SLAC-PUB-8363
February 2000

Experience with Multibunch Beam Stability at PEP-II

M. G. Minty *et al.*

Presented at International Workshop on Performance
Improvement of Electron-Positron Collider Particle Factories,
9/21/1999—9/24/1999, Tsukuba, Japan

Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309

Work supported by Department of Energy contract DE–AC03–76SF00515.

EXPERIENCE WITH MULTIBUNCH BEAM STABILITY AT PEP-II *

M.G. Minty†, W. Barry[1], Y. Cai, P. Corredoura, F.J. Decker, J. Fox, S. Heifets, T. Himel,
A. Kulikov, S. Prabhakar, R. Rimmer[1], J.T. Seeman, D. Teytelman, R. Tighe, U. Wienands
SLAC, Stanford, CA 94309, USA
[1] Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA

Abstract

In this report we discuss high-current, multibunch issues at the PEP-II B factory. To achieve the required beam currents, new techniques are used to help stabilize multibunch beams. In the longitudinal planes of both the low energy (LER) positron ring and the high energy (HER) electron ring, residual phase oscillations are damped with higher-order mode (HOM) absorbers on the accelerating cavities, direct and comb rf loops, a low-frequency 'woofer' link to correct multibunch modes supported by the cavity, and longitudinal bunch-by-bunch feedback. In the LER residual transverse motion has been successfully damped using transverse multibunch feedback up to the maximum current attained so far of 1.7 A. In the HER however a transverse instability has been observed at unexpectedly low beam currents. In this report we describe diagnostics used and summarize current 'thresholds' and compare these with expectation. Next we present measurements made in the HER to better understand the apparently low threshold. We also show selected data using short bunch trains. Practical issues associated with very high beam currents are discussed including gap transients and the stabilizing influence of the beam-beam interaction on multibunch beam stability.

1 INTRODUCTION

To obtain the highest possible luminosities, the PEP-II B factory must stably collide multiple, high-current bunches. Collective beam instabilities arising from interactions of the beam with its local environment, or more complicated processes involving intense synchrotron radiation and/or ions, for example, have the potential of limiting collider performance. Benefiting from knowledge gained at past and existing accelerators with relatively few, but high-current bunches, single-bunch beam instabilities have been avoided having carefully minimized the impedance seen by the beam. Beam stability with multiple, closely spaced bunches however has yet to be demonstrated at the high beam currents required by present-day collider factories.

Given the absence of any evidence of single-bunch beam instabilities at PEP-II, this report will focus on multibunch beam stability. The design parameters for PEP-II may be found in reference [1]. The organization is to a large extent chronological in that collider commissioning necessarily took place first with individual ring commissioning followed by collisions and then higher beam currents. At PEP-II (and elsewhere) beam stability was found to be strongly dependent on the particular fill pattern in the accelerator with evenly spaced fills (plus a small gap) being considerably more stable than with bunch trains. In section 2 we describe the diagnostics developed for analyzing multibunch beam stability. In section 3 we summarize observed thresholds with such even (plus gap) fills and compare these with expectation. The results are in reasonable agreement excepting most notably transverse beam stability in the HER which is discussed in section 4. Experimental data taken with bunch trains are presented in section 5. Beam stability in collision is discussed in section 6. High-current effects are described briefly in section 7 followed by a summary.

2 DIAGNOSTICS

A list of the primary diagnostics [2] used in multibunch stability studies is given in Table 1 along with the characteristic features of the instabilities measured. From a practical standpoint, data acquisition speed is of relevance; the table is ordered from left to right in order of decreasing data acquisition speed (including setup time) with current loss being the most obvious and growth rates having been the most time-consuming to determine.

There were two bunch current monitors (BCM) available: a (phase sensitive) bunch-by-bunch monitor [3] and a dc current transformer, which were used to detect loss along the fill pattern and changes in total current, respectively. The beam position monitors (BPMs [4]) were used to acquire turn-by-turn data which could be Fourier analyzed to determine characteristic frequencies. The BPM signal processors have a bandwidth of about 20 MHz so that each measurement encompasses the nearest 12 neighboring bunches in the design fill pattern (fewer if the bunches are more widely spaced). While multiple BPMs could be synchronously sampled turn-by-turn, the present controls architecture does not support synchronous measurement for all bunches, so the BPMs could not be used to determined mode frequencies.

