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REVIEW OF WEAK MIXING ANGLE RESULTS AT SLC AND LEP†

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In this paper, we review recent precise measurements of the weak mixing angle by the SLD experiment at SLC and by the ALEPH, DELPHI, L3, and OPAL experiments at LEP. If we assume that the Minimal Standard Model provides a complete description of the quark and lepton couplings to the Z boson, we find $\sin^2\theta_W^{\rm eff} = 0.23143 \pm 0.00028$. If this assumption is relaxed to apply to lepton couplings only, we find $\sin^2\theta_W^{\rm eff} = 0.23106 \pm 0.00035$. We compare these results with other precision electroweak tests.

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1 Introduction

The Minimal Standard Model (MSM) of electroweak interactions contains free parameters for 12 fermion masses, 4 quark mixing parameters, the Higgs mass, and 3 parameters to specify the $SU(2)_L$ X $U(1)$ gauge structure. The gauge structure parameters can be chosen to be the $SU(2)_L$ coupling constant g, the $U(1)$ coupling constant g', and the vacuum expectation value of the Higgs field. It is more common, however, to choose the experimentally well-measured values for the fine structure constant (α), the Fermi constant (G_F), and the Z^0 mass (M_Z). Three precise electroweak measurements are required to specify the gauge structure of the MSM, and a fourth precise measurement is needed to test it. This paper describes precise measurements of the weak mixing angle ($\tan\theta_W = g\prime/g$) for this purpose.

2 Z^0 Couplings and Asymmetry Experiments

At the $Zf\overline{f}$ vertex, the MSM gives the vector and axial vector couplings to be:

$$\begin{aligned} v_f &= I_f^3 - 2Q_f\sin^2\theta_W^{\rm eff} \\ a_f &= I_f^3 \end{aligned}$$

where I_f is the fermion isospin and Q_f is the fermion charge. Radiative corrections are significant and are treated as follows. First, vacuum polarization and vertex corrections are included in the coupling constants, and an effective weak mixing angle is defined to be $\sin^2\theta_W^{\rm eff} \equiv \frac{1}{4}(1 - v_e/a_e)$. Second, experimental measurements need to be corrected for initial state radiation and for $Z-\gamma$ interference to extract the Z-pole contribution.

Table 1: Fermion couplings and asymmetries.

Fermion	v_f	a_f	A_f	$\frac{\partial A_f}{\partial\sin^2\theta_W^{\rm eff}}$
ν	0.50	0.50	1.00	0
e, μ, τ	-0.04	-0.50	0.16	-7.9
u, c, t	0.19	0.50	0.66	-3.5
d, s, b	-0.35	-0.50	0.94	-0.6

One can define a fermion asymmetry parameter, A_f,

$$A_f = \frac{2v_f a_f}{v_f^2 + a_f^2}$$

Table 1 lists the vector and axial vector coupling constants, the fermion asymmetry parameter and its sensitivity to the weak mixing angle, for the different fermion species. (Table 1 assumes $\sin^2\theta_W^{\rm eff} = 0.23$.)

The cross-section for $e^+e^- \rightarrow Z^0 \rightarrow f\overline{f}$ can be expressed by

$$\frac{d\sigma^f}{d\Omega} \propto [v_f^2 + a_f^2]\left\{ \begin{array}{c} (1+\cos^2\theta)(1+PA_e)+ \\ 2\cos\theta A_f(P+A_e) \end{array} \right\} \qquad (1)$$

where θ is the angle of the outgoing fermion with respect to the incident electron, and P is the polarization of the electron beam (the positron beam is assumed to be unpolarized). We can then define *forward, backward,* and

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left, right cross-sections as follows:

$$\begin{aligned}
\sigma_F &= \int_0^1 \frac{d\sigma}{d\Omega} d(\cos\theta) \\
\sigma_B &= \int_{-1}^0 \frac{d\sigma}{d\Omega} d(\cos\theta) \\
\sigma_L &= \int_{-1}^1 \frac{d\sigma_L}{d\Omega} d(\cos\theta) \\
\sigma_R &= \int_{-1}^1 \frac{d\sigma_R}{d\Omega} d(\cos\theta) \qquad (2)
\end{aligned}$$

where σ_L (σ_R) is the cross-section for left (right) polarized electrons colliding with unpolarized positrons.

