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RESUMMATION OF THRESHOLD CORRECTIONS IN QCD TO POWER ACCURACY: THE DRELL-YAN CROSS SECTION AS A CASE STUDY[a]

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Resummation of large infrared logarithms in perturbation theory can, in certain circumstances, enhance the sensitivity to small gluon momenta and introduce spurious nonperturbative contributions. In particular, different procedures – equivalent in perturbation theory – to organize this resummation can differ by $1/Q$ power corrections. The question arises whether one can formulate resummation procedures that are explicitly consistent with the infrared behaviour of finite-order Feynman diagrams. We explain how this problem can be treated and resolved in Drell-Yan (lepton pair) production and briefly discuss more complicated cases, such as top quark production and event shape variables in the e^+e^- annihilation.

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1 Introduction

The potential to study "new physics" at the next generation of accelerators will to a large extent depend on the ability to control the strong interaction background. Hence the present interest in making QCD predictions for hard processes as quantitative as possible, with an increasing understanding of the importance to study higher-twist effects which are suppressed by powers of the large momentum. In general, higher-twist effects reflect the "leakage" of contributions from large distances into the process of interest. The theoretical status of these corrections is well-established in the total e^+e^- annihilation cross section and in deep inelastic scattering (DIS) (and in related quantities), where dispersion relations relate the physical observable to the operator product expansion (OPE) of a T-product of currents at small distances. The OPE does not allow us to calculate power corrections, but one learns from the OPE the particular suppression of higher twist effects – $1/Q^4$ for the total e^+e^- annihilation cross section versus $1/Q^2$ for the DIS structure functions – and the process-independence (universality) of the uncalculable higher twist matrix elements. The structure of power-suppressed effects in "geniune" Minkowskian quantities is much less understood. There are phenomenological and theoretical indications that in certain situations such corrections are large – of order $1/Q$ – and numerically important at all energies available today. In this talk we summarize the results of Ref. 1 on the structure of power corrections and renormalons in Drell-Yan (lepton pair) production, and speculate whether this example teaches us a lesson of general validity.

Drell-Yan production, apart from its phenomenological significance, is theoretically interesting because it is the simplest hard process with two large, but disparate scales, if we consider the production of the Drell-Yan pair (or heavy vector boson) close to the kinematical threshold $z \sim 1$ where $z = Q^2/s$, Q^2 being the mass of the pair and s the total cms energy of the colliding partons. In this situation gluon emission into the final state is suppressed by small phase space $(1-z)Q \ll Q$, and causes large perturbative corrections enhanced by Sudakov-type logarithms $\ln(1-z)$. Taking moments $\sigma(N, Q^2) = \int dz z^{N-1} d\sigma/dz$ and subtracting collinear divergences by forming the ratio of the Drell-Yan cross section and the quark distribution squared, one is left with the perturbative expansion for the logarithm of the "hard" cross section

$$\ln \omega_{DY}(N, Q^2) = \omega_1 \alpha_s \ln^2 N + \omega_2 \alpha_s^2 \ln^3 N + \ldots \omega_k \alpha_s^k \ln^{k+1} N + \ldots, \qquad (1)$$

where $\alpha_s = \alpha_s(Q)$ and we have shown terms with the leading power of the large logarithm at each order of α_s. In leading logarithmic approximation (LLA), these terms are resummed to all orders by the elegant formula

$$\ln \omega_{DY}(N,Q^2) = \frac{2C_F}{\pi}\int_0^1 dz \frac{z^N-1}{1-z}\int_{Q^2(1-z)}^{Q^2(1-z)^2}\frac{dk_\perp^2}{k_\perp^2}\,\alpha_s(k_\perp^2). \qquad (2)$$

Expansion of the running coupling under the integral in powers of $\alpha_s(Q^2)$ generates a perturbative series which correctly reproduces all coefficients in Eq. 1. Eq. 2 takes into account soft and collinear gluon emission, and can be improved systematically by including next-to-leading logarithms etc. The leading soft-collinear contribution has a geometrical interpretation in terms of the cusp (eikonal) anomalous dimension; it is universal and appears in resummation of threshold corrections to any hard QCD process.

