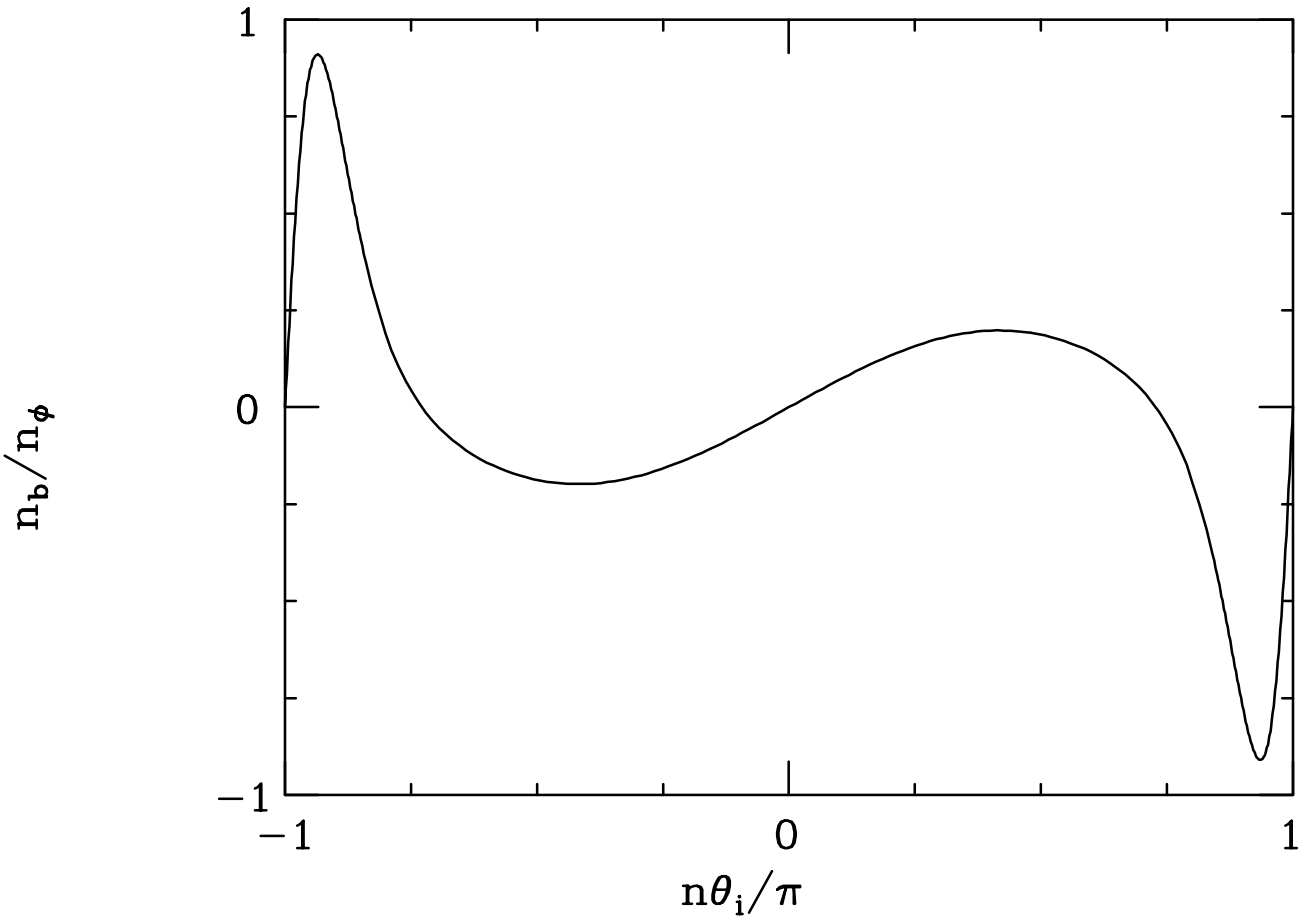
	Y	$B-L$	$N-(B-L)$
$\bar{e}$	2	1	0
LL	-2	-2	4
$H_u H_d$	0	0	2
LH_u	0	-1	3
$\bar{u}\bar{u}\bar{u}$	-4	-1	4
$\bar{u}\bar{u}\bar{d}$	-2	-1	4
$\bar{u}\bar{d}\bar{d}$	0	-1	4
$\bar{d}\bar{d}\bar{d}$	2	-1	4
$QL\bar{u}$	-2	-1	4
$QL\bar{d}$	0	-1	4
$QQ\bar{u}\bar{u}$	-2	0	4
$QQ\bar{u}\bar{d}$	0	0	4
$QQ\bar{d}\bar{d}$	2	0	4
$QQQL$	0	0	4
$QQQQ\bar{u}$	0	1	4
$QQQQ\bar{d}$	2	1	4
Q^6	2	2	4

Table 1: Combinations of fields for constructing flat directions.

	$B-L$
$H_u H_d$	0
$L H_u$	-1
$\bar{u}\bar{d}\bar{d}$	-1
$QL\bar{d}$	-1
$LL\bar{e}$	-1
$QQ\bar{u}\bar{d}$	0
$QQQL$	0
$QL\bar{u}\bar{e}$	0
$\bar{u}\bar{u}\bar{d}\bar{e}$	0
$QQQQ\bar{u}$	1
$QQ\bar{u}\bar{u}\bar{e}$	1
$LL\bar{d}\bar{d}\bar{d}$	-3
$\bar{u}\bar{u}\bar{u}\bar{e}\bar{e}$	1
$QLQL\bar{d}\bar{d}$	-2
$QQLL\bar{d}\bar{d}$	-2
$\bar{u}\bar{u}\bar{d}\bar{d}\bar{d}\bar{d}$	-2
$QQQQ\bar{d}LL$	-1
$QLQLQL\bar{e}$	-1
$QL\bar{u}QQ\bar{d}\bar{d}$	-1
$\bar{u}\bar{u}\bar{u}\bar{d}\bar{d}\bar{d}\bar{e}$	-1

Table 2: Renormalizable F and D flat directions in the standard model.


0.5
0.0
−0.5
−0.5
0
0.5
ϕ_I
ϕ_R


1
0
−1
−1
0
1
n_b/n_ϕ
$n\theta_i/\pi$