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TEV GRAVITY AND KALUZA-KLEIN EXCITATIONS IN e^+e^- AND e^-e^- COLLISIONS [a]

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We review the capability of e^+e^- and e^-e^- colliders to detect the virtual exchange of Kaluza-Klein towers of gravitons within the large extra dimension scenario of Arkani-Hamed, Dimopoulos and Dvali and in the localized gravity model of Randall and Sundrum with non-factorizable geometry.

1 Introduction

The exciting possibility that the gravity may become strong at the TeV scale due to the existence of extra dimensions offers some hope of solving the hierarchy problem. In this talk we review some of the experimental signatures for two such theories at $e^\pm e^-$ colliders. As we will see the predictions of these models are quite distinct but, in either case, lepton colliders will play a significant role in probing their detailed structure.

2 The ADD Model

In the model of Arkani-Hamed, Dimopoulos and Dvali(ADD) [1], gravity is allowed to live in n 'large' extra dimensions, *i.e.*, 'the bulk', while the Standard Model(SM) fields lie on a 3-D surface or brane, 'the wall'. Gravity then becomes strong in the full $4+n$-dimensional space at a scale $M_s \sim$ a few TeV which is far below the conventional Planck scale, $M_{pl} \sim 10^{19}$ GeV. The scales M_s and M_{pl} are simply related via Gauss' Law: $M_{pl}^2 = V_n M_s^{n+2}$, with V_n being the volume of the compactified extra dimensions. For n extra dimensions of the same size, $V_n \sim R^n$ and one finds that $R \sim 10^{30/n-19}$ meters assuming $M_s \sim 1$ TeV. Note that for separations between two masses less than R the gravitational force law becomes $1/r^{2+n}$. For $n = 1$, $R \sim 10^{11}$ meters and is thus excluded, but, for $n = 2$ one obtains $R \sim 0.1$ mm, which is at the edge of the sensitivity for existing experiments [2]. Astrophysics [3] requires that $M_s > 110$ TeV for $n = 2$ but only $\geq$ a few TeV for $n > 2$. The

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Feynman rules for this scenario can be found in Ref. [4]. Upon compactification, one finds that all of the members of the Kaluza-Klein(KK) tower of gravitons couples exactly as does the zero mode.

Outside of table top experiments that probe Newtonian gravity at short distances and astrophysics, the two ways of probing this scenario at colliders are via the emission of KK towers of gravitons in scattering processes or through the exchange of KK graviton towers between SM fields [4], with which we will be interested here. The virtual exchange of graviton towers either leads to modifications in SM cross sections and asymmetries or to new processes not allowed in the SM at the tree level. In the case of exchange the amplitude is proportional to the sum over the propagators of the entire KK tower which naively diverges when $n > 1$. This summation can either be regulated by a brute force cut-off, by the tension of the 3-brane [5], or through the finite extent of the SM fermion wave functions in the additional dimensions [6]. The differential cross sections then become relatively n insensitive functions of the effective cut-off scale, which we will here call M_H [7], and the overall sign of the dimension-8 operator induced by the KK tower, λ. We expect that M_H and the scale M_s are qualitatively similar, being exact equal in the case of a very stiff brane. In this virtual exchange, all of the gravitons act coherently and, due to their relatively tiny mass splittings, sum to a result which is only $M_H \sim 1$ TeV suppressed instead of Planck mass suppressed. Individual resonances associated with a given graviton exchange are smeared out and are not observable. A characteristic feature in all cases is the rapid growth with energy of the graviton contribution to the amplitude; relative to the pure SM, interference terms go as $\sim s^2/M_H^4$ whereas the pure gravity terms behave as $\sim s^4/M_H^8$. Thus we expect these KK contributions to cross section amplitudes to turn on rather rapidly.