However, the resummed cross section in the LLA now appears to be sensitive to the infrared (IR) region at the level of $1/Q$ corrections [2,3]. Indeed, remove gluons with energy $Q(1-z)/2$ less than $\mu \sim \Lambda_{QCD}$ by inserting the appropriate θ-function in the z-integral. A simple calculation shows that the cross section changes then by $\sim \mu N/Q$. Given this sensitivity to the IR region, one would suspect that geniune power corrections of this magnitude exist. The same conclusion can be obtained from considering divergences of the perturbative expansion of Eq. 2 in large orders (renormalons). One easily checks, however, that the dangerous IR contributions correspond to terms with less logarithms of N than are resummed by the LLA and thus are beyond the accuracy to which Eq. 2 has been derived. Thus this evidence is by itself not conclusive. A more accurate analysis will clarify two questions:

- Does the IR sensitivity (of order $1/Q$) of the LLA resummed cross section represent the 'true' magnitude of power corrections to the Drell-Yan process or is it artificially introduced by resummation, that is by the procedure that separates those regions of higher order Feynman diagrams which give rise to large logarithms?

- If the exact Drell-Yan cross section has no $1/Q$ IR contributions, can the resummation of large perturbative corrections to all orders be made consistent with the IR behaviour of finite-order Feynman diagrams? Or is the (spurious) $1/Q$ sensitivity found in LLA intrinsic and unavoidable for resummation of threshold corrections?

2 IR sensitivity of LLA and soft gluon emission at large angles

We provide evidence [1,4,5] that the Drell-Yan cross section is free from $1/Q$ IR contributions. To one-loop accuracy this statement can be checked by an explicit calculation with a small gluon mass λ as regulator. Nonanalytic terms in the small-λ expansion correspond to higher twist contributions [6,7]. A textbook calculation gives [1] at $N \gg 1$

$$\omega_{DY}(N,Q^2,\lambda^2) - \omega_{DY}(N,Q^2,0) = \frac{C_F\alpha_s}{\pi}\frac{N^2\lambda^2}{Q^2}\ln(\lambda^2/Q^2) + O\left(\frac{1}{N},\frac{N\lambda}{Q}\right). \tag{3}$$

Note that the suspected linear term $\sqrt{\lambda^2/Q^2}$ is absent; the leading IR contribution is of order $N^2\lambda^2/Q^2$. The N^2 enhancement signals that the power corrections are determined by the *smaller* of the two large scales: Q/N is the moment space analogue of the energy available for gluon emission. In terms of the energy fraction z the relevant scale is $(1-z)Q$ rather the mass of the Drell-Yan pair Q.

To understand the apparent $1/Q$ sensitivity of the LLA, it is instructive to consider the structure of the one-loop integral for soft gluon emission

$$\omega_{DY} \sim \int \frac{d^3k}{2k_0}\delta[(p_1+p_2-k)^2 - Q^2]\,|\mathcal{M}_{\mathcal{DY}}|^2 . \tag{4}$$

The matrix element (in LLA) is $|\mathcal{M}_{\mathcal{DY}}|^2 \sim 2Q^2/k_\perp^2$ and it is convenient to rewrite the phase-space integral as

$$\int \frac{d^3k}{2k_0} \sim \int \frac{dk_\perp^2}{\sqrt{k_0^2-k_\perp^2}} .$$

We now take a massless gluon, and to avoid collinear divergences introduce an explicit cutoff on the minimal transverse momentum $k_\perp > \mu$. Remembering that $k_0 = \sqrt{s}(1-z)/2$ and taking moments we get

$$\omega_{DY}^{soft} = \frac{2C_F}{\pi}\int_0^{1-2\mu/Q} dz\, z^{N-1}\int_{\mu^2}^{Q^2(1-z)^2/4}\frac{dk_\perp^2}{k_\perp^2}\frac{1}{\sqrt{(1-z)^2-4k_\perp^2/Q^2}} \tag{5}$$

The crucial point is now that the LLA corresponds to resummation of soft and *collinear* emission, that is the leading logarithms (and in fact the next-to-leading as well) come from the integration region of small gluon transverse momentum compared to its energy $k_\perp \ll k_0 = Q(1-z)/2 \ll Q$. Thus, for summation of large logarithms, it is safe to neglect the term $4k_\perp^2/Q^2$ under the square root, so that

$$\omega_{DY}^{soft+coll.} = \frac{2C_F}{\pi}\int_0^{1-2\mu/Q} dz\,\frac{z^{N-1}-1}{1-z}\int_{\mu^2}^{Q^2(1-z)^2/4}\frac{dk_\perp^2}{k_\perp^2}, \tag{6}$$

where we have replaced z^{N-1} by $z^{N-1}-1$ to take into account virtual gluon exchange. Taking the integrals, we get the expected double logarithm but also the linear term in μ/Q discussed in Sect. 1.

