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Observations Involving Broadband Impedance Modelling *

J. Scott Berg
Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309

Abstract

Results for single- and multi-bunch instabilities can be significantly affected by the precise model that is used for the broadband impedance. This paper discusses three aspects of broadband impedance modelling. The first is an observation of the effect that a seemingly minor change in an impedance model has on the single-bunch mode coupling threshold. The second is a successful attempt to construct a model for the high-frequency tails of an r.f. cavity. The last is a discussion of requirements for the mathematical form of an impedance which follow from the general properties of impedances.

1 Introduction

Computing instability thresholds and growth rates requires good models for broadband and narrow-band impedances.

In some cases seemingly minor changes in the broadband impedance model can cause significant changes in instability thresholds, as shown in section 2. Thus, the broadband impedances in the ring should be modelled more carefully. In section 3, this is done for the high-frequency tails of the r.f. cavities in the PEP-II B factory. Section 4 of this paper describes some of the mathematical properties that an impedance is required to satisfy, and some of the resulting constraints that these properties place on the form of a broadband impedance model.

2 Effect of Impedance Cutoff

The various small objects in the PEP-II ring have been modelled to have an impedance that is primarily inductive [1]. The impedance should be cut off so that the transverse impedance goes to zero at infinity. This can be done by writing the transverse inductive impedance as

$$\frac{-iL}{(1-i\omega/\omega_C)^{3/2}}, \tag{1}$$

where L is the low-frequency inductance, and ω_C is a cutoff frequency. The choice of 3/2 in the exponent is based on the assumption that the high-frequency tail of the impedance consists of a large number of resonances that give a roll-off similar to a single cavity [2].

The choice of the cutoff frequency ω_C has a strong effect on where transverse single-bunch mode coupling occurs, as shown in Fig. 1. The characteristic frequency cutoff of the bunch distribution $\beta_0 c/2\pi\sigma_\ell$ for this example is 4.77 GHz. Notice that there is a strong variation in the mode coupling threshold even for values of the ω_C well above this frequency.

This result demonstrates that seemingly minor details of the impedance model can have significant effects on the resulting analysis of coherent instabilities.

An important qualification to this example is that this analysis only included the $m = 0$ and $m = 1$ transverse modes (see [3, 4, 5]). As demonstrated in [6, 7, 8], when one includes impedances that have a high-frequency component, as we do here (the real part of (1) peaks at $\tan(\pi/5) \approx .727$ times the cutoff frequency), higher order modes should be included for a complete picture. As can be seen from the definition of K_k in [3, 4, 5], K_k peaks at $k\beta_0 c/4\pi\sigma_\ell$. Thus, m should be at least high enough so that K_{2k} (the diagonal term for row k in our eigenvalue system) has its peak at higher frequencies than any significant impedances for $k > m$. It is not clear whether m should be twice that value, so that even off-diagonal terms with significant overlap are considered even if their corresponding diagonal terms are negligible.