In e^+e^- collisions, as first shown by Hewett [4], due to the spin-2 nature of the gravitons in the tower, angular distributions (and polarization asymmetries) become particularly sensitive probes of this scenario. For example, the differential cross section for the process $e^+e^- \to f\bar{f}$ now contains both cubic as well as quartic terms in $\cos\theta$ and is shown in Fig.1 for $f = b$ at LEP II energies. In all such processes the interference between the SM and graviton KK tower exchanges is found to vanish when all angles are integrated over thus emphasizing the importance of examining differential distributions when trying to constrain M_H. Hewett also showed that the nature of the spin-2 graviton indirect exchange is sufficiently unique as to be easily distinguishable from other forms of new physics such as a Z' for values of M_H almost up to the search reach.

One can perform a combined fit for M_H by employing the angular distributions and polarization asymmetries of all kinematically accessible $f\bar{f}$ final states, as well as the polarization of the τ, to obtain a potential search reach for a linear collider with the results as shown in Fig.2. We see that the reach from such a combined channel analysis of this type is of order $M_H \sim 6 - 7\sqrt{s}$. Similar analyses can be performed for both Bhabha and Moller scattering but here one finds that systematic errors tend to dominate at large luminosities and, with only one channel each, these processes do not provide as good a sensitivity to M_H as does the combined fit.

e^+e^- annihilation into gauge boson pairs also can provide reasonable sensitivity to M_H as shown in Fig.3; as can be seen the KK towers do not lead to any

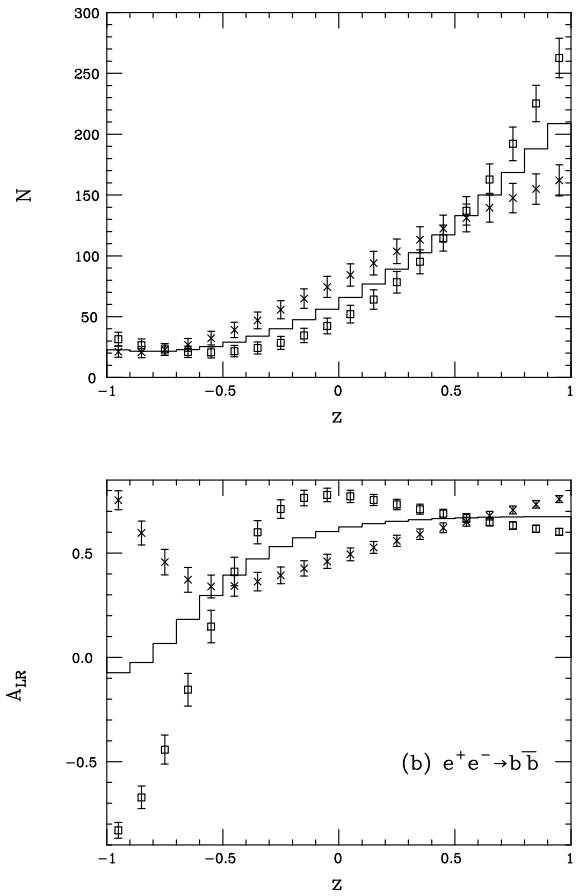

Figure 1: Distortions(top) in the bin integrated SM cross section(histogram) for $e^+e^- \to b\bar{b}$ at $\sqrt{s} = 200$ GeV with a luminosity of 1 fb^{-1} assuming $M_H = 0.6$ TeV. The two sets of 'data' correspond to $\lambda = \pm 1$. The corresponding distortions in the b Left-Right Asymmetry at a 500 GeV linear collider assuming $M_H = 1.5$ TeV are shown in the bottom panel assuming a luminosity of 75 fb^{-1}.

