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Transverse Kick in Misaligned Traveling Wave Structures Driven at the Fundamental Mode

David H. Whittum
Stanford Linear Accelerator Center, Stanford University, Stanford California, 94309

Heino Henke
Technical University Berlin, EN2, Einstufer 17, D-10587 Berlin, Germany

Abstract

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Transverse Kick in Misaligned Traveling Wave Structures Driven at the Fundamental Mode

D. H. Whittum* and H. Henke†
SLAC, Stanford University, Stanford CA, 94309

Abstract

Fabrication errors in traveling wave structures result in non-axissymmetric RF fields that couple to the rf drive at the fundamental mode frequency. We calculate the excitation of the dipole mode and the integrated effect on the beam, using the thin iris and small hole approximation.

1 INTRODUCTION

An advantage commonly claimed for structures with circular symmetry is that the accelerating kick is independent of transverse position, and no transverse deflection results from the fundamental accelerating mode. Here we calculate the correction to this statement resulting from misalignment of the accelerating cells during the fabrication process. The resulting tolerance on fabrication errors is qualitatively different from the well-known limit deriving from the loss in no-load voltage from cell detuning [1]. While the effect is small for conventional accelerator applications, it could be significant for applications requiring low-emittance beams, and for structures of small dimension, where mechanical alignment is difficult.

We consider an *n*-cell travelling wave structure as a stack of coupled cavities, as illustrated in Fig.1. To calculate the cell-to-cell coupling in the general case,

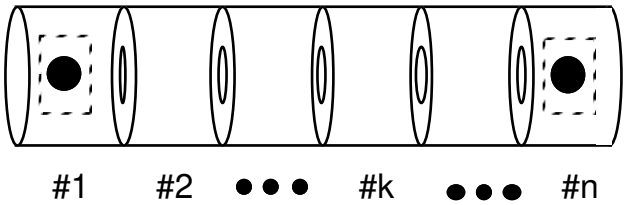

Figure: 1 An ideal, perfectly aligned *n*-cell structure.

we consider first a mode λ of a closed lossless cell, for which Maxwell's Equations may be expressed as, $\vec{\nabla}\times\vec{E}_\lambda = k_\lambda \vec{H}_\lambda$ and $\vec{\nabla}\times\vec{E}_\lambda = k_\lambda \vec{H}_\lambda$, with normalization of E_λ^2 to unit volume integral. With the introduction of the port coupling the cavity to another cavity, we consider the amplitude of the electric field in mode λ,

$$e_\lambda(t) = \int d^3\vec{r}\ \vec{E}(\vec{r},t)\bullet\tilde{E}_\lambda(\vec{r}),$$

for which Maxwell's Equations may be expressed as

$$\left(\frac{\partial^2}{\partial t^2}+\omega_\lambda^2\right)e_\lambda = -\omega_\lambda c\int_{port}\left(\hat{n}\times\vec{E}\right)\bullet\vec{H}_\lambda\, dA,$$

with $\omega_l = k_\lambda c$. The magnetic field amplitude

$$h_\lambda(t) = \int d^3\vec{r}\ \vec{H}(\vec{r},t)\bullet\tilde{H}_\lambda(\vec{r}),$$

may be computed from $\partial h_\lambda/\partial t = -\omega_\lambda e_\lambda / Z_0$, with Z_0~377Ω. When coupling is magnetic it is convenient to transform the coupling integral to the form,

$$\left(\frac{\partial^2}{\partial t^2}+\omega_\lambda^2\right)e_\lambda = \omega_\lambda Z_0\vec{H}_\lambda(0)\bullet\int_{port}\vec{r}\left(\hat{n}\bullet\frac{\partial\vec{H}}{\partial t}\right)dA.$$