There are a number of asymmetry experiments that can be performed at the Z resonance to determine the A_f parameters. With unpolarized beams one can measure the *forward-backward asymmetries* for different identified fermion final states. Using Equations 1 and 2, one finds

$$A_{FB}^f = \frac{\sigma_F^f - \sigma_B^f}{\sigma_F^f + \sigma_B^f} = \frac{3}{4} A_e A_f$$

The final state fermions from Z decay are polarized even for unpolarized collisions, and one can also extract the A_f parameters by measuring this polarization. In practice, this is only possible for the τ final state. One finds

$$P^\tau(\cos\theta) = \frac{2A_e\cos\theta + A_\tau(1+\cos^2\theta)}{(1+\cos^2\theta) + 2A_eA_\tau\cos\theta}$$

The asymmetries with unpolarized beams are well measured with the high statistics available at the 4 LEP experiments. There is a rich program at LEP to measure these asymmetries, and this is discussed in the next section. From these many measurements, a very precise determination of the weak mixing angle can be made.

At the SLC, the availability of a highly polarized electron beam allows additional asymmetry experiments to be performed which improve the determination of the A_f parameters. First, one can measure a *left-right forward-backward asymmetry*, defined by

$$A_{FB}^{LR} = \frac{(\sigma_F^L - \sigma_F^R) - (\sigma_B^L - \sigma_B^R)}{\sigma_F^L + \sigma_F^R + \sigma_B^L + \sigma_B^R} = \frac{3}{4} P_e A_f$$

This allows for a direct measurement of A_f, which is not accessible to the LEP experiments (except for the electron and tau). In particular, this is a very important measurement for the b final state. There is an apparent anomaly at the b vertex as evidenced by the large values for R_b measured by the LEP experiments. A_b is also very sensitive to new physics at the b vertex and can provide important additional information. We note that the SLD experiment currently measures $A_b = 0.841 \pm 0.053$,[3] which is somewhat lower than the MSM prediction of 0.935. The measurement of A_b, however, is not very sensitive to the weak mixing angle (see Table 1) and we will not consider the polarized forward-backward asymmetries further.

Table 2: Comparison of Z^0 Asymmetry Experiments.

Property	A_{LR}^{meas} $P_e = 80\%$	A_{FB}^l	A_{FB}^b	$\overline{P^\tau}$
MSM:				
Asymmetry	0.13	0.02	0.11	0.16
$\frac{\partial A}{\partial \sin^2\theta_W^{\mathrm{eff}}}$	6.3	1.5	5.6	7.9
Events:				
fraction of Z decays	0.96	0.12	0.19	0.04
sample size for equal statistical precision	1	135	120	250
Systematics:				
beam polarization	yes	no	no	no
efficiency/acceptance	no	yes	yes	yes
backgrounds	no	no	yes	yes
B mixing	no	no	yes	no
e-w interference correction	2%	100%	5%	2%

The most important asymmetry measurement at the Z resonance is the *left-right asymmetry*, A_{LR}, which is defined as

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = P_e A_e$$

This measurement, made with a polarized beam, allows a precise determination of A_e. All Z decay modes can be used, and this allows for a very simple analysis with great statistical power. Since A_e is very sensitive to $\sin^2\theta_W^{\mathrm{eff}}$, a precise measurement of A_{LR} yields a very precise determination of the weak mixing angle.

3 Analysis of Z^0 Asymmetry Experiments

A comparison of the different asymmetry measurements for measuring $\sin^2\theta_W^{\mathrm{eff}}$ is given in Table 2. Some of the properties considered for comparison are the size of the asymmetry and its dependence on the weak mixing angle, the relative Z sample size needed for an equivalent determination of $\sin^2\theta_W^{\mathrm{eff}}$, and the most significant sources of systematic errors. We begin by considering the experiments that determine the lepton asymmetry parameter, A_l.