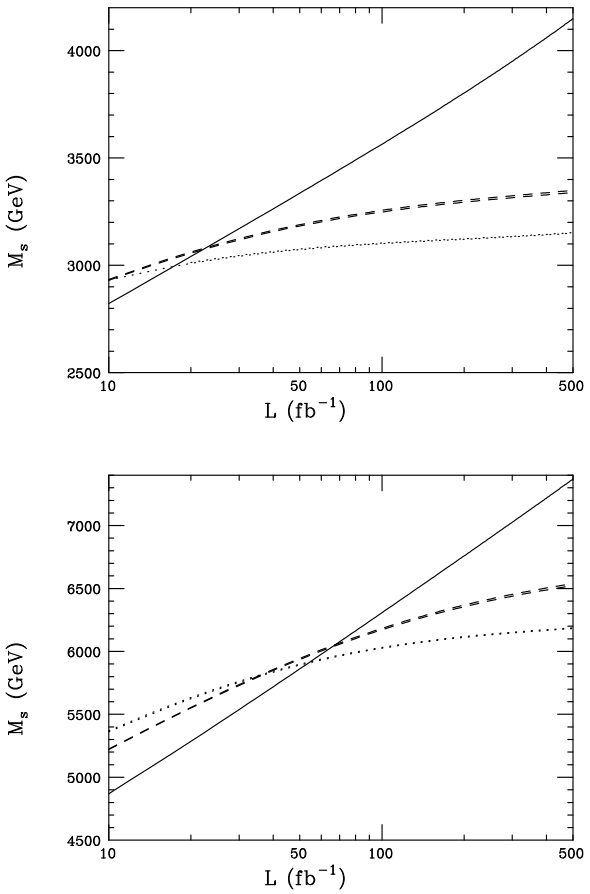

Figure 2: Search reaches for $M_s(M_H)$ at a 500 GeV(top) and 1000 GeV(bottom) e^+e^-/e^-e^- collider as a function of the integrated luminosity for Bhabha(dashed) and Moller(dotted) scattering for either sign of the parameter λ in comparison to the 'usual' search employing the combined channel $e^+e^- \to f\bar{f}$(solid) fit as described in the text.

Reaction	LEP II (2 fb^{-1})	LC (100 fb^{-1})
$e^+e^- \to ff$	1.15	$6.5\sqrt{s}$
$e^+e^- \to e^+e^-$	1.0	$6.2\sqrt{s}$
$e^-e^- \to e^-e^-$		$6.0\sqrt{s}$
$e^+e^- \to \gamma\gamma$	1.4	$3.2\sqrt{s}$
$e^+e^- \to WW/ZZ$	0.9	$5.5\sqrt{s}$
	Tevatron (2 fb^{-1})	LHC (100 fb^{-1})
$p^{(}\bar{p}^{)} \to \ell^+\ell^-$	1.4	5.3
$p^{(}\bar{p}^{)} \to t\bar{t}$	1.0	6.0
$p^{(}\bar{p}^{)} \to jj$	1.0	9.0
$p^{(}\bar{p}^{)} \to WW$	0.8	
$p^{(}\bar{p}^{)} \to \gamma\gamma$	1.4	5.4
	HERA (250 pb^{-1})	
$ep \to e+$ jet	1.0	
	$\gamma\gamma$ Collider (100 fb^{-1})	
$\gamma\gamma \to \ell^+\ell^-/t\bar{t}/jj$	$4\sqrt{s}$	
$\gamma\gamma \to \gamma\gamma/ZZ$	$(4-5)\sqrt{s}$	
$\gamma\gamma \to WW$	$11\sqrt{s}$	

Table 1: M_H search limits in TeV for a number of various processes.

appreciable modification of the $\gamma\gamma, ZZ$ angular distribution or in their total cross sections. We estimate that a combined fit to $f\bar{f}$ and gauge boson pair final states may have a reach as high as $M_H \sin 9 - 10\sqrt{s}$.

We also note briefly in passing that graviton exchange can lead to new processes which are forbidden at the tree level in the SM. A good example of this is $e^+e^- \to 2H$ where H is the SM or any one of the neutral MSSM Higgs bosons. The rate for such processes has been shown to be potentially significant[4] at linear colliders.

Table 1 provides a useful comparison of the various collider's capabilities to probe the value of M_H by employing a number of different processes. Note that the process $\gamma\gamma \to W^+W^-$ has the greatest reach of those so far examined.

