SLAC-PUB-7643
September 1997

Leptoquarks: Pride and Prejudice *

THOMAS G. RIZZO
Stanford Linear Accelerator Center
Stanford University, Stanford, CA 94309

Abstract

Attempts to understand the recent observation of an excess of events in the neutral and charged current channels at high$-Q^2$ at HERA has provided an excellent example of how experiments at both low and high energies can be used to simultaneously constrain scenarios which predict new physics beyond the Standard Model. In this talk I will discuss this subject from the point of view of the construction of new models of leptoquarks.

Talk given at the *Workshop on Physics Beyond the Standard Model: Beyond the Desert: Accelerator and Nonaccelerator Approaches*, Ringberg Castle, Germany, June 8-14, 1997.

*Work supported by the Department of Energy, Contract DE-AC03-76SF00515

1 Introduction: Physics Beyond the Standard Model

It is a widely held belief that new physics must exist beyond that predicted by the Standard Model(SM) if for no other reason than that it leaves us with too many unanswered questions and too many free parameters. Just when, where and how new physics will make its first appearance has been–and continues to be–a matter of some speculation. In the past few years we have seen a number of potential new physics signatures vanish as either more statistics was accumulated or the data and analysis thereof improved. These signatures have each in their turn induced some excitement in the community with the hope that a new window beyond the SM was finally opening. Though later vanishing, these affects have each taught us something new about what kinds of models can be constructed, even if they were not necessarily realized in nature, given the ever-tightening constraints provided by experiment. Still, new physics is out there somewhere waiting to be discovered; it is only a question of looking. We should learn to expect the unexpected.

Searching for new physics is a multi-pronged attack on the unknown. While the production of a non-SM particle at a collider would be the most obvious and undeniable signature that one can imagine, the first sign of something new maybe more subtle. For example, one can imagine a significant deviation from SM expectations in a precision measurement, *e.g.*, the W mass or polarized forward-backward asymmetry for $b-$quarks, A_b. Instead, one might imagine the observation of a process forbidden by the SM, such as $\mu \to e\gamma$. However, as is well known, new physics rarely contributes to only one of these scenarios. For example, SUSY leads to new particle production at colliders, a potentially observable shift in the W mass and/or $\sin^2\theta_{eff}$ and can enhance many rare decay processes beyond their SM expectations. The same is of course true with other forms of new physics. In fact, it likely that once new physics is found all three types of experiments will be necessary to unravel its detailed nature. Attempts to understand the excess of events at high$-Q^2$ recently observed at HERA

in both the neutral current(NC) and charged current(CC) channels[1] provides just such an example of the strong interplay between the various new physics search scenarios and the constraints that arise from the three classes of experiments.

2 Leptoquark Model Requirements

If the HERA excesses are real and non-resonant then possible explanations include, *e.g.*, the presence of higher dimensional operators[2] signalling compositeness or exotic modifications in the parton densities at large x[3]; both these proposed scenarios face some very serious difficulties. Instead, if the excess is resonant a popular[4, 5] explanation is the $s-$channel production of a $\simeq 200-220$ GeV scalar(*i.e.*, spin-0) leptoquark(LQ) with fermion number(F) equal to zero–the subject of the present work. (Given the most recent results[6] it appears that the H1 excess in the NC channel is apparently clustered at 201 ± 5 GeV while that for ZEUS is at 219 ± 9 GeV. Whether this is reconcilable with a single resonance is still unknown.) How did we so quickly deduce the LQ spin and F quantum number from the data?

Any discussion of LQ models has been historically based on the classic work by Buchmüller, Rückl and Wyler(BRW)[7]. Those authors provided not only a set of assumptions under which consistent LQ models can be constructed but then classified them according to their possible spins and fermion number thus leading to the 10 states displayed in Table 1. These assumptions may be stated as follows:

(a) LQ couplings must be invariant with respect to the SM gauge interactions

(b) LQ interactions must be renormalizable

(c) LQs couple to only a single generation of SM fermions

(d) LQ couplings to fermions are chiral

(e) LQ couplings separately conserve Baryon and Lepton numbers

(f) LQs only couple to the SM fermions and gauge bosons

If we strictly adhere to these rules then the requirements of gauge invariance and renormalizability fix all of the spin-1 LQ couplings and thus its production cross section at the Tevatron[8] is simply a function of its mass. The possibility that such particles can exist in the mass range below approximately 350 GeV can then be excluded based on the direct searches by both CDF and D0[9]. (As noted by Blümlein[4], the introduction of non-renormalizable anomalous couplings for the LQ may allow us to somewhat soften this conclusion but spin-1 LQs are still found to be excluded in the range of interest for the HERA excess.) If the LQ is a scalar but is of the $F = 2$ type then we would have expected to see an event excess show up in the e^-p NC channel and not the e^+p channel as is the case. This is demonstrated in Table 2 which shows the event rate for each of the BRW scalar LQs assuming a Yukawa coupling strength of $\tilde{\lambda} = \lambda/e = 0.1$ and a mass of 200 GeV normalized to a luminosity of 100 pb^{-1}. Even with the great disparity in integrated luminosity collected by the experiments in both channels and allowing for the free adjustment of the strength of the Yukawa coupling we must conclude that the LQ is of the $F = 0$ type. We note from Table 1 that all $F = 0$ scalar LQs must have $B_\ell = 1$ and lie in $SU(2)_L$ doublets, implying that more than one type of LQ must exist. Given the recent strengthening of the Tevatron search reach and the possible CC excess at HERA, this poses a serious challenge to the scalar LQ interpretation, although not as serious as was found in the case of vector LQs. Using the next-to-leading order cross section formulae of Krämer *et al.*[10], the 95% CL lower limit on the mass of a $B_\ell = 1$ scalar LQ is found by D0 to be 225 GeV. D0 has also performed a combined search for first generation leptoquarks by using the $eejj$, $e\nu jj$ and $\nu\nu jj$ channels. For fixed values of

Table 1: Quantum numbers and fermionic coupling of the leptoquark states. No distinction is made between the representation and its conjugate. B_ℓ is the branching fraction of the LQ into the ej final state and Q is its electric charge.

	Leptoquark	SU(5) Rep	Q	Coupling	B_ℓ
Scalars					
$F=-2$	S_{1L}	**5**	1/3	$\lambda_L\,(e^+\bar{u}),\ \lambda_L\,(\bar{\nu}\bar{d})$	1/2
	S_{1R}	**5**	1/3	$\lambda_R\,(e^+\bar{u})$	1
	$\widetilde{S}_{1R}$	**45**	4/3	$\lambda_R\,(e^+\bar{d})$	1
	S_{3L}	**45**	4/3	$-\sqrt{2}\lambda_L\,(e^+\bar{d})$	1
			1/3	$-\lambda_L\,(e^+\bar{u}),\ -\lambda_L\,(\bar{\nu}\bar{d})$	1/2
			−2/3	$\sqrt{2}\lambda_L\,(\bar{\nu}\bar{u})$	0
$F=0$	R_{2L}	**45**	5/3	$\lambda_L\,(e^+u)$	1
			2/3	$\lambda_L\,(\bar{\nu}u)$	0
	R_{2R}	**45**	5/3	$\lambda_R\,(e^+u)$	1
			2/3	$-\lambda_R\,(e^+d)$	1
	$\widetilde{R}_{2L}$	**10/15**	2/3	$\lambda_L\,(e^+d)$	1
			−1/3	$\lambda_L\,(\bar{\nu}d)$	0
Vectors					
$F=-2$	V_{2L}	**24**	4/3	$\lambda_L\,(e^+\bar{d})$	1
			1/3	$\lambda_L\,(\bar{\nu}\bar{d})$	0
	V_{2R}	**24**	4/3	$\lambda_R\,(e^+\bar{d})$	1
			1/3	$\lambda_R\,(e^+\bar{u})$	1
	$\widetilde{V}_{2L}$	**10/15**	1/3	$\lambda_L\,(e^+\bar{u})$	1
			−2/3	$\lambda_L\,(\bar{\nu}\bar{u})$	0
$F=0$	U_{1L}	**10**	2/3	$\lambda_L\,(e^+d),\ \lambda_L\,(\bar{\nu}u)$	1/2
	U_{1R}	**10**	2/3	$\lambda_R\,(e^+\bar{d})$	1
	$\widetilde{U}_{1R}$	**75**	5/3	$\lambda_R\,(e^+u)$	1
	U_{3L}	**40**	5/3	$\sqrt{2}\lambda_L\,(e^+u)$	1
			2/3	$-\lambda_L\,(e^+d),\ \lambda_L\,(\bar{\nu}u)$	1/2
			−1/3	$\sqrt{2}\lambda_L\,(\bar{\nu}d)$	0

Table 2: Expected number of events per $100^{-1}pb$ for each electron charge and state of polarization for a 200 GeV scalar leptoquark at HERA assuming $0.4 < y < 1$, $\tilde{\lambda} = 0.1$, and an electron-jet invariant mass $M_{ej} = 200 \pm 20$ GeV. These results have been smeared with a detector resolution of 5% in M_{ej}.

