NEW PHYSICS AND IMPROVEMENTS IN BEAM-BEAM TAIL SIMULATIONS†

T. Chen, J. Irwin, R. H. Siemann, Stanford Linear Accelerator Center,
Stanford University, Stanford, CA 94309

Abstract

Our beam-beam simulation code[1] has been improved and used to study lifetime phenomena under beam-beam interaction. One phenomenon observed is high order horizontal resonances (*e. g.*, 8th to 16th), which are found to be exceptionally strong in the horizontal tail and have significant impact on lifetime. A first order perturbation calculation gives good agreement with the simulation on resonance width and tune. Gas scattering is a new feature has been added to the program. The lifetime result is compared with analytical calculation and agrees well. The other effect studied is parasitic crossings in PEP-II. The simulation demonstrates a dramatic tail blow-up when the horizontal separation is reduced to about 7 σ.

1 HIGH ORDER HORIZONTAL RESONANCES IN BEAM-BEAM INTERACTION

1.1 Analysis

A perturbation method has been developed to study beam-beam physics[2]. In this section, we concentrate on horizontal resonances. Assuming a linear storage ring and a beam-beam interaction as a perturbation, one can find the island tune Q_r and resonance full width ΔA_x[3].

The island tunes for different resonance orders are given in Figure 1 for $A_y = 1$ and $R = 0.04$, where R is the ratio of vertical beam size and horizontal beam sizes. One can see that the island tune is large for these parameters even for high order resonances. For example, for $A_x = 6$, Q_r ranges from $0.05\xi_x$ to $0.20\xi_x$. For $\xi_x = 0.0455$, $Q_r = 9.1\times10^{-3}$ to 2.3×10^{-3}. Usually in a storage ring, the inverse of the damping time is in the order of 2×10^{-4}, which means that a particle trapped by the resonance circulates many times during a damping time. The effective aperture has been moved to the minimum amplitude of the resonance island. The full widths of resonances are plotted in Figure 2. Since the aperture being reduced by the resonance is the full width of the resonance, a wide resonance is certainly more dangerous. The impact of the lifetime could be significant.

1.2 Simulation

Simulations have been performed to check the results of the analytical calculations. The simulation is a simple weak-strong model that consists of a linear transfer matrix and a beam-beam kick. To obtain the island tunes, a single particle is launched near the resonance center and tracked for 1024×p turns. Then, the island tunes can be found by an FFT of the coordinates.

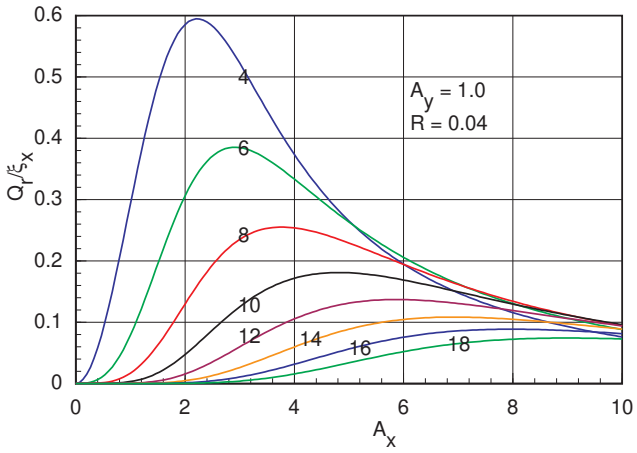

Figure 1: Island tunes normalized by horizontal tune shift for p = 4, ..., 18.