However, this IR contribution of order $1/Q$ comes from the end-point integration region $1-z \sim \mu/Q$ (where $k_\perp \sim k_0$) in which neglect of the $k_\perp^2/Q^2$ term under the square root is not justified. In fact, the square root cannot even be expanded in $k_\perp^2/(Q^2(1-z)^2)$ since this would generate increasingly singular contributions. Instead, the integral must be taken exactly. When this is done [1], all μ/Q terms disappear.

The physical reason for the enhanced IR sensitivity of the resummed cross section in LLA is that we neglected soft gluon radiation at large angles $k_\perp \sim k_0$. To recover the correct IR behavior, the phase space integral for soft gluon emission has to be taken exactly; the common collinear approximation is sufficient for summing logarithms to leading and next-to-leading logarithmic accuracy but is misleading for the analysis of power-suppressed effects.

3 Resummation of soft emission and Wilson lines

Exponentiation of large logarithms occurs for both, collinear emission and soft gluon emission, including emission at large angles. Thus, after a complete treatment of resummation of soft gluon emission, the apparent $1/Q$ sensitivity in LLA should be compensated by other conributions to the exponent. The best-known generalization of the LLA formula Eq. 2 was given by Sterman [8]

$$\begin{aligned}\ln \omega_{DY}(N,\alpha_s) &= \int_0^1 dz\, \frac{z^{N-1}-1}{1-z}\left\{2\int_{Q^2(1-z)}^{Q^2(1-z)^2} \frac{dk_\perp^2}{k_\perp^2}\,\Gamma_{\text{cusp}}(\alpha_s(k_\perp))\right. \\ &\qquad \left. +B(\alpha_s(\sqrt{1-z}Q)) + C(\alpha_s((1-z)Q)\right\} + \mathcal{O}(1) \qquad (7)\end{aligned}$$

This expression involves three "anomalous dimensions" Γ_{cusp}, B and C. The LLA corresponds to taking into account the leading term $O(\alpha_s)$ in the expansion of Γ_{cusp} and neglecting B, C. The next-to-leading logarithmic accuracy requires two terms in Γ_{cusp} and the leading $O(\alpha_s)$ terms in B, C. In general, higher order corrections to $\Gamma_{\text{cusp}}, B, C$ generate contributions with less and less powers of $\ln N$ for a given power of α_s.

The three terms in Eq. 7 have the following origin: The function "B" comes from subtracting the DIS cross section (squared) and is irrelevant for our discussion. The term with a double integral resums soft and collinear gluon radiation to all orders. This contribution is universal for all hard processes. Finally, "C" corrects for soft gluon radiation at large angles and is process-dependent. This term starts with $O(\alpha_s^2)$ in agreement with the common wisdom that the radiation at large angles is suppressed by two powers of $\ln N$, but taking it into account can be crucial to recover the correct IR behavior.

Thus, the $1/Q$ IR sensitivity of the generalized LLA expression given by the first term in Eq. 7 should be cancelled by the "C"-term. To test how this happens, one needs some approximation to calculate the anomalous dimensions to all orders in perturbation theory. A convenient formal parameter is N_f, the number of light fermion flavors. The leading contribution in the large-N_f limit corresponds to a chain of fermion loops inserted into the single gluon line. Taking into account the chain of loops has two effects: First, it generates the correct argument of the running coupling in Eq. 7 which is the gluon transverse momentum. [As usual, we tacitly assume that N_f can be used to reconstruct the full one-loop running, determined by β_0.] Second, counterterms for individual fermion loops produce non-trivial anomalous dimensions $\Gamma_{\rm cusp}, B, C$ to all orders in α_s. The corresponding calculation is technical and can be found in Ref. 1. The result is that all $1/Q$ IR contributions generated by the generalized LLA term are cancelled by the IR contributions related to factorial divergence of the perturbation expansion of "C". This tells us that although the resummation formula Eq. 7 is valid, it involves significant cancellations between different contributions and the true IR behaviour is restored only after summation of an *infinite* number of terms in the expansion of "C". This contradicts the logic of resummation to resum an infinite number of large logarithms by calculating only a finite number of terms in the anomalous dimensions.