For illustration, consider the case of two right-cylindrical pillboxes operated in the TM_{010} mode. One has, for the unperturbed mode,

$$\vec{E}_{01} = \hat{z}J_0(\beta_{01}r)E_{01},\ \vec{H}_{01} = -\hat{\phi}J_0'(\beta_{01}r)E_{01},$$

where $\beta_{01} = \omega_{01}/c = j_{01}/R$, the cavity radius is R, the length is L, and j_{01}~2.405. The normalization is $E_{01} = \left(\pi L R^2 J_1^2(j_{01})\right)^{-1/2}$, with $J_1(j_{01})$~0.5192. In the presence of a centered aperture of radius $a<<R$, the electric field lines are deformed, some terminating on the iris edge, with the result that in the iris there is a tangential electric field [1]

$$\vec{E}_\perp = \tfrac{1}{\pi}\tfrac{\vec{r}}{\sqrt{a^2-r^2}}\Delta E_z,$$

where $\Delta E_z = \left(e_{01}^1 - e_{01}^2\right)E_{01}$ is the discontinuity in normal electric field across the closed iris, and e_λ^k is the amplitude of mode λ, in cell #k. Evaluating the port integral, one finds

$$\left(\frac{\partial^2}{\partial t^2}+\omega_{01}^2\right)e_{01}^1 = -\tfrac{1}{2}\kappa_{01}\omega_{01}^2\left(e_{01}^1 - e_{01}^2\right),$$

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† On leave from Technische Universität Berlin.

$$\left(\tfrac{\partial^2}{\partial t^2}+\omega_{01}^2\right)e_{01}^2 = -\tfrac{1}{2}\kappa_{01}\omega_{01}^2\left(e_{01}^2 - e_{01}^1\right),$$

with coupling constant

$$\kappa_{01} = \tfrac{4}{3\pi J_1^2(j_{01})}\tfrac{a^3}{LR^2} = 1.575\tfrac{a^3}{LR^2}.$$

In the general case of multiple cells, one has a chain of coupled oscillators. Including the perturbation due to wall-losses, one finds that the *k*-th interior cell satisfies,

$$\left(\tfrac{\partial^2}{\partial t^2}+\tfrac{\omega_{01k}}{Q_{wk}}\tfrac{\partial}{\partial t}+\omega_{01k}^2\right)e_{01}^k = \tfrac{1}{2}\omega_{01k}^2\left(\kappa_{01k-}e_{01}^{k-1}+\kappa_{01k+}e_{01}^{k+1}\right).$$

For a perfectly tuned structure driven at the design frequency ω, the mode amplitude, takes the form, $e_{01}^k = \tilde{e}_{01}e^{-\gamma(k-1)}$, with $\gamma=j\theta+\Gamma$, θ the design phase-advance per cell, and Γ the damping decrement per cell. The accelerating or "no-load" voltage may be expressed as $V_{NL} = \Re\tilde{V}_{01}e^{j\omega t}$, with t the arrival time at the first cell, and

$$\tilde{V}_{01} = \left(E_{01}LT\right)e^{j\varphi/2}\sum_k e^{j(k-1)\varphi}\tilde{e}_{01}^k,$$

with $T = \sin(\varphi/2)/(\varphi/2)$ and $\varphi=\omega L/c$ the transit angle at the drive frequency. By design, travelling-wave structures are operated at synchronism $\theta=\varphi$, so that $\tilde{V}_{01} = \left(E_{01}LT\right)e^{j\varphi/2}\tilde{e}_{01}\left(1-e^{-n\Gamma}\right)\left(1-e^{-\Gamma}\right)^{-1}$. For a constant impedance structure $n\Gamma=\tau$ is the attenuation parameter for the structure.

Next, let us perform a similar calculation for the TM_{110} mode in this geometry. Considering the *x*-coupled polarization, the mode basis functions are

$$\vec{E}_{11} = \hat{z}E_{11}J_1(\beta_{11}r)\cos\phi,$$
$$\vec{H}_{11} = E_{11}\left(\hat{r}\tfrac{J_1(\beta_{11}r)}{\beta_{11}r}\sin\phi + \hat{\phi}J_1'(\beta_{11}r)\cos\phi\right),$$

where $\beta_{11} = \omega_{11}/c = j_{11}/R$, $j_{11}\tilde{}3.832$ and the normalization is $E_{11} = \left(\tfrac{\pi}{2}LR^2J_2^2(j_{11})\right)^{-1/2}$, where $J_2(j_{11})\tilde{}-0.4028$. Near the axis $\tilde{H}_\lambda \approx \tfrac{1}{2}\hat{y}E_{11}$.