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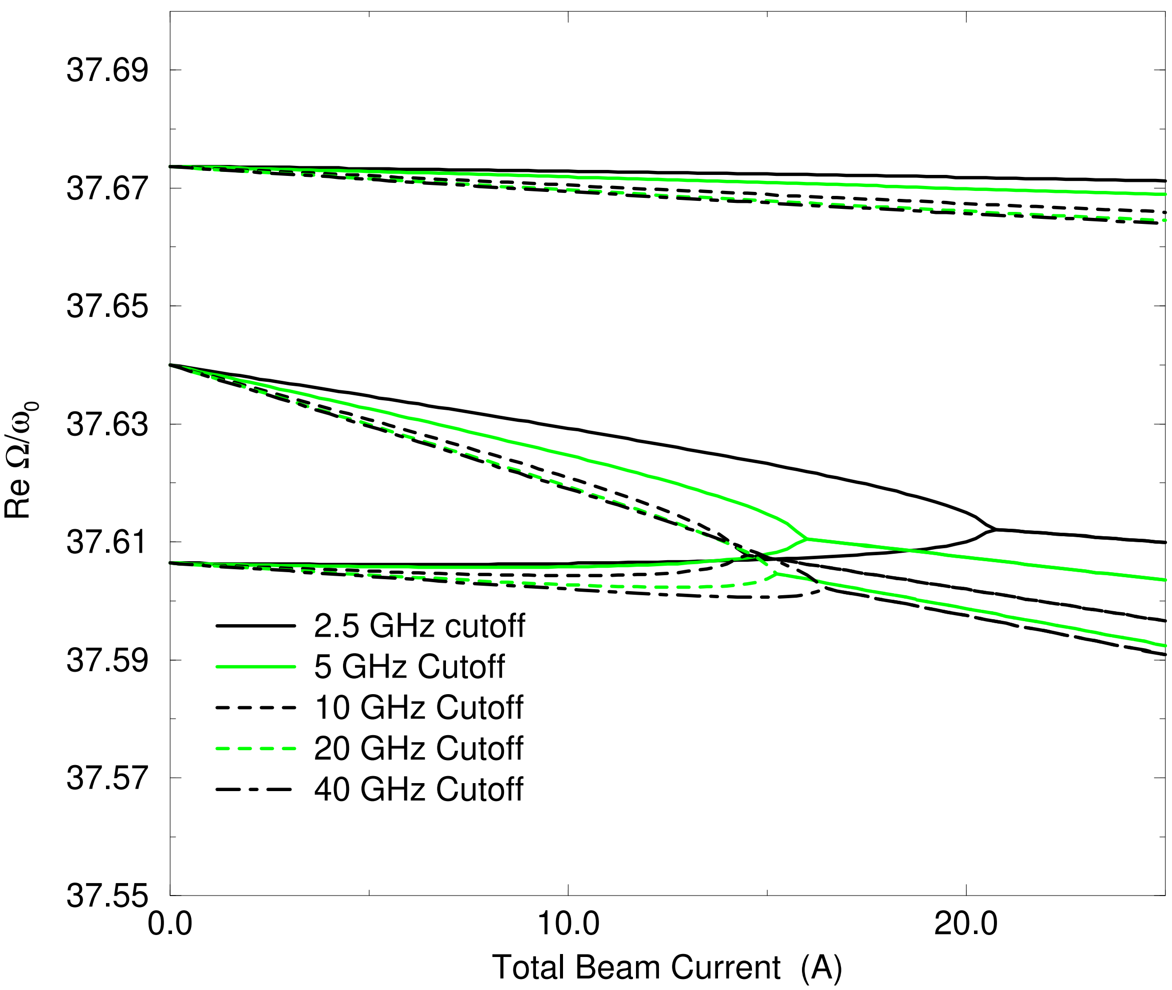

FIGURE 1: Real parts of mode frequencies for a single bunch, plotted versus total beam current. The parameters for the PEP-II B factory are used. The beam becomes unstable at the point where two mode frequencies coincide. Different sets of lines represent different values of inductance cutoff ω_C as described in the text. The impedance is otherwise the model used in [3].

3 Model of High-Frequency Cavity Tails

Section 2 suggests that a good model for the high-frequency impedance behavior is important. An important part of that behavior can come from the high-frequency impedance due to r.f. cavities.

The impedance of the r.f. cavities in PEP-II has been modelled to consist of the known higher-order modes plus a high-frequency tail. The real longitudinal wake of any device must satisfy the following physical considerations [9]:

- It must be real
- It must be causal
- It must be energy-conserving
- It must have no effect on a DC beam.

Since the total wake for the cavity satisfies these properties, then if the higher-order modes used are the real resonant modes of the cavity, it is expected that the remaining wake should also satisfy these properties.

The longitudinal impedance due to a single isolated cavity rolls off as $\omega^{-1/2}$ at high frequencies [10]. Nearly the simplest possible impedance that satisfies the above properties and rolls off as $\omega^{-1/2}$ is

$$iA\left[\left(1+\frac{\omega}{\omega_0+i\alpha}\right)^{-1/2}-\left(1-\frac{\omega}{\omega_0-i\alpha}\right)^{-1/2}\right]. \tag{2}$$

This model is "simple" in the sense that it consists of only a pair of symmetrically placed branch cuts in the lower half plane. The only simpler model would be one where there was a single branch cut along the negative imaginary axis. It turns out that such a model does not have enough freedom to describe the cavity tails.

The model (2) will be used to describe the cavity tails; the parameters A, ω_0, and α will be found based on the actual cavity properties. Other models have been proposed for an impedance with an $\omega^{-1/2}$ roll-off. See [11, 12].