Leptoquark	N_L^-	N_R^-	N_L^+	N_R^+
SM background	51.7	28.7	9.98	20.0
S_{1L}	121.	28.7	9.98	20.4
S_{1R}	51.7	167.	10.8	20.0
$\widetilde{S}_{1R}$	51.7	63.0	11.5	20.0
S_{3L}	190.	28.7	9.98	23.5
R_{2L}	52.4	28.7	9.98	158.
R_{2R}	51.7	29.4	148.	20.0
$\widetilde{R}_{2L}$	53.2	28.7	9.98	54.4

the leptoquark mass below 225 GeV, these search constraints can be used to place an upper limit on B_ℓ. For M_{LQ}=200(210,220) GeV, D0 obtains the constraints $B_\ell \leq 0.45(0.62, 0.84)$ at 95% CL. Of course if CDF and D0 combine their searches in the future, then the 225 GeV bound may rise to $\simeq 240$ GeV, in which case even stronger upper bounds on B_ℓ will be obtained. Allowing the LQ to have decays into the νj final state with a reasonable branching fraction would solve this problem and would yield the desired CC signal at HERA. However the models in Table 1 do not allow for this possibility.

How do we interpret these conflicting demands? It is clear that the BRW structure must be too restrictive and so conditions (a)-(f) must be critically re-examined. While the assumptions of gauge invariance and renormalizability are unquestionable requirements of LQ model building, it is possible that the other conditions one usually imposes are much too strong–unless they are specifically demanded by data. This observation implies that for

LQs to be experimentally accessible now, or anytime soon, their couplings to SM fermions must be essentially purely chiral and must also separately conserve both Baryon and Lepton numbers. The condition that LQs couple to only a single SM generation is surely convenient by way of avoiding the numerous low energy flavor changing neutral current constraints[11] but is far from natural in the mass eigenstate basis. A short analysis indicates[5] that the natural imposition of this condition in the original weak basis for the first generation LQ and then allowing for CKM-like intergenerational mixing does not obviously get us into any trouble with experimental constraints especially in lepton generation number is at least approximately conserved. However, this does not give us the flexibility we need to avoid the Tevatron bounds or to induce an excess in the CC channel. Clearly then, to obtain a new class of LQ models the LQs themselves must be free to couple to more than just the SM fermions and gauge fields. Note that assumption (f) effectively requires that the LQ be the only new component added to the SM particle spectrum which seems quite unlikely in any realistic model; this assumption must be dropped.

What kind of LQ interaction do we want? In order to satisfy the HERA and Tevatron constraints it is clear that we need to have an $F = 0$ scalar LQ as before, preferably an isosinglet so that we do not have several LQ states of various masses to worry about, but now with an effective coupling to SM fermions such as

$$\mathcal{L}_{wanted} = [\lambda_u \nu u^c + \lambda_d e d^c] \cdot LQ + h.c.\,, \tag{1}$$

with comparable values of the effective Yukawa couplings λ_u and λ_d thus fixing the LQ's electric charge, $Q(LQ) = \pm 2/3$. An alternative possibility, allowing for either Dirac neutrinos or a ν^c which is light and appears as missing p_T in a HERA or Tevatron detector, is the interaction

$$\mathcal{L}'_{wanted} = [\lambda'_u \nu^c u + \lambda'_d e^c d] \cdot LQ' + h.c.\,. \tag{2}$$

[It is important for later analyses to note that we cannot have these two interactions si-

multaneously as we would then strongly violate assumption (d).] It is easy to see that in either case the LQs of any other charge assignment cannot simultaneously couple to both ej and νj as is required by the HERA and Tevatron data. Unfortunately, either of the above Lagrangians as they stand violate assumption (a) in that they are not gauge invariant with respect to $SU(2)_L$. This implies that the desired Yukawa couplings are only effective ones and must arrived at from some more fundamental theory. Even if we are successful in obtaining one or both of these Lagrangians, is it clear that we can find values of λ_u and λ_d which are compatible with all of the data?

3 Constraining Leptoquark Couplings

What are the existing constraints on LQ Yukawa couplings? If the LQ has only the couplings described by one of the above Lagrangians then we can, *e.g.*, trade in λ_u for $B_\ell = \lambda_d^2/(\lambda_d^2+\lambda_u^2)$, since we are assuming that the LQ has no other decay modes. As discussed above, the Tevatron searches place a λ_d-independent constraint on B_ℓ for any fixed value of the LQ mass. Similarly, as promised, low energy measurements play an important role here as well. The recent constraints on the size of any allowed deviation of the weak charge from its SM value in Atomic Parity Violation(APV) in Cesium[12], $\Delta Q_W = 1.09 \pm 0.93$, places B_ℓ-independent bounds on λ_d[13] for fixed M_{LQ}. Similarly, $\mu - e$ universality in π decay, expressed through the ratio $R = \Gamma(\pi \to e\nu)/\Gamma(\pi \to \mu\nu) = 0.9966 \pm 0.0030$, constrains the product of couplings $\lambda_u\lambda_d$[14]. The observed rate of NC events at HERA itself essentially constrains instead the product $\lambda_d^2 B_\ell$; in the later case QCD and efficiency corrections are quite important[15]. Putting all of these together defines an approximate allowed region in the $B_\ell - \tilde{\lambda}_d$ plane shown in Fig.1 for different values of M_{LQ}. Here, we define $\tilde{\lambda} = \lambda/e$, with e the conventional proton charge. (This scaling of the coupling to e follows earlier

tradition[16].) We note that these allowed regions are compatible with the cross section required to explain the HERA CC excess.

There are other means to probe LQ couplings but they are somewhat more indirect. There are several ways in which LQs may make their presence known in e^+e^- collisions[5, 17]. At center of mass energies below the threshold for pair production, the existence of LQs can lead to deviations in both the cross section and angular distributions for $e^+e^- \to q\bar{q}$. This may be particularly relevant when $\sqrt{s}$ is comparable to the leptoquark mass as would be the case at LEP II if a 200-220 GeV LQ did exist. The origin of these modifications is due to the $t-$channel LQ exchange and is thus proportional in amplitude to the square of the unknown Yukawa coupling. However, in [5] it was shown that even with large data samples it is unlikely that LEP II will have the required sensitivity to probe couplings as small as those suggested by the HERA data as shown in Fig.2. The OPAL Collaboration[18] has recently performed this analysis with real data at somewhat lower energies but with comparable results.

Turning this process around, we can imagine that the Drell-Yan production of either e^+e^- or $e^\pm\nu$ channel at the Tevatron Main Injector may show some sensitivity[19] to LQ exchange in the $t-$channel. In the e^+e^- case the observables are the invariant mass distribution and the forward-backward asymmetry. In the $e^\pm\nu$ channel, the corresponding observables are the transverse mass distribution on the electron rapidity asymmetry. Figs. 3 and 4 show the result of these considerations; neither channel has the sensitivity to probe Yukawas in the desired range if only 2 fb^{-1} of luminosity is available.

