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Phenomenology, Astrophysics and Cosmology of Theories with Sub-Millimeter Dimensions and TeV Scale Quantum Gravity

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Abstract

We recently proposed a solution to the hierarchy problem not relying on low-energy supersymmetry or technicolor. Instead, the problem is nullified by bringing quantum gravity down to the TeV scale. This is accomplished by the presence of $n \geq 2$ new dimensions of sub-millimeter size, with the SM fields localised on a 3-brane in the higher dimensional space. In this paper we systematically study the experimental viability of this scenario. Constraints arise both from strong quantum gravitational effects at the TeV scale, and more importantly from the production of massless higher dimensional gravitons with TeV suppressed couplings. Theories with $n > 2$ are safe due mainly to the infrared softness of higher dimensional gravity. For $n = 2$, the six dimensional Planck scale must be pushed above ~ 30 TeV to avoid cooling SN1987A and distortions of the diffuse photon background. Nevertheless, the particular implementation of our framework within type I string theory can evade all constraints, for any $n \geq 2$, with string scale $m_s \sim 1$ TeV. We also explore novel phenomena resulting from the existence of new states propagating in the higher dimensional space. The Peccei-Quinn solution to the strong CP problem is revived with a weak scale axion in the bulk. Gauge fields in the bulk can mediate repulsive forces $\sim 10^6 - 10^8$ times stronger than gravity at sub-mm distances, as well as help stabilize the proton. Higher-dimensional gravitons produced on our brane and captured on a different "fat" brane can provide a natural dark matter candidate.

1 Introduction

In a recent paper [1], we have proposed a framework for solving the hierarchy problem which does not rely on supersymmetry or technicolor. Rather, the problem is solved by removing its premise: the fundamental Planck scale, where gravity becomes comparable in strength to the other interactions, is taken to be near the weak scale. The observed weakness of gravity at long distances is due to the presence of n new spatial dimensions large compared to the electroweak scale. This can be inferred from the relation between the Planck scales of the $(4+n)$ dimensional theory $M_{Pl(4+n)}$ and the long-distance 4-dimensional theory $M_{Pl(4)}$, which can simply be determined by Gauss' law (see the next section for a more detailed explanation)

$$M^2_{Pl(4)} \sim r_n^n M^{n+2}_{Pl(4+n)} \tag{1}$$

where r_n is the size of the extra dimensions. Putting $M_{Pl(4+n)} \sim 1$ TeV then yields

$$r_n \sim 10^{30/n-17}\text{cm} \tag{2}$$

For $n = 1$, $r_1 \sim 10^{13}$ cm, so this case is obviously excluded since it would modify Newtonian gravitation at solar-system distances. Already for $n = 2$, however, $r_2 \sim 1$ mm, which is precisely the distance where our present experimental measurement of gravitational strength forces stops. As n increases, r_n approaches $(\text{TeV})^{-1}$ distances, albeit slowly: the case $n = 6$ gives $r_6 \sim (10\text{MeV})^{-1}$. Clearly, while the gravitational force has not been directly measured beneath a millimeter, the success of the SM up to ~ 100 GeV implies that the SM fields can not feel these extra large dimensions; that is, they must be stuck on a wall, or "3-brane", in the higher dimensional space. Summarizing, in our framework the universe is $(4 + n)$

dimensional with Planck scale near the weak scale, with $n \geq 2$ new sub-mm sized dimensions where gravity perhaps other fields can freely propagate, but where the SM particles are localised on a 3-brane in the higher-dimensional space.

An important question is the mechanism by which the SM fields are localised to the brane. In [1], we proposed a field-theoretic implementation of our framework based on earlier ideas for localizing the requisite spin 0,1/2[2] and 1 [3] particles. In [4] we showed that our framework can naturally be embedded in type I string theory. This has the obvious advantage of being

formulated within a consistent theory of gravity, with the additional benefit that the localization of gauge theories on a 3-brane is automatic [5]. Further interesting progress towards realistic string model-building was made in [6].

The most pressing issue, however, is to insure that this framework is not experimentally excluded. This is a concern for two main reasons. First, quantum gravity has been brought down from 10^{19} GeV to $\sim$ TeV. Second, the structure of space-time has been drastically modified at sub-mm distances. The main objective of this paper is to examine the phenomenological, astrophysical and cosmological constraints on our framework. Subsequently, we discuss a number of new phenomena which emerge in theories with large extra dimensions.

The rest of the paper is organized as follows. In section 2, we derive the exact relationship between the Planck scales of the $(4+n)$ and 4-d theories in three ways in order to gain some intuition for the physics of higher dimensional theories. Of course, roughly speaking, if $M_{Pl(4+n)} \sim 1$ TeV, we expect new physics responsible for making a sensible quantum theory of gravity at the TeV scale. There is a practical difference between the new physics occurring at ~ 1 TeV versus ~ 10 TeV, as far as accessibility to future colliders is concerned. In section 3, we therefore give a more careful account of the relationship between the scale of new physics and $M_{Pl(4+n)}$ in the particular case where gravity is embedded in type I string theory. As a set-up for the discussion of phenomenological constraints,in section 4 we identify and discuss the interactions of new light particles in the effective theory beneath the TeV scale: higher dimensional graviton, and possibly Nambu-Goldstone bosons of broken translation invariance. In section 5 we begin the discussion of phenomenological constraints in earnest, beginning with laboratory experiments. The most stringent bounds are not due to strong gravitational effects at $\sim$ TeV energies, but rather due to the possibility of producing massless particles, the higher dimensional gravitons, whose couplings are only 1/TeV suppressed. In section 5 we discuss potential problems this can cause with rare decays, and in sections 6 and 7 we consider astrophysical and cosmological constraints. Remarkably, due primarily to the extreme infrared softness of higher dimensional gravity, we find that for $n > 2$ all experimental limits are comfortably satisfied. The case $n = 2$ is quite tightly constrained, with a lower bound $\gtrsim 30$ TeV on the 6-d Planck scale. Nevertheless, precisely for $n = 2$, this Planck mass can still be consistent with string excitations at the TeV scale, and therefore may still provide a natural solution to the

hierarchy problem. Not only are cosmological constraints satisfied, there are new cosmological possibilities in our scenario. In particular, we discuss the possibility that gravitons produced on our brane and captured on a different, "fat" brane in the bulk, can form the dark matter of the Universe. The following two sections illustrate further possibilities for new physics in this framework. In section 9, we show that the Peccei-Quinn axion can solve the Strong-CP problem and avoid the usual astrophysical bounds if the axion field lives in the bulk. In section 10, we note that a gauge field living in the bulk can naturally have a miniscule gauge coupling $\sim 10^{-16}$ to wall states and pick up a mass $\sim 1\text{mm}^{-1}$ through spontaneous breaking on the wall. If these gauge fields couple to B or $B-L$, they can mediate repulsive forces $\sim 10^6 - 10^8$ times stronger than gravity at sub-mm distances. This gives a spectacular experimental signature may be observed in the near future. Finally, in section 11, we turn to the important question of the determination of the radii of the extra dimensions. While we do not offer any dynamical proposal, we parametrize the potential for the radius modulus and consider cosmological constraints coming from the requirement that the radius is not significantly altered since before the era of Big-Bang Nucleosynthesis (BBN). We draw our conclusions in section 12. Appendix 1 discusses the somewhat subtle issue of the Higgs phenomenon for spontaneously broken translational invariance, and appendix 2 presents a toy model illustrating some aspects of moduli stabilization.

2 Relating Planck Scales

2.1 Gauss Law

Here we will derive the exact relationship between the Newton constants $G_{N(4+n)}, G_{N(4)}$ of the full $(4+n)$ and compactified 4 dimensional theories, which are defined by the force laws

$$\begin{aligned} F_{(4+n)}(r) &= G_{N(4+n)} \frac{m_1 m_2}{r^{n+2}} \\ F_{(4)}(r) &= G_{N(4)} \frac{m_1 m_2}{r^2}. \end{aligned} \tag{3}$$

We will carry out this simple exercise in three different ways. The easiest derivation is a trivial application of Gauss' Law. Let us compactify the n new

dimensions y_α by making the periodic identification $y_\alpha \sim y_\alpha + L$. Suppose now that a point mass m is placed at the origin. One can reproduce this situation in the uncompactified theory by placing "mirror" masses periodically in all the new dimensions. Of course for a test mass at distances $r \ll L$ from m, the "mirror" masses make a negligible small contribution to the force and we have the $(4+n)$ dimensional force law. For $r \gg L$, on the other hand, the discrete distance between mirror masses can not be discerned and they look like an infinite n spatial dimensional "line" with uniform mass density. The problem is analogous to finding the gravitational field of an infinite line of mass with uniform mass/unit length, where cylindrical symmetry and Gauss' law give the answer. Following exactly the same procedure, we consider a "cylinder" C centered around the n dimensional line of mass, with side length l and end caps being three dimensional sphere's of radius r. We now apply the $(4+n)$ dimensional Gauss' law which reads

$$\int_{\text{surface}\,C} F dS = S_{(3+n)} G_{N(4+n)} \times \text{Mass in}\, C \tag{4}$$

where $S_D = 2\pi^{D/2}/\Gamma(D/2)$ is the surface area of the unit sphere in D spatial dimensions (recall that the usual Gauss law has a 4π factor on the RHS). In our case, the LHS is equal to $F(r) \times 4\pi \times l^n$, while the total mass contained in C is $m \times (l^n/L^n)$. Equating the two sides, we find the correct $1/r^2$ force law and can identify

$$G_{N(4)} = \frac{S_{(3+n)}}{4\pi} \frac{G_{N(4+n)}}{V_n} \tag{5}$$

where $V_n = L^n$ is the volume of compactified dimensions.

We can also derive this result directly by compactifying the Lagrangian from $(4+n)$ to 4 dimensions, from which we can also motivate a definition for the "reduced" Planck scale in both theories. In the non-relativistic limit and in $(4+n)$ dimensions, the action for the interaction of the (dimensionless) gravitational potential $\phi = g^{00} - 1$, with a mass density ρ, is given by

$$I_{(4+n)} = \int d^{4+n}x \frac{1}{2} \hat{M}^{n+2}_{(4+n)} \phi \nabla^2_{(3+n)} \phi + \rho_{(4+n)} \phi + \cdots \tag{6}$$

where ∇^2_D is the D spatial dimensional Laplacian, and we define $\hat{M}_{(4+n)}$ as the reduced Planck scale in $(4+n)$ dimensions. Note that if we wish to work with canonically normalized ϕ field, we rewrite $\phi = \hat{M}^{-1/2}_{(4+n)} \phi_{can}$, and

the Lagrangian becomes

$$I_{(4+n)} = \int d^{4+n}x \frac{1}{2}\phi_{can}\nabla^2_{(3+n)}\phi_{can} + \frac{1}{\sqrt{\hat{M}^{n+2}_{(4+n)}}}\rho_{(4+n)}\phi_{can} + \cdots \tag{7}$$

showing that the interaction of the canonically normalized field are suppressed by $\sqrt{\hat{M}_{(4+n)}}$. Upon integrating out ϕ, we generate the potential

$$\int dt d^{(3+n)}x d^{(3+n)}y \frac{1}{\hat{M}^{n+2}_{(4+n)}}\rho_{(4+n)}(x)\nabla^{-2}_{(3+n)}(x-y)\rho_{(4+n)}(y). \tag{8}$$

Using

$$\nabla^{-2}_D(x-y) = \frac{1}{(D-2)S_D}\frac{1}{|x-y|^{D-2}} \tag{9}$$

we have for the force between two test masses

$$F_{(4+n)}(r) = \frac{1}{\hat{M}^{n+2}_{(4+n)}S_{(3+n)}}\frac{m_1 m_2}{r^{n+2}} \tag{10}$$

from which we find the relationship between the reduced Planck scale and Newtons constant

$$\hat{M}^{n+2}_{(4+n)} = \frac{G^{-1}_{N(4+n)}}{S_{(3+n)}}. \tag{11}$$

We can compactify from $(4+n)$ to 4 dimensions by restricting all the fields to be constant in the extra dimensions; integrating over the n dimensions then yields the 4 dimensional action

$$I_4 = \int d^4x \frac{1}{2}(V_n \hat{M}^{n+2}_{(4+n)})\phi\nabla^2_3\phi + \rho_3\phi + \cdots \tag{12}$$

and therefore the reduced Planck scales of the two theories are related according to

$$M^2_{(4)} = \hat{M}^2_{(4+n)}V_n, \tag{13}$$

which using eqn.(11) reproduces the relation between the Newton constants in eqn.(5). An interesting string theoretic application of this result was made in [7], where it was used to low the string scale to the GUT scale, choosing the radius of 11'th dimension in M-theory to be $\sim 10^{-27}$cm. Attempts to reduce

the string scale much further were considered in [8], but their conclusions were basically negative.