3 The RS Model

Randall and Sundrum(RS)[8] have proposed a new scenario wherein the hierarchy is generated by an exponential function of the compactification radius, called a warp factor. Unlike the ADD model, they assume a 5-dimensional non-factorizable geometry, based on a slice of AdS_5 spacetime. Two 3-branes, one being 'visible' with the other being 'hidden', with opposite tensions rigidly reside at S_1/Z_2 orbifold fixed points, taken to be $\phi = 0, \pi$, where ϕ is the angular coordinate parameterizing the extra dimension. It is assumed that the extra-dimensional bulk is only populated by gravity and that the SM lies on the brane with negative tension. The solution to Einstein's equations for this configuration, maintaining 4-dimensional Poincare invariance, is given by the 5-dimensional metric $ds^2 = e^{-2\sigma(\phi)}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2 d\phi^2$, where the Greek indices run over ordinary 4-dimensional spacetime, $\sigma(\phi) = kr_c|\phi|$

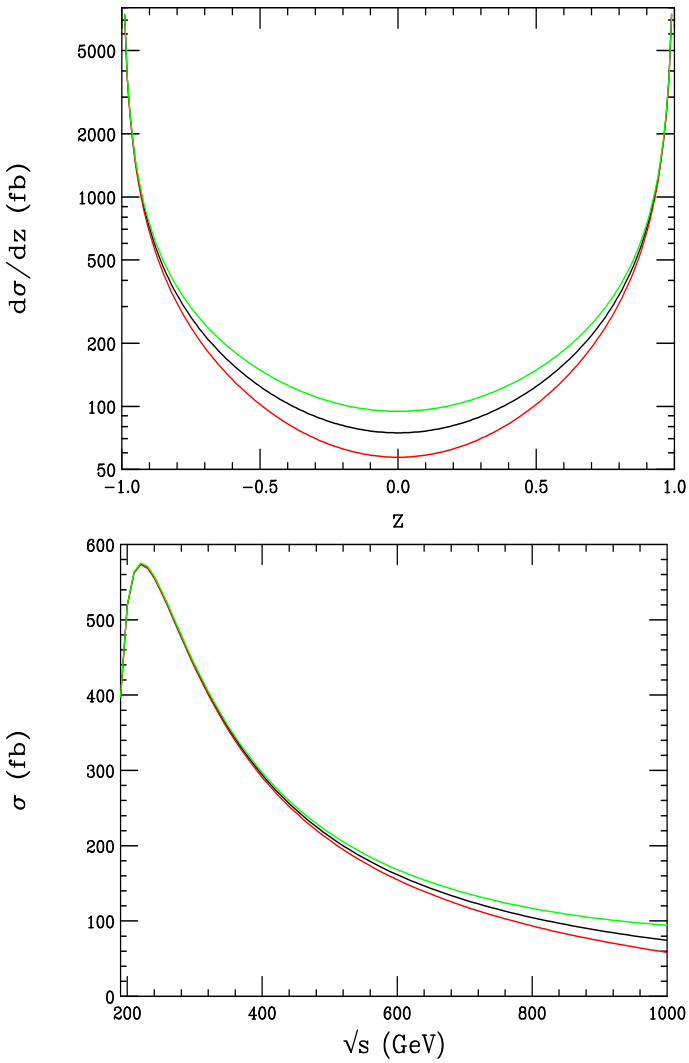

Figure 3: (Top)Differential cross section for $e^+e^- \to \gamma\gamma$ at 1 TeV for the SM (center curve) and for $M_H = 3$ TeV with $\lambda = \pm 1$. (Bottom) Z-pair cross section for the same cases as in the Top panel. Note $z = \cos\theta$.

with r_c being the compactification radius of the extra dimension, and $0 \leq |\phi| \leq \pi$. Here k is a scale of order the Planck mass and relates the 5-dimensional Planck scale M to the cosmological constant. Examination of the action in the 4-dimensional effective theory in the RS scenario yields the relationship $\overline{M}_{Pl}^2 = M^3/k$ for the reduced effective 4-D Planck scale.