The simplest remedy would be to use the exact phase space factor for the one-loop gluon emission and replace the first term in Eq. 7 by

$$\ln\omega_{DY}(N,\alpha_s) = 2\int_0^1 dz\,[z^{N-1}-1]\int_{Q^2(1-z)}^{Q^2(1-z)^2}\frac{dk_\perp^2}{k_\perp^2}\,\frac{\Gamma_{\rm cusp}(\alpha_s(k_\perp))}{\sqrt{(1-z)^2+4k_\perp^2/Q^2}}+\ldots \tag{8}$$

With this substitution the $1/Q$ IR sensitivity disappears and the functions B, C are modified starting $O(\alpha_s^2)$ so that the undesired behavior of C in large orders is removed.

A different approach to soft gluon resummation emphasizes the renormalization of Wilson lines [9]. Its theoretical advantage is the operator language that avoids the separation of small-angle and large-angle soft emission. $1/Q$ IR contributions never appear. The starting point is the well-known fact that soft gluon emission from a fast quark can be described by a Wilson line operator along the classical trajectory of the quark. The product of Wilson lines for the annihilating quark and antiquark is denoted by $U_{\rm DY}(x)$, where x is the annihilation space-time point. Up to corrections which vanish as $z \to 1$, the partonic Drell-Yan cross section is given by [9]

$$\omega_{\rm DY}(z,\alpha_s) = H_{\rm DY}(\alpha_s)\, W_{\rm DY}(z,\alpha_s)\,. \tag{9}$$

$H_{\rm DY} = 1 + \mathcal{O}(\alpha_s)$ is a short-distance dominated function, independent of z. $W_{\rm DY}$ is the square of the matrix element $\langle n|TU_{\rm DY}(0)|0\rangle$, summed over all final states:

$$W_{\rm DY}(z,\alpha_s) = \frac{Q}{2}\int_{-\infty}^{\infty} \frac{dy_0}{2\pi}\, e^{iy_0Q(1-z)/2}\, \langle 0|\bar{T}\, U^\dagger_{\rm DY}(y)\, T\, U_{\rm DY}(0)|0\rangle \tag{10}$$

The Fourier transform is taken with respect to the energy of soft partons and $y = (y_0, \vec{0})$.

The crucial observation is that the Wilson line depends only on the ratio $(\mu N)/(QN_0)$ (taking moments of $W_{\rm DY}(z,\alpha_s)$), where μ is a cutoff separating soft and hard emission (the renormalization scale for the Wilson line) and N_0 is a suitable constant. Hence the N-dependence of the Drell-Yan cross section in the soft limit can be obtained from the μ-dependence of Wilson lines, which is given by the renormalization group equation (here $\alpha_s = \alpha_s(\mu)$)

$$\left(\mu^2\frac{\partial}{\partial\mu^2} + \beta(\alpha_s)\frac{\partial}{\partial\alpha_s}\right) \ln W_{\rm DY}\left(\frac{\mu^2N^2}{Q^2N_0^2},\alpha_s\right) = \Gamma_{\rm cusp}(\alpha_s)\, \ln\frac{\mu^2N^2}{Q^2N_0^2} + \Gamma_{\rm DY}(\alpha_s)\,. \tag{11}$$