Leptoquarks can also be produced singly at hadron colliders through their Yukawa couplings. Compared to pair production, this mechanism has the advantage of a larger amount of available phase space, but has the disadvantage in that it is directly proportional

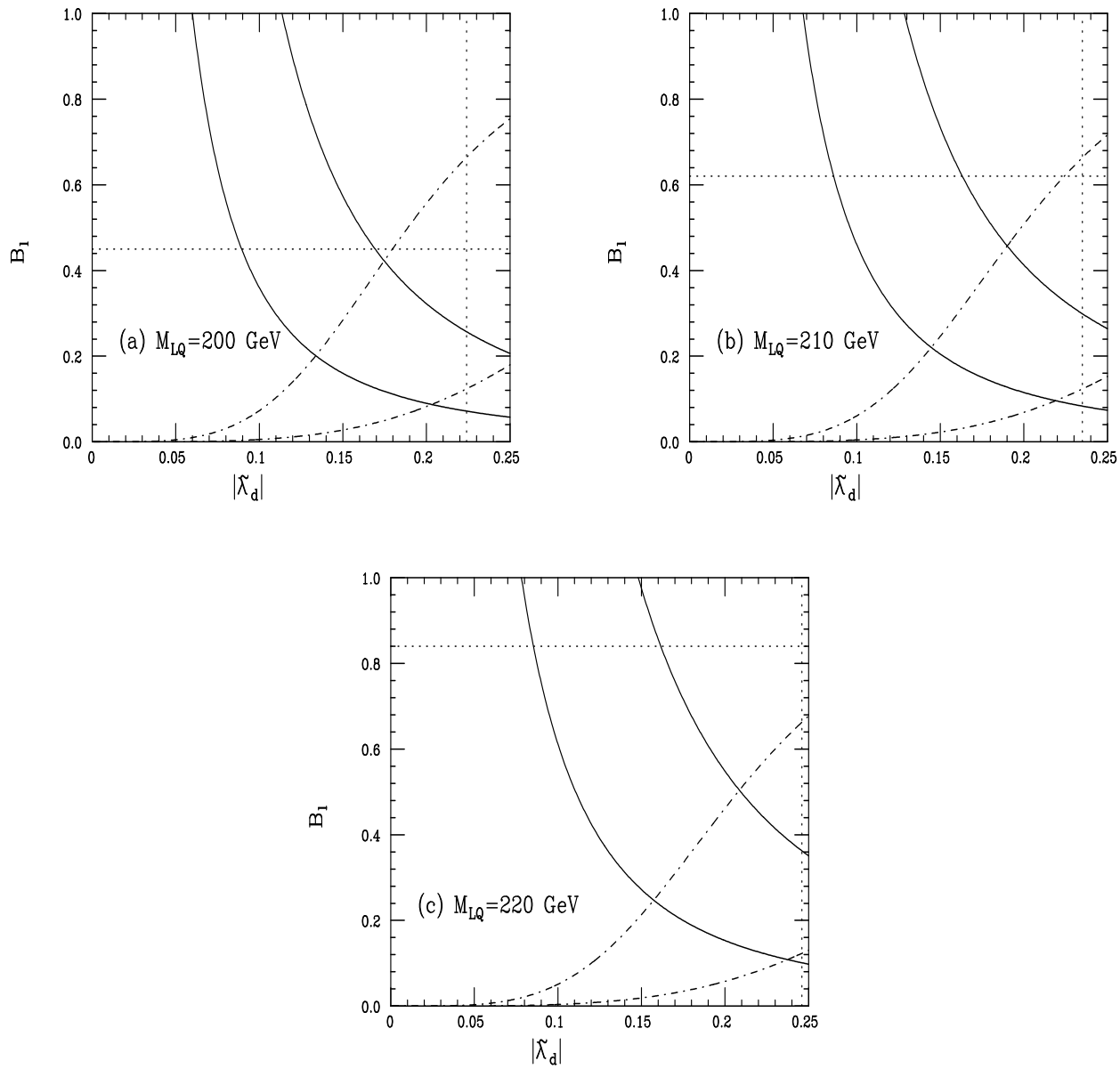

Figure 1: Allowed parameter space region in the $B_\ell - \tilde{\lambda}_d$ plane for a LQ with mass 200 GeV(top left), 210 GeV(top right) or 220 GeV(bottom). The region allowed by the direct Tevatron searches is below the horizontal dotted line while that allowed by APV data in Cesium is to the left of the vertical dotted line. The region inside the solid band is required to explain the HERA excess in the NC channel. The region above the dash-dotted curve is allowed by π decay universality: the lower(upper) curve corresponds to the case where $\lambda_u \lambda_d > (<) 0$.

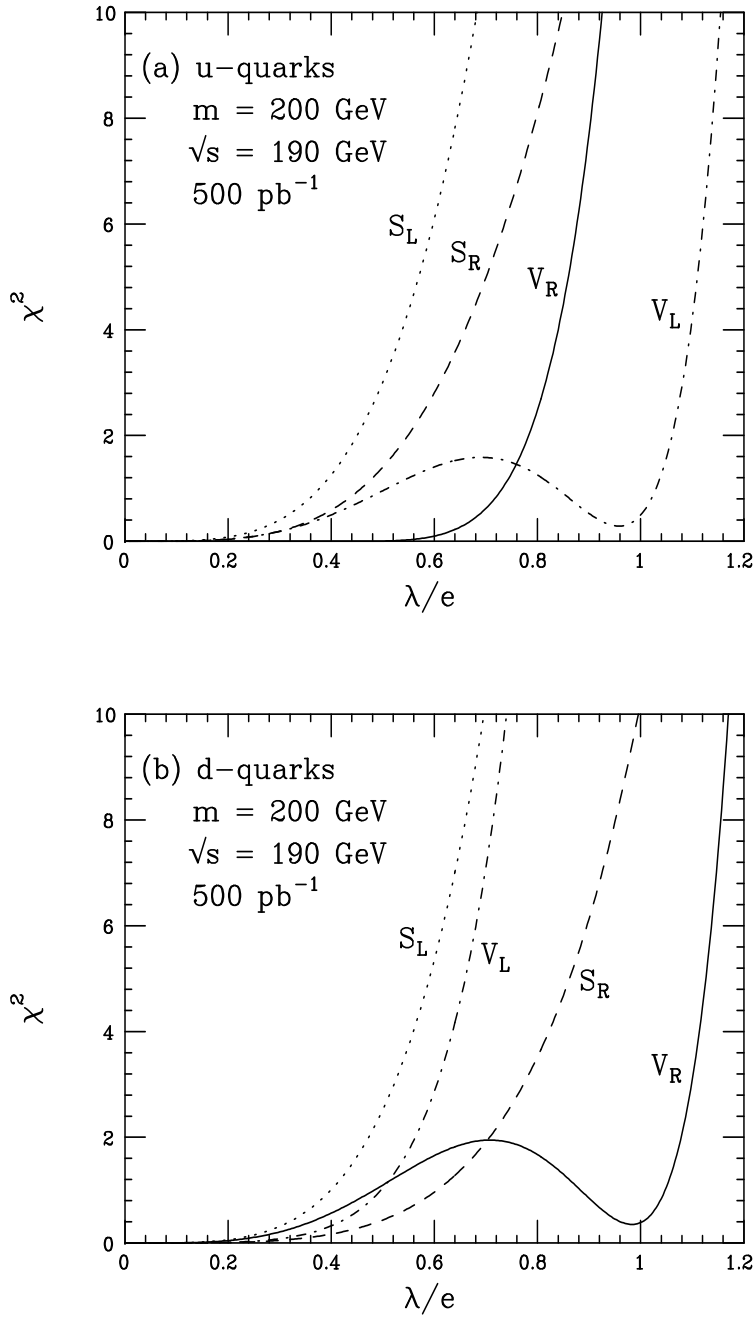

Figure 2: χ^2 fits to the SM angular distribution for $e^+e^- \to q\bar{q}$ at 190 GeV including the effects of a 200 GeV LQ coupling to (a) u- or (b) d-quarks. S or V labels spin-0 or spin-1 type LQs. In both cases the dotted(dashed) curve corresponds to a scalar LQ with a left(right)-handed coupling while the dash-dotted(solid) curve corresponds to the vector LQ case with left(right)-handed couplings.