Finally, we can understand this result purely from the 4-dimensional point of view as arising from the sum over the Kaluza-Klein excitations of the graviton. From the 4-d point of view, a $(4+n)$ dimensional graviton with momentum $(q_1, \cdots, q_n)$ in the extra n dimensions looks like a massive particle of mass $|q|$. Since the momenta in the extra dimensions are quantized in units of $2\pi/L$, this corresponds to an infinite tower of KK excitations for each of the n dimensions, with mass splittings $2\pi/L$. While each of these KK modes is very weakly coupled ($\sim 1/M_{(4)}$), their large multiplicity can give a large enhancement to any effect they mediate. In our case, the potential between two test masses not only has the $1/r$ contribution from the usual massless graviton, but also has Yukawa potentials mediated by all the massive modes as well:

$$\frac{V(r)}{m_1 m_2} = G_{N(4)} \sum_{(k_1,\cdots,k_n)} \frac{e^{-(2\pi|k|/L)r}}{r}. \tag{14}$$

Obviously, for $r \ll L$, only the ordinary massless graviton contributes and we have the usual potential. For $r \gg L$, however, roughly $(L/r)^n$ KK modes make unsuppressed contributions, and so the potential grows more rapidly as L^n/r^{n+1}. More exactly, for $r \gg L$,

$$\begin{aligned} \frac{V(r)}{m_1 m_2} &\rightarrow G_{N(4)} r \times (\frac{L}{2\pi R})^n \times \int d^n u e^{-|u|} \\ &= \frac{G_{N(4)} V_n}{r^{n+1}} \times \frac{S_n \Gamma(n)}{(2\pi)^n}. \end{aligned} \tag{15}$$

This yields the same relationship between $G_{N(4)}$ and $G_{N(4+n)}$ found in eqn.(5) upon using the Legendre duplication formula

$$\Gamma(\frac{n}{2})\Gamma(\frac{n}{2}+\frac{1}{2}) = \frac{\sqrt{\pi}}{2^{n-1}}\Gamma(n). \tag{16}$$

We will encounter this phenomenon repeatedly in this paper: the interaction of the higher dimensional gravitons can be understood in two ways. Directly from the (4+n) dimensional point of view, the graviton couplings are suppressed $1/\sqrt{\hat{M}_{4+n}}$, which can be understood from the 4 dimensional point of view as arising from a sum over a large multiplicity of KK excitations each

of which has couplings suppressed by $1/M_{(4)}$. Note that for higher n, the couplings of $(4+n)$ dimensional gravitons are suppressed by more powers of the $(4+n)$ dimensional Planck scale $\hat{M}_{(4+n)}$, and so their interactions become increasingly soft in the infrared (the flip-side of the worse UV problems!). As already mentioned, and as will be seen in detail in many examples, this IR softness is crucial to the survival of the theory when the fundamental Planck scale is taken to be near the TeV scale.

We make one last comment on 2π factors. If we express $V_n = L_n^n = (2\pi r_n)^n$, the first KK excitation has a mass r_n^{-1}, and r_n (not L_n) more correctly describes the physical size of the extra dimensions. For instance, the potential between two test masses a distance L_n apart is only modified at $O(e^{-2\pi}) \sim 10^{-2}$, whereas the change is $O(1)$ for a distance r_n apart. In terms of r_n, the relation eqn.(13) becomes

$$M_{(4)}^2 = M_{(4+n)}^{(n+2)} r_n^n,\ M_{(4+n)}^{n+2} \equiv (2\pi)^n \hat{M}_{(4+n)}^{n+2}. \tag{17}$$

We will see below that the experimental bounds most directly constrain $M_{(4+n)}$, and that it is $M_{(4+n)}$ which is required to be close to the weak scale for solving the hierarchy problem. Putting in the numbers, we find for r_n

$$r_n = 2 \times 10^{31/n - 16} \text{mm} \times \left(\frac{1\text{TeV}}{M_{(4+n)}} \right)^{1+2/n} \tag{18}$$

For $n = 2$ and $M_{(6)}$= 1 TeV, $r_2 \sim 1$ mm, which is precisely the distance at which gravity is currently measured directly. For $n = 6$, $r_6 \sim (10\ \text{Mev})^{-1}$, and for very large n, r_n approaches $M_{(4+n)}^{-1}$.

3 Relating the Planck scale to the String Scale

In this subsection we wish to be more precise about the various scales in our problem. Namely, we wish to quantify what exactly what we mean by

"gravity gets strong at the weak scale". Of course, we are really interested in relating the scale m_{grav} at which the new physics responsible for making a sensible quantum theory of gravity appears, to parameters of the low-energy theory such as e.g. $G_{N(4+n)}$ or $M_{(4+n)}$. There is a practical reason for finding determining this relationship. Both theoretically and experimentally, $m_{grav} \lesssim 1$ TeV is most desirable, on the other hand, the most stringent

experimental constraints we will discuss directly constrain the interactions of the $(4+n)$ dimensional gravitons and hence put a bound on $M_{(4+n)}$. It is therefore important to determine how this bound translates into a constraint on m_{grav}.

Without a specific theory in mind, it is difficult to relate m_{grav} to $M_{(4+n)}$, other than the expectation that they are "closeby". To be more concrete, we suppose that the theory above m_{grav} is a string theory, specifically the realization of our scenario within type I' string theory outlined in [4], which we briefly review here. The low-energy action of type-I string theory in 10-dimensions reads

$$S = \int d^{10}x \left(\frac{m_s^8}{(2\pi)^7 \lambda^2} \mathcal{R} + \frac{1}{4} \frac{m_s^6}{(2\pi)^7 \lambda} F^2 + \cdots \right). \tag{19}$$

where $\lambda \sim e^{\phi}$ is the string coupling, and m_s is the string scale, which we can identify with m_{grav}. Compactifying to 4 dimensions on a manifold of volume V_6, we can identify the resulting coefficients of R and $(1/4)F^2$ with $M_{(4)}^2$ and $1/g_4^2$, from which we can find

$$\begin{aligned} M_{(4)}^2 &= \frac{(2\pi)^7}{V_6 m_s^4 g_4^2} \\ \lambda &= \frac{g_4^2 V_6 m_s^6}{(2\pi)^7}. \end{aligned} \tag{20}$$

Putting $m_s \sim 1$ TeV and $g_4 \sim 1$ fixes a very small value for λ and a compactification volume much *smaller* than the string scale. A more appropriate description is obtained by $T-$ dualising, where we compactify on a manifold of volume V_6' with a new string coupling λ' given by

$$\begin{aligned} V_6' &= \frac{(2\pi)^{12}}{V_6 m_s^{12}}, \\ \lambda' &= \frac{(2\pi)^6}{m_s^6 V_6} \lambda = \frac{g_4^2}{2\pi}. \end{aligned} \tag{21}$$

In this $T-$dual description, the KK excitations of the open strings in the type-I picture become winding modes of type-I' open strings stuck to a D3 brane, while only the closed string (gravitational) sector propagates in the

bulk. Thus our scenario for solving the hierarchy problem can naturally be embedded in this picture. The 4-dimensional Planck scale

$$M_4^2 = \frac{2\pi}{g_4^4} m_s^8 \left(\frac{V_6}{(2\pi)^6} \right) \tag{22}$$

can then be much larger than the string scale if V_6 is much bigger than m_s^{-6}. To make contact with our framework, we assume that of the six compact dimensions, $(6-n)$ have a size $L_{(6-n)} = (2\pi r_{(6-n)})$ with the "physical" size $r_{(6-n)} \sim m_s^{-1}$, while the remaining n dimensions of size $L_n = 2\pi r_n$ are the "large" ones we previously discussed. Then $V_6/(2\pi)^6 = r_{(6-n)}^{(6-n)} r_n^n$, and combining eqns(22,17) we obtain

$$\frac{M_{(4+n)}}{m_s} = \left(\frac{2\pi}{g_4^4} \right)^{\frac{1}{n+2}} (r_{(6-n)} m_s)^{\frac{6-n}{n+2}}. \tag{23}$$

It is clear that for the higher values of n, the possible enhancements of $M_{(4+n)}/m_s$ from the first two factors are negligible and we should expect $M_{(4+n)} \sim m_s \lesssim 1$ TeV. For the case $n = 2$, however, the first factor can range from 2 to 3 depending on which of the SM gauge couplings are chosen to represent g_4, and we can choose r_4 to be somewhat larger than the string scale perhaps as low as $\sim (300\text{GeV})^{-1}$. These factors can be enough to push $M_{(4+n)}$ to somewhat higher values ~ 10 TeV while keeping $m_s \sim 1$ TeV. As will see later, the strongest constraints occur for the lowest values of n and in some cases will indeed push $M_{(4+n)}$ above ~ 10 TeV. It is reassuring to know that even in this case, new string physics may be seen at ~ 1 TeV.

4 Couplings of Bulk Gravitons and Nambu-Goldstones of Broken Space-Time Symmetries.

In this section, we wish to describe the light degrees of freedom which exist in the effective theory beneath the scale of quantum gravity m_{grav} and the tension f of the wall. In our scenario it is most natural to assume $f \sim m_{grav}$. This sort of effective theory is interesting because some states (such as the SM fields) live on a wall in the extra dimensions, while other fields (such

as the gravitons) can freely propagate in the higher dimensional space. Of course, the presence of the wall breaks translational invariance in the extra n dimensions. Part of our discussion depends on whether this is a spontaneous or explicit breaking of the $(4+n)-$d Poincare symmetry. Let the position of a point x on the wall, in the higher dimensions $a = 4, \cdots 3+n$, be given by $y^a(x)$. In the case where the breaking is spontaneous, wall configurations which differ from each other by a uniform translation $y^a(x) \to y^a(x) + c$ are degenerate in energy. The $y^a(x)$ are then dynamical fields, Nambu-Goldstone bosons of spontaneously broken translation invariance. The fields in the effective theory consist of the $y^a(x)$, together with the SM fields on the wall and gravity in the full higher dimensional bulk. The interactions of this effective theory are constrained by the requirement that the full $(4+n)-d$ Poincare invariance be realized non-linearly on these fields. A very nice analysis of the structure of this effective theory together with the leading terms in its energy expansion has recently been given by Sundrum [11]. We will not repeat this analysis here, as many of the details are unimportant for phenomenological constraints we consider. We will instead study the form of the least suppressed interactions to the y^a and the bulk gravitons.

Before turning to this, we raise a puzzling question not addressed in [11]. Since gravity can be thought of as gauging translation invariance, and since translation invariance is spontaneously broken, why are the $y^a(x)$ not "eaten" by the corresponding "gauge field" $g^{\mu a}$, which would become massive? We analyze this question in appendix 1. The conclusion is that the $y^a(x)$ are indeed eaten and the corresponding 4-d "gauge" field gets a mass $\sim f^2/M_{(4)} \sim (1\text{ mm})^{-1}$ for $f \sim 1$ TeV. Notice that, if $M_{(4+n)}$ is held fixed and $r_n \to \infty$, this mass goes to zero since $M_{(4)} \to \infty$, and so that the analysis of [11], which was implicitly done in this limit, is unaffected. Furthermore,this mass is so small that almost processes we consider will involve energies $\gg 1(\text{mm})^{-1}$, and so by the equivalence theorem, it is much more convenient to think in terms of the original picture of massless gravitons and Nambu-Goldstone fields. Nevertheless, as we will see later, a $\sim(\text{mm})^{-1}$ mass is generically be generated for any "bulk" gauge field when the gauge symmetry is broken on the wall, and can lead to very interesting experimental consequences.

We now turn to the leading couplings, first to goldstone fields ignoring gravity, then to gravity ignoring the goldstones. To begin with, note that the y^a have mass dimension -1, and are therefore written in terms of the canonically normalized goldstone fields π^a as $y^a = \pi^a/f^2$. This is in analogy to the

usual case of Goldstone boson of internal symmetries, where the analogue of y^a is an angle θ^a of the group transformation, related to the physical pion fields as $\theta^a = \pi^a/f$. This immediately means that the interactions with the y^a, for $f \sim 1$ TeV, are always weaker than neutrino interactions which are suppressed only by $\sim 1/m_W^2$. In fact, it is easy to see that for interactions with scalars or vectors or a single Well fermion, the leading operators must involve *two* $y's$ and are therefore even more suppressed $\sim 1/f^4$. This follows from a completely straightforward operator analysis, but can also be simply understood as follows. The fluctuations in the wall given by $\partial_\mu y^a(x)$ induce a non-trivial metric on the wall, inherited from the metric of the bulk space. Ignoring gravity, the bulk metric is flat and the induced metric on the wall is

$$g_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu y^a \partial_\nu y^a \tag{24}$$

which is symmetric under $y^a \to -y^a$, so the interactions of y which result from non-trivial g involve pairs of $y's$. Following [11], an operator involving a single y interacting with vector-like Well fermions (ψ, ψ^c) can also be written

$$\mathcal{O}_{1y} = c\psi\partial^\mu\psi^c\partial_\mu y \tag{25}$$

Of course, since this operator violates chirality, we expect that the coefficient c is suppressed by $\sim m_\psi/m_{grav}$, up to a model-dependent coefficient.

Next consider the coupling of the SM fields to gravitation, but without exciting the y^a. If the bulk metric is G_{MN} where $M, N = 0, \cdots, 3+n$, the induced metric on the wall is trivially

$$g_{\mu\nu}(x) = G_{\mu\nu}(x, y^a = 0). \tag{26}$$

The bulk gravitons are the perturbations of G_{MN} about η_{MN},

$$G_{MN} = \eta_{MN} + \frac{H_{MN}}{\sqrt{M_{(4+n)}^{n+2}}} \tag{27}$$

and the linear interactions with SM wall fields are given by

$$\int d^4x T^{\mu\nu} \frac{H_{\mu\nu}(x, y^a = 0)}{\sqrt{M_{(4+n)}^{n+2}}} \tag{28}$$

where $T^{\mu\nu}$ is the 4-d energy momentum tensor for the SM fields. Two things are immediately obvious from this coupling. First, there is no coupling to

the $H_{a\mu}, H_{ab}$ gravitons. This is intuitively clear: without changing the shape of the wall (i.e. exciting the y^a), the wall fields make zero contribution to $T^{\mu a}, T^{ab}$ and the the couplings to the corresponding H's vanish. Second, the interaction clearly violates translation invariance in the extra dimensions, and therefore the extra dimensional momenta p^a need not be conserved however in the interactions between the wall and bulk states, while energy is still conserved because time translational invariance still holds. More intuitively we can think of the wall as being infinitely heavy, so that it can recoil to absorb extra-dimensional momentum without absorbing energy. This can also be seen explicitly by expanding $H_{\mu\nu}(x, y^a = 0)$ into Kaluza-Klein modes

$$H_{\mu\nu}(x, y^a = 0) = \sum_{n^a} \frac{1}{\sqrt{r^n_n}} H^{n^a}_{\mu\nu} \tag{29}$$

which shows that the wall $T^{\mu\nu}$ couples to all KK modes with equal strength $1/M_{(4)}$. Of course, there are many other couplings involving combinations of the y^a and gravitons, but they are all suppressed by further powers of $1/M_{(4+n)}$ and/or $1/f^2$.