For transverse stability studies, using as input the position measurement from transverse feedback [5], both a high-bandwidth spectrum analyzer (SA) and the bunch-by-bunch, turn-by-turn data-recording capabilities of the longitudinal feedback (LFBDA [6]) were also used. For both instruments, detection of current loss is parenthetically indicated since an independent current measurement was used for normalization of the input signal. By vary-

* Work supported by the Department of Energy, contract DE-AC03-76SF00515; † email: MINTY@slac.stanford.edu

ing the total beam current the onset of undamped motion was measured. While both instruments could be used in determining the instabilities' mode frequencies and modal growth rates (the SA being used in zero-span mode for the latter), only the LFBDA was time-synchronized to turning on and off the multibunch feedback loops. For longitudinal beam stability studies, the LFBDA was the primary diagnostic using as inputs the sum signal of a dedicated BPM and total current measured using a dc current transformer. In addition, measurements from IQ-based beam monitors were used to evaluate low-order longitudinal mode frequencies and growth rates. The difference in the individual bunch phases measured using these monitors allowed monitoring and correction of collision time differences across the beam current distributions arising from the gap in the fill pattern.

measurement	BCM	BPM	SA	LFBDA
current loss	y	y	(y)	(y)
threshold	n	y	y	y
mode frequency	n	n	y	y
modal growth rate	n	n	y	y

Table 1: Primary diagnostics used in multibunch beam stability studies: bunch current monitors (BCM), beam position monitors (BPM), a spectrum analyzer (SA), and the data acquisition capabilities of the longitudinal feedback system (LFBDA). Here 'y' (yes) and 'n' (no) denote applicability of each diagnostic for the intended measurement.

3 STABILITY THRESHOLDS

Listed in Table 2 is a summary of typical single-beam thresholds obtained with multibunch feedback loops off (in the respective plane of interest) and with the indicated current distributions, which typically has evenly spaced bunches and a $5-10\%$ gap for ion clearing and for the beam abort kicker. By current threshold we mean the beam current at which residual beam motion is detected. Such motion ensues when the instability growth rate is balanced by all possible (Landau, head-tail, radiation, etc.) damping mechanisms. The source of longitudinal beam instability is primarily cavity HOMs. Transversely, in the HER the limits are presumed to be given by a possible HOM in the interaction region (IR) and/or ions while in the LER they are dominated by resistive wall and, at high currents, by a possible multipactoring instability [7].

In the HER, the measured longitudinal threshold [8] of 550 mA with close to the design fill pattern is higher than the 320 mA estimate, which assumed design report parameters with an impedance budget including 20 rf cavities and took into account radiative damping as being the only damping mechanism. The numbers cited here exclude fundamental-mode motion; that is, for the cavity impedance, only HOMs are considered. The increased threshold is due possibly to increased Landau damping arising from variations in the synchrotron tune [8] along the bunch fill pattern due to the cavity transient caused by the ion clearing gap – measured with a single beam, a 3.6% variation in synchrotron tune was measured with the design fill pattern having a 10° phase variation from the head to the tail of the bunch train at 370 mA total current[8].

plane	HER (e^-)	LER (e^+)
long.	550 mA (>650 mA)	310 mA (>1.7 A)
horiz.	<20 mA (270 mA)	>>200 mA (>1.7 A)
vert.	<20 mA (270 mA)	>>200 mA (>1.7 A)

Table 2: Single-beam thresholds for beam stability achieved to date with multibunch feedback off for evenly spaced bunch distributions with a small gap. Longitudinally the data were obtained with 1623 bunches with a 7% gap (HER) and with 786 bunches with a 10% gap (LER). Transversely the fill pattern used corresponds to the present running conditions consisting of 1/2 the design number of bunches (830) with a 5% gap. Shown in parentheses are the single-beam thresholds with feedback on and the design fill pattern (1658 bunches with a 5% gap) in all cases excepting transversely in the HER for which the results given were obtained with 291 evenly spaced bunches minus a 5% gap. With beams in collision, the transverse stability thresholds are significantly higher (see section 6).

In the LER, there are presently 4 rf cavities. The estimated threshold of 385 mA is slightly higher than the measured [9] value of 310 mA, which was determined by noting where the growth rate of the strongest cavity modes (in the range of modes 780 to 800) became positive. The detected strong HOMS agree with those predicted [10].

At high beam current and with the rf cavities correspondingly detuned, we do not observe longitudinal coupled-bunch instabilities driven by the accelerating mode of the rf cavities [11]. These modes have predicted growth rates well exceeding those of all other transverse and higher-order longitudinal cavity modes and are successfully suppressed using direct and comb rf feedback loops and a woofer link [11].

