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New Constraints on Chiral Gauge Theories

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Abstract

Recently, a new constraint on the structure of a wide class of strongly coupled field theories has been proposed. It takes the form of an inequality limiting the number of degrees of freedom in the infrared description of a theory to be no larger than the number of underlying, ultraviolet degrees of freedom. Here we apply this inequality to chiral gauge theories. For some models we find that it is always satisfied, while for others we find that the assumption of the validity of the inequality implies a strong additional restriction on the spectrum of massless composite particles.

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I. INTRODUCTION

Strongly coupled quantum field theories play a central role in the description of nature at the most fundamental level. Quantum chromodynamics describes the observed strong interactions, and other strongly interacting theories are part of many efforts to extend the standard model. But strongly coupled field theories are notoriously difficult to analyze directly. Because of this, general constraints on their behavior can be very useful.

In a recent paper [1], a new general constraint on strongly coupled field theories was proposed. It takes the form of an inequality limiting the number of massless degrees of freedom in the infrared description of a field theory to be no larger than the number of ultraviolet degrees of freedom. This inequality was conjectured to apply to all asymptotically free theories (those governed by a free ultraviolet fixed point), whose infrared behavior is also governed by a fixed point, not necessarily free. It was noted that the inequality can also apply to certain theories with interacting ultraviolet fixed points.

The inequality is formulated using finite temperature as a device to probe all energy scales, with the degree-of-freedom count defined using the free energy of the field theory. The zero-temperature theory of interest is characterized using the quantity f_{IR}, related to the free energy by

$$f_{IR} \equiv -\frac{90}{\pi^2} \lim_{T\to 0} \frac{\mathcal{F}}{T^4} \ , \tag{1}$$

where T is the temperature and $\mathcal{F}$ is the conventionally defined free energy per unit volume (which is equal to minus the pressure). This limit will be well defined if the theory has an infrared fixed point. For the special case of an infrared-free theory, f_{IR} is simply the number of massless bosonic degrees of freedom plus 7/8 times the number of massless fermionic degrees of freedom. The corresponding expression in the large T limit is

$$f_{UV} \equiv -\frac{90}{\pi^2} \lim_{T\to \infty} \frac{\mathcal{F}}{T^4} \ . \tag{2}$$

Just as in the infrared, this limit will be well defined if the theory has an ultraviolet fixed point. For an asymptotically free theory, f_{UV} counts the underlying, ultraviolet degrees of freedom in a similar way.

In terms of these quantities, the conjectured inequality for asymptotically-free theories is

$$f_{IR} \leq f_{UV} \ . \tag{3}$$

In Ref. [1], the inequality was compared to known results and then used to derive new results

for several strongly coupled, vector-like gauge theories. [1]

In the present paper, we extend this study to chiral gauge theories. These theories, in which the left- and right- handed fermions couple differently to the gauge fields, are potentially important examples of strongly coupled gauge field theories. The motivation for the effort to understand better the nonperturbative behavior of chiral gauge theories stems not just from their field-theoretic interest, but also from their possible application to physics beyond the standard model, including (i) models of particle substructure that can produce massless composite fermions and hence account for the fact that the observed fermions have masses much smaller than the lower bound on a hypothetical compositeness scale, and (ii) dynamical symmetry breaking of electroweak and higher symmetries. A key feature is that the fermion content is subject to an additional constraint not present in vectorial gauge theories, namely the absence of gauge and global π_4 (Witten) anomalies (and the absence of mixed gauge-gravitational anomalies, if one includes gravity). This greatly reduces the number of models that one can consider. On the other hand, a number of powerful techniques that one can use for vectorial gauge theories are absent for chiral gauge theories; these include correlation function inequalities (since the fermion measure is not positive). There are also complications in trying to formulate chiral gauge theories on the lattice because of fermion doubling. Nevertheless, one can still use the 't Hooft global anomaly matching conditions [2] as well as large-N methods.

For asymptotically free chiral theories, a variety of infrared phenomena can be consistent with global anomaly matching. One possibility is that as in QCD the gauge symmetry remains intact but the theory confines and the global flavor symmetries break spontaneously. Another possibility is that confinement sets in but that the global symmetries are unbroken. This is realized by the formation of gauge singlet, massless composite fermions, along with other possible degrees of freedom. It is also possible that the theory does not confine but is governed in the infrared by an interacting fixed point, possibly weak. The symmetries will again remain unbroken. Yet another well-known possibility is that these theories dynamically break their own gauge symmetries, e.g., by the formation of fermion condensates. Each of these possibilities except the last will play a role here.