It involves two anomalous dimensions $\Gamma_{\rm cusp}(\alpha_s)$ and $\Gamma_{\rm DY}(\alpha_s)$ related to the cusp and to presence of light-like segments on the Wilson line, respectively. The general solution of Eq. 11 is given by

$$\begin{aligned} \ln W_{\rm DY}\left(\frac{N^2}{N_0^2},\alpha_s(Q)\right) &= \ln W_{\rm DY}(1,\alpha_s(QN_0/N)) + \\ &+ \int_{Q^2N_0^2/N^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2}\left[\Gamma_{\rm cusp}(\alpha_s(k_\perp)\, \ln\frac{k_\perp^2N^2}{Q^2N_0^2} + \Gamma_{\rm DY}(\alpha_s(k_\perp))\right], \end{aligned} \tag{12}$$

where we set $\mu = Q$. The inhomogeneous second term in Eq. 12 can be rewritten (identically) in a more familiar form, which resembles the first term in Eq. 7:

$$2\int_0^{1-N_0/N} \frac{dz}{1-z}\left[\int_{(1-z)^2Q^2}^{Q^2}\frac{dk_\perp^2}{k_\perp^2}\,\Gamma_{\rm cusp}(\alpha_s(k_\perp)) + \Gamma_{\rm DY}(\alpha_s((1-z)Q)\right] \tag{13}$$

Note presence of the *initial condition* $W_{\rm DY}(1,\alpha_s(QN_0/N))$. Its expansion in α_s produces subdominant logarithms $\alpha_s^k \ln^{k-1} N$ which can be absorbed into a redefinition of $\Gamma_{\rm DY}$:

$$\Gamma_{\rm DY}(\alpha_s) \longrightarrow \tilde{C}(\alpha_s) = \Gamma_{\rm DY}(\alpha_s) - \beta(\alpha_s)\frac{d}{d\alpha_s}\ln W_{\rm DY}(1,\alpha_s). \qquad (14)$$

$\tilde{C}(\alpha_s)$ starts at order α_s^2. It does not affect resummation of large logarithms in N to next-to-leading accuracy $\alpha_s^k \ln^k N$.

It remains to subtract the DIS cross section, which can also be implemented in the language of Wilson lines, see Ref. 9 for details. Finally, we get

$$\begin{aligned}\ln\omega_{\rm DY}(N,\alpha_s) \;\; &= \;\; -\int_0^{1-N_0/N} dz\,\frac{1}{1-z}\left\{2\int_{Q^2(1-z)}^{Q^2(1-z)^2}\frac{dk_\perp^2}{k_\perp^2}\,\Gamma_{\rm cusp}(\alpha_s(k_\perp))\right.\\ &\left. \quad + \tilde{B}(\alpha_s(\sqrt{1-z}Q)) + \tilde{C}(\alpha_s((1-z)Q))\right\} + \mathcal{O}(1)\,. \qquad (15)\end{aligned}$$

($N_0 = \exp(-\gamma_E)$ in the $\overline{\rm MS}$ scheme.) This form of the resummed cross section is as legitimate in the framework of the perturbation theory as the more conventional expression in Eq. 7. They have different properties, however, as far as sensitivity to the IR behavior of the coupling is concerned, which becomes important when the anomalous dimensions $\Gamma_{\rm cusp},\ldots$ are truncated to finite order. Since the region of very large $z \to 1$ is removed in Eq. 15, this expression shows no IR sensitivity at all unless $N > Q/\Lambda_{\rm QCD}$. Loosely speaking, this is so because the Wilson line approach treats small- and large-angle gluon emission in a coherent way. Because this technique can also treat subleading logarithms in a systematic way, it is preferred over, for example, the modification of the phase space as in Eq. 8.

It is natural to expect (and explicit calculation [1] in the large-N_f limit confirms this) that the anomalous dimensions $\Gamma_{\rm cusp}(\alpha_s)$ and $\Gamma_{\rm DY}(\alpha_s)$ in the $\overline{MS}$ scheme are analytic functions of the coupling at $\alpha_s = 0$. Then, all power corrections to the resummed cross section (to all orders in $N\Lambda_{\rm QCD}/Q$) originate exclusively from the initial condition for the evolution equation for the Wilson line, and are not created (or modified) by the evolution, i.e. by soft gluon resummation. Thus, if the resummation of soft gluon emission is done coherently for all angles, the only effect of soft gluons on power corrections is a *change of scale*, the replacement $Q \to Q/N$ as the parameter of the power expansion. This suggests that, in general, there is no reason to suspect new nonperturbative contributions in resummed cross sections as compared to finite-order calculations. The conclusion that power corrections to Drell-Yan production are suppressed by $1/Q^2$ is then consistent with the analysis of power corrections at tree level by Qiu and Sterman [10].