In the transverse planes, the threshold estimates consider resistive wall as being the primary source of multibunch motion. The estimates therefore differ for both rings as the arc chambers in the HER are constructed of copper while the LER, with shorter arcs, are made of aluminum. Both rings have straight-section vacuum chambers made of stainless steel. Transversely, the observed thresholds in the LER are higher than the estimated values of 115 mA horizontally and 75 mA vertically. Since the estimate includes radiation damping only, the higher thresholds may indicate additional damping from head-tail and/or Landau damping. We note that the predicted growth time due to resistive wall is independent of the number of bunches provided that the gap length and total current are fixed. In the HER, the observed thresholds are considerably less than the 120 mA (horizontal) and 125 mA (vertical) threshold estimates (see next section). Multibunch beam stability thresholds with beams in collision (see section 6) are apparently much higher: the record luminosity to date (through 1999) is $1.43\times\ 10^{33}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$ achieved with 910 mA LER and 640 mA HER beam currents in 830 equally spaced bunches including a 5% gap.

4 TRANSVERSE BEAM STABILITY IN THE HER

To date there is no single interpretation which explains all the single-beam measurements [12] in the HER. An example current threshold measurement is shown in Fig. 1. Plotted on the vertical axes are the measured root mean square (rms) of the position distribution; i.e. the standard deviation of the beam centroid motion, obtained from 100 turn-by-turn BPM measurements. From these data the threshold was about 50 mA both horizontally and vertically.

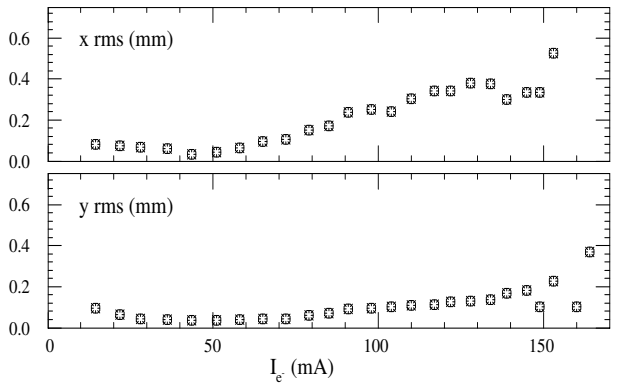

Figure 1: Threshold ($\sim$50 mA) measurement in the HER with transverse multibunch feedback off in the horizontal (top) and vertical (bottom) planes with about 90 evenly spaced bunches and a 10% gap.

Numerous experiments, summarized in Table 3, were performed to better understand this apparently low threshold. In general, while beam stability with bunch trains was highly reproducible, experiments with well-separated bunches were hampered by day-to-day irreproducibility. As a consequence, while absolute measurements of the instabilities' properties proved difficult, relative changes could be studied. The experimental results seem to suggest more than one instability mechanism; experiments with bunch trains (see next section) evidenced large amplitude bunch oscillations resulting in beam loss while this was not the case with more evenly spaced fill patterns. Whether or not the dynamics observed with bunch trains is important with more evenly-spaced bunches, as in the design fill pattern, has yet to be seen.

From the experiments of Table 3, the most likely cause of beam instability seemed to be an effective impedance in the interaction region (IR) near the interaction point (IP). The first hint of a possible impedance source was observed by measuring the rms of transverse beam motion using single corrrectors to make a global orbit oscillation. Shown in Fig. 2 are the measured rms beam positions versus amplitude of applied vertical orbit oscillation. Three different correctors were used to fully span the 60 degree lattice of the HER. The full scale of the applied perturbation (horizontal axis) ranged from $\pm$5 mm peak-to-peak as measured independently using fits to multiple BPMs. Of the three betatron phases tested, one phase showed a significant change while the intermediate phase only hinted at an orbit dependence to the transverse instability. The beam was insensitive to

variable	focus of study	observation
no. bunches[1]	characterization	reference data
bunch spacing	characterization	thres increase with increased spacing
bunch length	wakefields	no effect observed
chromaticity	wakefields	thres increase
collimation	wakefields	no effect observed
RF cavity res frequency	wakefields	no effect observed
IR cooling H_2O temp.	wakefields	no effect observed
IR vacuum	ions	some thres increase
IP β-function	ions/wakefields	mode shift
emittance	ions/wakefields	increased thres with coupling
global orbit	ions/wakefields	thres sensitivity
IP orbit	ions/wakefields	thres sensitivity

Table 3: Summary of single-beam multibunch stability experiments in the PEP-II HER. [1]measured at fixed single-bunch current and/or fixed total beam current

the third phase or to any of the three applied changes to the horizontal closed orbit.