We should also mention that if translation invariance is *explicitly* broken in the extra dimensions, as in the case where the wall is "stuck" to a point in the higher dimensions, the modes y^a corresponding to the fluctuations of the wall become massive and are irrelevant to low energy physics.

5 Lab bounds

5.1 Macroscopic gravity

Given that the gravitational interaction is unchanged over distances bigger than the size of the extra dimensions, and that gravity is only significant on much larger scales, the change in gravity at distances smaller than ~ 1 mm is harmless. One may wonder about systems where gravity is known to be important, but where the typical inter-particle separation is smaller than ~ 1 mm, e.g. in the sun. It is clear, however, that all effects due to the new gravity beneath ~ 1 mm must be suppressed by powers in the ratio of the size of the new dimensions over the typical size R of the gravitating body. The reason is that, if we divide the body into ~ 1 mm balls, these balls have normal gravitational interactions. Since important gravitational effects are

bulk effects, the only error incurred in splitting the body into $\sim$ mm sized balls can at most be power suppressed in $(1\text{mm}/R)$. For instance, let us compute for the gravitational self energy per unit mass of a ball of radius R and density ρ:

$$\frac{E_{grav}}{\rho R^3} \sim \int_0^{r_n} d^3r \frac{G_{N(4+n)}\rho}{r^{(n+1)}} + \int_{r_n}^{R} d^3r \frac{G_{N(4)}\rho}{r} \tag{30}$$

where the first integral uses the $(4+n)$ dimensional gravitational potential and the second is the usual piece. Now, the usual piece is dominated by large distances and gives a contribution $\sim G_{N(4+n)}\rho R^2$. However, for $n = 2$, the new contribution is log divergent and is cutoff off at short distances by the typical inter-particle separation r_{min} and at long distances by R, and for $n > 2$, the new contribution is dominated by short distances and is cutoff by r_{min}. The fractional change in the gravitational energy due to the new interaction is then

$$\frac{\Delta E_{grav}}{E_{grav}} \sim \frac{G_{N(4+n)}}{G_{N(4)} r_{min}^{n-2} R^2} \sim \frac{r_n^n}{r_{min}^{n-2} R^2}. \tag{31}$$

Note that for $G_{N(4+n)} \sim (\text{TeV})^{-(n+2)}$ and for r_{min} larger than $\sim (\text{TeV})^{-1}$, this contribution is largest for $n = 2$, and

$$\frac{\Delta E_{grav}}{E_{grav}} \lesssim \left(\frac{1\text{mm}}{R}\right)^2. \tag{32}$$

which is completely irrelevant for the sun. The smallest objects for which the gravitational self-energy plays any role is the neutron star which has $R \sim 10$ km, giving an unobservably small fractional change $\sim 10^{-12}$ in gravitational energy.

5.2 Mesoscopic gravity

While the normal Newtonian gravitation is unaffected on distances larger than r_n, the gravitational attraction between two objects grows much more quickly $\sim 1/r^{n+2}$ at distances smaller than r_n. This is of course a reflection of the fact that, in this scenario, gravity "catches up" with the other interactions at $\sim 10^3$ GeV rather than at 10^{19} GeV. The flip side of this is that, even though gravity is much stronger than before, it is still much weaker than the

other forces at distances appreciably larger than the weak scale. Consider for instance the ratio of the new gravitational force to the electromagnetic force between a proton and an electron a distance r apart

$$\frac{F_{grav}}{F_{em}} \sim \frac{G_{N(4+n)} m_e m_p}{\alpha r^n} \sim 10^{-7} \left(\frac{10^{-17}\mathrm{cm}}{r} \right)^n . \tag{33}$$

The smallest value of r where electromagnetic effects are dominant are atomic sizes $r \sim 10^{-8}$ cm, and even then for the worst case $n = 2$, the above ratio is unobservably small $\sim 10^{-25}$. Of course on larger distances the electromagnetic interactions are screened due to average charge neutrality, while gravity is not. Even here, however, the residual electromagnetic forces still dominate over the new gravity. As an example consider the Van der Waals (VdW) force between two hydrogen atoms, in their ground state, a distance $r \gg r_{\mathrm{bohr}}$ apart from each other. This arises due to the dipole-dipole interaction potential, i.e. the energy of the dipole-moment of atom 1 in the electric field set up by the dipole moment of atom 2:

$$V_{int.} \sim \frac{d_1 d_2}{4\pi r^3}. \tag{34}$$

The first order energy shift due to this interaction vanishes in the ground state since the ground state expectation value of each dipole moment vanishes by rotational invariance. The second order perturbation then gives the usual VdW $1/r^6$ potential,

$$\begin{aligned} \Delta V(r) &\sim \sum_{n,n'} \frac{d_{10n}^2 d_{20n'}^2}{2E_0 - E_n - E'_n} \frac{1}{16\pi^2 r^6} \\ &\sim \alpha \left(\frac{r_{bohr}}{r} \right)^5 \frac{1}{r}. \end{aligned} \tag{35}$$

The ratio of this VdW force to the ordinary gravitational attraction between the hydrogen atoms is

$$\frac{F_{VdW}}{F_{\text{ord. grav.}}} \sim \left(\frac{1\mathrm{mm}}{r} \right)^5 , \tag{36}$$

and we see that while electrostatic effects are irrelevant for distances larger than ~ 1 mm, the VdW force dominates over ordinary gravity at sub-mm

distances. This is in fact the central obstacle to the sub-mm measurements of gravitational strength forces. Even in our scenario with much stronger gravity, VdW forces dominate down to atomic scales, (where the electromagnetic effects are no longer even shielded). For the case of $n = 2$ new dimensions, the new dimensions open up near the mm scale, and the gravitational force only increases as $1/r^4$ at smaller distances, which is still overwhelmed by VdW. Already for $n = 3$, the new dimensions open at $\sim 10^{-7}$ cm and VdW dominates still further.

Of course in the case $n = 2$, we expect a switch from $1/r^2$ to $1/r^4$ gravity roughly beneath $r_2 \sim 1$ mm. There are no direct measurements of gravity at sub-mm distances. The best current bound on sub-mm $1/r^3$ potentials actually comes from experiments measuring the Casimir forces at ~ 5 microns [9]. Parametrizing the force between two objects composed of N_1, N_2 nucleons separated by a distance r as

$$V(r) = CN_1N_2\frac{(10^{-15}\mathrm{m})^2}{r^3} \tag{37}$$

the best current bound is $C \lesssim 7 \times 10^{-17}$[9]. If we assume that the only gravitational strength forces beneath r_2 is the 6−d Newtonian potential, this corresponds to

$$\begin{aligned} C &= \frac{G_{N(6)}(m_N \times 10^{15}\mathrm{m}^{-1})^2}{3} = \frac{1\mathrm{GeV}^4}{50M_{(6)}^4} \\ &\rightarrow M_{(6)} \gtrsim 4.5\mathrm{TeV}. \end{aligned} \tag{38}$$

If we take this indirect bound seriously, then from eqn.(18), r_2 shrinks to ~ 30 microns, which is however still well within the reach of the planned experiments directly measuring gravity at sub-mm distances. There may be contributions to the long-range force beneath r_2 beyond those from the KK excitations of the ordinary graviton, which may compensate the gravitational force and the and the force at the ~ 5 micron distances probed in the Casimir experiments may not be as strong we have considered, with correspondingly weaker bounds. If the $1/r^4$ force is canceled at short distances, a sub-leading $1/r^3$ force may remain. In this case, the transition from $1/r^2 \rightarrow 1/r^3$ could be observed for r_2 as large as $\sim .5$mm. It is interesting that this potential could also be interpreted as Newtonian gravity in 5 space-time dimensions, with a new dimension opening up at the millimeter scale!

5.3 "Compositeness" bounds

We next discuss laboratory bounds. Since we have quantum gravity at the TeV scale, in theory above a TeV will generate higher dimension operators involving SM fields, suppressed by powers of $\sim$ TeV. Of course, operators such as these which lead to proton decay or large flavor-violations in the Kaon system must somehow be adequately suppressed as we have discussed in previous papers[1, 4]. However, the majority of higher dimension operators suppressed by $\sim$ TeV are safe. Their effects can show up either in modifying SM cross-sections (and are therefore constrained by "compositeness" searches), or they can give corrections to precisely measured observables such as the electron/muon $(g-2)$ factors or the S-parameter. Since we do not know the exact theory above a TeV, the coefficients of these higher dimension operators are unknown, but we will estimate their order of magnitude effects to show that they do not provide significant constraints on the framework. We discuss "compositeness" constraints first. The strongest bounds on 4-fermion operators of the form

$$\mathcal{O}_{4-\text{fermi}} = \frac{2\pi^2}{\Lambda^2}(\bar{\psi}\psi)^2 \tag{39}$$

are from LEP searches in the lepton sector, which require at most $\Lambda \gtrsim 3.5$ TeV. If the this operator is generated with coefficient $1/m_{grav}^2$, it is safe for $m_{grav} \gtrsim 1$ TeV.

While most of these operators have unknown coefficients, some have contributions from physics beneath the scale m_{grav} which are in principle calculable. For instance, the tree-level exchange of the $(4+n)$ dimensional gravitons can give rise to local 4–fermion operators[4]. We can understand this from the 4-dimensional viewpoint as follows. If the typical external energy for the fermions is $\sim E$, then the exchange of a KK excitation of the graviton labeled by momenta $(k_1, \cdots, k_n)r_n^{-1}$ with mass $|k|r_n^{-1} \gtrsim E$ generates a local 4-fermion operator. Summing over the KK modes yields an operator of the form

$$\mathcal{O} = C \sum_{|k|r_n^{-1} \gtrsim E} \times \frac{E^2}{M_{(4)}^2} \times \frac{1}{|kr_n^{-1}|^2} \times (\bar{\psi}\psi)^2 \tag{40}$$

where C is an $O(1)$ coefficient to be determined by an exact computation. For $n = 2$, the sum over KK modes is log-divergent in the UV, while for $n > 2$

it is power divergent. Of course, this sum must be cutoff for the KK modes heavier than m_{grav}, where new physics sets in. For $n = 2$, the logarithm is not large enough to significantly enhance the operator; however, for $n > 2$, the power divergence changes the $1/M_{(4)^2}$ suppression to an E^2/m_{grav}^4 effect:

$$\mathcal{O} = C\left(\frac{m_{grav}}{M_{(4+n)}}\right)^{n+2} \times \frac{E^2}{m_{grav}^4}(\bar{\psi}\psi)^2. \tag{41}$$

Of course the precise bound on m_{grav} depends on the relationship between m_{grav} and $M_{(4+n)}$. If we take the string scenario and identify m_{grav} with m_s, then this relationship is given in eqn.(23). Even in the worst case where the the "small" radii are not larger than the string scale ($r_{6-n}m_s = 1$, the bound on m_s coming from equating the coefficient of the four-fermi operator with $2\pi^2/\Lambda^2$ yields

$$m_s > \left(\frac{Cg_4^4}{4\pi^3}\right)^{1/4} \sqrt{\Lambda E}. \tag{42}$$

Since the strongest bounds on Λ come from LEP where the energy is at most 100 GeV, we are safe for all n as long as $m_s \gtrsim$ TeV.

5.4 Cosmic rays

While colliders have not yet attained the energies required to probe new strong quantum gravitational effects at the TeV scale, one can wonder whether very high energy cosmic rays place any sort of bounds on our scenario. Indeed, there are very high energy cosmic rays (nucleons) of energies up to $\sim 10^{20}$ eV $= 10^8$ TeV, eight orders of magnitude more energetic than the fundamental Planck scale. Furthermore, when these nucleons impinge on a stationary nucleon, the center of mass energy can be as high as ~ 1000 TeV. This raises two questions. First, is there anything wrong with having a particle with energy so much larger than the fundamental Planck scale? And second, do interactions with such high energies probe post-Planckian physics? The answer to both questions is no, and we address them in turn.

It is obvious that there is nothing wrong with having a particle of arbitrarily high energy, since energy is not Lorentz invariant. The question is however, whether a particle can be accelerated from rest to a Post-Planckian energy. There is certainly no problem with accelerating a particle to post-TeV

energies, as long as the acceleration is sufficiently small (but over large enough distances) so that energy loss to ordinary radiation is negligible. Note that relevant acceleration scales will be so much smaller than the weak scale that the couplings to ordinary radiation vastly dominate the coupling to higher dimension gravitons, so that as long as ordinary radiation is negligible, the gravitational radiation energy loss is even smaller. It is interesting to note that, in the context of normal gravitational theory, there have been speculations that it may be impossible to accelerate a particle to post-Planckian energies; at least many acceleration mechanisms fail for a variety of reasons [10]. As a typical example, suppose that the acceleration is provided by a constant electric field E acting over a region of size R. In order to accelerate a charge e to energy $\mathcal{E}$, we must have $eER \sim \mathcal{E}$. On the other hand, there is an energy $V \sim E^2R^3$ stored inside the region, which would give a black hole of event horizon size $R_{\rm hor} \sim V/M_P l^2 = (\mathcal{E}/M_{Pl})^2 R$. For $\mathcal{E} \gg M_{Pl}$, the horizon size is much larger than R and the system would collapse into a black hole. These sorts of arguments have led to speculations that perhaps for reasons related to fundamental short-distance physics, post-Planckian energies are inaccessible. Our example suggests otherwise: while may be difficult to accelerate to energies beyond the effective four dimensional Planck scale, energies beyond the fundamental short-distance Planck scale can easily be attained.