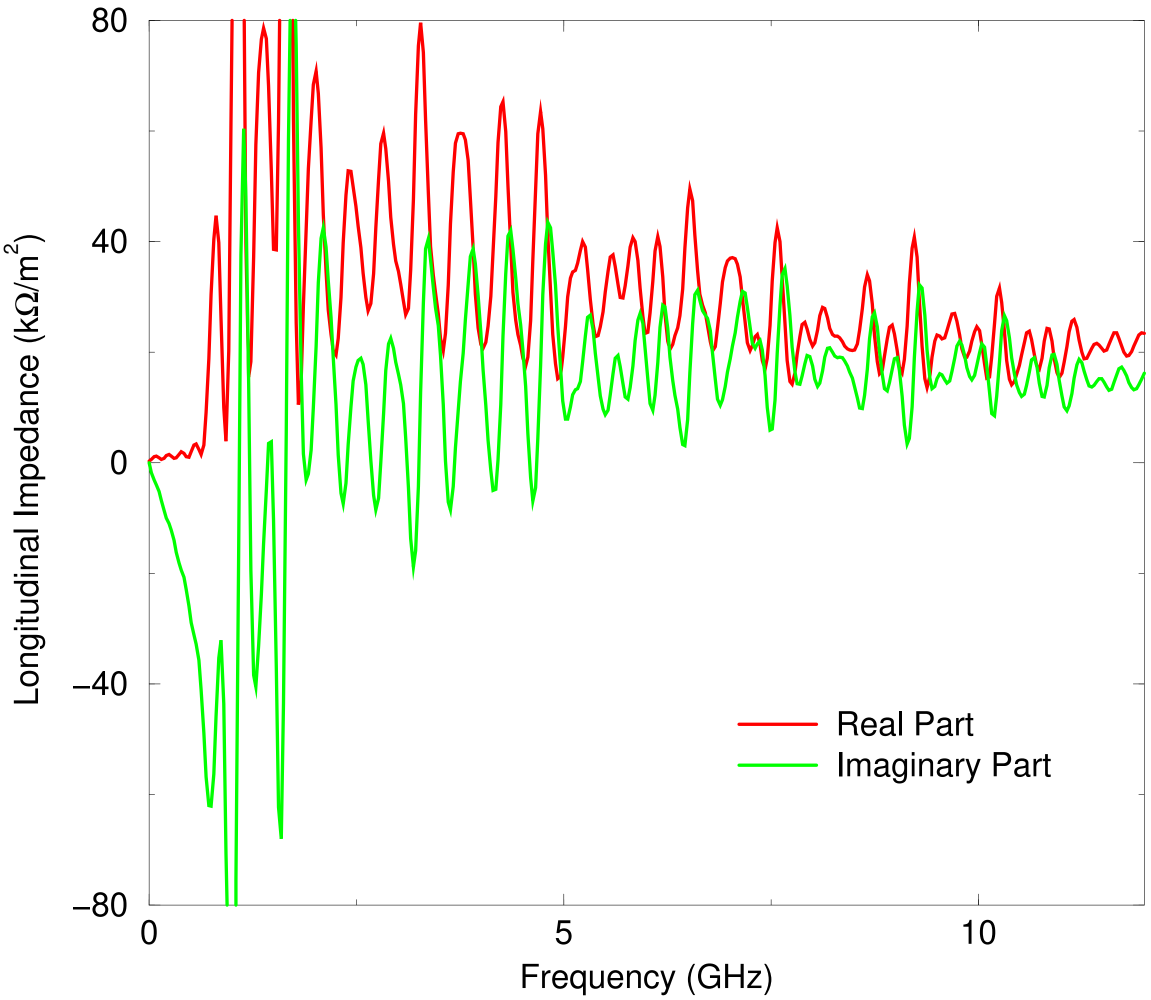

FIGURE 2: Impedance of a PEP-II r.f. cavity model found with ABCI.

The total wake of the cavity is obtained by running a model of the PEP-II r.f. cavity through the program ABCI for $m = 1$ (to get the transverse wake) [1, 13]. The resulting impedance is shown in Fig. 2. The narrow-band impedances from this rough model of the cavity are expected to be similar, but not quite the same, as the known resonant modes of the cavity. Because of this discrepancy in the resonant modes (and the fact that the integration time is relatively short compared to the resonant mode decay times), the impedance of the known cavity modes cannot simply be subtracted from the total cavity impedance. Instead, the wake due to the known cavity modes is subtracted from the total wake given by ABCI, and the result is Fourier transformed to obtain the impedance. This impedance is shown in Fig. 3.