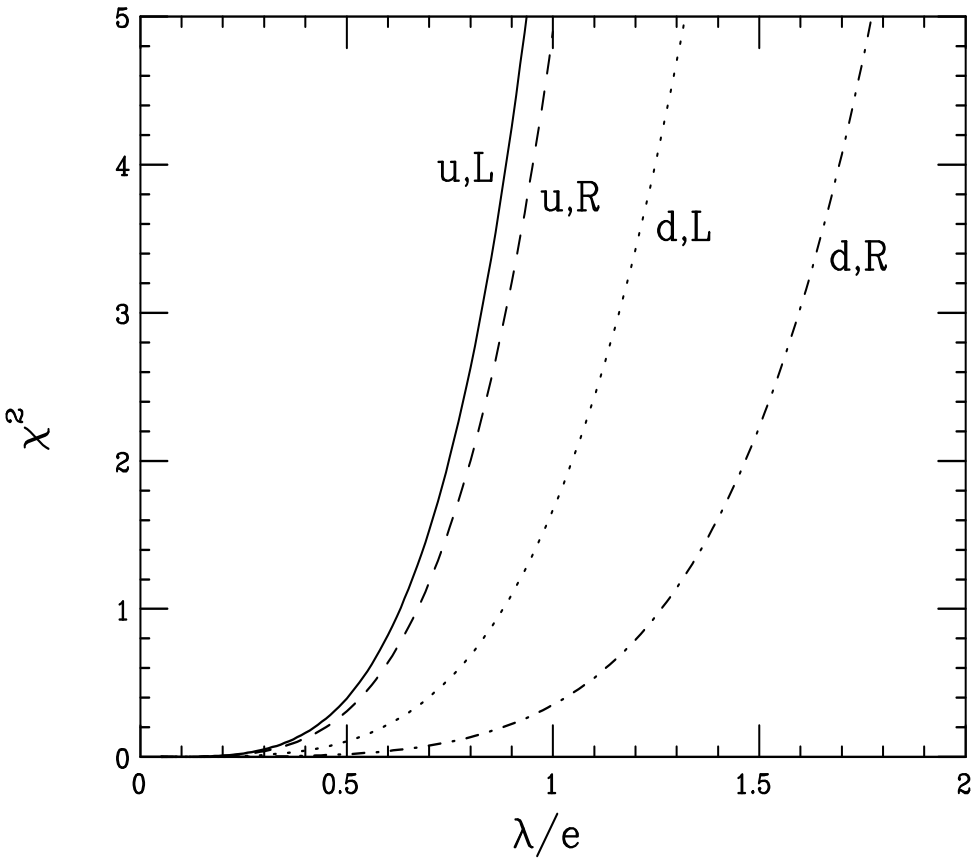

Figure 3: χ^2 fits to Drell-Yan production at the Tevatron Main Injector assuming a luminosity of 2 fb^{-1} including the effects of a 200 GeV scalar leptoquark for each type of leptoquark coupling as labeled.

to the small Yukawa coupling. For a general 200 GeV scalar leptoquark with a coupling strength of $\tilde{\lambda} = 0.15$ calculations show that we would obtain approximately $\sim 88\,,196$ events from $gd + g\bar{d}\,, gu + g\bar{u}$ fusion, respectively, assuming 10 fb^{-1} of integrated luminosity at the Main Injector/TeV33. This event rate should be marginally sufficient to provide a very rough determination of the value of Yukawa coupling $\tilde{\lambda}_d$ for the models of specific interest.

Additional LQ coupling information can possibly be obtained from the sum of the squares of the first row of the CKM matrix, $\sum_i |V_{ui}|^2$. This involves combining experiments at low, medium and high energies and it an essential test of quark-lepton universality and CKM unitarity. In the SM this sum is, of course, unity, but LQ exchange can yield either an apparent upward or downward shift in the extracted value of $|V_{ud}|$:

$$|V_{ud}|^2_{eff} \simeq |V_{ud}|^2_{true} - 1.52 \times 10^{-3} \left(\frac{200\ GeV}{M_{LQ}}\right)^2 \left(\frac{\tilde{\lambda}_u}{0.15}\right) \left(\frac{\tilde{\lambda}_d}{0.15}\right) , \tag{3}$$

so that it would appear experimentally as if a unitarity violation were occurring. Interestingly, the value of the above sum has recently been discussed by Buras[20], who reports $\sum_i |V_{ui}|^2 = 0.9972 \pm 0.0013$, which is more than 2σ below the SM expectation. Clearly, if $\tilde{\lambda}_u \tilde{\lambda}_d > 0$, the LQ exchange provides one possible additional contribution which, for $\tilde{\lambda}_u = \tilde{\lambda}_d = 0.15$ and $M_{LQ} = 200$ GeV, would increase the sum to 0.9987, now only 1σ low. This "same sign" possibility is clearly preferred by the combined set of present data as shown in Fig.2. This situation requires watching in the future.

A last possibility is the observation of LQ exchange through radiative corrections. Unfortunately, it has been shown[5] that LQs rapidly decouple and give only tiny contributions to the oblique parameters[21]. In addition it has also been shown that LQ exchange does not significantly modify Z-pole physics[22] through vertex corrections.

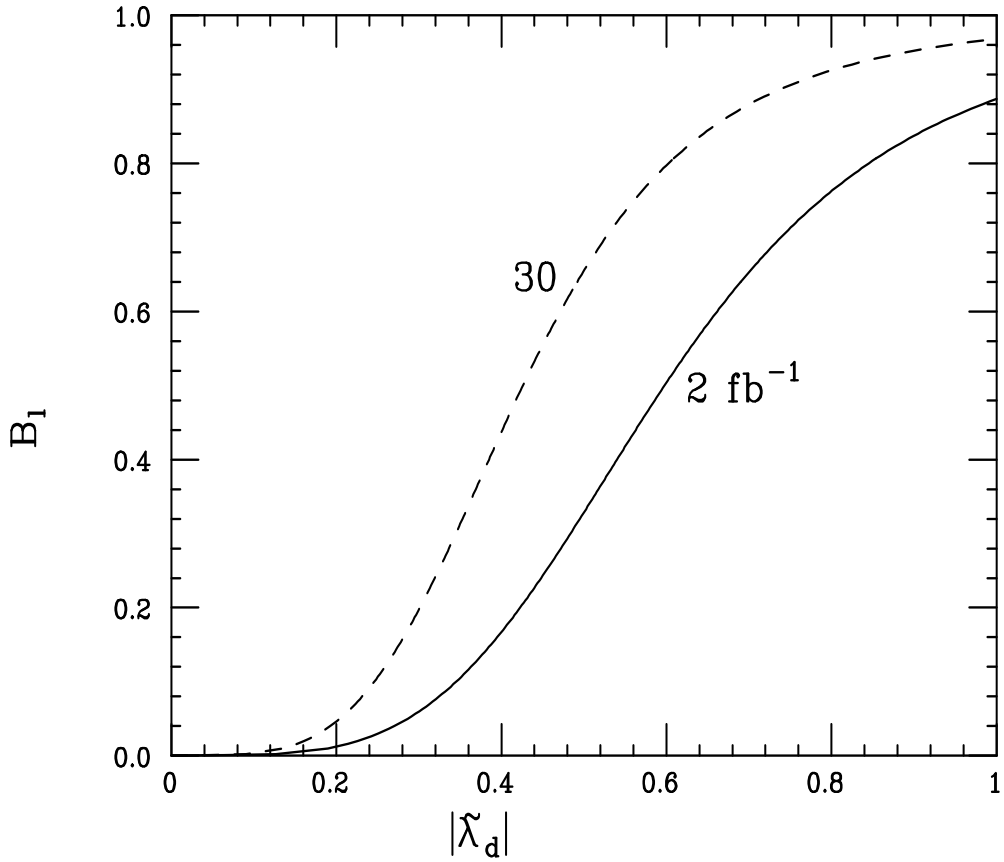

Figure 4: 95% CL bound on B_ℓ as a function of $\tilde{\lambda}_d$ from a fit to both the M_T distribution and lepton rapidity asymmetry, $A(\eta_\ell)$, at the Tevatron Main Injector for two integrated luminosities as indicated. The area below and to the right of the curves are excluded, a LQ of mass 200 GeV is assumed.