The redefinition in Eq. 14 transforms the IR sensitivity (and potential power corrections) of the initial condition for the evolution equation for the Wilson line into IR sensitivity of the function $\tilde{C}$ in Eq. 15. As mentioned above, this function becomes important precisely when one starts to be sensitive to gluon radiation at large angles, and the conclusions on power corrections depend sensitively on this region. Because of this, we are sceptical that universality of nonperturbative corrections to resummed cross sections could be deduced from the universality of soft-collinear gluon emission as embodied by the eikonal (cusp) anomalous dimension, an idea originally put forward in Refs. 3,11. In the Wilson line technique the solution of the evolution equation never involves the QCD coupling integrated over the IR Landau pole as long as $N < Q/\Lambda_{QCD}$. Since this inequality sets the boundary for a perturbative treatment anyway, Eq. 15 (which coincides with the resummation procedure used in Ref. 12 to next-to-leading logarithmic accuracy) is suited for all moments that can be treated in a power expansion in $1/Q$.

4 Top quark production at the TEVATRON

In the light of our discussion, let us consider recent results for the resummed top quark production cross section, which we summarize in Table 1. We concentrate on the comparison of two new calculations [13,14]. Both assumed $m_t = 175$ GeV and used the same parametrisations for the structure functions. Hence the difference is entirely due to different resummation prescriptions. The difference in central values is of order 15%, compared to $\sim$ 10% renormalization scale dependence and $\sim$ 5% due to uncertainty in the structure functions. Apparently, resummation causes the largest ambiguity. Note that the resummed cross section of Ref. 14 practically coincides with the strict $O(\alpha^2)$ result. Thus, in Ref. 14 the resummation of $\ln N$ terms has a negligible effect, while the resummation in Ref. 13 produces a $10-15\%$ enhancement. Since both procedures sum all leading logarithms (in the sense of Eq. 2), the difference is entirely due to terms with less powers of logarithms which are beyond the accuracy of the resummation in the strict sence. Unless we can prefer one particular resummation procedure, the difference would have to be considered as the present theoretical uncertainty. Our discussion of Drell-Yan production suggests the criterium that resummation procedures should not introduce power corrections (factorial divergence in large orders) which are not already present in finite order approximations. From this point of view, we are led to prefer the prescription used in Ref. 14, which starts from Eq. 15.

Table 1: Resummed cross section for the $t\bar{t}$ production at the TEVATRON. $m_t = 175\,\mathrm{GeV}$, MRSA' parton distributions for the central value quoted.

Ref.	$\sigma_{t\bar{t}}$, pb	Uncertainty
LSN [15]	4.95	4.53 – 5.66
BC [13]	5.32	4.93 – 5.40
CMNT [14]	4.75	4.25 – 5.00

The major numerical difference between Ref. 13 and Ref. 14 comes from a different procedure to implement the inverse Mellin transformation from moment space to momentum space. The subtle problem here is to which extent one can avoid contributions of very large moments $N \geq Q/\Lambda_{QCD}$, which strictly speaking can not be treated by short-distance methods. This problem is somewhat similar to the problem of analytically continuing perturbative QCD predictions from Euclidian in Minkowski space, relevant, for example, in connection with the τ-lepton hadronic width. The particular way of performing the analytic continuation becomes important when one uses a *resummed* coupling constant, and the guiding principle proves to be to avoid the region $Q < \Lambda_{QCD}$ in the complex Q-plane, where no short-distance treatment is possible. If the region $Q < \Lambda_{QCD}$ is not avoided, one may introduce spurious $1/Q^2$ power corrections to the decay width [7], which are absent in the OPE. Similarly, Catani *et al.* find [14] that the inverse Mellin transform of the resummed cross section in moment space has to be done by exact numerical integration in the complex N plane, avoiding the region $\mathrm{Re}\,N \to \infty$ where the IR singularity in the running coupling becomes important. Failure to avoid this region may result in spurious effects of order $(\Lambda_{QCD}/Q)^{\sim 0.3}$.

Within the approach of Ref. 13 this problem is somewhat masked by using a resummation formula similar to Eq. 7, in which the sensitivity to the IR behavior of the coupling is of order $1/Q$ for *any* N. As explained above, this IR sensitivity is an artifact of truncating the anomalous dimensions to finite order. One also notes that applying the prescription of Ref. 13 to Drell-Yan production requires to introduce a phenomenological $1/Q$-correction, that mainly seems to cancel the $1/Q$-effects generated by the resummation prescription [16].