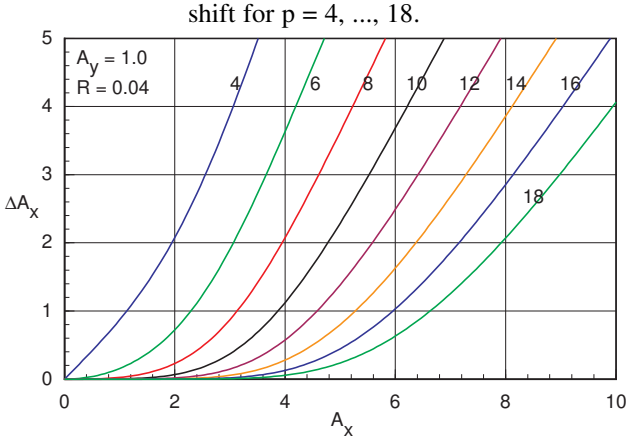

Figure 2: Full width of resonance islands for p = 4, ..., 18

The resonances p=8, ..., 16 are checked by simulation. For each resonance, the horizontal tune is set such that the resonance is located at about 7 σ_x. The vertical tune Q_y is chosen as 0.64, and the results are not sensitive to the choice. The horizontal beam-beam tune shift is 0.0455. The analytical calculation and simulation results for Q_r are listed in Table 1 for comparison. The agreement is good at high order resonances but diverged at low order. The discrepancy may be caused by the strength of the resonance which is so strong that the perturbation method is not a good approximation. However, the conclusion that the island tune is much higher than radiation damping rate is still true.

The island width can be found by showing the beam tail distributions generated by the simulation program that was written to study beam-beam lifetime[1]. This will not only give the island widths, but also the

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physical impact to the beam. Figure 3 gives the distribution contour plots in amplitude space. The conditions are the same as mentioned previously. The contour lines are the equal logarithm density locations. One can easily see the bulge in the horizontal tail, which resulted from the resonance. In Table 2, the analytical results of island full widths at 7 σ_x are listed for the different resonances. This is closely comparable to the width of the bulges in Figure 3.

The comparison between the analytical calculation and simulation proves that the perturbation method of the beam-beam interaction is very effective, especially the agreement is good for very high order terms, which is not usual in most perturbation methods. The impact of the resonances on lifetime gives stopbands in tune plane. This is discussed in detail in Ref. [3].

Table 1: Comparison of island tunes from the analytical calculation and simulation

Resonance order	p=8	p=10	p=12	p=14	p=16
Q_x	0.6236	0.5986	0.5819	0.57	0.5611
Analytical calculation	0.0073	0.0066	0.0059	0.0049	0.0041
Simulation	0.0047	0.0054	0.0055	0.0050	0.0044

Table 2: Analytical calculation of island widths

Resonance order	p=8	p=10	p=12	p=14	p=16
Full island width	7.5	5.2	3.8	2.7	1.9

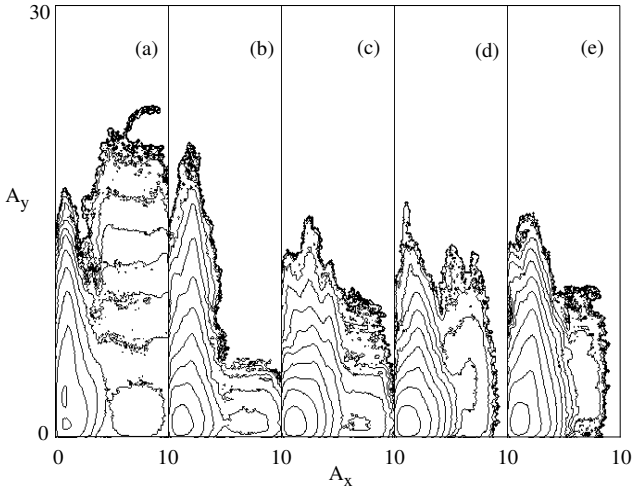

Figure 3: Beam tail distributions in transverse amplitude space: a) q_x=0.6236 for p=8 resonance, b) q_x=0.5986 for p=10 resonance, c) q_x=0.5819 for p=12 resonance, d) q_x=0.57 for p=14 resonance, and e) q_x=0.5611 for p=16 resonance.

2 GAS SCATTERING SIMULATION

2.1 Analytical Calculation and Simulation

At very large amplitude where beam-beam kick is weak, the lifetime is dominated by gas scattering or other physics effects rather than beam-beam. This was not included in previous tail simulation code, which gives unrealistic long lifetime at very large amplitudes. Gas scattering effect was included in a similar code[4], and is now included in our code.

Small angle nuclear Rutherford scattering is well understood physics. The cross section of an electron scattered into angle θ is given by

$$\frac{d\sigma}{d\Omega} = \left(\frac{2Zr_e}{\gamma}\right)^2 \frac{1}{\theta^2}. \qquad (1)$$

where Z is the atomic number of gas atom, γ is the energy of incident electron, and r_e is the classical electron radius.