4 New Leptoquark Model Building

We now turn to dealing with the construction of models that lead to one of the Lagrangians above. Given the fixed gauge structure of the SM the most likely new interactions that LQs may possess are with the Higgs field(s) responsible for spontaneous symmetry breaking and with new vector-like fermions that are a common feature in many extensions of the SM. In this lecture we will consider and classify all models wherein heavy vector-like fermions(VLFs) are used to generate the effective interactions $\mathcal{L}_{wanted}$ or $\mathcal{L}'_{wanted}$ at low energies. The emphasis of our approach will be to keep the VLFs as active participants in our models and not auxiliary devices to produce the desired coupling structure. We will assume that the fundamental LQ coupling is that between a VLF and a SM fermion. a LQ couples to both a VLF and that ordinary SSB induces a a mixing between the two sets of fermions. SSB is thus the true source of the desired LQ interactions and explains how effective interactions can arise that are not obviously gauge invariant. The small size of the effective Yukawa couplings in the above Lagrangians, $\mathcal{L}_{wanted}$ or $\mathcal{L}'_{wanted}$, will then be subsequently explained by the same mechanism that produces the ordinary-exotic fermion mixing and sets the scale of the VLF masses in the TeV region. We note that the use of VLFs in this role is particularly suitable since in their unmixed state they make essentially no contribution to the oblique parameters[21], they are automatically anomaly free and they can have bare mass terms which are SM gauge invariant. (Alternatively their masses can be generated by the vacuum expectation value of a SM singlet Higgs field.)

To proceed with the analysis, we first construct the six 'skeleton' models that are obtainable by simply coupling one of the usual SM fermion representations, $(L, Q, u^c, d^c, e^c, \nu^c)$, with an appropriate VLF, X_i(or X_i^c), and the LQ field. [We use $X_i(X_i^c)$ to denote the VLF fields with $F > (<)0$.] Note that we have allowed for the possibility if right-handed neutri-

nos. To this we must add the bare mass term for X_i as well as any gauge invariant terms that can be constructed using the remaining SM fermion fields, X_i^c(or X_i), and the SM Higgs doublet fields H and H^c. In each case gauge invariance tells us the the quantum numbers of the VLFs under the assumption that they are either in singlets or in fundamental representations under $SU(2)_L$ and $SU(3)_C$. We strictly adhere to BRW constraints (a)-(e) in forming our constructions. These six 'skeletons' are:

$$
\begin{aligned}
\mathcal{L}_A &= \lambda_A L X_1^c \cdot LQ + a_u X_1 u^c H + a_d X_1 d^c H^c - M_1 X_1 X_1^c - M_Q' Q X_1^c \,, \\
\mathcal{L}_B &= \lambda_B Q X_2^c \cdot LQ + a_e X_2 e^c H^c + a_\nu X_2 \nu^c H - M_2 X_2 X_2^c - M_L' L X_2^c \,, \\
\mathcal{L}_C &= \lambda_C X_3 u^c \cdot LQ + a_1 L X_3^c H - M_3 X_3 X_3^c - M_N' N \nu^c \,, \\
\mathcal{L}_D &= \lambda_D X_4 d^c \cdot LQ + a_2 L X_4^c H^c - M_4 X_4 X_4^c - M_E' E e^c \,, \\
\mathcal{L}_E &= \lambda_E X_5 e^c \cdot LQ + a_3 Q X_5^c H^c - M_5 X_5 X_5^c - M_D' D d^c \,, \\
\mathcal{L}_F &= \lambda_F X_6 \nu^c \cdot LQ + a_4 Q X_6^c H - M_6 X_6 X_6^c - M_U' U u^c \,,
\end{aligned}
\tag{4}
$$

We will assume that all of the Yukawa couplings that appear in these 'skeletons' are of order unity and that no fine-tuning is present. These constructs have a number of obviously desirable features but they do not yet have all the necessary ingredients. For example, the LQ in 'skeleton' C(D) only couples to the $u(d)$ quark while that in 'skeleton' E(F) only couples to $e(\nu)$. For 'skeletons' A and B we see that the LQs couple to both u, d and e, ν, respectively. The solution to this problem is to combine the various 'skeletons' into full models that have all of the desired couplings. This procedure is straightforward but when doing so we must take care not to violate the assumption that the LQ couplings are chiral. Given this very strong constraint, the entire list of models that can be constructed in this fashion are only ten in number: A, B, CD, EF, AC, AD, ACD, BE, BF and BEF. The combination of letters symbolizes that we add the respective Lagrangians and identify the LQ field as common. We note that models A, CD, AC, AD and ACD produce the interaction $\mathcal{L}_{wanted}$, while models B,

EF, BE, BF and BEF produce instead $\mathcal{L}'_{wanted}$. As we will see below in each case the color, isospin and electric charge quantum numbers are completely fixed by gauge invariance and the assumption that the LQ is a $|Q| = 2/3$ isosinglet with $F = 0$. It is very important to remember that the five models leading to $\mathcal{L}'_{wanted}$ would be *excluded* if the neutrino is not a Dirac field of if ν^c cannot appear as missing energy or p_T in a detector.

Having said all this we have yet to explicitly see how even one of these models works so we now examine model A in detail. Here, we have coupled an exotic fermion, denoted as X_1, to L plus a leptoquark. In this case gauge invariance requires that X_1 be an isodoublet, with member charges of $2/3, -1/3$ since the leptoquark charge and fermion number are fixed, as well as an $SU(3)_C$ triplet. We can thus write $X_1^T = (U^0, D^0)$, where the superscript denotes the weak eigenstate fields. When H and H^c receive vevs (which we denote by v and v^c, respectively), the $a_{u,d}$ terms in the above Lagrangian induce off-diagonal couplings in both the $Q = -1/3$ and $Q = 2/3$ quark mass matrices. Neglecting the u- and d-quark masses, these are given in the $\bar{\psi}^0_L M \psi^0_R$ weak eigenstate basis by

$$\bar{\psi}^0_L M_u \psi^0_R = (\bar{u}^0, \bar{U}^0)_L \begin{pmatrix} 0 & -M'_Q \\ a_u v & -M_1 \end{pmatrix} \begin{pmatrix} u^0 \\ U^0 \end{pmatrix}_R , \tag{5}$$

$$\bar{\psi}^0_L M_d \psi^0_R = (\bar{d}^0, \bar{D}^0)_L \begin{pmatrix} 0 & -M'_Q \\ a_d v^c & -M_1 \end{pmatrix} \begin{pmatrix} d^0 \\ D^0 \end{pmatrix}_R . \tag{6}$$

Both $M_{u,d}$ can be diagonalized by a bi-unitary transformation which becomes simply bi-orthogonal under the assumption that the elements of $M_{u,d}$ are real, resulting in the diagonal mass matrices $M^{diag}_{u,d} = U_L(u,d) M_{u,d} U_R(u,d)^\dagger$. Since $U_{L,R}(u,d)$ are simple 2×2 rotations they can each be parameterized by a single angle $\theta^{u,d}_{L,R}$. Assuming that both M_1, M'_Q are large in comparison to either $a_u v$ or $a_d v^c$ we find $\theta^{u,d}_R \simeq a_{u,d} v(v^c) M_1/(M_1^2 + M'^2_Q)$. With

M_1, M'_Q of order 1 TeV, v, v^c of order 100 GeV and $a_{u,d}$ of order unity this implies that $\theta_R^{u,d} \simeq 0.05$. Writing $U^0 \simeq U + \theta_R^u u$ in terms of the mass eigenstate fields, and similarly for D^0, the interaction involving the SM fermions and the leptoquark thus becomes

$$\mathcal{L}_{light} = \left[\left(\frac{\lambda_A a_u v M_1}{(M_1^2 + M_Q'^2)} \right) \ \nu u^c + \left(\frac{\lambda_A a_d v^c M_1}{(M_1^2 + M_Q'^2)} \right) \ e d^c \right] \cdot LQ + h.c.\,, \tag{7}$$

which is the exact form we desired in Eqn. (1). For λ_A again of order unity this naturally leads to a reasonable relative branching fraction for the $LQ \to \nu j$ decay mode, and gives acceptable values for $\lambda_{u,d}$ in Eqn. (1) for M_1, M'_Q in the TeV range. Note that $\theta_R^{u,d} \neq 0$ leads to a modification of both the u and d quark couplings to the Z and induces W-mediated right-handed charged current interactions as well.

What about $\theta_L^{u,d}$? $\theta_L^{u,d} \neq 0$ does not contribute to the LQ couplings or influence Z couplings of u, d since the left-handed SM fermions and the VLFs have the same quantum numbers. With M_1 and M'_Q of comparable size both $\theta_L^{u,d}$ are found to be large and of almost identical magnitude. However, modifications to the left-handed CC couplings of u and d to the W are only sensitive to the deviation $\Delta = 1 - \cos(\theta_L^u - \theta_L^d)$. Here, for $M_1 = M'_Q$, the difference $\theta_L^u - \theta_L^d \approx (a_u^2 v^2 - a_d^2 v^{c2})/4M_1^2$ is found to be very small, of order $\sim \theta_R^2 \simeq (0.05)^2$. This implies that Δ itself is of only order 10^{-5} or less–practically invisible.