We examine several anomaly-free chiral gauge theories. Some of these are automatically

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[1] While the fixed points are necessary to ensure the existence of the limits Eqs. (1,2), one can imagine applying the inequality when the ultraviolet fixed point is only approximate, that is when it is a feature of an effective low energy theory. f_{UV} would then count degrees of freedom for energy scales much larger than masses and confinement or symmetry breaking scales in the effective theory, but much smaller than the scale of new physics.

asymptotically free. For the others, we restrict the number of fermion representations so that they are asymptotically free. The quantity f_{UV} may then be computed perturbatively. We consider the possible infrared realizations of the theories as allowed by global anomaly matching and large-N methods, and compute f_{IR}. For some models, the inequality is automatically satisfied, while for others the assumption of the validity of the inequality provides a strong restriction on the infrared realization.

II. SU(N) MODELS

We begin with models based on the gauge group SU(N) [3,4]. For all the models considered in this paper, the beta function is generically written as

$$\beta = \mu \frac{d\alpha}{d\mu} = -\beta_1 \Big(\frac{\alpha^2}{2\pi}\Big) - \beta_2 \Big(\frac{\alpha^3}{4\pi^2}\Big) + O(\alpha^4) \ , \tag{4}$$

where the terms of order α^4 and higher are scheme-dependent.

The first model contains massless fermions transforming according to (i) a symmetric tensor representation of SU(N), $S = \psi_L^{(ab)}$, and (ii) $N+4$ conjugate fundamental representations $F_{iL}^c = \psi_{iL}^a$, where a, b are SU(N) group indices, $i = 1, .., N+4$ is a flavor index, and all fermion fields are written as left-handed Weyl fields. The β-function coefficients are $\beta_1 = 3N - 2$, so that the theory is asymptotically free, and $\beta_2 = (1/4)(13N^2 - 30N + 1 + 12N^{-1})$. Since the theory is asymptotically free, f_{UV} is given by the free-field count of the thermodynamic degrees of freedom:

$$f_{UV} = 2(N^2 - 1) + (7/4)[(1/2)N(N+1) + (N+4)N] \ . \tag{5}$$

To determine f_{IR}, we assume that the theory confines and apply the 't Hooft anomaly matching conditions. The global flavor symmetry group is $G_f = \mathrm{SU}(N+4) \times \mathrm{U}(1)$, where the SU($N+4$) mixes the $(N+4)$ F_i^c fields and the U(1) is the linear combination of the original U(1)'s generated by $S \to e^{i\theta_S} S$ and $F^c \to e^{i\theta_{F^c}} F^c$ that is left invariant by instantons.

The anomaly matching conditions are consistent with the hypothesis that the global flavor symmetry group G_f is unbroken and the massless spectrum is comprised of gauge-singlet composite fermions transforming according to the antisymmetric second-rank tensor representation of G_f. In the large-N limit, it has in fact been argued [4] that this *is* the infrared spectrum of the theory. From this spectrum one can determine that

$$f_{IR} = \frac{7}{4} \frac{(N+4)(N+3)}{2} \ . \tag{6}$$

whence

$$\Delta f \equiv f_{UV} - f_{IR} = (1/4)[15N^2 + 7N - 50] \ , \tag{7}$$

which is positive for all $N \geq 2$. The inequality is automatically satisfied.

The second model contains massless fermions in (i) the antisymmetric tensor representation of SU(N), $A = \psi_L^{[ab]}$ and, if $N \geq 5$, also (ii) $N-4$ conjugate fundamental representations $F_{iL}^c = \psi_{iL}^a$, where $i = 1, ..., N-4$. For the beta function, we have $\beta_1 = 3N + 2 > 0$, and $\beta_2 = (1/4)(13N^2 + 30N + 1 - 12N^{-1})$. Since the theory is asymptotically free, we find

$$f_{UV} = 2(N^2 - 1) + \frac{7}{4}\Big[\frac{N(N-1)}{2} + (N-4)N\Big] \ . \tag{8}$$