3.1 Measurement of A_{LR}

For the A_{LR} analysis, all Z decay modes can be used, though in practice the SLD experiment excludes the *ee* mode to avoid the added complexity of correcting for t-channel interference. Currently SLD uses a calorimetric analysis only, and makes cuts on total observed energy, energy imbalance and number of clusters. This results in a very low background to the Z sample of about 0.2%. The only significant systematic error is the determination of the beam polarization. This is measured precisely with a Compton polarimeter, which for the 1994-95 SLD run measured $P_e = (77.34 \pm 0.61)\%$. Including all the SLD data from 1992-95 (approximately 150K Z decays), SLD measures $A^0_{LR} = 0.1551 \pm 0.0040$. This corresponds to a determination of the weak mixing angle of $\sin^2\theta_W^{\rm eff} = 0.23049 \pm 0.00050$.

3.2 Measurement of A^l_{FB}

As part of the Z lineshape analysis, the LEP experiments fit for A^e_{FB}, A^μ_{FB}, and A^τ_{FB}. These asymmetries have a strong dependence on $\sqrt{s}$, and hence are very sensitive to initial state radiation and $\gamma - Z$ interference. These electroweak radiative corrections are comparable to the size of the measured asymmetries, and this results in the large corrections noted in Table 2. However, within the framework of the MSM, these corrections are well determined and do not pose a significant source of uncertainty. If one assumes lepton universality, one can combine the results for $A^e_{FB}, A^\mu_{FB}, A^\tau_{FB}$ to obtain A^l_{FB}. Doing this, and combining the results for the 4 LEP experiments, one finds $A^{0,l}_{FB} = 0.0172 \pm 0.0012$. This results in a value for the weak mixing angle of $\sin^2\theta_W^{\rm eff} = 0.23096 \pm 0.00068$.[2]

3.3 Measurement of P^τ

Parity violation in the tau decay allows for a measurement of its polarization. Five decay modes of the tau are used by the LEP experiments: $\tau \to \pi\nu(BR = 12\%)$; $\tau \to \rho\nu(BR = 24\%)$; $\tau \to a_1\nu(BR = 8\%)$; $\tau \to e\nu\nu(BR = 18\%)$; $\tau \to \mu\nu\nu(BR = 18\%)$. The tau polarization can be determined from the kinematics of its decay once the particular decay mode is determined. The $\cos\theta$ dependence of the tau polarization can be used to determine A_e and A_τ separately. Combining the results from the 4 LEP experiments, one finds $A_e = 0.1390 \pm 0.0089$ and $A_\tau = 0.1418 \pm 0.0075$. These yield values for the weak mixing angle of $\sin^2\theta_W^{\rm eff} = 0.2325 \pm 0.0011$ (P^τ_{FB} result from A_e) and $\sin^2\theta_W^{\rm eff} = 0.23218 \pm 0.00095$ (P^τ result from A_τ).[2]

3.4 Measurement of A^b_{FB}, A^c_{FB}

For the heavy quark asymmetries it is necessary to obtain an enriched sample of b or c quark events of known purity. Additionally, one needs to tag an event hemisphere as quark or anti-quark. With the high LEP statistics available, it is possible to determine this tagging efficiency directly from the data by comparing the rates of single and double tags. Three techniques are used in the heavy quark analyses. First, a *lepton tag* can be used, and this is done by all 4 LEP experiments. The P and P_T distributions of leptons can be used to assign a probability that the lepton is a b or a c quark. Then the sign of the lepton tags the quark charge. The second method, referred to as *jet charge tag*, utilizes a lifetime tag to enrich the b sample and then uses a momentum-weighted jet charge to tag the quark charge. This analysis is performed by the ALEPH, DELPHI and OPAL collaborations. The third method uses a D^* *tag*. Fast D^*s are used to enrich the c sample, and then the sign of the pion in the D^* decay ($D^{*\pm} \to D^0\pi^\pm$) tags the quark charge. This analysis is also performed by the ALEPH, DELPHI and OPAL collaborations. The results from these 3 techniques are shown separately for the individual experiments in Figure 1. In this figure, the first 4 entries in the A^b_{FB} category use a lepton tag and the last 3 entries use a jet charge tag; the first 4 entries in the A^c_{FB} category use a lepton tag and the last 3 entries use a D^* tag. Combining the results from the 4 LEP experiments, one finds $A^{0,b}_{FB} = 0.0997 \pm 0.0031$ and $A^{0,c}_{FB} = 0.0729 \pm 0.0058$. Extracting the weak mixing angle from these results yields $\sin^2\theta_W^{\rm eff} = 0.23209 \pm 0.00055$ (from $A^{0,b}_{FB}$) and $\sin^2\theta_W^{\rm eff} = 0.2318 \pm 0.0013$ (from $A^{0,c}_{FB}$).[3]