The other models above work more or less in a similar fashion except that in most cases mixing is taking place between a number of different SM fermions and their VLF partners. The effective couplings in Eqns. 1 and 2 then derive from more than a single source. For completeness Table 3 identifies the VLFs which are present in each of these models.

Model	Vector-like Fermions
A	$\begin{pmatrix} U \\ D \end{pmatrix}_{L,R}$
CD	$N_{L,R}; E_{L,R}$
AC	$\begin{pmatrix} U \\ D \end{pmatrix}_{L,R}; N_{L,R}$
AD	$\begin{pmatrix} U \\ D \end{pmatrix}_{L,R}; E_{L,R}$
ACD	$\begin{pmatrix} U \\ D \end{pmatrix}_{L,R}; N_{L,R}; E_{L,R}$
B	$\begin{pmatrix} N \\ E \end{pmatrix}_{L,R}$
EF	$U_{L,R}; D_{L,R}$
BE	$\begin{pmatrix} N \\ E \end{pmatrix}_{L,R}; D_{L,R}$
BF	$\begin{pmatrix} N \\ E \end{pmatrix}_{L,R}; U_{L,R}$
BEF	$\begin{pmatrix} N \\ E \end{pmatrix}_{L,R}; U_{L,R}; D_{L,R}$

Table 3: Listing of models and the new vector-like fermions which are contained in them.

5 Tests

To directly test the proposed models we must look for new physics signatures *beyond* those suggested by the LQ interactions described Eqns. 1 and 2. One possibility is to probe for the additional interactions between the VLFs and the LQ; this is obviously difficult since the VLFs are so massive. A second possibility is to directly look for the influences of the VLFs themselves. For the time being this must be an indirect search since the LHC will be required to directly produce the VLFs in the range of interest to us here. Once the LHC is available, however, a 1 TeV color triplet will produce 1000 events/yr even with a luminosity of 10 fb^{-1}.

In the first category a potential new process of interest is the pair production of like sign LQs at the Tevatron through $t-$ and $u-$channel N exchange (in those models where it is present) when N is a Majorana field: $uu \to 2LQ$. This rate for this process goes as the fourth power of the λ_B or λ_C Yukawa coupling but these are assumed to be of order unity so that potentially large cross sections are obtainable. One finds the subprocess cross section to be

$$\frac{d\sigma}{d\hat{t}} = \frac{\lambda^4}{64\pi\hat{s}} \left[\frac{M_N(\hat{t} + \hat{u} - 2M_N^2)}{(\hat{t} - M_N^2)(\hat{u} - M_N^2)} \right]^2 , \tag{8}$$

where M_N is the mass of the N. Note that as $M_N \to 0$ the rate vanishes as one might expect for a Majorana fermion induced process. The cross section for this reaction at the Tevatron Main Injector for $\lambda = 1$ and $M_{LQ} = 200$ GeV is approximately 50 fb for M_N=1 TeV and falls off quickly with increasing as M_N increases. Since the signature for this process is 2 jets plus like-sign leptons there is little SM backgrounds and so it may be observable during Run II.

The best indirect tests for the presence of VLFs which mix with the conventional fermions are searches for new physics associated with deviations in couplings from SM

expectations[16, 23]. Two of the best tests here are our old friends quark-lepton universality in the guise of V_{ud} and the leptonic decays of the π discussed above. To clarify this point we note that in models where, *e.g.*, the u and d mix with isodoublet VLFs the W couples as $\bar{u}d_L \cos(\theta_L^u - \theta_L^d) + \bar{u}d_R \sin\theta_R^u \sin\theta_R^d$ whereas if the VLFs are isosinglets the corresponding coupling is $\bar{u}d_L \cos\theta_L^u \cos\theta_L^d$. (The corresponding couplings can be also written down for the case of leptonic mixing.) We note also the general feature we find is that in models where the VLFs are in isodoublets $\theta_R' s \sim 0.05$ and differences in $\theta_L' s \sim (0.05)^2$, whereas the converse is true when the relevant VLFs are isosinglets.

Leptonic mixing will never show up as a shift in the value of $|V_u d|^2$ since the modification in the amplitude occurs not only in the process $n \to pe\bar{\nu}$ but also in μ decay so that it is absorbed into the definition of G_F. On the otherhand if quark mixing occurs $|V_u d|^2$ will experience an apparent small shift(to leading order in the mixing angles) $\sim -(\theta_L^u - \theta_L^d)^2 + (\theta_R^u \theta_R^d)^2$ in models with isodoublet VLFs and $\sim -(\theta_L^u)^2 - (\theta_L^d)^2$ in the isosinglet VLF case. For the ratio R in π decay any modification of the hadronic matrix element will factor out so that there is no sensitivity to quark mixing. However, leptonic mixing no longer factorizes and we find a shift in R by an amount $\sim -(\theta_L^\nu - \theta_L^e)^2 - (\theta_R^\nu \theta_R^e)^2$ in the case of isodoublet mixing and $\sim -(\theta_L^\nu)^2 - (\theta_L^e)^2$ for isosinglets. Recall that isodoublet leptonic mixing is only viable in models where the neutrino is Dirac or the right-handed neutrino appears as an ordinary neutrino. In these same isodoublet models, the presence of both left- and right-handed CC couplings and heavy VLFs can lead to a contribution to the $g-2$ of the electron and ν which are typically both or order $a\ few \cdot 10^{-11}$, neither of which are far from the present level of sensitivity.

The mixing of the SM fermions with the VLF modify their couplings to the Z. These are difficult to observe particularly in the case of quarks due to the small size of the effect

and QCD correction uncertainties. In the case of leptonic mixing there is not only the contribution due to mixing but there is an overall normalization change in the couplings due to our redefinition of G_F. This mixing can lead to a shift in $Z \to e^+e^-$ width by $\simeq 0.2$ MeV and an apparent shift in $\sin^2\theta_{eff}$ from the asymmetries of $\simeq 0.0006$. Again, shifts of this size are near the present limit of experimental sensitivity. In a similar manner SM expectations for Q_W in APV measurements may also be modified if SM mixing with the VLF occurs. However, in this case it is easy to show that the fractional change in Q_W due to these effects is only at the level of $\sim 10^{-3}$.

6 Grand Unification with Leptoquarks

If LQs are indeed real and we also believe that there is experimental evidence for coupling constant unification then we must begin to examine schemes which contain both ingredients as pointed out in[5]. In the scenarios presented here the SM quantum numbers of the LQ are fixed but new VLFs have now been introduced as well, all of which will alter the usual RGE analysis of the running couplings.

Before discussing SUSY models we note with some curiosity that coupling unification *can* occur in LQ models containing exotic fermions even if SUSY is not introduced as was shown many years ago in [24, 25]. Of course in the work of Murayama and Yanagida [24], the LQ was an isodoublet and one of the particular models on the BRW list, now excluded by the combined HERA and Tevatron data. In the scenarios presented above the LQ is now a $Q = 2/3$ isosinglet so that the Murayama and Yanagida analysis does immediately apply. Fortunately, we see from the results of Ref. [25] that a second possibility does exist for just this case: one adds to the SM spectrum the LQ and its conjugate as well as a vector-like pair of color-triplet, isodoublets together with the field H^c. This is the just particle content

of the model A. To verify and update this analysis, let us assume for simplicity that all the new matter fields are introduced at the weak scale and take $\sin^2\theta_w = 0.2315$ as input to a two-loop RGE analysis. We then obtain the predictions that coupling unification occurs at 3.5×10^{15} GeV and $\alpha_s(M_Z)$ is predicted to be 0.118. If unification does indeed occur we can estimate the proton lifetime[26] to be $\tau_p = 1.6 \times 10^{34\pm1}$ years, safely above current constraints[14]. We find this situation to be rather intriguing and we leave it to the reader to further ponder.