For $N \geq 6$, the global symmetry group is $G_f = \mathrm{SU}(N-4) \times \mathrm{U}(1)$, where the SU($N-4$) mixes the $N-4$ fields F_i^c and the U(1) is the linear combination of the original U(1)'s generated by $A \to e^{i\theta_A} A$ and $F^c \to e^{i\theta_F^c} F^c$ that is left invariant by instantons. The anomaly matching conditions are consistent with the conclusion that the massless spectrum consists of gauge-singlet composite fermions transforming according to the symmetric second-rank tensor representation of G_f. (In the degenerate case, $N = 4$, there are no massless fermions.) From this spectrum, we deduce that

$$f_{IR} = \frac{7}{4}\frac{(N-4)(N-3)}{2} \ . \tag{9}$$

Whence

$$\Delta f = (1/4)[15N^2 - 7N - 50] \ , \tag{10}$$

which is positive for the relevant range $N \geq 4$. The inequality is again satisfied.

The third model is an extension of model 1 with the same gauge group and fermions transforming as (i) a symmetric tensor representation $S = \psi_L^{(ab)}$; (ii) $N+4$ conjugate fundamental representations: $F_i^c = \psi_{iL}^c$, where $i = 1, ..., N+4$; and (iii) p pairs of fundamental and conjugate fundamental representations $F_{iL},\ F_{iL}^c, i = 1, ..., p$.

We have $\beta_1 = 3N - 2 - (2/3)p$ and $\beta_2 = (1/4)\{13N^2 - 30N + 1 + 12/N - 2p((13/3)N - 1/N)\}$. Hence, the theory is asymptotically free if

$$p < (9/2)N - 3 \ . \tag{11}$$

We shall restrict p so that this condition is satisfied. We then find

$$f_{UV} = 2(N^2 - 1) + \frac{7}{4}\Big[\frac{N(N+1)}{2} + (N+4)N + 2pN\Big] \ . \tag{12}$$

The global symmetry group is

$$G_f = SU(r) \times SU(p) \times U(1) \times U(1)' \tag{13}$$

where

$$r = N + 4 + p \ . \tag{14}$$

The first U(1) in (13) is generated by $F_{iL} \to e^{i\omega} F_{iL}$ and $F^c_{iL} \to e^{-i\omega} F^c_{iL}$, which is a vectorial symmetry and hence is not affected by instantons; the U(1)$'$ is the one left invariant by instantons. The anomaly matching conditions are consistent with the suggestion [5] that the spectrum of the model consists of gauge-singlet massless composite fermions transforming according to the representations

$$(\ \square\!\!\square\ ,\ 1) + (\ \overline{\square}\ ,\ \square\ ) + (1,\ \overline{\square\!\square}\ ) \tag{15}$$

of $SU(r) \times SU(p)$ in G_f.

For simplicity, we will restrict our discussion to the large-N limit. It was noted in Ref. [4] that the above composite-fermion realization of G_f is not possible if this limit is taken with fixed p. Assuming confinement, G_f would have to be broken to a smaller group. If, on the other hand, we take the limit

$$N \to \infty, \quad p \to \infty, \quad \frac{p}{N} \sim O(1) \tag{16}$$

(so that loops of the p fermions are not suppressed), the above realization with unbroken G_f is possible.

We therefore assume the limits in Eq. (16), with

$$\frac{p}{N} \equiv \lambda < \frac{9}{2} \tag{17}$$

for asymptotic freedom, and calculate that

$$f_{IR} = \frac{7}{4}\Big[\frac{r(r-1)}{2} + rp + \frac{p(p+1)}{2}\Big] \ , \tag{18}$$

where only the leading terms in the limit (16) are relevant. It follows that, keeping only these leading terms,

$$\Delta f = (15/4)N^2 - (7/2)p^2 \ . \tag{19}$$

Hence, in the large-N limit, $\Delta f > 0$ only if λ (is finite and) satisfies

$$\lambda \le \Big(\frac{15}{4}\Big)^{1/2} = 1.04 \ . \tag{20}$$

which is a considerably stronger upper bound on p than (17). (For very small $\lambda \sim const./N$, we have already noted that this realization is impossible and that G_f must break.) Thus there is a finite λ range leading to composite fermions and an unbroken G_f.