Assuming that we live on the 3-brane located at $|\phi| = \pi$, it is found that a field on this brane with the fundamental mass parameter m_0 will appear to have the physical mass $m = e^{-kr_c\pi} m_0$. TeV scales are thus generated from fundamental scales of order $\overline{M}_{Pl}$ via a geometrical exponential factor and the observed scale hierarchy is reproduced if $kr_c \simeq 11 - 12$. Hence, due to the exponential nature of the warp factor, no additional large hierarchies are generated.

Recent analyses[9] examined the phenomenological implications and constraints on the RS model that arise from the exchange of weak scale towers of gravitons. There it was shown that the masses of the KK graviton states lie at the weak scale are given by $m_n = kx_n e^{-kr_c\pi}$ where x_n are the roots of $J_1(x_n) = 0$, the ordinary Bessel function of order 1. It is important to note that these roots are *not* equally spaced, in contrast to most KK models with one extra dimension, due to the non-factorizable metric. This is an important phenomenological distinction. Expanding the graviton field into the KK states one finds the interaction

$$\mathcal{L} = -\frac{1}{\overline{M}_{Pl}} T^{\alpha\beta}(x) h^{(0)}_{\alpha\beta}(x) - \frac{1}{\Lambda_\pi} T^{\alpha\beta}(x) \sum_{n=1}^{\infty} h^{(n)}_{\alpha\beta}(x) \,. \tag{1}$$

Here, $T^{\alpha\beta}$ is the stress energy tensor on the brane and we see that the zero mode separates from the sum and couples with the usual 4-dimensional strength, $\overline{M}_{Pl}^{-1}$; however, all the massive KK states are only suppressed by Λ_π^{-1}, where we find that $\Lambda_\pi = e^{-kr_c\pi}\overline{M}_{Pl}$, which is of order the weak scale. This implies that the tower of weak scale gravitons also couple with weak strength and so may have important phenomenological impact. In particular, the KK gravitons may now appear as $s-$channel resonances in a large number of processes. Unfortunately, no resonances occur in the e^-e^- channel; there the physics of the ADD and RS models will appear to be quite similar in character.

The RS model has essentially 2 free parameters which we can take to be the mass of the first KK graviton mode and the ratio $c = k/\overline{M}_{Pl}$; the later quantity is restricted to be less than ~ 0.1 to maintain the self-consistency of the scenario (to prevent a radius of curvature smaller than the Planck scale in 5 dimensions) and if it is taken too small another hierarchy is formed so that very small values must also be avoided. The present and future bounds on the parameters of this model can be found in Ref. [9] assuming no signal is observed. Fig. 4 shows the cross section and A_{FB} for the process $e^+e^- \to \mu^+\mu^-$ as a function of $\sqrt{s}$ in the presence of KK graviton resonances for several values of the parameter c in the range 0.01-1. Note that the width of the resonance grows quadratically with the value c; for fixed c the width increases with the third power of the KK resonance mass. Sitting on any one of these KK resonances, in the case of small values of c, will immediately reveal the unique quartic angular distribution corresponding to spin-2 graviton exchange, *e.g.*, for the case of fermions in the final state one obtains a decay distribution

$\sim 1 - 3\cos^2\theta + 4\cos^4\theta$. The branching fractions of these KK states are also quite distinctive as is shown in Fig.5.

From the above discussion it is clear that lepton colliders provide an excellent means to probe new theories of gravity.