Next we turn to the second issue: do cosmic ray collisions with center of mass energies far above the TeV significantly probe the physics at distances smaller than $\sim (\mathrm{TeV})^{-1}$? The answer to this is obviously no; the huge fraction of the cross-section for nucleon-nucleon scattering is diffractive, arising from the finite size of the nucleon, giving a typical cross section ~ 30 millibarn. The point is of course that it is not enough for the c.o.m. energy to be large, after all two particles traveling in opposite direction with large energies but infinitely far apart have huge c.o.m. energy but do not interact! In order to probe short distance physics at distances r, it is necessary to have a momentum transfer $\sim r^{-1}$; but the vast majority of nucleon-nucleon interactions only involve $\sim$GeV momentum transfers. In fact, cosmic rays lose energy in the atmosphere not through diffractive QCD scattering but by creating electromagnetic showers, where the effective momentum transfer per interaction is still smaller.

5.5 Precision observables

Corrections to electron and muon $(g-2)$ are expected to be naturally small for a very general and well-known reason. The higher dimension operators which can contribute to e.g. the electron $(g-2)$ are of the form

$$\mathcal{L}_{g-2} \sim \frac{c_5}{m_{grav}} e^c \sigma^{\mu\nu} F_{\mu\nu} e + \text{higher dimensional operators.} \tag{43}$$

Since the lowest dimension operator violates electron chirality, we parametrize $c_5 = d_5 m_e/m_{grav}$, and since the QED contribution to $(g-2)$ generates the same operator with coefficient $\alpha/(\pi m_e)$, the fractional change in $(g-2)$ is of order

$$\frac{\delta(g-2)}{g-2} \sim d_5 \frac{\pi}{\alpha} \left(\frac{m_e}{m_{grav}}\right)^2 \tag{44}$$

which even for $d_5 \sim 1$ and $m_{grav} \sim 1$ TeV is $\sim 10^{-10}$, smaller than the experimental uncertainty $\sim 10^{-8}$. The contribution to the muon $(g-2)$ is similarly safe. Of course, there are contributions to d_5 which can be computed in the low energy theory involving loops of the light $(4+n)$ dimensional graviton, in which case d_5 is further suppressed by a loop factor, and the fractional change in $(g-2)$ is correspondingly smaller. Furthermore, since all other operators have higher dimension, they will at most make comparable contribution to $(g-2)$. Note that the chirality suppression of the dimension 5 operator was crucial: a $c_5 \sim 1$ is grossly excluded. The correct estimate given above indicates why the anomalous magnetic moment measurements, in spite of their high precision, do not significantly constrain new weak scale physics.

Similar arguments apply to the corrections to precision electroweak observables. Consider the graviton loop correction to the S parameter. Again from the 4−d viewpoint, we are summing over the contributions of the towers of KK gravitons. We consider contributions from modes heavier and lighter than m_Z respectively. Recall that each KK mode has $1/M_{(4)}$ suppressed couplings. For the modes lighter than m_Z, each contributes $\sim (m_Z/M_{(4)})^2$ to S. We therefore estimate

$$\begin{aligned} S_{m_{KK}<m_Z} &\sim (m_Z/M_{(4)})^2 \times (m_Z r_n)^n \\ &\sim (m_Z/M_{(4+n)})^{n+2} \end{aligned} \tag{45}$$

which is a tiny $\lesssim 10^{-3} - 10^{-4}$ contribution even for the worst case $n = 2$, $M_{(4)} = 1$ TeV. For the contribution from a KK mode heavier than m_Z, S also vanishes in the limit $m_{KK} \to \infty$, so the contribution to S from each mode is $\sim m_Z^4/(M_{(4)}^2 m_{KK}^2)$. Therefore, the contribution to S from these states is

$$S_{m_{KK}>m_Z} = \sum_{|k|r_n^{-1}>m_Z} \frac{m_Z^4}{M_{(4)}^2(|k|r_n^{-1})^2}. \tag{46}$$

This is precisely the same sum as was encountered in the compositeness section, and it is power divergent in the UV for $n \geq 3$. Cutting the power divergence off at m_{grav}, we find

$$S_{m_{KK}>m_Z} \sim (\frac{m_Z}{m_{grav}})^4 \tag{47}$$

which even for $n = 6$ is $\lesssim 10^{-4}$ for $m_{grav} \sim 1$ TeV.

5.6 Rare decays to higher dimension gravitons

A far more important set of constraints follow from the fact that the $(4+n)$ dimensional graviton is a massless particle with couplings to SM fields suppressed by powers of $\sim 1/\text{TeV}$. In this respect, it is similar to other light particles like axions or familons. These are known to be in disastrous conflict with experiment for decay constants in the $\sim$ TeV region, for familons because they give rise to large rates for rare flavor-changing processes, for axions because they can take away too much energy from stellar objects through their copious production. We must check that the analogous processes do not rule out a $(4+n)$ dimensional graviton with 1/TeV-suppressed couplings. Another way of stating the problem is as follows. As we have remarked several times, from the 4-dimensional point of view, the graviton spectrum consists of the ordinary massless graviton, together with its tower of KK excitations spaced by r_n^{-1}. While the coupling of each of these KK modes is suppressed by $1/M_{(4)}$, there is an enormous number $\sim (Er_n)^n$ of them available with mass lower than energy E, and there combined effects are much stronger than suppressions of $\sim 1/M_{Pl}$. This large multiplicity factor is responsible for converting $1/M_{(4)}$ effects to stronger $/M_{(4+n)}$ effects, as we have already seen explicitly in the conversion between $1/r^2$ to $1/r^{n+2}$ Newtonian force law. However, as we have mentioned, the infrared softness

of higher dimension gravity will allow this scenario to survive. We begin with bounds from rare decays of SM particles involving the emission of gravitons into the extra dimensions, beginning with the decay $K \to \pi +$ graviton (the analogous familon process $K \to \pi +$ familon puts the strongest bound on familon decay constants $\sim 10^{12}$ GeV). Recall that even though the emission of a single graviton into the extra dimensions violates conservation of extra-dimensional momentum, it is nevertheless allowed, since the presence of the wall on which SM fields is localised breaks translational invariance in the extra dimensions. However, since time translational invariance is still good, energy must still be conserved. Notice also that this process will proceed through e.g. the spin-0 component of the massive KK excitations of the graviton in order to conserve angular momentum. A tree-level diagram for the process can be obtained by attaching a $(4+n)$ dimensional graviton to any of the legs of the Fermi interaction $\bar{s}d\bar{u}d$. Again, on dimensional grounds, the decay width for the decay into any single KK mode is at most

$$\Gamma_{KK} \sim \left(\frac{1}{16\pi}\frac{m_K^5}{M_W^4}\right) \times \frac{m_K^2}{M_{(4)}^2} \tag{48}$$

where the first factor has been isolated as roughly the total KK decay width. However, there is a large multiplicity factor from the number of KK modes with mass $\lesssim m_K$ which are energetically allowed, $\sim (m_K r_n)^n$. The total width to gravitons is then

$$\Gamma_{K\to\pi+\text{graviton}} \sim \left(\frac{1}{16\pi}\frac{m_K^5}{M_W^4}\right) \times \left(\frac{m_K}{M_{4+n}}\right)^{n+2} \tag{49}$$

yielding a branching ratio

$$B(K \to \pi + \text{graviton}) \sim (m_K/M_{(4+n)})^{n+2} \tag{50}$$

Even in the most dangerous case $n = 2, M_{(6)} \sim 1TeV$, this branching ratio is $\sim 10^{-12}$ and is safely smaller than the bound, although a more careful calculation is required for this case. As we will see in the next sections, astrophysics and cosmology seem to require $M_{(6)} \gtrsim 10$ TeV for $n = 2$, in which case the branching ratio in Kaon decay goes down another four orders of magnitude to $\sim 10^{-16}$. Note that the scaling for the branching ratio could have also been derived directly from the $(4+n)$ dimensional point of view.

As we have remarked earlier, the couplings of the graviton are dimensionless when expressed in terms of the $(4+n)$ dimensional metric G_{MN}, which can be expanded about flat space-time as

$$G_{MN} = \eta_{MN} + \frac{h_{MN}}{M_{(4+n)}^{(n+2)/2}} \tag{51}$$

where h_{AB} is the canonically normalized field (of mass dimension $1 + n/2$) in $(4 + n)$ dimensions. Therefore, there is a factor of $1/M_{(4+n)}^{(n+2)/2}$ in the amplitude and $1/M_{(4+n)}^{(n+2)}$ in the rate. Inserting factors of the only other scale, m_K to make a dimensionless branching ratio, we arrive at the same estimate for $B(K \to \pi +$ graviton). We see explicitly that it is the infrared softness of the interactions of the higher-dimensional theory which is responsible for insuring safety, although this was certainly not guaranteed for relatively low n.

Analogous branching fractions for flavor-conserving and violating decays for B quarks are also safe, with branching ratios $\sim 10^{-8}$ for the worst case $n = 2, M_{(6)} = 1$TeV, and further down to $\sim 10^{-12}$ for the $M_{(6)} \gtrsim 10$TeV favored by astrophysics and cosmology. The largest branching fractions are for the heaviest particles, the most interesting being for Z decays. The decay $Z \to f\bar{f} +$ graviton can occur at tree-level, with a branching fraction

$$B(Z \to f\bar{f} + \text{graviton}) \sim (\frac{m_Z}{M_{4+n}})^{n+2} \tag{52}$$

which can be as large as $\sim 10^{-4}$, still not excluded by Z-pole data. Other decays like $Z \to \gamma +$ graviton are only generated at loop level, with unobservably small branching ratios.

6 Astrophysics

We now turn to astrophysical constraints on our scenario, analogous to bounds on the interaction of other light particles such as axions. In our case, the worry is that, since the gravitons are quite strongly ($\sim 1/$) TeV coupled, they are produced copiously and escape into the extra dimensions, carrying away energy. Having escaped, the gravitons have a very small probability to return and impact with the wall fields: this is intuitively obvious

since the wall only occupies a tiny region of the extra dimensions. We can also understand this from the point of view of producing graviton KK excitations. As usual, even though each KK mode is $1/M_{(4)}$ coupled, significant energy can be dumped into the KK gravitons because of their large multiplicity. However, each single KK mode, once produced, has only its $1/M_{(4)}$ coupled interactions with wall fields. In the next section we will quantify this correspondence, finding that the higher dimensional gravitons have a mean-free time for interaction with wall exceeding the age of the universe for all graviton energies relevant here. the upshot is that the gravitons carry away energy without returning energy, thereby modifying stellar dynamics in an unacceptable way. For the axion, the strongest such bounds come from SN1987A, which constrain the axion decay constant $f_a \gtrsim 10^9$ GeV. This naively spells doom for our 1/ TeV coupled gravitons. However, since the gravitons propagate in extra dimensions and have interactions that are softer in the infrared, our scenario survives the astrophysical constraints.

We will do a more detailed analysis below; however, in order to get an idea of what is going on we estabilsh a rough dictionary between rates for axion and graviton emission. Since any axion vertex is suppressed by $1/f_a$, any rate for axion emission is proportional to

$$\text{Rate of axion prod.} \propto \frac{1}{f_a^2} \tag{53}$$

Now consider graviton production. The first point is that if the temperature $T$of the star is much smaller than r_n^{-1}, none of the KK excitations of the graviton can be produced and the only energy loss is the miniscule one to the ordinary graviton. If $T \gg r_n^{-1}$, on the other hand, a very large number $\sim (Tr_n)^n$ of KK modes can be produced. Since each of these modes has couplings suppressed by $1/M_{(4)}$, the rate for graviton production goes like

$$\text{Rate of graviton prod.} \propto \frac{1}{M_{(4)}^2} \times (Tr_n)^n \sim \frac{T^n}{M_{(4+n)}^{n+2}} \tag{54}$$

Note that this is exactly analogous to what happened with e.g. $K \to \pi +$ graviton, and that this dependence could have been inferred directly from the $(4+n)$-dimensional viewpoint just as in eqn.(51). We can now establish the rough dictionary between f_a and $M_{(4+n)}$:

$$\frac{1}{f_a^2} \to \frac{T^n}{M_{(4+n)}^{n+2}}. \tag{55}$$

This dictionary contains the essence of what will be found by more detailed analysis below. The strongest bounds come from the hottest systems (where the bounds on f_a are also the strongest). However, even for the SN where the average kinetic energy corresponds $T \sim 30$ MeV, $f_a \gtrsim 10^9$ GeV requires that for $n = 2$ that $M_{(6)} \gtrsim 10$ TeV, whereas already for $n \geq 3$, $M_{(4+n)}$ can be $\lesssim 1$ TeV. Recall also that an $M_{(6)} \sim 10$ TeV can be consistent with new physics (for instance string excitations) at the ~ 1 TeV scale and is therefore not unnatural as far as the gauge hierarchy is concerned. The constraints from other systems such as the Sun (where $T \sim$ KeV or Red giants (where $T \sim 100$ KeV) are weaker and are satisfied for $M_{(4+n)} \lesssim 1$ TeV.

We now move to a somewhat more detailed analysis. This is necessary because there are some qualitative differences between the axion and graviton couplings; for instance, the axion coupling to photons is suppressed not only by $1/f_a$ but also by an "anomaly factor" $\alpha/4\pi$, while there is no corresponding anomaly price for gravitons. Furthermore, there are some effects that can not be determined from dimensional analysis alone, for instance, in some systems, most of the gravitational radiation comes from non-relativistic particles, and the energy emission rate depends on the small ratio $\beta = v/c$ in a way that can not be fixed by dimensional analysis. It is easy to deduce the dependence on β from the couplings to the physical gravitons. A non-relativistic particle of mass m, moving with some velocity β_i has an energy momentum tensor $T_{\mu\nu} = m(dx_\mu/d\tau)(dx_\nu/d\tau)$ which in the non-relativistic limit $\beta_i \ll 1$ becomes

$$T_{00} = m,\ T_{0i} = p_i,\ T_{ij} = \frac{p_i p_j}{m}. \tag{56}$$

Therefore, the coupling to the physical graviton polarizations, which come from the transverse,traceless components h^{ij}, has a factor $\sim p^2/m \sim T$ in the amplitude. Therefore, there is no dependence on β from the gravitational vertex. The situation is different for couplings to photons; there the fundamental coupling is $eA^\mu(dx_\mu/d\tau)$, and in the non-relativistic limit the coupling to the physical photons (the transverse part of A_i) is suppressed by β_i. Of course, for relativistic particles $\beta \sim 1$ and dimensional analysis is all that is needed to estimate the relevant cross sections.