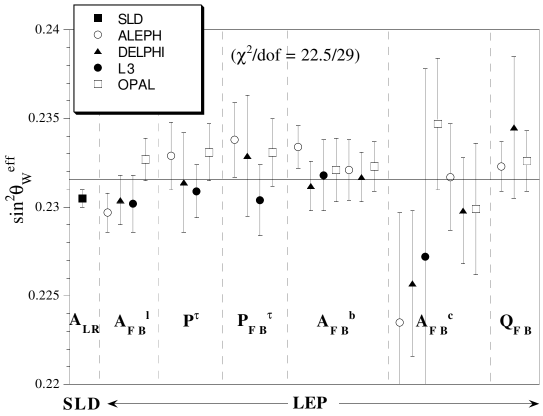

Figure 1: Weak Mixing Angle Measurements

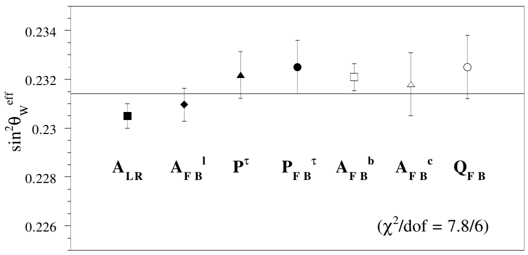

Figure 2: Weak Mixing Angle Results by Technique

3.5 Measurement of Q_{FB}

The last of the Z asymmetry experiments for determining the weak mixing angle utilizes all hadronic decay modes of the Z. The *jet charge technique* described above is again used, and one relies on MonteCarlo simulation to determine the relative abundances of the different quark species. This analysis is performed by the ALEPH, DELPHI, and OPAL experiments. Combining their results yields $\sin^2\theta_W^{\rm eff} = 0.2325 \pm 0.0013$.[2]

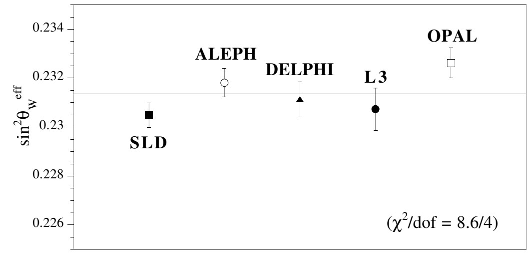

Figure 3: Weak Mixing Angle Results by Experiment

4 Weak Mixing Angle Results

The weak mixing angle results from the SLD, ALEPH, DELPHI, L3 and OPAL experiments are shown in Figures 1, 2, and 3.[1] Figure 1 gives the results by individual experiment for each of the 7 techniques described above. Figure 2 summarizes the results by technique and Figure 3 summarizes the results by experiment. The results are all consistent, though the χ^2 for consistency is somewhat worse for Figures 2 and 3.

If we assume that the MSM provides a complete description of the quark and lepton couplings to the Z boson, then all these results can be combined to give

$$\sin^2\theta_W^{\rm eff} = 0.23143 \pm 0.00028$$

If this assumption is relaxed to apply to lepton couplings only, we find $\sin^2\theta_W^{\rm eff} = 0.23106 \pm 0.00035$.