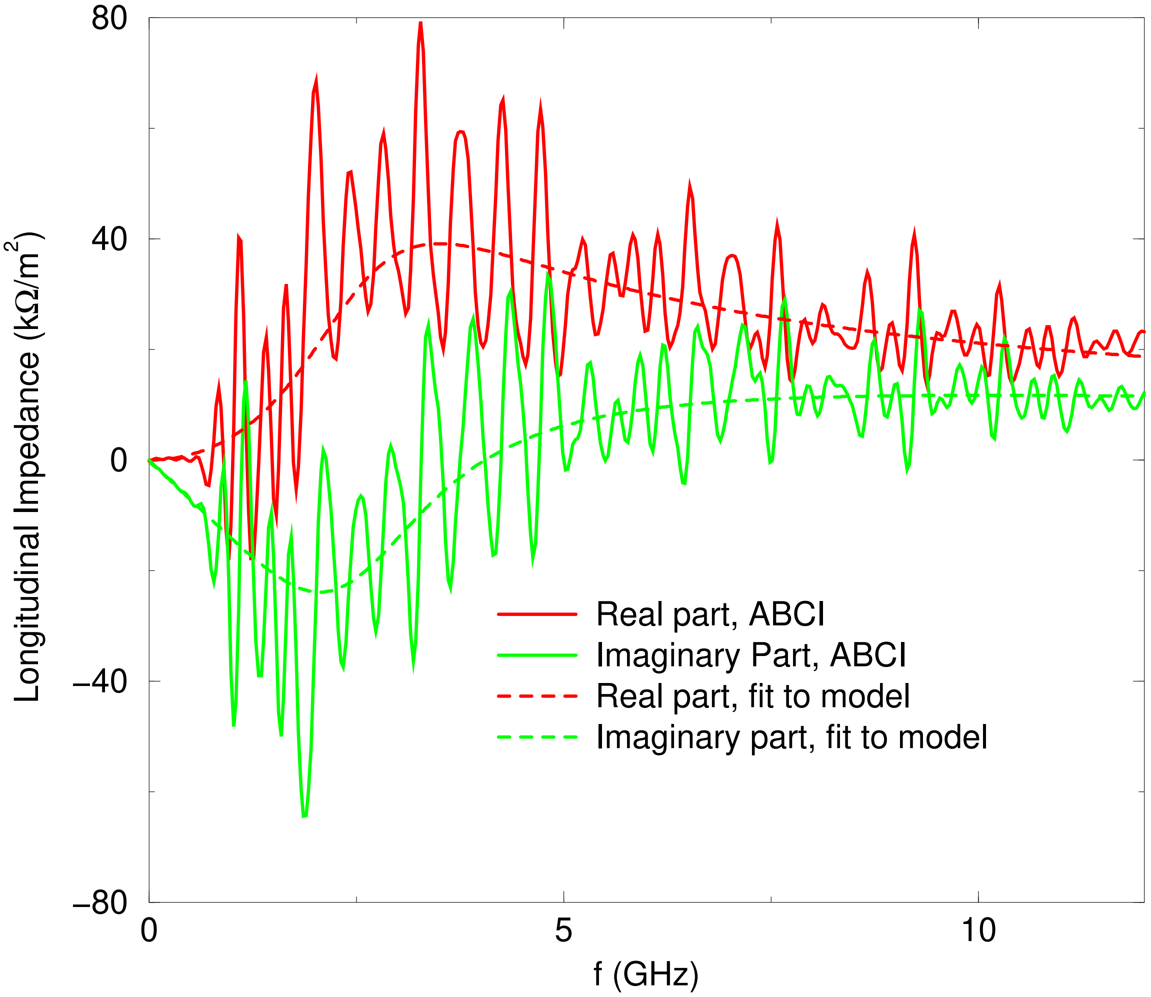

FIGURE 3: Impedance of the PEP-II r.f. cavity with higher-order modes subtracted. Also shown is the model [Eq. (2)] that was fit to this impedance. The parameters of the model are $A = 45.1344$ kΩ/m^2, $\alpha = 1.34722$ GHz, and $\omega_0 = 2.4$ GHz.

The parameters of the model (2) are fit by matching the low-frequency limit of the model to the low frequency limit of the imaginary part of the impedance, and matching the high-frequency limit of the model to the average of the real and imaginary parts of the impedance. The value of ω_0 is fixed for this fit. An initial guess of $\omega_0 = 2$ GHz is made, since this frequency is just above the last of the known cavity modes. The value of ω_0 is adjusted, while maintaining the matching of the low- and high-frequency limits, to give the best fit (as judged by eye). The resulting ω_0 turned out to be 2.4 GHz. A comparison of the fit model and actual impedance (after removal of known higher-order modes) is shown in Fig. 3. The agreement can be seen to be very good.

To obtain the good agreement that was found here, all three parameters in the model were necessary. This is an advantage of this model over the models used in [11, 12], which have fewer degrees of freedom. Note that the cavity tails give a small but non-negligible contribution to the inductance.

This analysis could be improved in the following ways: the value of the parameters could be found by a nonlinear fit instead of fitting one parameter by eye; the wakes could be computed with ABCI for longer times; a better way of subtracting the higher-order modes could be found; and better windowing could be done on the data (to smooth out the oscillations).