To compute the cell-to-cell coupling of the TM_{110} mode one requires the normal component of the magnetic field in the iris [2],

$$H_z = \tfrac{2}{\pi}\tfrac{r}{\sqrt{a^2-r^2}}\Delta H_y\sin\phi,$$

where $\Delta H_y = \tfrac{1}{2}E_{11}\left(h_{11}^1 - h_{11}^2\right)$ is the discontinuity in tangential magnetic field across the conducting boundary in the absence of the port. Evaluating the port integral

$$\vec{H}_\lambda(0)\bullet\int_{port}\vec{r}\left(\hat{n}\bullet\vec{H}\right)dA = \tfrac{2}{3}a^3E_{11}\Delta H_y,$$

and, eliminating h_λ^k in favor of e_λ^k, one finds,

$$\left(\tfrac{\partial^2}{\partial t^2}+\omega_{11}^2\right)e_{11}^1 = \tfrac{1}{2}\omega_{11}^2\kappa_{11}\left(e_{11}^1 - e_{11}^2\right),$$
$$\left(\tfrac{\partial^2}{\partial t^2}+\omega_{11}^2\right)e_{11}^2 = \tfrac{1}{2}\omega_{11}^2\kappa_{11}\left(e_{11}^2 - e_{11}^1\right),$$

where the coupling constant is

$$\kappa_{11} = \tfrac{4}{3\pi J_2^2(j_{11})}\tfrac{a^3}{LR^2}.$$

In the normal development of the subject of dipole modes, it would be natural at this point to extend the formulation to include excitation by the beam in a multi-cell structure. However, our interest is not beam-excitation of dipole modes, but excitation by the externally driven fundamental mode.

2 CELL MISALIGNMENT

We consider next a multi-cell structure with misalignments, parameterized as illustrated in Fig. 2. Depending on the fabrication technique, additional constraints may apply. For example, for the Mark III structure [1], all $c_k\tilde{}0$ to within the straightness of the pipe, and the x_k are roughly independent. For the DDS structure [3], all $c_k\tilde{}x_k$ to the machining accuracy of the cups, while the cell-to-cell offsets are roughly independent. Note that each of these approximate equalities places the emphasis on *post-machining* assembly, where the largest errors occur in practice.

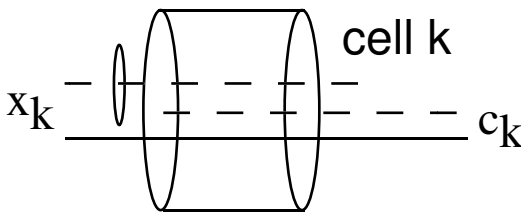

Figure: 2 With respect to a reference axis (solid line), cell k has offset c_k, while the kth iris (on the upstream side of cell k) is offset by x_k from the axis.

To compute the dipole mode excitation in cell k, let us first fix attention on the downstream port. We will work to lowest order in misalignments, neglecting quadrupole and higher modes, and neglecting too the perturbation to the fundamental mode from the dipole

mode. In the absence of a port the discontinuity in tangential magnetic field from the TM_{01} mode alone is

$$\Delta H_y = \tfrac{1}{2} E_{01}\beta_{01}\left(\left[x-c_k\right]h_{01}^k - \left[x-c_{k+1}\right]h_{01}^{k+1}\right).$$

It is the zeroth Fourier component in iris-centered azimuth of this term that determines the normal magnetic field threading the port, and this is just the average over the coupling iris, $\left\langle \Delta H_y \right\rangle = \tfrac{1}{2} E_{01}\beta_{01}\left(\varepsilon_k h_{01}^k - \delta_{k+1} h_{01}^{k+1}\right)$, with $\varepsilon_k = x_{k+1} - c_k$, and $\delta_k = x_k - c_k$. The resulting drive term for the *k*th-cell TM_{11} mode is then

$$\left(\tfrac{\partial^2}{\partial t^2} + \omega_{11}^2\right) e_{11}^k \Big|_{upstream,01} = -\omega_{11}^2 \eta\left(\varepsilon_k e_{01}^k - \delta_{k+1} e_{01}^{k+1}\right),$$

where the coupling constant is

$$\eta = \tfrac{1}{3} a^3 E_{01} E_{11} \beta_{01} = 1.73 \tfrac{a^3}{LR^3}.$$

Including all other perturbations one has

$$\left(\tfrac{\partial^2}{\partial t^2} + \tfrac{\omega_{11-k}}{Q_k}\tfrac{\partial}{\partial t} + \omega_{11-k}^2\right) e_{11}^k = \tfrac{1}{2}\omega_{11-k}^2\left(\kappa_{11k-} e_{11}^{k-1} + \kappa_{11k+} e_{11}^{k+1}\right) - \omega_{11-k}^2 D_k,$$

with

$$D_k = \eta_{k+}\left(\varepsilon_k e_{01}^k - \delta_{k+1} e_{01}^{k+1}\right) + \eta_{k-}\left(\delta_k e_{01}^k - \varepsilon_{k-1} e_{01}^{k-1}\right),$$