Since we are concerned with the energy lost to gravitons escaping into the extra dimensions, it is convenient and standard [14] to define the quantities $\dot{\epsilon}_{a+b\to c+grav}$ which are the rate at which energy is lost to gravitons via the

process $a + b \to c + graviton$, per unit time per unit mass of the stellar object. In terms of the cross-section $\sigma_{a+b\to c+grav}$ the number densities $n_{a,b}$ for a,b and the mass density ρ, $\dot{\epsilon}$ is given by

$$\dot{\epsilon}_{a+b\to c+grav.} = \frac{\langle n_a n_b \sigma_{a+b\to c+grav} v_{rel} E_{grav} \rangle}{\rho} \tag{57}$$

where the brackets indicate thermal averaging. We can estimate the cross-sections for all graviton production processes as follows. From the graviton vertex alone, we get the usual $T^n/M_{(4+n)}^{n+2}$ dependence which already has the correct dimensions for a cross-section. The dependence on dimensionless gauge couplings etc. are trivially obtained, while the appropriate factors of β for non-relativistic particles are dealt with as in the previous paragraph. Finally, we insert an overall factor $\delta \sim 1/16\pi$ to approximately account for the phase-space. The relevant processes and estimated cross-sections are shown below

• Gravi-Compton scattering: $\gamma + e \to e + grav$

$$\sigma v \sim \delta e^2 \frac{T^n}{M_{(4+n)}^{n+2}} \beta^2 \tag{58}$$

• Gravi-brehmstrahlung: Electron- Z nucleus scattering radiating a graviton $(e + Z \to e + Z + grav)$

$$\sigma v \sim \delta Z^2 e^2 \frac{T^n}{M_{(4+n)}^{n_2}} \tag{59}$$

• Graviton production in photon fusion: $\gamma + \gamma \to grav$

$$\sigma v \sim \delta \frac{T^n}{M_{(4+n)}^{n+2}} \tag{60}$$

• Gravi-Primakoff process: γ + EM field of nucleus Z $\to grav$

$$\sigma v \sim \delta Z^2 \frac{T^n}{M_{(4+n)}^{n+2}} \tag{61}$$

• Nucleon-Nucleon Brehmstrahlung: $N + N \to N + N + grav$ (relevant for the SN1987A where the temperature is comparable to m_π and so the strong interaction between N's is unsuppressed)

$$\sigma v \sim (30 \text{ millibarn}) \times (\frac{T}{M_{(4+n)}})^{n+2} \tag{62}$$

Armed with these cross-sections, we can proceed to discuss the energy-loss problems in the Sun, Red Giants and SN1987A.

6.1 Sun

The temperature of the sun is ~ 1 KeV, and the relevant particles in equilibrium are electron, protons and photons. The number densities $n_e = n_p$ and n_γ are roughly comparable, $\sim n_{e,p,\gamma} \sim (\text{ Kev})^3$. The electrons and protons are non-relativistic. The observed rate at which the sun releases energy per unit mass per unit time is

$$\dot{\epsilon}_{normal} \sim 1\text{erg g}^{-1}\text{s}^{-1} \sim 10^{-45}\text{TeV}. \tag{63}$$

We must therefore demand that the rate of energy loss to gravitons is less than this normal rate. We will consider the processes in turn. Begin with the Gravi-Compton scattering. Using $n_e/\rho = n_p/\rho = 1/m_p$ and $n_\gamma \sim T_{sun}^3$, we find

$$\dot{\epsilon} \sim 4\pi\alpha\delta \frac{T_{sun}^{n+5}}{m_p m_e M_{(4+n)}^{n+2}} \tag{64}$$

and therefore

$$M_{(4+n)} \gtrsim 10^{\frac{16-6n}{n+2}}\text{GeV}. \tag{65}$$

Even the worst case $n = 2$ only requires $M_{(4+n)} \gtrsim 10$ GeV. Gravi-brehmstrahlung is not relevant since there are no high-Z nuclei present in the sun. Photon pair fusion into graviton is more important than the the analogous process $\gamma + \gamma \to$ axion, which is highly suppressed by the "anomaly price" $\alpha/4\pi$. For the case of graviton, the rate is given by

$$\dot{\epsilon} \sim \delta \frac{T_{sun}^{n+7}}{\rho M_{(4+n)}^{n+2}}. \tag{66}$$

This places a lower bound on $M_{(4+n)}$,

$$M_{(4+n)} \gtrsim 10^{\frac{18-6n}{n+2}} GeV. \tag{67}$$

For $n = 2$, this is a stronger bound $M_{(6)} \gtrsim 30$ GeV, but certainly no problem.

The Gravi-Primakoff (with photons scattering off the electric field of the protons) is sub-dominant to the last bound because, while protons and photons have roughly equal number density, the electric field surrounding a proton is proportional to the electric charge $e/4\pi$ and so the Gravi-Primakoff rate is suppressed relative to the photon-photon fusion by rate by $\sim \alpha$.

Finally, nucleon-nucleon brehmstrahlung is is irrelevant because at these temperatures, the collisions of nucleons can not probe the strong interaction core.

It is clear that the situation with the sun is so safe because it's temperature is so low. Because electrons, protons and photons occur in equal abundance, but the cross-sections involving photons and electrons are suppressed by α and β^2 effects, the dominant process is the photon-photon fusion, which yields even for the worst case $n = 2$, $M_{(6)} \gtrsim 30$ GeV. For red giants, the temperature is somewhat larger, $T \sim 10$ KeV, and the constraints are somewhat different, but the temperature is still so low that certainly $M_{(4+n)} \sim 1$ TeV is safe for all n. Clearly the strongest bounds will come from SN1987A where the temperature is significantly higher ~ 30 MeV. We turn there now.

6.2 SN1987A

During the collapse of the iron core of SN1987A, about 10^{53} ergs of gravitational binding energy was released in a few seconds; the resulting neutron star had a core temperature ~ 30 MeV. We must ensure that the graviton luminosity does not exceed the liberated 10^{53} erg s^{-1}:

$$\mathcal{L}_{grav} = \dot{\epsilon} M_{SN} \lesssim 10^{53} ergs^{-1} \sim (10^{16}\text{GeV})^2 \tag{68}$$

There are two dominant processes here: nucleon-nucleon brehmstrahlung (which is the dominant process for axions), together with the Gravi-Primakoff process (which is again sub-dominant in the axion case because the "anomaly factor" $\alpha/4\pi$). The graviton luminosity from the nucleon-nucleon brehmstrahlung is roughly

$$\mathcal{L}_{grav} \sim M_{SN} \times \frac{n_N^2}{\rho} \times 30\text{millibarn} \times \left(\frac{T}{M_{4+n)}}\right)^{n+2}. \tag{69}$$

For $M_{SN} \sim 1.6 M_{sun} \sim 10^{57}$ GeV, $n_N \sim 10^{-3}$ GeV^3 and $\rho \sim 10^{-3}$ GeV^4, we find the following bound on $M_{(4+n)}$

$$M_{(4+n)} \sim 10^{\frac{15-4.5n}{n+2}} \text{TeV}. \tag{70}$$

For $n = 2$, this is quite a strong bound, requiring $M_{(6)} \gtrsim 30$ TeV. We next estimate the graviton luminosity from the Gravi-Primakoff process. Using $n_{Fe}/\rho \sim 1/m_{Fe}$ and $Z \sim 50$, we have

$$\mathcal{L}_{grav} \sim 10^{57}\text{GeV}\delta Z \frac{T_{SN}^{n+4}}{M_{(4+n)}^{n+2}} \tag{71}$$

which requires

$$M_{(4+n)} \gtrsim 10^{\frac{12-4.5n}{n+2}}\text{TeV}. \tag{72}$$

This is a somewhat weaker bound than for nucleon-nucleon brehmstrahlung. The basic reason is that while again in the SN, nucleon and photon abundances are comparable (actually nucleons are somewhat more abundant), the nucleon-nucleon brehmstrahlung cross-section is enhanced by strong-interaction effects.

In summary, we have found as expected that the strongest astrophysical bounds come from the hottest system, SN1987A. The bounds for $n = 2$ were quite strong, requiring $M_{(6)} \gtrsim 30$ TeV. This illustrates that the phenomenological viability of our scenario is not an immediate consequence of localizing the SM particles on a wall. Nevertheless, for $n > 2$, the infrared softness of higher dimensional gravity was enough to evade the constraints for $M_{(4+n)} \sim 1$ TeV. Even for $n = 2$, $M_{(6)} \sim 30$ TeV is consistent with a string scale $\sim few$ TeV, and therefore this case is still viable for solving the hierarchy problem and accessible to being tested at the LHC.

7 Cosmology

It is clear that in our scenario, early universe cosmology is drastically different than the current picture. Since the fundamental short distance scale is ~ 1 TeV, the highest temperature at which we can conceivably think about a reasonable space-time where the universe is born is $\sim m_{grav} \sim TeV$ rather than $M_{(4)} \sim 10^{19}$ GeV. Even beneath these temperatures, however, the dynamics of the extra dimensions is critical to the behavior of the universe on the wall. In the absence of any concrete mechanism for stabilizing the radius of the extra dimensions, we can not track the history of the universe starting from TeV temperatures. Of course, nothing is known directly about the universe at TeV temperatures. The only aspect of the early universe which

we know about with some certainty is the era of Big-Bang Nucleosynthesis (BBN) which begins at temperatures $\sim$ 1 MeV. The successful predictions of the light element abundances from BBN implies that the expansion rate of the universe during BBN can not be modified by more than $\sim 10\%$. Since the size of the extra dimensions determines $G_{N(4)}$ and hence the expansion rate of the $4-d$ universe on the wall, we know that whatever the mechanism for stabilizing the extra-dimensional radii, they must have settled to their current size before the onset of BBN. Note that the radii must be fixed with size $\lesssim$ mm, which is much smaller than the Hubble size $\sim 10^{10}$ cm at BBN. Therefore, the expansion of the 4-d universe can be described by the usual 4-d Robertson-Walker metric. This is analogous to the analysis of macroscopic gravity in section 5.1, where we saw that even when inter-particle separations are smaller than r_n, the large-distance gravitational energetics are unaffected. Furthermore, the extra-dimensions must be relatively empty of energy-density, since this would also contribute to the expansion rate of the 4-d universe.

This leads us to parametrize our ignorance about the physics determining the radius as follows. Extrapolating back in time from BBN, we assume that the universe is "normal" from BBN up to some maximum temperature T_* for the wall states. By "normal" we mean that the extra dimensions are essentially frozen and empty of energy density. One possible way this initial condition can come about is if T_* is the re-heating temperature after a period of inflation on the wall. The inflaton is a field localised on the wall and its decays re-heat predominantly wall-states while not producing significant numbers of gravitons.

We will test the consistency and cosmological viability of such a starting point. The main reason this will be non-trivial is due again to the presence of light modes other than SM particles- namely the extra-dimensional gravitons and, for the case where the wall is free to move, the goldstones describing the position of the wall.

It is easy to see that the goldstones are not especially problematic: they have a very small mass $\sim 10^{-3}eV$, and since they are their own antiparticles, they would count as $n/2$ extra neutrinos during Nucleosynthesis if they have thermal abundance. For $n = 2$, this is marginally consistent with BBN, whereas for $n > 2$ we have to insure that they are not thermal during BBN. This puts some upper bound on the "normalcy" temperature T_*. If the (model-dependent) coupling $\sim \lambda\psi\partial^\mu\psi^c\partial_\mu g/f^2$ is responsible for thermal-

ization, the goldstone drops out of equilibrium when

$$\begin{aligned} T_* &\lesssim \frac{f^{4/3}}{M_{Pl}^{1/3}\lambda_{max}^{2/3}(T_*)} \\ &\rightarrow \lambda^{2/3}(T_*)T_* \lesssim 10^{-2}\text{GeV} \end{aligned} \tag{73}$$

where $\lambda_{max}(T_*)$ is the largest Yukawa coupling of a SM particles thermal at temperature T_*.This roughly translates to $T_* \lesssim 1$ GeV. If instead the model-independent couplings suppressed by $1/f^4$ are keeping equilibrium, decoupling happens when

$$T_* \lesssim \frac{f^{8/7}}{M_{Pl}^{1/7}} \sim 10\text{GeV}. \tag{74}$$

This is a weak bound for obvious reasons: the goldstones are essentially massless, with smaller interaction cross-sections than neutrinos, and so it is guaranteed that they decouple before BBN, where neutrinos decouple. Furthermore, since they are so light, these goldstones can not over-close the universe.

Gravitons provide further cosmological challenges.