For larger λ values, up to the asymptotic freedom limit $\lambda = 9/2$, we expect the theory to be in the nonabelian Coulomb phase, with an interacting infrared fixed point. Near the upper end, the fixed point will be weakly interacting as determined by the first two terms in the β function. The fixed point is then given by $\alpha_* = -2\pi\beta_1/\beta_2$, and since $\beta_1 > 0$, it exists only if $\beta_2 < 0$. In the large-N limit, one finds that the two-loop fixed point exists if

$$\frac{3}{2} < \lambda < \frac{9}{2} \ , \tag{21}$$

and its value is

$$\alpha_* N = \frac{8\pi(9 - 2\lambda)}{13(2\lambda - 3)} \ . \tag{22}$$

Of course, the lower end of this range is not reliable since the fixed-point coupling, as determined by the two-loop β function, approaches infinity. For λ near 9/2, the infrared and ultraviolet degrees of freedom are the same, and Δf is positive due to the (negative) perturbative correction to f_{IR}. Whether the nonabelian Coulomb phase persists down to the value in Eq. (20), $\lambda = 1.04$, or even lower, is unknown. The inequality says simply that the confined phase with massless composite fermions and unbroken global symmetry group G_f cannot exist if $\lambda > 1.04$. The full exploration of the phases of this model as a function of λ is an interesting and unsolved strong-coupling problem.

III. DIRECT PRODUCT GAUGE GROUPS

We next examine a class of models first discussed by Georgi [6] with direct product gauge groups G_k composed by alternating SU(N) and SU(M) k times, so that G_2 = SU(N)$\times$ SU(M), G_3 = SU(N)$\times$ SU(M)$\times$ SU(N), etc. Letting $M(N,1)$ denote M copies of the representation $(N,1)$, we take the fermion content to be

$$M(N,1), \quad (\bar{N},\bar{M}), \quad N(1,M) \quad \text{for} \quad k = 2 \ , \tag{23}$$

$$M(N,1,1), \quad (\bar{N},\bar{M},1), \quad (1,M,N), \quad M(1,1,\bar{N}) \quad \text{for} \quad k = 3 \ , \tag{24}$$

$$M(N,1,1,1), \ (\bar{N},\bar{M},1,1), \ (1,M,N,1), \ (1,1,\bar{N},\bar{M}), \ N(1,1,1,M) \ \text{ for } \ k = 4 \ , \tag{25}$$

etc. For the i'th SU(N), $\beta(g_i) = -(g_i^3/(48\pi^2))(11N - 2M) + O(g_i^5)$ and for the j'th SU(M), $\beta(g_j) = -(g_j^3/(48\pi^2))(11M - 2N) + O(g_j^5)$, so that the theory is asymptotically free if

$$\frac{2}{11} < \frac{N}{M} < \frac{11}{2} \ , \tag{26}$$

which will be assumed henceforth. Then

$$f_{UV} = 2[\ell_k(N^2-1) + \ell(M^2-1)] + (7/4)(k+1)MN \ , \tag{27}$$

where $\ell_k = \ell$ if $k = 2\ell$ is even and $\ell + 1$ if $k = 2\ell + 1$ is odd.

Again assuming confinement so that the massless, physical states transform as singlets under G_k, one can apply the 't Hooft anomaly matching conditions. Consider first even k; then the global flavor symmetry group is $G_{f,k\ \mathrm{even}}$=SU(M)$\times$ SU(N)$\times$ U(1), where the SU(M) mixes the M copies of $(N, 1, ..., 1)$, the SU(N) mixes the N copies of $(1, ..., M)$, and the U(1) is the one left invariant by instantons. The anomaly matching conditions for the SU(N)^3, SU(M)^3, SU(M)^{2}U(1), and SU(N)^{2}U(1) anomalies are consistent with the conclusion, also supported by large N, M arguments [4] (with N/M fixed in the interval (26)) that the spectrum consists of massless composite fermions that transform according to the (M, N) representation of G_f. Hence, $f_{IR,\ k\ \mathrm{even}} = (7/4)MN$ and

$$\Delta f_{k\ \mathrm{even}} = 2\ell(N^2 + M^2 - 2) + (7/4)kMN > 0 \ . \tag{28}$$