To estimate the lifetime limited by the aperture in one transverse direction, we found the number of electrons scattered by an angle larger than $\theta_{x,\min}$:

$$N_{\theta>\theta_{x,\min}} = \frac{\pi}{2} N_e N_{gas} C \left(\frac{2Zr_e}{\gamma}\right)^2 \frac{1}{\theta_{x,\min}^2}. \qquad (2)$$

where N_e is the number of electrons, N_{gas} is the gas density, and C is the circumference of the storage ring.

The projected angle θ_x is related with aperture A_x in average by $\theta_x^2 = A_x^2 \frac{\varepsilon_x}{\overline{\beta_x}}$, where ε_x is the emittance and $\overline{\beta_x}$ is the average β-function. Therefore, the lifetime from gas scattering can be written as a function of aperture:

$$\tau = \frac{N_e}{N_{\theta>\theta_{x,\min}}} T_0 = \frac{2\varepsilon_x A_x^2}{\pi N_{gas} c \overline{\beta}_x^2 \left(2Zr_e/\gamma\right)^2}, \qquad (3)$$

where c is the speed of light. Take the PEP-II LER as an example, the average horizontal β-function is 15 meters, the emittance is 64 nm, the energy γ=6066, and the gas density at 10^{-8} torr, 300K is 3.22×10^{14} m^{-3}. With Z=7.5, we have:

$$\tau = 582.5 A_x^2. \qquad (4)$$

In our beam-beam lifetime simulation program[1], boundaries are introduced to track large amplitude particles. The flux of particles crossing the boundaries due to nonlinearity was obtained from previous tracking and used as the boundary condition. In modeling beam-gas scattering, the scattered particle is inserted at boundaries as an additional part of boundary flux. Typically, the scattering flux is much smaller than the flux due to nonlinearities. Therefore, there is no impact on the original part. In the code, we first calculate the probability of a particle scattering across the boundary. Then, at each turn, a random number is generated to determine whether a particle scattered across the boundary. When a scattering event

happened, four more random numbers are generated. Two of them provide the scattering angle coordinates, and the other two determine the betatron phase advances to the IP. Finally, the coordinates are transferred to the IP and inserted as a boundary-crossing particle.

2.2 Simulation Result

Figure 4 shows the simulated and analytically calculated lifetime from gas scattering only. They have a good agreement. Under 5σ, the simulation shows the lifetime due to the aperture cutting into the Gaussian distributed core which the analytical calculation does not include.

Figure 5 gives the simulated lifetime versus vertical aperture curves for the scattering only, beam-beam only and both beam-beam and scattering cases. It is obvious, under this condition, the two effects are simply combined. At small amplitudes, the beam-beam dominates, while at the large amplitudes, the gas scattering dominates. This is contradictory with other studies[4]. The source of the contradiction is not yet understood. So far we haven't observed any case that the two effects interfere and enhance lifetime problem. However, we have found when a nonlinear lattice with a small dynamic aperture is involved, the combination of the three effects results in a worse lifetime.

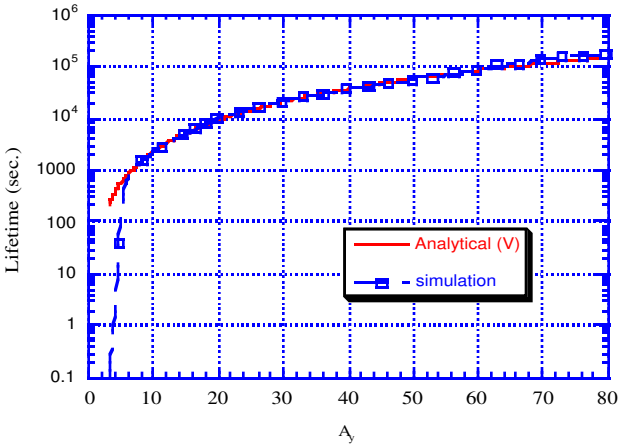

Figure 4. Comparison of lifetime calculated by simulation and analytical calculation. Plots show the lifetime versus vertical apertures.