• **Expansion dominated cooling**

The energy density of the radiation on the wall cools in two ways. The first is the normal cooling due to the expansion of the universe:

$$\frac{d\rho}{dt}|_{\text{expansion}} \sim -3H\rho \sim -3\frac{T^2}{M_{Pl}}\rho \tag{75}$$

The second is cooling by "evaporation" into the extra dimensions, by producing gravitons which escape into the bulk. Notice again that this sort of cooling does not occur if the SM fields couple to some generic 1/ TeV coupled but 4-dimensional particle X, since the rates for the forward and backward reactions would proceed at the same rate and X would thermalize. The rate for graviton production is proportional to the usual factor $1/M_{(4+n)}^{n+2}$, and the rate for evaporative cooling can be determined by dimensional analysis to be

$$\frac{d\rho}{dt}|_{\text{evap.}} \sim -\frac{T^{n+7}}{M_{(4+n)}^{n+2}} \tag{76}$$

The expansion rate of the universe can only be normal if the rate for normal expansion by cooling is greater than the that from evaporation. This put an upper bound on the temperature T_* at where the universe can be thought of as normal:

$$T_* \lesssim \left(\frac{M_{(4+n)}^{n+2}}{M_{Pl}}\right)^{1/(n+1)} \sim 10^{\frac{(6n-9)}{(n+1)}}\text{MeV} \times \left(\frac{M_{(4+n)}}{1\text{TeV}}\right)^{(n+2)/(n+1)} \tag{77}$$

For the worst case $n = 2$, this is $T_* \lesssim 10$ MeV for $M_{Pl(4+n)} \sim 1$ TeV. However, the astrophysical constraints prefer $M_{(4+n)} \sim 10$ TeV, in which case T_* moves up to $\lesssim 100$ MeV, while for $n = 6$, $T_* \lesssim 10$ GeV. Of course,as $n \to \infty, T_* \to M_{(4+n)}$. It is reassuring that in all cases, $T_* \gtrsim 1$ MeV, so that BBN will not be significantly perturbed.

We can understand this constraint in another way. The rate of production of $(4+n)$ dimensional gravitons produced per relativistic species ("photons") on the wall, is given by

$$\frac{d}{dt}\frac{n_{grav}}{n_\gamma} = \langle n_\gamma \sigma_{\gamma\gamma\to grav} v\rangle \sim \frac{T^{n+3}}{M_{(4+n)}^{n+2}} \tag{78}$$

so that the total number density of gravitons produced during a Hubble time starting at temperature T_* is

$$\frac{n_{grav}}{n_\gamma} \sim \frac{T_*^{n+1} M_{Pl}}{M_{(4+n)}^{n+2}} \tag{79}$$

The "cooling" bound we have given corresponds to requiring $n_{grav} << n_\gamma$.

• **BBN constraints**

We must ensure that the produced gravitons do not significantly affect the expansion rate of the universe during BBN. The energy density in gravitons red-shifts away as R^{-3} rather than R^{-4}. This is because, from the 4 dimensional point of view, the gravitons produced at temperature T are massive KK modes with mass $\sim T$. Alternately, from the $(4+n)$ dimensional point of view, while the graviton is massless, the extra radii are frozen and not expanding, so the component of the graviton momentum in the extra dimensions is not red-shifting. The ratio of the energy density in gravitons versus photons by the time of BBN is then

$$\frac{\rho_{grav.}}{\rho_\gamma}|\text{BBN} \sim \frac{T_*}{1\text{MeV}} \times \frac{T_*^{n+1} M_{Pl}}{M_{(4+n)}^{n+2}} \tag{80}$$

Therefore, to insure normal expansion rate during BBN, the bound on T_* is slightly stronger

$$T_* \lesssim 10^{\frac{6n-9}{n+2}} \times \frac{M_{(4+n)}}{1\text{TeV}}. \tag{81}$$

• **Over-closure by gravitons**

The constraints we have discussed above would equally well apply to the production of purely purely 4-d particles with 1/TeV suppressed couplings of the appropriate power. The production of- gravitons is, however, qualitatively different since they escape into the bulk, with a very low probability of returning to interact with the SM fields on the wall. Consider the width Γ for a graviton propagating with energy E in the bulk, to decay into two photons on the wall. This interaction can only take place if the graviton is within its Compton wavelength $\sim E^{-1}$ from the wall. The probability that this is the case in extra dimensions of volume r_n^n is

$$P_{\text{grav. near wall}} \sim (E r_n)^{-n} \tag{82}$$

On the other hand, when it is near close to the wall, it decays into photons with a coupling suppressed by $\sim M_{(4+n)}^{-(n+2)/2}$, and therefore the width is

$$\Gamma_{\text{near wall}} \sim \frac{E^{n+3}}{M_{(4+n)})^{n+2}} \tag{83}$$

The total width Γ is

$$\Gamma = P_{\text{grav. near wall}} \times \Gamma_{\text{near wall}} \sim \frac{E^3}{r_n^n M_{(4+n)}^{n+2}} \sim \frac{E^3}{M_{(4)}^2} \tag{84}$$

This simple result could have also been understood directly from the KK point of view: the coupling of any KK mode is suppressed by $1/M_{(4)}$, so the width for any individual KK mode to go into SM fields is suppressed by $1/M_{(4)}^2$ and the above width follows from dimensional analysis. Of course significant amounts of energy can be lost to these KK modes, despite their weak coupling, for the usual reason of their enormous multiplicity. Among other things, eqn.(84) implies that the gravitons can be very long-lived, since they can not decay in the empty bulk . This is because, as long as the momenta in the extra dimensions is conserved, the graviton (which is massless from the (4+n) dimensional point of view) can not decay into two other

massless particles. Of course, interaction with the wall breaks translational invariance and allows momentum non-conservation in the extra dimensions, but this requires that the decay take place on the wall. The lifetime of a graviton of energy E is then

$$\tau(E) \sim \frac{M_{(4)}^2}{E^3} \sim 10^{10}\text{yr} \times \left(\frac{100\text{MeV}}{E}\right)^3 . \tag{85}$$

The gravitons produced at temperatures beneath ~ 100 MeV have lifetimes of at least the present age of the universe. The ratio n_{grav}/n_γ which was constrained to be $\lesssim 1$ in the above analysis must be in fact much smaller in order for the gravitons not to over-close the universe. As we have mentioned, most of the gravitons are "massive" with mass $\sim T_*$ from the 4-d point, they dramatically over-close the universe if their abundance is comparable to the photon abundance at early times.

The energy density stored in the gravitons produced at temperature T_* is

$$\rho_{grav} \sim T_* \times n_{grav} \sim \frac{T_*^{n+5} M_{Pl}}{M_{(4+n)}^{n+2}} \tag{86}$$

which then red-shifts mostly as R^{-3}. The ratio ρ_{grav}/T^3 is invariant. The critical density of the universe today corresponds to to $(\rho_{crit}/T^3) \sim 3 \times 10^{-9}$ GeV. For the gravitons not to over-close the universe, we therefore require for critical density at the present age of the universe. We therefore require

$$\begin{aligned} 3 \times 10^{-9}\text{GeV} \gtrsim \quad & \rho_{grav}/T_*^3 \quad \sim \frac{T_*^{n+2} M_{Pl}}{M_{(4+n)}^{n+2}} \\ \rightarrow \quad & T_* \lesssim 10^{\frac{6n-15}{n+2}}\text{MeV} \times \frac{M_{(4+n)}}{\text{TeV}} \end{aligned} \tag{87}$$

This is a serious constraint. For $n = 2$, we have to push $M_{(4+n)}$ to the astrophysically preferred ~ 10 TeV, to even get $T_* \sim 1$ MeV, although of course in this case a much more careful analysis has to be done. For $n = 6$, we need $T_* \lesssim 300$ MeV.

•Late decays to photons

Finally, we discuss the bounds coming from the late decay of gravitons into photons which would show up today as distortions of the diffuse photon spectrum. For $T_* \lesssim 100$ MeV, the graviton lifetime is longer than the age of the universe by $\sim (100\text{ MeV}/T_*)^3$, but a fraction $\sim (T_*/100\text{ MeV})^3$ of them

have already have already decayed, producing photons of energy $\sim T_*$. The flux of these photons (i.e. the number passing through a given solid angle $d\Omega$ per unit time) is then roughly

$$\frac{d\mathcal{F}(\mathcal{T}_*)}{d\Omega} \sim n_{0grav} H_0^{-1} \times (\frac{T_*}{100 \text{ MeV}})^3. \tag{88}$$

This is to be compared with the observational bound on the diffuse background radiation at photon energy E, which can be fit approximately by

$$\frac{d\mathcal{F}(\mathcal{E})}{d\Omega} \lesssim \frac{1\text{MeV}}{E} \text{cm}^{-2}\text{sr}^{-1}s^{-1} \tag{89}$$

Using the previously derived expressions for the present n_{grav}, this gives us a bound on T_*,

$$T_* \lesssim 10^{\frac{6n-15}{n+5}} \text{MeV} \times \left(\frac{M_{(4+n)}}{\text{TeV}}\right)^{\frac{n+2}{n+5}} \tag{90}$$

Again, for $n = 2$, even pushing $M_{(4+n)}$ to $\sim$ 10 TeV pushes T_* up to only $\lesssim 1$ MeV. On the other hand, for $n = 6$ and $M_{10} \sim 1$ TeV, $T_* \lesssim 100$ MeV is safe.

Notice that the bound from photons always demands a T_* which is lower than that which critically closes the universe. Therefore, in this minimal scenario, the KK gravitons can not account for the dark matter of the universe. Of course this is not a problem, the dark matter can be accounted for by other states in the theory. Given the inevitability of graviton production, however, graviton dark matter would certainly be attractive. There is a way out of the bound from decay to photons which can make this possible.

• **Fat-branes in the bulk**

The problem arose because we assumed that, once the graviton is emitted into the extra dimensions, it must eventually return to our 4-d wall in order to decay. Suppose however that there was another brane in the bulk, of perhaps a different dimensionality. Since gravity couples to everything, it could in particular couple to the matter on this new wall and lower the branching ratio for decaying on our wall. In fact, if the new wall has more than three spatial dimensions, the branching ratio to decay into photons on our wall would be drastically reduced. This can be seen in a number of ways. Suppose that the new wall has $(3+p)$ spatial dimensions with $p \leq n$. Note that since the extra dimensions are compactified, the extra p spatial dimensions are not infinite

but have size $\sim r_n$. We will call this new wall a fat-branes. Now, a graviton propagating in the bulk with energy $E \gg r_n^{-1}$ cannot resolve the difference between this new wall and stacks of $(Er_n)^p$ normal $3-d$ walls spaced E^{-1} apart. But then, the branching ratio for the graviton to decay on our wall is reduced greatly by $(Er_n)^{-p}$. The width for gravitons to decay on the new wall is

$$\Gamma \sim \frac{T^3}{M_{Pl}^2} \times (Tr_n)^p \sim \frac{T^{p+3}}{M_{Pl(4+p)}^{p+2}} \tag{91}$$

where we have used the relationship between Planck scales of different dimensionalities in the final expression. This also gives another interpretation of the result. From the viewpoint of a graviton of energy $E \gg r_n^{-1}$, the fat-brane may as well be infinite in all $3+p$ dimensions. Therefore, just as the width to decay on our wall is small because the interaction of any single graviton KK mode is suppressed by $1/M_{(4)}$, so the width to decay on the other wall is suppressed by $M_{(4+p)}^{-(p+2)/2}$. The branching ratio is then bigger because the higher dimensional Planck scale relevant to the $(3+p)$-brane is smaller. The lifetime for the graviton to decay on the fat-brane can easily be much smaller than the age of the universe. What is the fate of gravitons which decay on the fat-brane?

• **Dark matter on the fat-brane**

In order to understand the evolution of the universe after the decay of gravitons on the fat-brane, it is important to understand the cosmology of the fat-brane itself. There are two important points. First, just as for our 3-brane, at distances larger than r_n gravity on the fat-brane is normal and four-dimensional. This is because on scales larger than r_n, the "thickness" of the fat-brane can not be resolved. Second, the energy densities on all branes contribute to the 4-d expansion rate of both our brane and the fat-brane. Therefore, there is effectively a single energy density and a common 4-d expansion rate for the two branes. Consequently, the way that the expansion rate is affected after the gravitons are captured on the fat-brane depend on the nature of the decay products there. If they are non-relativistic, their energy density red-shifts away like R^{-3} and they may provide a dark matter candidate. Notice that this dark matter may actually "shine" on its own brane; it is only dark to us. This allows any mass range for the dark matter candidates, since they can never into ordinary photons.

8 TeV Axion in the bulk and the strong CP problem

As we have remarked, the main reason our scenario remains phenomenologically viable is that the couplings to states that can propagate in the bulk are suppressed. This observation can also be used to revive the TeV axion as a solution to the strong CP problem, if the axion is taken to be a bulk field. Without specifying the origin of the axion, the relevant terms in the low-energy effective theory are

$$\mathcal{L}_{eff} \supset \int d^{4+n}x(\partial a)^2 + \int d^4x \frac{a(x;x^a=0)}{f_a^{(n+2)/2}} F\tilde{F}. \tag{92}$$

where $a = 4, \cdots, 3+n$ runs over the extra dimensions. Just as always, QCD will generate a potential for $a(x, x^a = 0)$. In order to minimize energy, $a(x, x^a = 0)$ will prefer to sit at the minimum of this potential, solving the strong CP problem on the wall. Furthermore, in order to minimize kinetic energy, a will take on this vev uniformly everywhere in the bulk. From the 4 dimensional point of view, we can expand a into KK excitations. After going to canonical normalization, each of these has $1/M_{(4)}$ suppressed couplings to $F\tilde{F}$ for $f \sim M_{(4+n)} \sim$ TeV. The potential that is generated by QCD is then minimized with the zero mode acquiring the appropriate vev and all the massive modes having zero vev.