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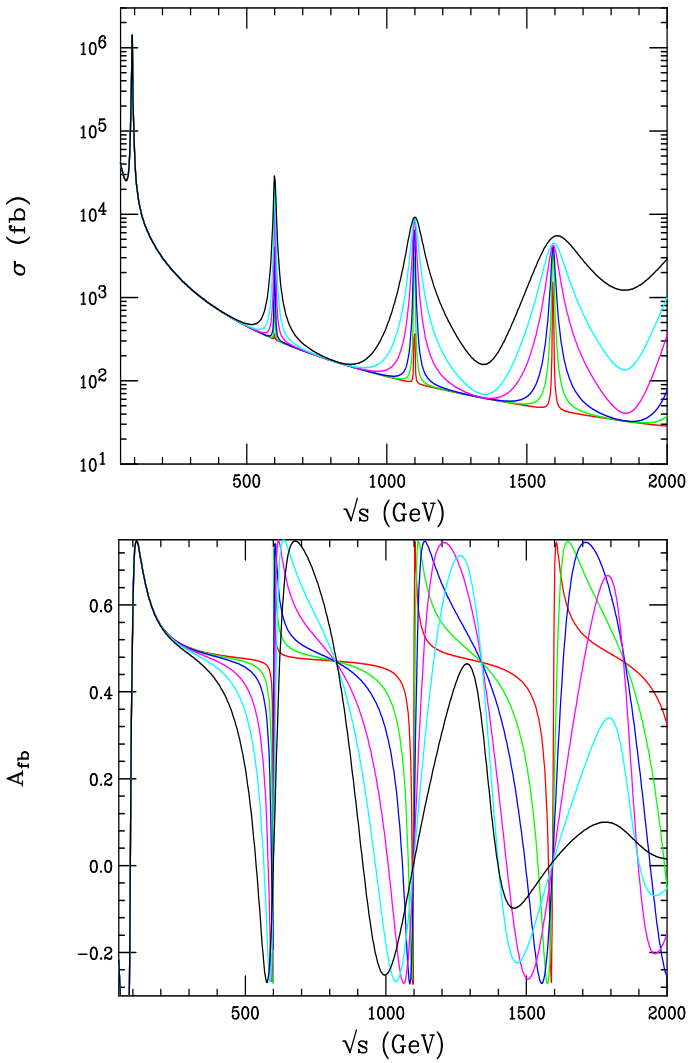

Figure 4: Cross section(top) and A_{FB}(bottom) for $e^+e^- \to \mu^+\mu^-$ including the exchange of KK gravitons, taking the mass of the first mode to be 0.6 TeV, as a function of energy. From outside in the curves correspond to c=0.1, 0.07, 0.05, 0.03, 0.02, and 0.01, respectively, with the same labeling in the bottom panel.

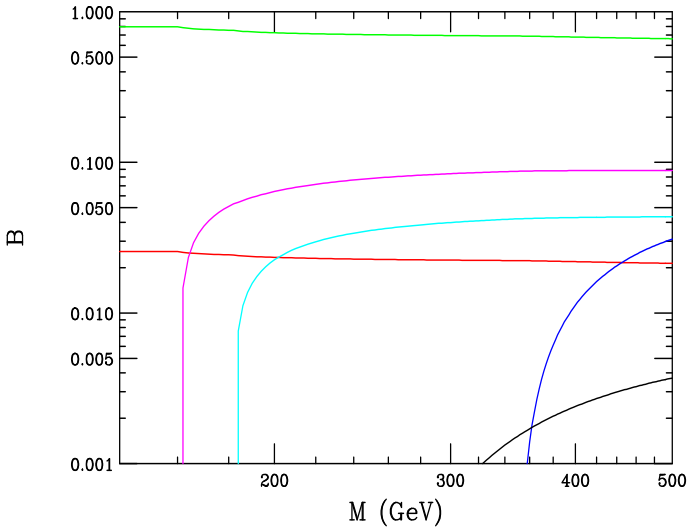

Figure 5: Graviton branching fractions in the RS model as a function of their mass. From top to bottom on the right-hand side, the final states are jets(gluons and light quarks), W-pairs, Z-pairs, $t\bar{t}$, lepton pairs and Higgs pairs for M_H=120 GeV. The branching fraction for photon pairs is twice that for leptons.