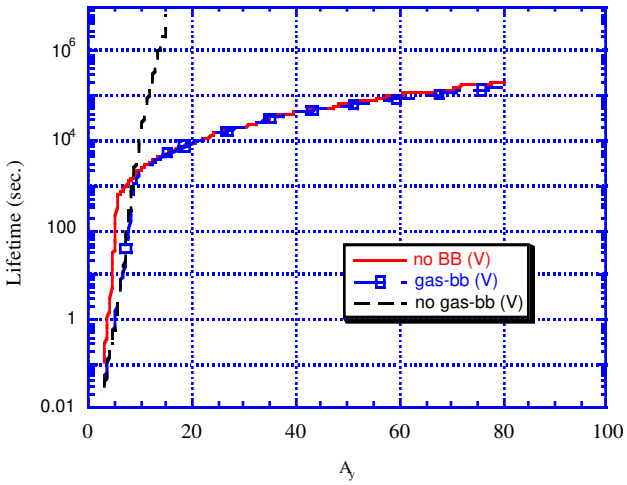

Figure 5. The lifetime versus vertical apertures. The lifetime is computed under the condition of: a) with gas scattering but no beam-beam interaction;b) with both beam-beam and gas scattering; and c) with beam-beam but no gas scattering. The tunes are q_x=0.57 and q_y=0.64. When beam-beam turned on, the beam-beam tune shifts are ξ_x=0.037 and ξ_y=0.033.

3 PARASITIC CROSSINGS AT PEP-II

PEP-II is a multi-bunch collider with head-on collisions. In each ring, bunches are spaced by 4 nsec. At the interaction region, two beams are separated by the bending magnets near the IP, as well as other magnets, taking advantage of the two beams having different energies. However, due to the close bunch spacing, the bunches will meet at places other than IP before they move into the separate rings. The parasitic crossings give strong kicks to large amplitude particles of the other beam, which results in a lifetime problem. The simulation program is basically the same as used in previous sections, but two single slice kicks are added on both sides of the main beam-beam kicks.

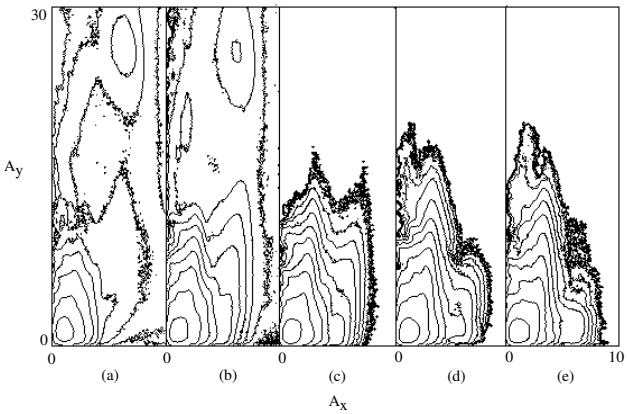

Figure 6. Tail distributions with parasitic separations: (a) 2 mm; (b) 2.3 mm; (c) 2.5 mm; (d) 3 mm; (e) 3.5 mm.

The tail distributions with different separations are shown in Figure 6. The lifetime becomes bad at 2.3 mm separation. In terms of beam size, this separation corresponds to 7.7 σ. The designed separation is 3.5 mm, which is equivalent to 11 σ in horizontal

direction. The simulation results conclude that the current design is safe in terms of the parasitic crossings.

REFERENCES

[1]. T Chen, J. Irwin, and R. H. Siemann, Phys. Rev. E **49**, 2323 (1994)

[2]. F. M. Izrailev and I. B. Vasserman, Proceedings of the 7th All Union Conference on Charged Particle Accelerators, Dubna, USSR, 288 (1980). S. Krishnagopal and R. H. Siemann, Phys. Rev. D **41**, 2312 (1990)
R. H. Siemann, Frontiers of Particle Beams: Factories with e^+e^- Rings (Springer-Verlag, Lecture Notes in Physics, Volume 425 edited by M. Dienes, M. Month, B. Strasser and S. Turner), 327 (1994).

[3]. T Chen, J. Irwin, and R. H. Siemann, SLAC-PUB-96-7185(1996).

[4]. D. N. Shatilov and A. Zholents, Proc. of Part. Acc. Conf., Dallas, TX (1995), p9671.