An explicit field theoretic model producing such an axion field can be easily constructed. Let u^c, d^c and Q be the weak doublet and singlet quark fields respectively and H be a electroweak Higgs doublet. In our theory these states are the four-dimensional modes on the 3-brane. Let χ be a bulk complex scalar field whose spatially constant vev will break PQ symmetry.

$$\langle\chi\rangle \sim M_{(4+n)}^{1+\frac{n}{2}} \tag{93}$$

The $4+n$ dimensional axion field is defined as

$$a = \langle\chi\rangle arg\chi = \frac{a_4(x_\mu)}{\sqrt{r_n^n}} + \text{KK- modes} \tag{94}$$

where we have expanded into KK modes. As already mentioned, the zero mode a_4 is a genuine four-dimensional axion field, with the $1/r_n^{\frac{n}{2}}$ insuring

that its 4-d kinetic term is canonically normalized. The coupling of χ with matter on the 3-brane can be written as

$$\int d^{n+4}x\delta(x^a)\frac{\chi}{M_{(4+n)}^{1+\frac{n}{2}}}(HQu^c + H^*Qd^c) \tag{95}$$

It is straightforward to see that an effective coupling of the genuine axion to $F\tilde{F}$ is

$$\sim \frac{a_4}{\langle\chi\rangle r_n^{n/2}}F\tilde{F} \sim \frac{a_4}{M_4}F\tilde{F} \tag{96}$$

and thus from the point of view of the four-dimensional theory it is effectively a Planck-scale axion. While the bulk axion field a has only 1/TeV suppressed couplings, it is safe from all astrophysical constraints we have considered for the same reason gravitons are safe. Of course, 4-d axions with such high decay constants ordinarily suffer from the usual cosmological moduli problem [15]; we have nothing to add to the early cosmology which needs to drive the axion to the origin. However,as long as the axion is at its origin at temperature T_*, it will not be significantly excited during the subsequent evolution of the universe, again for the same reason gravitons were not significantly excited.

9 Gauge Fields in the Bulk

For a variety of reasons, it seems unlikely that $SU(3)\otimes SU(2)\otimes U(1)$ is the only gauge group under which the SM fields are charged. Normally, the non-observation of additional gauge particles is attributed to a very high scale of symmetry breaking $\gtrsim$ TeV and comparably high masses for the gauge bosons. The impact of these heavy gauge bosons on low energy physics is then very limited. By contrast, in this section we will see that the situation can change dramatically in theories with large extra dimensions. This can happen if if the new gauge bosons can freely propagate in the bulk *, while matter charged under the gauge group, including scalars which may spontaneously break the symmetry, live on a 3-brane. The following features emerge: independent of the number of extra dimensions, the gauge field can mediate a repulsive force more than a million times stronger than gravity at distances smaller than a millimeter. This raises the exciting possibility that

*The gravi-photons are model-independent examples of this sort.

these forces will be discovered in the measurements of sub-mm gravitational strength forces [16].

Consider for simplicity a $U(1)$ gauge field propagating in the bulk. The free action is

$$\mathcal{L} = \int d^{4+n}x \frac{1}{4g_{4+n}^2} F^2,\ F_{MN} = \partial_M A_N - \partial_N A_M \tag{97}$$

where we take the scale of the dimensionful $(4+n)$ dimensional gauge coupling to be $\sim$ the ultraviolet cutoff $M_{(4+N)}$:

$$g_{(4+n)}^2 \sim M_{(4+n)}^{-n} \tag{98}$$

This gauge field interacts with matter fields living on a 3-brane via the induced covariant derivative on the brane

$$D_\mu \phi = (\partial_\mu + i q_\phi A_\mu(x, x^a = 0))\phi. \tag{99}$$

Expanding A_μ in KK modes, only the zero mode $A^0(x)$ transforms under wall field gauge transformations $\phi \to e^{i\theta(x)}\phi$; the rest of the KK modes are massive starting at r_n^{-1}. At distances much larger than r_n^{-1}, only the zero mode is relevant, and the action becomes

$$\mathcal{S} = \int d^4x \frac{1}{g_4^2}(\partial_\mu A_\nu^0 - \partial_\nu A_\mu^0)^2 + \mathcal{L}_{\rm matter}(\phi, D_\mu^0 \phi) \tag{100}$$

where the effective 4-dimensional gauge coupling is

$$g_4^2 \sim \frac{1}{r_n^2 M_{(4+n)}^n} \sim \frac{M_{(4+n)}^2}{M_{(4)}^2}. \tag{101}$$

The first interesting point is that this is a miniscule gauge coupling $g_4 \sim 10^{-16}$ for $M_{(4+n)} \sim 1$ TeV, independent of the number of extra dimensions n. Suppose this gauge field couples to protons or neutrons. The ratio of the repulsive force mediated by this gauge field to the gravitational attraction is

$$\frac{F_{\rm gauge}}{F_{grav}} \sim \frac{g_4^2}{G_N m_p^2} \sim 10^6 \left(\frac{g_4}{10^{-16}}\right)^2 \tag{102}$$

Clearly, the corresponding gauge boson can not remain massless. If the gauge symmetry is broken by the vev of a field χ on the wall, the gauge boson will

get a very small mass, which is however exactly in the interesting range experimentally:

$$m_{A^0}^{-1} = (g_4 q_\chi \langle\chi\rangle)^{-1} \sim 1\text{mm} \times \left(\frac{10^{-16}}{g_4}\right)\left(\frac{1\text{TeV}}{q_\chi\langle\chi\rangle}\right) \tag{103}$$

Of course there are a number of undetermined parameters so a hard prediction is difficult Nevertheless, it is reasonable that $g_4 \sim 10^{-16}$ is a lower bound, and so we can expect repulsive forces between say $10^6 - 10^8$ times gravitational strength at sub-mm distances. For all $n > 2$, the mass of the KK excitations of the gauge field are too large to give a signal at the distances probed by the next generation sub-mm force experiments. The case $n = 2$ has still richer possibilities since the KK excitations will have comparable masses to the lowest mode, and may contribute significantly to the measured long-range force.

The most interesting possibility is to relate this new gauge field with the global Baryon (B) or Lepton (L) number symmetries of the standard model. The gauging of the anomaly-free $B-L$ symmetry has a definite experimental signal: since atoms are neutral, the $B - L$ charge of an atom is its neutron number. Thus, the hydrogen atom will not feel this force, while it will be isotope dependent for other materials. Gauging other combinations of B and L, e.g. either B or L separately, is very interesting as well. Let us consider the case of gauging baryon number. Of course we have to worry about canceling anomalies; the most straightforward way out is to add chiral fermions canceling the anomaly which become massive when $SU(2)_L \times U(1)_Y$ is broken. For instance, we can add three extra generations with opposite baryon numbers (ignoring the obvious problem with the S-parameter). The interest in this exercise is that it may provide a mechanism for suppressing proton decay. Although baryon number must be broken, dangerous proton decay operators may be tremendously suppressed if the higgs that breaks B lives on a different brane.

10 Cosmological Stability of Large Radii

We have said nothing about what fixes the radii of the extra dimensions at their large values, this is an outstanding problem. The largeness of the

extra begs another, more dramatic question: why is our 4-dimensional universe so much larger still? It is not considered a failing of the SM that it offers no explanation of why the universe is so much larger than the Planck scale. Indeed, this is equivalent to the cosmological constant problem. If the density of the universe at the Planck time was $O(1)$ in Planck units, there would be no other time scale than the Planck time and the universe would not grow to be 10^{10} years old. This was only possible because the energy density is so miniscule compared to M_{Pl}^4, which is precisely the cosmological constant problem. It may be that once the cosmological constant problem is understood, whatever makes the enormity of our 4-d universe natural can also explain the (much milder) largeness of the extra n dimensions.

In this context we would like to make the side remark that the usual cosmological constant problem is in some sense less severe in our framework. Suppose for instance that there is a string theory with string tension $m_s \sim$TeV, but where the SUSY is primordially broken only on our wall at the scale m_s. As we argued in [4], the SUSY breaking mass splittings induced for bulk modes is then highly suppressed $\sim$ (1 mm)$^{-1}$ at most. However, there is nothing that can be done about the $\sim$ (TeV)^4 vacuum energy on the wall, and we have to imagine canceling it by fine-tuning it away against a bare cosmological constant

$$\int d^{4+n}x\sqrt{-g}\Lambda_{0(4+n)} \to \int d^4x\sqrt{-g}(r_n^n\Lambda_{0(4+n)}). \tag{104}$$

Since the radii r_n are large compared to (TeV)$^{-1}$, the mass scale $\Lambda_{0(4+n)}^{1/(4+n)}$ does not have to be as large as the TeV scale to cancel the cosmological constant. Note that since the SUSY splittings in the bulk are so small, there is no worry of an $\sim$(TeV)$^{4+n}$ cosmological constant being generated.

We do not, however, have to hide behind our ignorance about the cosmological constant problem. It may be that the radii are large for more mundane reasons: for some reason, some of the radius moduli have a potential energy with a minimum at very large values of rm_{grav}. Even without knowing anything about the origin on such a potential, we can place phenomenological constraints on $V(r)$ by requiring that the field was not significantly perturbed from its minimum by interacting with the hot universe from the time of BBN to the present. Since the modulus is a bulk field, $V(r)$ should be a bulk energy density, and $U(r) = V(r)r^n$ should have a minimum at large r. Suppose that at temperature T_*, the modulus was already stabilised at its minimum

r_*. How significantly is it perturbed as the wall fields dump energy into the extra dimensions? We estimate this by first computing the total amount of energy dumped into the extra dimensions. Any bulk field must have at least a $1/M_{(4+n)}^{(n+2)/2}$ suppression for its coupling, and so the maximum rate for dumping energy into the extra dimensions, per unit time is

$$\dot{E}_{wall-} \sim \frac{T^{n+7}}{M_{(4+n)}^{n+2}} \times V_3 \tag{105}$$

where V_3 is the three-volume of the region of the wall losing the energy. At worst, this energy gets entirely transfered to changing the potential of the radius modulus,

$$-\dot{E}_{wall} = \dot{E}_{rad.} = \dot{U}(r)V_3 \tag{106}$$

and so the change in $U(r)$ over a Hubble time is

$$\delta U(r) \sim \frac{T_*^{n+5} M_{Pl}}{M_{(4+n)}^{n+2}} \tag{107}$$

Translating this change as $\delta U(r) \sim (\delta r)^2 U''(r_*)/2$, we obtain a bound on $U''(r_*)$ from the requirement that $\delta r/r \lesssim 10^{-1}$:

$$U''(r_*) r_*^2 \gtrsim \frac{T_*^{n+5} M_{Pl}}{M^{n+4}} \tag{108}$$

Any theory where this inequality can not be satisfied for $T_* \gtrsim 1$ MeV is ruled out by cosmology during and after BBN.

Of course we do not have a theory predicting a $U(r)$ which naturally generates a large radius. Nevertheless, we can speculate on what sort of $U(r)$ can produce minima at large value r_*. In analogy with dimensional transmutation, a large hierarchy can be generated if log$r$ is determined to be , say, $O(10)$. We can in any case parametrize $U(r)$ so that

$$U(r) = g(r, m_{grav}) f(\log(r m_{grav})) \tag{109}$$

where $f(x)$ is a dimensionless function, and $g(r, m_{grav})$ has dimensions mass^4. One natural assumption on the form of g is that the fully decompactified theory $r m_{grav} \to \infty$ should be a minimum of the potential; certainly in string theory, there is a vacuum as the string coupling goes to zero with all

ten dimensions large. In that case, it must be that $g(r, m_{grav}) \to 0$ at least as fast as a power law as $r \to \infty$. We will also consider the case where $g(r)$ is essentially flat in analogy with the "geometric hierarchy" potential. The question is now whether the theory can develop a minimum, not for infinitely large radius but for finite but large values of rm_{grav}. Since we are interested in the limit of a large rm_{grav} anyway, we can approximate $g(r)$ at large r with its leading power law behavior

$$g(r, m_{grav}) \to c\frac{m_{grav}^{4-a}}{r^a}. \tag{110}$$

Requiring $U(r)$ to be stationary then gives

$$\frac{m_{grav}^{4-a}}{r^{a+1}} \times (f' - af) = 0 \tag{111}$$

so there is a minimum at a value $x_* = \log(r_* m_{grav})$ where

$$\frac{f'(x_*)}{f(x_*)} = a. \tag{112}$$

Note that the condition for the existence of a local minimum at large r is completely determined by f. It is certainly not implausible that the there are dimensionless ratios of $O(10)$ in f , leading to a value of x_* also of $O(10)$, leading to a very large (but not infinite) radius.

Consider first the "geometric hierarchy" scenario where $a = 0$. In this case,

$$U''(r_*) = \frac{m_{grav}^4 f''(x_*)}{r_*^2}, \tag{113}$$

and the bound from eqn.(108) translates to

$$T_* \lesssim 10^{\frac{3n}{n+5}}\text{GeV} \times f''^{\frac{1}{n+5}} \times \left(\frac{M_{(4+n)}}{\text{TeV}}\right)^{\frac{n+6}{n+5}} \tag{114}$$

We do not expect $f''(x_*)$ to be larger than $O(10)$, and even if it is larger, it is raised to small fractional power. For $n = 2$ and $M_{(6)} \sim 10$ TeV, this requires $T \lesssim 100$ GeV, certainly the weakest of all cosmological bounds we have considered. For all $n > 2$, the bound is easily met with $T_* \lesssim 10$ GeV and $M_{(4+n)} \sim 1$ TeV.

The cases of intermediate "hardness", $4 > a > 0$ are also less constraining than the other cosmological bounds for obvious reasons: $U''(r_*)r_*^2$ is enhanced by a positive power of m_{grav}, so it cost significant energy to excite fluctuations in the radius.

Finally, consider the "soft" case of $n = 4$. Here, the scale of the potential is determined by its very size, making it "soft" for large radii. Here,

$$U''(r_*)r_*^2 = \frac{f''(x_*) - 16f(x_*)}{r_*^4} \tag{115}$$

and T_* is bounded as

$$T_* \lesssim 10^{(\frac{3n-120/n}{n+5})}\text{GeV}\left(\frac{M}{\text{TeV}}\right)^{\frac{6+n+8/n}{n+5}} \tag{116}$$

Clearly for $n = 2$, and even for $M_{(6)} \sim 10$ TeV, $T_* \lesssim 100$ eV,and so the radius could no have settled by the time of BBN, where T ~ 1 MeV.The case $n = 3$ is marginal $T_* \lesssim 1$ MeV for $M_{(7)} \sim$1 TeV, but is fine already for $M_7 \sim 10$ TeV. For $n \geq 4$, however, even this "soft" scenario can be accomadated with $T_* \lesssim 10$ MeV and $M_{(4+n)} \sim 1$ TeV.