Of course there are other reasons to introduce SUSY beyond that of coupling constant unification. This subject has been discussed at some length in [5] from which we extract several important observations: (i) To *trivially* preserve the successful unification of the SUSY-SM, only complete $SU(5)$ representations can be added to the MSSM spectrum. As is well-known, the addition of extra matter superfields in complete $SU(5)$ representations delays unification and brings the GUT scale closer to the string scale. Of course, there still remains the rather unnatural possibility of adding incomplete, but 'wisely chosen', split representations. Employing split representations certainly allows for more flexibility at the price of naturalness but still requires us to choose sets of $SU(3)_C \times SU(2)_L \times U(1)_Y$ representations which will maintain asymptotic freedom and perturbative unification. An example of this rather bizarre scenario is the possibility of adding a $(2,3)(1/6)$ from a $\mathbf{15}$ and a $(1,1)(1) \oplus (1,\bar{3})(-2/3)$ from a $\mathbf{10}$ to the low energy spectrum[5]. Here the notation refers to the $(SU(3)_C, SU(2)_L)(Y/2)$ quantum numbers of the representation. We remind the reader that the LQ itself transforms as $(1,3)(2/3)$; the smallest standard $SU(5)$ representation into which the LQ + LQ^c can be embedded is a $\mathbf{10}\oplus\overline{\mathbf{10}}$ while in flipped-$SU(5) \times U(1)$[27], it can be placed in a $\mathbf{5}\oplus\overline{\mathbf{5}}$. (ii) Since we are using VLFs in these models, it is clear that only pairs of representations, $\mathbf{R}+\overline{\mathbf{R}}$, can be added to the MSSM spectrum in order to maintain anomaly cancelation. Of course this is also true for the LQ superfield in that both LQ and

LQ^c fields must now be present as discussed above. (*iii*) To preserve perturbation theory and asymptotic freedom up to the GUT scale when adding complete representations, at most one $\mathbf{10}+\overline{\mathbf{10}}$ or three $\mathbf{5}+\overline{\mathbf{5}}$ can be appended to the low energy spectrum of the MSSM apart from SM singlets. The reason for this is the general observation that if one adds more than three, vector-like, color triplet superfields to the MSSM particle content then the one-loop QCD beta function changes sign. Recall that the LQ itself *already* accounts for one of these color triplets. This same consideration also excludes the introduction of light exotic fields in higher dimensional $SU(3)_C$ representations. Complete $SU(5)$ representations larger than $\mathbf{10}+\overline{\mathbf{10}}$ are found to contribute more than this critical amount to the running of the QCD coupling.

These are highly restrictive constraints on the construction of a successful GUT scenario containing both VLFs and LQs and we see than none of the models discussed above can immediately satisfy them *unless* the LQ and VLF superfields can be placed into a single $SU(5)$ representation. In the standard $SU(5)$ picture, we can then place $(U, D)^T$, an isosinglet E^c and LQ^c into a single $\mathbf{10}$ with the corresponding conjugate fields in the $\overline{\mathbf{10}}$. This would form a hybrid of model A with the 'skeleton' model D, which we've denoted by AD above. Of course we pay no penalty for also including 'skeleton' model C here as well, which then yields model ACD. Instead, when we consider the flipped-$SU(5) \times U(1)$ case, it would appear that we can place $(N, E)^T$ and LQ^c into a $\overline{\mathbf{5}}$ with the conjugate fields in the $\mathbf{5}$; this is exactly model B. It would also seem that no penalty is paid as far as unification is concerned for including the 'skeleton' model C here as well *except* that this would violate assumption (*iii*) about the chirality of LQ couplings to fermions. However, this model is no longer truly unified since the hypercharge generator is not fully contained within the $SU(5)$ group itself and lies partly in the additional $U(1)$. While the $SU(3)_C$ and $SU(2)_L$ couplings will unify, $U(1)_Y$ will not join them even when arbitrary additional vector-like singlet fields are added.

Thus unification no longer occurs in this scenario so that this possibility is now excluded.

The LQ embedding situation becomes more perplexing if the LQ and VLFs cannot occupy the same GUT multiplet. In this case unification and asymptotic freedom constraints become particularly tight and we are forced to consider the split multiplet approach mentioned above. This means that we add the fields $(2,3)(1/6) \oplus (1,1)(1) \oplus (1,\bar{3})(-2/3)$ and their conjugates at low energies but constrain them to be from different $SU(5)$ representations. In this case the combination $(1,3)(2/3) \oplus (1,\bar{3})(-2/3)$ corresponds to the isosinglet LQ and its conjugate so what remains can only be the VLF fields. Note that we have again arrived back at models AD and ACD. Are these the only solutions? We have performed a systematic scan over a very large set of VLFs with various electroweak quantum numbers under the assumption that they are either color singlets or triplets, demanding only that (i) QCD remains asymptotically free and (ii) the model passes the so-called "B-test"[28] which is highly non-trivial to arrange. Essentially the B-test takes advantage of the observation that if we know the couplings at the weak scale and we demand that unification takes place *somewhere* then the values of the one-loop beta functions must be related. Note that it is a necessary but not sufficient test on our choice of models but is very useful at chopping away a large region of parameter space. Using the latest experimental data[29], we find that

$$B = \frac{b_3 - b_2}{b_2 - b_1} = 0.720 \pm 0.030 \,, \tag{9}$$

where the ± 0.030 is an estimate of the corrections due to higher order as well as threshold effects and the b_i are the one-loop beta functions of the three SM gauge groups. Note that $B_{MSSM} = 5/7 \simeq 0.714$ clearly satisfies the test. If we require that (i) and (ii) be satisfied and also require that the unification scale not be too low then only the solutions described above survive after examining $> 7 \times 10^7$ combinations of matter representations. While not completely exhaustive this search indicates the solutions above are fairly unique. It is

interesting to observe that models constructed around model A produce successful grand unification both with and without SUSY.

7 Conclusion and Outlook

In this talk we have seen how a wealth of data from low and medium energy experiments as well as high energy colliders can be combined to point us in a fixed direction for LQ model building. I have also discussed a general framework for the construction of new $F = 0$ scalar LQ models which go beyond the original classification by Buchmüller, Rückl and Wyler. This approach is based on the observation that in any realistic extension of the SM containing LQs it is expected that the LQs themselves will not be the only new ingredient. This construction technique is, of course, far more general than that required to address the specific issue of the HERA excess. While the assumptions of gauge invariance and renormalizability are unquestionable requirements of model building, it is possible that the other conditions one usually imposes are much too strong–unless they are clearly demanded by data. This observation implies that for LQs to be experimentally accessible their couplings to SM fermions must be essentially chiral and separately conserve both Baryon and Lepton numbers. The assumption that LQs couple to only a single SM generation is surely a convenient way of avoiding numerous low energy flavor changing neutral current constraints but is far from natural in the mass eigenstate basis. What is required to obtain a new class of LQ models is that the LQs themselves must be free to couple to more than just the SM fermions and gauge fields.

Given the fixed gauge structure of the SM the most likely new interactions that LQs may possess are with the Higgs field(s) responsible for spontaneous symmetry breaking and with new VLFs that are a common feature in many extensions of the SM. In the discussion above it has been shown how two new forms of the effective interactions of LQs with the

SM fermions, consistent with Tevatron searches, the HERA excess in both the NC and CC channels and low-energy data, can arise through the action of VLFs and ordinary symmetry breaking. The typical VLF mass was found to lie in the low TeV region and they could thus be directly produced at future colliders with known rates.

We saw that we could construct ten new models which fell into two broad classes according to the chirality of the resulting LQ couplings to the SM fermions. The VLFs themselves were shown to lead to a number of model-dependent effects which are close to the boundary of present experimental sensitivity. LQs within the framework of models containing VLFs were also shown to be consistent with Grand Unification in both a supersymmetric *and* non-supersymmetric context. The common feature of both schemes is the structure associated with model A, *i.e.*, the VLFs are color triplet, weak isodoublets in a $(2,3)(1/6)$ representation and both H and H^c Higgs fields are required to be present as is LQ^c field. In both scenarios the GUT scale is raised appreciably from the corresponding model wherein LQs and vector-like fermions are absent. In the SUSY case a $(1,1)(1)$ field is also required with the optional addition of a SM singlet, corresponding to models AD and ACD. In some sense, ACD is the "anti-E_6" model in that the color triplet VLFs are in isodoublets while the color singlet fields are all isosinglets. Interestingly, in this scenario there is a vector-like fermion corresponding to every type of SM fermion.