Next consider odd k; then the global flavor symmetry group is $G_{f,k\ \mathrm{odd}}$=SU(M)$\times$ SU(M)$\times$ U(1), where the two SU(M)'s mix the M copies of $(N, 1, ..., 1)$ among themselves and the M copies of $(1, ..., N)$ among themselves, respectively, and the U(1) is as before, with appropriate charges. Anomaly matching conditions and large M, N arguments imply that G_f is spontaneously broken to $G_{diag.}$ = SU(M)$_V\times$ U(1). Hence, the massless spectrum consists of $M^2 - 1$ Goldstone bosons, so that $f_{IR,\ k\ \mathrm{odd}} = M^2 - 1$ and

$$\Delta f_{k=2\ell+1} = 2(\ell+1)(N^2-1) + (2\ell-1)(M^2-1) + (7/2)(\ell+1)MN > 0 \ . \tag{29}$$

Thus, the global symmetry is believed to remain unbroken with massless composite fermions for even k, and is believed to break, producing Goldstone bosons, for odd k. In both cases, $\Delta f > 0$. The inequality provides no new information about these models.

IV. SUPERSYMMETRIC MODELS

There are a number of chiral $\mathcal{N} = 1$ supersymmetric gauge theories for which dual descriptions are known which are weakly coupled in the IR. It is a nontrivial test of the

inequality to check that f_{IR} as computed from these duals is indeed less than f_{UV} . We have performed this check for several chiral supersymmetric theories and found the predictions of the inequality to be in agreement with the conjectured duals in all cases. The theories we studied include (i) all so-called s-confining theories as listed in Ref. [7]; (ii) an $SO(10)$ theory with chiral superfields transforming according to a spinor and N vector representations [8], and (iii) an $SU(N)$ theory with chiral superfields comprised of a symmetric tensor S and $N+4$ antifundamental representations $\bar{Q}$ [9].

The theory (iii) is the $\mathcal{N}=1$ supersymmetric version of the model we discussed in the beginning of Section II (with no tree-level superpotential). This theory is asymptotically free for all N, and we obtain

$$f_{UV} = \frac{15}{4}\left(N^2 - 1 + \frac{1}{2}N(N+1) + N(N+4)\right) \ . \tag{30}$$

In [8] a dual $\mathcal{N}=1$ supersymmetric description of the IR dynamics of this theory was proposed. The dual theory is non-chiral and has gauge group $SO(8)$, $N+4$ copies of the vector representation of $SO(8)$, a spinor of $SO(8)$, and $(N+4)(N+5)/2+1$ gauge singlets. For $N \geq 13$ the dual is IR free and we can calculate f_{IR} by simply counting dual fields

$$f_{IR} = \frac{15}{4}\left(28 + 8(N+4) + 8 + \frac{1}{2}(N+4)(N+5) + 1\right) \ . \tag{31}$$

The constraint $f_{IR} \leq f_{UV}$ then becomes $N \geq 9$; thus the inequality is satisfied for all numbers of flavors for which we can reliably compute the quantities f_{UV} and f_{IR}.

Although we have concentrated on chiral gauge theories in this paper, we add some remarks here on a particular vectorial gauge theory for which no dual description is known. As we will see, applying the conjectured inequality yields a new constraint on the IR spectrum of this theory. The model is an $\mathcal{N}=1$ supersymmetric $SU(N)$ gauge theory with the same matter content as $\mathcal{N}=2$ supersymmetric QCD: F chiral superfields Q_i and $\bar{Q}_i$ transforming in the fundamental and antifundamental representations of $SU(N)$ and one chiral superfield A in the adjoint representation. The difference with the $\mathcal{N}=2$ theory is that we do not include a tree level superpotential term $W = gQA\bar{Q}$. Setting the superpotential coupling to zero breaks the extended supersymmetry to $\mathcal{N}=1$ and the resulting theory is much less well understood. Our goal is to investigate how the inequality constrains the infrared dynamics of this theory.