Other examples of situations where a large vev for a field can be generated while the excitations about the minimum have a mass uncorrelated with the vev can be constructed. In appendix 2, we present a supersymmetric toy example of this type.

11 The large n limit

Finally, we wish to comment on an interesting limit of our framework, where the number of new dimensions becomes very large. This case may be excluded by theoretical prejudices about string theory being the true theory of gravity, which seems to limit $n \leq 6$ or 7, but we will ignore this prejudice here. The $n \to \infty$ limit is interesting for many reasons. The main point is that in this limit, the size of the new dimensions does not have to be much larger than $M_{(4+n)}$, solving the remaining "hierarchy" problem in our framework. For instance, for $n = 100$ new dimensions, the correct $M_{(4)}$ can be reproduced with $M_{Pl(4+n)} \sim 2$ TeV and extra dimensional radii $\sim$ (1 TeV $)^{-1}$ in size. Since the extra dimensions are now in the TeV range, no

special mechanism is required to confine SM fields to a wall in the extra dimensions. Furthermore, all the KK excitations of the graviton are at ~ 1 TeV and are therefore irrelevant to low-energy physics. All the low-energy lab and astrophysical constraints involving emission of gravitons into the extra dimensions are gone. Indeed, the theory beneath a TeV is literally the SM, and all that is required is that dangerous higher-dimension operators suppressed by a TeV are forbidden, a feature required even for small n. The cosmology of this framework is also completely normal at temperatures $\lesssim 1$ TeV, with no worries about losing energy by emitting gravitons into the extra dimensions. Apart from the usual strong-gravitational signals at colliders, in this limit the KK excitations of SM fields for each of the n new dimensions may also be observed.

12 Discussion and Outlook

Over the last twenty years, the hierarchy problem has been one of the central motivations for constructing extensions of the SM, with either new strong dynamics or supersymmetry stabilizing the weak scale. By contrast, in [1] we proposed that the problem simply does not exist if the fundamental short-distance cutoff of the theory, where gravity becomes comparable in strength to the gauge interactions, is near the weak scale. This led immediately to the requirement of new sub-mm dimensions and SM fields localised on a brane in the higher-dimensional space. Unlike the other solutions to the hierarchy problem, our scenario does not require any special dynamics to stabilised the weak scale. On the other hand, it leads to one of the most exciting possibilities for new accessible physics, since in this scenario the structure of the quantum gravity can be experimentally probed in the near future. Given the amount of new physics brought down to perhaps dangerously accessible energies, it is crucial to check that this framework is not already experimentally excluded.

In this paper, we have systematically studied experimental constraints on our framework from phenomenology, astrophysics and cosmology. Because of the power law decoupling of higher-dimension operators, there are no significant bounds from "compositeness" or precision observables, which in any case do not tightly constrain generic new weak scale physics. Rather, the most dangerous processes involve the production of unavoidable new

massless particles in our framework–the higher dimensional gravitons–whose couplings are only suppressed by 1/TeV. Analogous light 4-dimensional particles with 1/TeV suppressed couplings, such as axions or familons, are grossly excluded. Nevertheless, we find that for all $n > 2$, the extreme infrared softness of higher dimension gravity allows the $(4+n)$ dimensional Planck scale $M_{(4+n)}$ to be as low as ~ 1 TeV. The experimental limits are not trivially satisfied, however, and for $n = 2$, energy loss from SN1987A and distortions of the diffuse photon background by the late decay of cosmologically produced gravitons force the 6 dimensional Planck scale to above ~ 30 TeV. For precisely $n = 2$, however, there can be an $O(10)$ hierarchy between the 6-dimensional Planck scale and the true cutoff of the low-energy theory, as was discussed in section 3 for the particular implementation of our scenario within type I string theory. A natural solution to the hierarchy problem together with new physics at the accessible energies can still be accomadated even for $n = 2$.

Of course it is possible that we have overlooked some important effects which exclude our framework. Nevertheless, these theories have evaded the quite strong experimental limits we have considered in a quite general way. The strongest bounds were evaded since higher-dimensional theories are soft in the infrared. Alternately, as n grows, r_n decreases and the number of available KK excitations beneath a given energy E also decreases. In fact, as $n \to \infty$, $r_n^{-1} \to$ TeV, all KK excitations are at a TeV, and effective low energy theory is simply the SM with no additional light states. This shows that this sort of new physics can not be excluded simply because it is exotic, the theory becomes safest in the limit of infinitely extra dimensions.

On the theoretical front, perhaps the most important issues to address are the mechanism for generating large radii, and early universe cosmology. Other issues include proton stability, SUSY breaking in the string theory context, and gauge coupling unification. The latter is the one piece of indirect evidence that suggests the existence of a fundamental energy scale far above the weak scale [17]. It has been pointed out that the existence of intermediate scale dimensions larger than the weak scale, into which the SM fields can propagate, can speed up gauge coupling unification due to the power law running of gauge couplings in higher dimensional theories [18]. In [4], the proposal of [1] was combined with the mechanism of [18] in a string context, thereby achieving both gravity and gauge unification near the TeV scale. A similar proposal was later made in [19]. Alternately, [6] suggested

that different gauge couplings may arise from different branes, leading to a possibly different picture for gauge coupling unification.

Experimentally, this framework can be tested at the LHC, and on a shorter scale, can be probed in experiments measuring sub-mm gravitational strength forces. If the scale of quantum gravity is close to a TeV as motivated by the hierarchy problem, at least two types of signatures will be seen at the LHC [1, 4]. The first involve the unsuppressed emission of gravitons into the higher dimensional bulk, leading to missing energy signatures. The second involve the production of new states of the quantum gravitational theory, such as Regge-recurrences for every SM particle in the string implementation of our framework. Clearly the detailed characteristics of these signatures must be studied in greater detail.

There is also the exciting possibility that the upcoming sub-mm measurements of gravity will uncover aspects of our scenario. There are at least three types of effects that may be observed.

• Transition from $1/r^2 \to 1/r^4$ Newtonian gravity for $n = 2$ extra dimensions. In view of our astrophysical and cosmological considerations, which push the 6-dimensional Planck mass to ~ 30 TeV, the observation of this transition will be especially challenging.

• On the other hand, particles with sub-mm Compton wavelengths can naturally arise in our scenario, for instance due to the breaking of SUSY on our 3-brane [4]. These will mediate gravitational strength Yukawa forces.

• A new possibility pointed out in this paper is that gauge fields living in the bulk and coupling to a linear combination of Baryon and Lepton number can mediate repulsive forces which are $\sim 10^6 - 10^8$ times gravity at sub-mm distances.

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Appendix 1

In this appendix we consider the Higgs effect for the breaking of "gauged" translation invariance, that is spontaneous translation invariance breaking in the presence of gravity. In the usual KK picture (some of the) $g_{\mu A}$ ($\mu = 0, \cdots, 3$, $a = 4, \cdots, 3+n$ components of the higher-dimensional metric are viewed as massless gauge fields in 4 dimensions, with the gaugessymmetry being translations in the extra n dimensions. The result we find is simple and easy to as the exact analogue of the result we found for the small mass of bulk gauge fields when the gauge symmetry is broken on the wall. The zero modes of the $g^{\mu a}$ eat the y^a goldstone bosons to get a mass

$$m^2 \sim \frac{f^4}{M_{(4)}^2} \tag{117}$$

where f is the wall tension. Intuitively, the large radius means that the zero mode of the "gauge field" $g^{\mu a}$ has a very small "gauge coupling" $\sim 1/M_{(4)}$. The "vev" which breaks translation invariance is nothing but the localised energy density of the wall f^4, and the above formula follows. For completeness, however, we will consider this effect in somewhat more detail. We will refer to the 'wall' as to a localized, stable configuration independent of the coordinate (x^a), that minimizes the action. One can imagine the wall as some sort of topological defect in higher dimensions.

First turn off gravity and let $\Phi(x^a)$ be the vev of the real scalar field forming the wall. Consider the action S for the field configuration $\Phi(x^a + y^a(x))$. Translation invariance in x^a demands that

$$S[\Phi(x^a + y^a(x))] = \int d^{4+n}x f(x^a)\partial_\mu y^a \partial^\mu y^a + \cdots \tag{118}$$

where no linear term is present since S is stationary at Φ, and $f(x^a)$ is some function localised around the position of the wall $x^a = 0$. At distances large compared to the "thickness" of the wall, we can approximate $f(x^a) = f^4\delta(x^a)$ where f has units of mass, and

$$S[\Phi(x^a + y^a(x))] \to \int d^{4+n}x \delta(x^a) f^4 \partial^\mu y^a \partial_\mu y^a. \tag{119}$$

As expected, the y^a are massless dynamical degrees of freedom living on the wall, the Nambu-Goldstone bosons of spontaneously broken translation

invariance. Global translations in x^a are realized non-linearly on the y^a via $y^a(x) \to y^a(x) + c$. The quantity f^4 can be interpreted as the tension of the wall.

Now turn on gravity, specifically the $g^{\mu a}$ "gauge" fields which gauge local translations in x^a:

$$y^a(x) \to y^a(x) + c(x),\ g^{\mu a} \to g^{\mu a} + \partial^\mu c. \tag{120}$$

As usual, we can go to a unitary gauge where $y^a(x)$ are everywhere set to zero. In this gauge, the $g^{\mu a}$ obtain a position dependent mass term

$$\mathcal{L}_{\rm mass} = \int d^{4+n}x \delta(x^a) f^4 (g^{\mu a})^2. \tag{121}$$

That the mass term should be position dependent is intuitively obvious. Far from the wall, no local observer knows that translation invariance has been spontaneously broken; the graviton masses should therefore vanish away from the wall.

Let us expand $g^{\mu a}$ in canonically normalized KK modes $h^{\mu a}_{n_\alpha}$, recalling that each individual KK mode will come suppressed by $1/M_{(4)}$. The KK modes have already have a mass $\sim (n/r_n)^2$, and the position dependent mass term from symmetry breaking becomes

$$\mathcal{L}_{\rm break} = \int d^4x \frac{f^4}{M^2_{(4)}} (\sum_{n_\alpha} h^{\mu a}_{n_\alpha})^2. \tag{122}$$

As long as $f^2/M_{(4)}$ is smaller than $1/r_n$, the masses of the heavy KK excitations are not significantly perturbed by the breaking term. The zero mode does not have any mass in the absence of symmetry breaking, however, so it gets a mass

$$m^2_{h^{\mu a}_0} = \frac{f^4}{M^2_{(4)}}. \tag{123}$$

Note that, for $f \sim$ TeV, this mass is $\sim (\text{mm})^{-1}$, and at least for $n > 2$ the assumption than the mass is much smaller than r_n^{-1} is justified. For $n = 2$, the first few KK modes can not be completely decoupled, and some linear combination of them eat the y^a. We however still expect the lightest graviton mode to have mass $\sim (\text{mm})^{-1}$ in this case as well.

Appendix 2

As discussed in the text, a vague worry about having very large dimensions comes from the impression that the potentials responsible fro stabilizing the radius modulus will be very "soft", and therefore the modulus will be very light, possibly giving cosmological problems. In this example we present an explicit counter-example to this intuition, albeit in a toy model. We will write down a theory where (a) field S is a flat direction to all orders in perturbation theory, b) a potential for S is generated by non-perturbative effects leading to distinct minima very far separated from each other, while (c) the curvature of the potential for S around its minima are completely uncorrelated with the sizes of $\langle S \rangle$. The model is supersymmetric and the these features will be generated without any fine-tuning.

Consider first, an $SU(N)$ QCD with N flavors $Q, \bar{Q}$ and a singlet field S, coupled with a tree-level superpotential

$$W_{tree} = \lambda S Tr(Q\bar{Q}) \tag{124}$$

This model has been discussed many times and has found a variety of applications. At the classical level and to all orders in perturbation theory, $S \neq 0$ and $Q, \bar{Q} = 0$ is a flat direction. For $S \gg \Lambda$, $Q, \bar{Q}$ can be integrated out, and gaugino condensation in the low energy theory gives

$$W_{eff}(S) = \lambda \Lambda^2 S \tag{125}$$

This is of course the only superpotential consistent with all the symmetries, and gives rise (at lowest order) to an exactly flat potential $V(S) = |\lambda \Lambda^2|^2$. Of course, the potential is modified by corrections to the Kahler potential of S, and most generally

$$V(S) = \frac{|\lambda \Lambda^2|^2}{Z(S)} \tag{126}$$

where Z_S is the wavefunction renormalisation of S. For $S \gg \Lambda$, the potential remains approximately flat since the corrections to $Z(S)$ are perturbative, however for $S \sim \Lambda$, this description breaks down. We are guaranteed, however, that there is a *supersymmetric* minimum at $S = 0$. The exact superpotential including the quantum modified constraint (see [20] for a review) for this case is

$$W = \lambda S Tr(M) + X(detM - B\bar{B} - \Lambda^{2N}). \tag{127}$$

This superpotential admits supersymmetric vacuum with $\langle S \rangle = 0$, while the curvature of the effective potential for S is $\sim \Lambda^2$.

We can find variations on this model with copies of the gauge group and matter to produce multiple minima for S. Consider e.g. the group $SU(N) \times SU(N')$ with respectively N, N' flavors, and still a single singlet S, and consider the tree superpotential

$$W = \lambda S Tr(Q\bar{Q}) + (S - m) Tr(Q'\bar{Q}') \tag{128}$$

where m is some arbitrary dimensionful scale. It is easy to see that, for $S \ll m$, the potential looks like what we discussed previously, with a SUSY minimum around $S = 0$ with curvature $\sim \Lambda^2$, while there is also a SUSY minimum at $S = m$ with curvatures $\sim \Lambda'$, with a flat potential separating the minima. In this example, the (classically flat) field S can obtain an arbitrary vev, completely uncorrelated with the curvatures of the potential around the minima.

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