4 General Properties of Impedances

As stated earlier, there are four properties that longitudinal impedances must satisfy [9]:

- The impedance must correspond to a real wakefield. Thus, $[Z(-\omega^*)]^* = Z(\omega)$.
- The impedance must correspond to a causal wake. Thus, it must be analytic in the upper half plane (i.e., no poles or branch cuts).
- The impedance must be energy-conserving. Thus, the real part of $Z(\omega)$ must be non-negative for real ω.
- A DC beam should be unaffected by wakefields. Thus, $Z(\omega)$ must be zero at $\omega = 0$.

This section discusses the constraints that these facts, in particular energy conservation, put on the impedance models that one can construct.

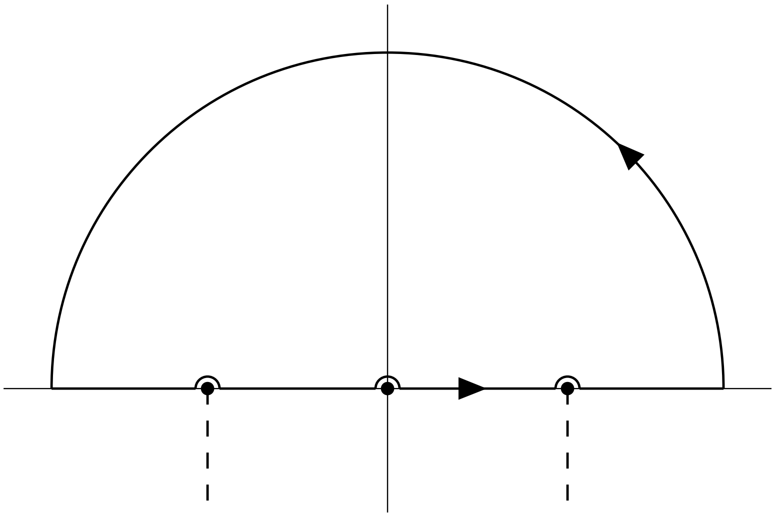

FIGURE 4: Integration contour for $Z'(\omega)/Z(\omega)$. Dashed lines represent branch cuts; the single dot at the origin is a pole of $Z'(\omega)/Z(\omega)$.

Assume that one has an impedance that rolls off like $(-i\omega)^\nu$ as $\omega \longrightarrow \infty$ in the complex plane. Integrate $Z'(\omega)/Z(\omega)$ around the contour shown in Fig. 4. The contour should pass over any zeros or poles of Z on the real axis, or any branch cuts that touch rhe real axis, as shown.

The integral around the contour is

$$i\left(\pi\nu - \pi\sum_{k=0}^{N-1}\alpha_k + \Delta\phi\right), \tag{3}$$

where N zeros, poles, or branch cuts intersect the real axis, each of order α_k respectively (e.g., $\alpha_k = -1$ for a simple pole). The change in phase along the segments on the real axis is $\Delta\phi$. Equation (3) is equal to $2\pi i$ times the number of zeros, including multiplicity, in the upper half plane, n_Z. Note that n_Z must be a nonnegative integer, since no poles or branch cuts can be in the upper half plane for Z. Since the real part of $Z(\omega)$ must be positive along the real axis, the absolute value of the change in phase must be less than or equal to π along any given segment between zeros, poles, or branch cuts. Thus, since there are $N+1$ such segments, $-(N+1)\pi \le \Delta\phi \le (N+1)\pi$, resulting in the inequality

$$-(N+1) \le 2n_Z + \sum_{k=0}^{N-1}\alpha_k - \nu \le N+1. \tag{4}$$

As an example, say no branch cuts or poles of the impedance touch the real axis. Then the α_k are integers that are greater than or equal to 1. Thus, the sum of the α_k is an integer greater than or equal to N. Since n_Z is a non-negative integer, equation (4) then constrains $\nu \geq -1$, but places no constraints on the upper limit for ν.

5 Conclusion

This paper has shown how in some cases, knowing the details of the broadband impedance of a machine can be important. Because of this, a model of the high-frequency tails of the PEP-II r.f. cavities was constructed. This model gave a good approximation to the behavior due to those tails for a model of the cavity run through the program ABCI. The general properties of impedances place constraints on the model for the broadband impedance that one can use. These constraints contributed to the decision to use the model of equation (2) for the cavity tail impedance.

Much of this work, especially that in sections 2 and 3 is preliminary and requires more research, as indicated in comments in those sections.

Finally, thanks to Sam Heifets for many helpful discussions.

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