The difference between various resummation procedures should also be perceptible in high-$p_\perp$ jet production at the TEVATRON, and a theoretical understanding of this difference might be one aspect in understanding the apparent excess of large-$p_\perp$ jets seen by CDF.

5 $1/Q$ IR sensitivity of thrust

The Drell-Yan cross section appears to have no $1/Q$ power corrections. This is not generally the case for any quantity. There are good reasons to suspect the existence of $1/Q$ nonperturbative hadronization corrections to event shapes observables in e^+e^- annihilation [17]. Unlike the Drell-Yan cross section, these observables cannot be expressed directly in terms of Feynman diagrams, and are obtained by integrating the QCD amplitudes with certain weight functions such as to emphasize a particular final state configuration. These weight functions generally destroy the balance of gluon emission at small and large angles, and make these observables sensitive to nonperturbative momentum flow at large angles. As a consequence $1/Q$ corrections are invariably expected for event shapes. For example, the average thrust $\langle 1-T\rangle$ of the final state is computed to leading order in α_s by inserting the factor

$$1 - T = (k_0 - \sqrt{k_0^2 - k_\perp^2})/Q \tag{16}$$

into the phase-space integral for gluon emission, which has a structure identical to the Drell-Yan cross section in moment space, see (5). The above factor suppresses small-angle emission but causes $1/Q$ IR sensititvity.

An interesting speculation is whether $1/Q$ corrections to event shapes are universal in the sense that they can be related to a single nonperturbative parameter [18,11,19]. Although, due to importance of large angle emission, this parameter would not be related to the universal cusp anomalous dimension, the hypothesis makes sense so long as the underlying physical process is the same for all event shapes. Strictly speaking, the answer seems to be negative, since $1/Q$ corrections also occur outside the two-jet region[20] and higher-order corrections are not suppressed, because α_s is evaluated at low scale, so that it is not counted. One may still argue in favour of *approximate* universality [4,21], if the coupling stays finite and reasonably small in the infrared. This purely phenomenological hypothesis could in principle be subjected to experimental tests. This, in fact, seems very hard in practice, because of poor control of higher orders of the perturbative series. One may suspect that the largest part of the hadronization correction to event shapes estimated by Monte Carlo event generators is in fact related to the perturbative parton cascade which can indeed be universal to the extent that the event shape variable is dominated by the two-jet kinematics.

To illustrate the difficulty in testing universality consider the average thrust $\langle 1-T\rangle$. The existing experimental data are well described by [17]

$$\langle 1-T\rangle(Q) = 0.335\,\alpha_s(Q) + 1.02\,\alpha_s(Q)^2 + \frac{1GeV}{Q} \tag{17}$$

where the first two terms give the QCD calculation (to $O(\alpha_s^2)$ accuracy). The power correction is needed to gain agreement with the data (over the entire range of Q). The second order perturbative correction has a large coefficient, indicating that the adopted scale Q is in fact inadequate for this process. The scale setting problem for event shapes is difficult. However, as a natural first guess one can try to take α_s at a scale of order of the jet mass M, which is related to thrust in the two-jet limit by $M^2 = (1-T)\,Q^2/2$ [We take the scale $Q_*^2 = (1-T)\,Q^2/4$, since for fixed T this is the upper limit on the gluon transverse momentum.]. Taking $\alpha_s(m_Z) = 0.12$ and using the fixed-order QCD result $\langle 1-T\rangle \sim 0.07$, we get $Q_* \sim 0.13Q$. Fitting again a $1/Q$ correction to the same data, we get

$$\langle 1-T\rangle(Q) = 0.335\,\alpha_s(Q_*) + 0.19\,\alpha_s(Q_*)^2 + \frac{0.4GeV}{Q} \tag{18}$$

The second-order coefficient in the perturbative series has become much smaller and the size of the required hadronization correction has also decreased. One might actually think of writing the power correction as $0.05GeV/Q_*$. Viewed this way, the issue of universality becomes inseparable from the problem of determining the most appropriate scale for the process.

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