As described in the introduction the terms $\tilde{e}_{01}^k$ are known; thus the amplitudes $\tilde{e}_{11}^k$ may be obtained in the frequency domain, with the numerical solution of a tri-diagonal matrix equation. The impulse to an electron enetering the first cell at time *t* can then be computed from $\Delta p_x = \Re \tilde{P}_{11} e^{j\omega t}$, where ω is the frequency of the drive and the integral of vertical magnetic field yields

$$\tilde{P}_{11} = \tfrac{j}{2}\tfrac{e}{c}\tfrac{\omega}{\omega_{11}} E_{11} L T e^{j\varphi/2} \sum_k e_{11}^k e^{j(k-1)\varphi}.$$

To make some analytic progress, note that it is typically the case that the fundamental mode drive frequency lies in a stop-band for the dipole modes and thus to a good approximation, adjacent TM_{110} excitations are decoupled. Thus $\tilde{e}_{11}^k \approx \tfrac{\omega_{11-k}^2}{\Delta_k} D_k$, with $\Delta_k = \omega^2 - \omega_{11-k}^2 + j\omega_{11-k}\omega / Q_k$. For illustration let us simplify to the case of a constant impedance structure. Suppressing subscripts, and substituting $e_{01}^k = \tilde{e}_{01} e^{-\gamma(k-1)}$, we have

$$e_{11}^k e^{j(k-1)\varphi} \approx \tfrac{\omega_{11-k}^2}{\Delta_k}\eta\left\{\varepsilon_k + \delta_k - \delta_{k+1} e^{-\gamma} - \varepsilon_{k-1} e^{\gamma}\right\}$$

Performing the sum, we have,

$$\tfrac{1}{mc}\tilde{P}_{11} = \tfrac{1}{2}\tfrac{e}{mc^2}\tilde{V}_{01} j \tfrac{\omega_{11}\omega}{\Delta}\eta \tfrac{E_{11}}{E_{01}}\left(\tfrac{1-e^{-\Gamma}}{1-e^{-n\Gamma}}\right) n\xi$$

where,

$$n\xi = \sum_{k=1}^{n-1}\varepsilon_k + \sum_{k=2}^{n}\delta_k - e^{-\gamma}\sum_{k=1}^{n-1}\delta_{k+1} - e^{\gamma}\sum_{k=1}^{n-1}\varepsilon_{k-1} = \left(e^{\gamma}-1\right)\left\{\left(1-e^{-\gamma}\right)\sum_{k=2}^{n}\left(x_k - c_k\right) + \left(c_n - c_1\right)\right\}.$$

This phasor ξ is, in magnitude, roughly an average of misalignments over the structure. However, remarkably, for cup-type cells and assuming the first and last cells are centered on the beam-axis, there is no deflection to lowest order in misalignments. For the Mark III-type assembly errors add. We may extract from this result a simple rule of thumb for the maximum deflection Δp for a short ($n\Gamma << 1$) structure imparting maximum energy ΔE,

$$\Delta p \approx \tfrac{1}{c}\Delta E \tfrac{\xi}{d},$$

with

$$\tfrac{1}{d} \approx 1.6 \tfrac{a^3}{LR^3}.$$

In this form the result permits simple, practical estimates. For example, taking an 11.4GHz structure, operated in $2\pi/3$-mode, with $a/\lambda \tilde{}0.16$, we find $L \tilde{}0.88$cm, $R \tilde{}1$cm, $a \tilde{}0.42$cm, and $d \tilde{}15$cm. Thus a 1μm average offset between iris and cell center, in a 100MeV accelerating section, can produce a maximum kick of $7\text{x}10^{-4}$MeV/c, or a 0.6 μrad deflection for a 1GeV beam. This kick is reduced to zero as the beam approaches the accelerating crest.

3 CONCLUSIONS

We have quantified a new alignment tolerance on accelerating structures, arising from cross-talk between the fundamental and dipole modes, occuring when cylindrical symmetry is broken due to fabrication errors.

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