Realistic LQ models provide a rich source of new physics beyond the Standard Model.

Acknowledgments

The author would like to thanks J.L. Hewett for collaboration on the work on which this talk is based. The author also appreciates comments, discussions and input from S. Eno(D0), G. Landsberg(D0), J. Conway(CDF) and H. Frisch(CDF) regarding the current Tevatron

constraints. In addition, he thanks Y. Sirois(H1) and D. Krakauer(ZEUS) for discussions of the HERA data.

References

[1] C. Adloff *et al.*, H1 Collaboration, Z. Phys. **C74**, 191 (1997); J. Breitweg *et al.*, ZEUS Collaboration, Z. Phys. **C74**, 207 (1997).

[2] For a comprehensive analysis of this possibility and original references, see V. Barger *et al.*, hep-ph/9707412. The analysis by these authors apparently excludes this possibility as the source of the HERA excess in the NC channel.

[3] S. Kuhlman, H.L. Lai and W.K. Tung, hep-ph/9704338; K.S. Babu, C. Kolda and J. March-Russell, hep-ph/9705399; J.F. Gunion and R. Vogt, hep-ph/9706252; S. Rock and P. Bosted, hep-ph/9706436; W. Melnitchouk and A.W. Thomas, hep-ph/9707387.

[4] G. Altarelli *et al.*, hep-ph/9703279; J. Blümlein, hep-ph/9703287; J. Kalinowski *et al.*, hep-ph/9703288; K.S. Babu *et al.*, hep-ph/9703299; M. Suzuki, hep-ph/9703316; G.K. Leontaris and J.D. Vergados, hep-ph/9703338; I. Montvay, hep-ph/9704280; S.F. King and G.K. Leontaris, hep-ph/9704336; J. Elwood and A. Faraggi, hep-ph/9704363; B. Dutta, R.N. Mohapatra and S. Nandi, hep-ph/9704428; M. Heyssler and W.J. Stirling, hep-ph/9705229; J. Ellis, S. Lola and K. Sridhar, hep-ph/9705416; S. Jadach, B.F.L. Ward and Z. Was, hep-ph/9705429; A. Blumhofer and B. Lampe, hep-ph/9706454; E. Keith and E. Ma, hep-ph/9707214; N.G. Deshpande and B. Dutta, hep-ph/9707274.

[5] J.L Hewett and T.G. Rizzo, SLAC-PUB-7430, hep-ph/9703337, to appear in Physical Review **D** and SLAC-PUB-7549, hep-ph/9708419.

[6] B. Straub, ZEUS Collaboration, talk given at the *XVIII International Symposium on Lepton and Photon Interactions*, July 28-August 1, 1997, Hamburg, Germany. For the most recent results, see E. Elsen, H1 Collaboration, talk given at the *International Europhysics Conference on High Energy Physics*, Jerusalem, August 19-26, 1997.

[7] W. Buchmüller, R. Rückl, and D. Wyler, Phys. Lett. **B191**, 442 (1987).

[8] J.L. Hewett, T.G. Rizzo, S. Pakvasa, H.E. Haber, and A. Pomarol, in *Proceedings of the Workshop on Physics at Current Accelerators and the Supercollider*, June 1993, ed. J.L. Hewett, A. White, and D. Zeppenfeld.

[9] F. Abe *et al.*, CDF Collaboration, hep-ex/9708017, submitted to Phys. Rev. Lett.; B. Abbott *et al.*, D0 Collaboration, hep-ex/9707033, submitted to Phys. Rev. Lett. .For the most recent results, see B. Klima, D0 Collaboration, talk given at the *International Europhysics Conference on High Energy Physics*, Jerusalem, August 19-26, 1997.

[10] M. Krämer, T. Plehn, M. Spira and P. Zerwas, hep-ph/9704322.

[11] S. Davidson, D. Bailey, and B.A. Campbell, Z. Phys. **C61**, 613 (1994); M. Leurer, Phys. Rev. **D50**, 536 (1994), and **D49**, 333 (1994).

[12] C.S. Wood *et al.*, Science **275** 1759 (1997).

[13] For a recent review and updated, see J.L. Rosner, hep-ph/9704331.

[14] R.M. Barnett *et al.*, (Particle Data Group), Phys. Rev. **D54**, 1 (1996).

[15] Z. Kunszt and W.J. Stirling, hep-ph/9703427; T. Plehn, H. Spiesberger, M. Spira and P.M. Zerwas, hep-ph/9703433. For an updated discussion on the effects of QCD and initial and final state radiation, see C. Freiberg, E. Norrbin and T. Sjöstrand, hep-ph/9704214.

[16] J.L. Hewett and T.G. Rizzo, Phys. Rep. **183**, 193 (1989).

[17] J.L. Hewett and T.G. Rizzo, Phys. Rev. **D36**, 3367 (1987); J. Blümlein and R. Rückl, Phys. Lett. **B304**, 337 (1993); J. Blümlein and E. Boos, *Nucl. Phys.*(Proc. Suppl.) *37B*,

181 (1994) ; J. Blümlein, E. Boos, and A. Kryukov, Phys. Lett. **B392**, 150 (1997); D. Choudhury, Phys. Lett. **B346**, 291 (1995); J.E. Cieza-Montalvo and O.J. Eboli, Phys. Rev. **D47**, 837 (1993); T.G. Rizzo, Phys. Rev. **D44**, 186 (1991); H. Dreiner *et al.*, Mod. Phys. Lett. **3A**, 443 (1988); G. Bhattacharyya, D. Choudhury, and K. Sridhar, Phys. Lett. **B349**, 118 (1995); M.S. Berger, hep-ph/9609517.

[18] K. Ackerstaff *et al.*, OPAL Collaboration, hep-ex/9708024.

[19] R. Rückl and P. Zerwas, in *Proceedings of the LaThuile Workshop on Physics at Future Accelerators*, LaThuile, Italy, 1987, ed. J.H. Mulvay, CERN Report 87-07 (1987).

[20] A. Buras, talk given at the *Seventh International Symposium on Heavy Flavor Physics*, University of California, Santa Barbara, July 7-11, 1997. This analysis relies heavily on the work presented in E. Hagberg *et al.*, nucl-ex/9609002.

[21] M. Peskin and T. Takeuchi, Phys. Rev. Lett. **65**, 964 (1990), and Phys. Rev. **D46**, 381 (1992); W. Marciano and J. Rosner, Phys. Rev. Lett. **65**, 2963 (1990); G. Altarelli and R. Barbieri, Phys. Lett. **B253**, 161 (1990); D. Kennedy and P. Langacker, Phys. Rev. Lett. **65**, 2967 (1990), and Phys. Rev. **D44**, 1591 (1991); I. Maksymyk, C.P. Burgess, and D. London, Phys. Rev. **D50**, 529 (1994); C.P. Burgess *et al.*, Phys. Lett. **B326**, 276 (1994).

[22] O. Eboli, hep-ph/9508342; O.J. Eboli, M.C. Gonzalez-Garcia, and J.K. Mizukoshi, hep-ph/9612254, and Nucl. Phys. **B443**, 20 (1995); G. Bhattacharyya, J. Ellis, and K. Sridhar, Phys. Lett. **B336**, 100 (1994), and Erratum **B338**, 522 (1994).

[23] See, A. Djouadi, J. Ng, and T.G. Rizzo, in *Electroweak Symmetry Breaking and Beyond the Standard Model*, ed. T. Barklow *et al.*, (World Scientific, Singapore), hep-ph/9504210, and references therein.

[24] H. Murayama and T. Yanagida, Mod. Phys. Lett. **A7**, 147 (1992).

[25] T.G. Rizzo, Phys. Rev. **D45**, 3903 (1992).

[26] W.J. Marciano, talk given at the *1983 International Symposium on Lepton and Photon Interactions at High Energies*, Ithaca, NY, August 1983.

[27] I. Antoniadis, J. Ellis, J.S. Hagelin and D.V. Nanopolous, Phys. Lett. **B194**, 321 (1987).

[28] M. Peskin, SLAC-PUB-7479, 1997.

[29] A. Bohm, L3 Collaboration, talk presented at the 32nd *Recontres de Moriond: Electroweak Interactions and Unified Theories*, Le Arces, France, 15-22 March, 1997.