5 Comparison with MSM and with other Precision Electroweak Measurements

A convenient framework for analyzing the consistency of the precise $\sin^2\theta_W^{\rm eff}$ measurement with the MSM and with other precision electroweak measurements is given by the Peskin-Takeuchi parametrization[4] for extensions to the MSM. This parametrization assumes that vacuum polarization effects dominate and expresses new physics in terms of the parameters S and T, which are defined in terms of the self-energies of the gauge bosons. In S-T space, a measurement of an electroweak observable corresponds to a band with a given slope. And the MSM, for a given range of top mass and Higgs mass, corresponds to a parallelogram. Figure 4 shows[5] the S-T plot for measurements of the weak mixing angle ($\sin^2\theta_W^{\rm eff}$), the Z width (Γ_Z),[2] the W mass (M_W),[6] and the ratio of neutral current to charged current cross-sections in neutrino-nucleon scattering (R_ν).[7] The experimental bands shown correspond to one sigma contours. For the MSM, the Higgs mass is allowed to be in the range from 60 GeV to 1000 GeV, and the top mass is allowed to be in the range 167 GeV to 192 GeV. The right (left) edge of the MSM box corresponds to $M_H = 1000GeV$ ($M_H = 60GeV$), and the upper (lower) edge corresponds to $M_t = 192GeV$ ($M_t = 167GeV$). We see that all the data is consistent and that it prefers a Higgs mass around 100 GeV.

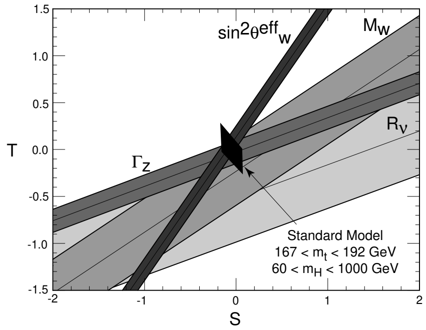

Figure 4: S-T Trajectories of Standard Model Tests

6 Conclusions

The ALEPH, DELPHI, L3 and OPAL experiments at LEP presented results this summer based on approximately 3.5 million Z decays per experiment, and the SLD presented results based on approximately 150 thousand Z decays using a polarized electron beam at SLC. Together, these experiments have determined the weak mixing angle to be $\sin^2\theta_W^{\rm eff} = 0.23143 \pm 0.00028$. This result is consistent with the MSM and with other precision electroweak measurements. The LEP experiments have now completed their datataking at the Z resonance with a total of about 4.5 million Z decays per experiment, while the SLD experiment plans to run through 1998 and accumulate an additional 500 thousand Zs.

References

1. The results from the 4 LEP experiments for A^l_{FB}, $P^\tau(\cos\theta)$, and Q_{FB} are taken from reference 2. The results for A^b_{FB} and A^c_{FB} for the LEP experiments are obtained using as input the data in reference 3. This heavy quark data is then analyzed using a consistent set of 13 input parameters. These 13 parameters are taken to be: $R_b = 0.2219$, $R_c = 0.154$, $Br(b \to l) = 11.12\%$, $Br(b \to c \to l) = 7.76\%$, $\overline{\chi} = 0.1145$, $A^b_{FB}(-2) = 5.5\%$, $A^c_{FB}(-2) = -7.9\%$,

$A^b_{FB}(pk) = 9.39\%, A^c_{FB}(pk) = 6.44\%$, $A^b_{FB}(+2) = 11.2\%$, $A^c_{FB}(+2) = 8.9\%$, $A_b = 0.841$, $A_c = 0.606$. We are grateful to the LEP Heavy Flavour Group and to Su Dong for providing the programs to generate the heavy flavour LEP data which is used in Figures 1 and 3.
2. LEP Electroweak Working Group, *A combination of preliminary LEP electroweak results for the 1995 summer conferences,* LEPEWWG-95-02, 1995.
3. LEP Heavy Flavor Group and SLD Heavy Flavor Group, *Combined LEP and SLD Electroweak Heavy Flavour Results for Summer 1995 Conferences,* LEPHF-95-02, SLD Physics Note 39, 1995.
4. M.E. Peskin and T. Takeuchi, *Phys. Rev.* **D46** (1992) 381.
5. This figure was produced by M. Swartz and updates a plot from M. Swartz, "Tests of the Electroweak Standard Model", in *Proceedings of the XVIth International Symposium on Lepton and Photon Interactions*, Ithaca NY, 1993, Eds. P. Drell and D. Rubin, pp.381-424.
6. CDF Collaboration, F Abe et al., *Measurement of the W Boson Mass,* FERMILAB-PUB-95-033-E, 1995.
7. The result used is taken from an average by G. Altarelli, R. Barbieri, and F. Caravaglios, *Nucl. Phys.* **B405** (1993) 3.