For $F < 2N$ the theory is free in the ultraviolet, and therefore

$$f_{UV} = \frac{15}{4}\left(2(N^2-1) + 2FN\right) \ . \tag{32}$$

A full description of the IR theory is not known, but a few general statements can be made. The classical scalar potential of this theory has a large number of flat directions; the theory has a moduli space of inequivalent vacua. It is well known that the classical moduli space can be parametrized in terms of the independent gauge invariant polynomials which in this case are

$$T_k = \mathrm{Tr} A^k, \;\; k = 2, ..., N \quad \text{and} \quad M_i = Q A^i \bar{Q} \;, \;\; i = 0, ..., N-1 \;, \tag{33}$$

in addition to several baryonic gauge invariants. Here the T_k are singlets, whereas the M_i have F^2 components.

No infrared-free description to this theory has been found, and there are arguments [10] that the theory at the origin of moduli space remains interacting and is conformal in the infrared for any value of $0 < F < 2N$. Still, the inequality can provide useful information, as we will now discuss.

An interesting picture for the infrared dynamics has been put forward in [11], based on a study of a closely related $Sp(2N)$ theory. If one assumes that the infrared fixed point corresponds to an interacting superconformal theory, then the superconformal algebra must contain an anomaly-free $U(1)_R$ symmetry. It follows from the superconformal algebra that the scaling dimensions of all chiral composites are equal to $3/2$ times their charge under the $U(1)_R$. Unfortunately, the R-symmetry of our theory is not unique; there exists a one-parameter family of R-symmetries of which one is the $U(1)_R$ in the superconformal algebra. However one can still derive relations between the scaling dimensions of the chiral composites M_i and T_k. The result of such an analysis is that for $F < N/2$, the scaling dimensions of some composites, as determined from the superconformal R-charges, drop below the unitarity bound. This then implies that these composites decouple from the interacting conformal theory. Similar behavior is anticipated for $N/2 \leq F < 2N$. This picture for the dynamics is also supported by duality [11]. In summary, we expect that in the deep infrared this theory splits into disjoint sectors: one sector with an interacting conformal theory, and other sectors consisting only of a set of m free composite fields M_i.

We now apply the inequality to get a constraint on this scenario. To calculate f_{IR} we add the contributions from each of the disjoint sectors. We cannot calculate the contribution from the conformal sector, but we know that it is positive.[2] This allows us to compute a lower bound on f_{IR} by adding only the contributions from the m free fields $M_0, ..., M_{m-1}$:

[2]It follows from the definition in Eq. (1) together with $\mathcal{F} = -p$, that f_{IR} has the same sign as the pressure, which is always positive.

$$f_{IR} \geq \frac{15}{4} m F^2 \ . \tag{34}$$

Demanding that $f_{IR} \leq f_{UV}$ then gives $m \leq 2(\frac{N^2}{F^2} + \frac{N}{F} - \frac{1}{F^2})$. Since only the first N of the M_i are independent gauge invariants, we already have that $m \leq N$, so that the inequality provides new information only for $F > 2$. The constraint then simplifies to

$$m < 2\frac{N}{F}(\frac{N}{F} + 1) \ . \tag{35}$$

We see that for larger F this constraint can be very significant. For example, for F near the asymptotic freedom bound at $F = 2N$ we find $m < 3/2$, implying that only the first in the series of mesons, M_0, can decouple from the conformal theory and become a free field.

The case of $SU(2)$ is somewhat special and leads to a tight constraint. For $SU(2)$ both Q and $\bar{Q}$ transform in the fundamental doublet representation of $SU(2)$, and the global flavor symmetry is enhanced to $SU(2F)$. The mesons $M_{0,1}$ transform as tensors under this enlarged flavor symmetry. M_0 is an antisymmetric tensor, whereas M_1 is a symmetric tensor. The constraint then allows the following scenario: for $F \geq 4$ the theory is not asymptotically free and we do not get a constraint; for $F = 2, 3$ only the meson M_0 can decouple and become free; and for $F = 1$ both M_0 and M_1 may be free.

V. SUMMARY

In summary, we have performed a quantitative comparison of the ultraviolet and infrared degrees of freedom, as measured by $\Delta f = f_{UV} - f_{IR}$, for several chiral gauge theories and for one vector-like, supersymmetric gauge theory. We have shown that in some models Δf is automatically positive, whereas in others, such as the third model of Section II and the vector-like, supersmmetric model (Section IV), the conjectured inequality $\Delta f \geq 0$ places interesting restrictions on the low energy structure of the theory.

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