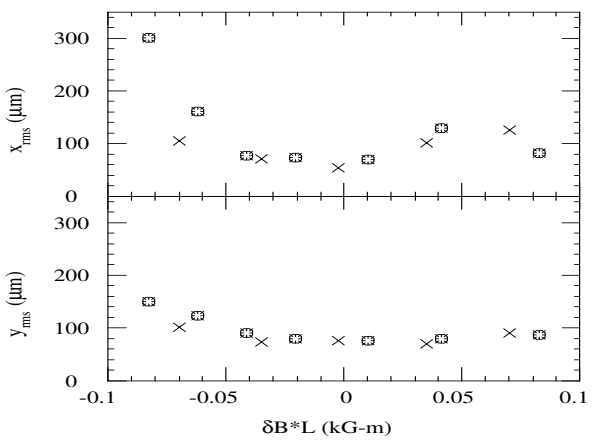

Figure 2: Measured x (top) and y (bottom) BPM rms in the HER versus two different phases of vertical global orbit distortion (shown as crosses and circles) with about 90 evenly spaced bunches plus a 10% gap. Transverse multibunch feedback was turned off.

The instability source was later localized to the immediate vicinity of the IR in measurements made using closed bumps. Shown in Fig. 3 is a difference of two orbits taken with and without a 400 μrad closed x'-bump at the IP. In Fig. 4 is shown the horizontal mode-0 betatron amplitude as a function of this angle. Clearly, it was possible to induce beam instability where peak-to-peak oscillations of up to 2 mm were documented.

The motion along the fill pattern was recorded using a pattern of 415 evenly distributed bunches minus a 10% gap for various total beam currents. These data are shown in

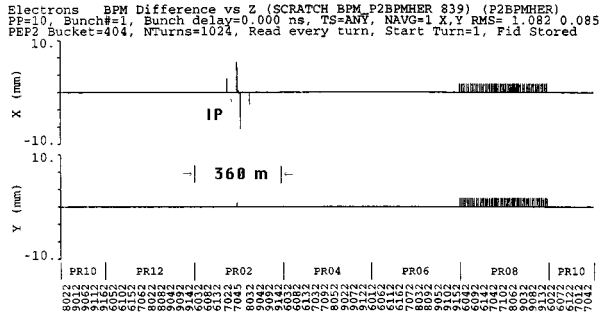

Figure 3: Difference orbit showing closed 400 μrad x'-bump at the HER IP.

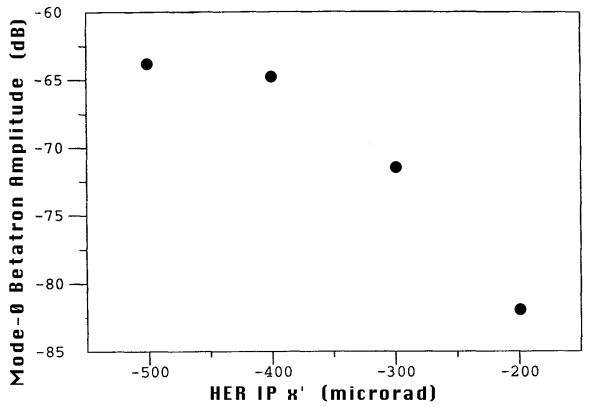

Figure 4: Spectrum analyzer measurement of mode-0 amplitude as a function of (closed) IP x'-bump amplitude in the HER.

Fig. 5. Interestingly, at certain settings of the IP angle, it was impossible to inject and store beam into certain buckets as shown in Fig. 6. The observed restoration of beam stability roughly halfway along the fill pattern was not expected; it seems to suggest an increased spread in betatron tunes giving a damping rate faster than the instabilities' growth rate.

5 TRANSVERSE BEAM STABILITY WITH BUNCH TRAINS

Multibunch beam dynamics observed with closely-spaced bunches in short bunch trains may or may not be important with the design PEP-II fill pattern. Multibunch beam dynamics with bunch trains were first noted in the HER as an inability to inject sequential high-current bunches (of about 1 mA compared to the design single-bunch beam current of 0.45 mA) with the design bunch spacing and transverse feedback turned off. Shown in Fig. 7 are measurements of the charge along the train for the indicated total current obtained using sequential-pulse filling. Interestingly, the same current distribution resulted after filling the train uniformly and then turning off the horizontal feedback loop [12].

To better understand the cause of beam loss, the BPMs were used to measure the transverse motion for selected

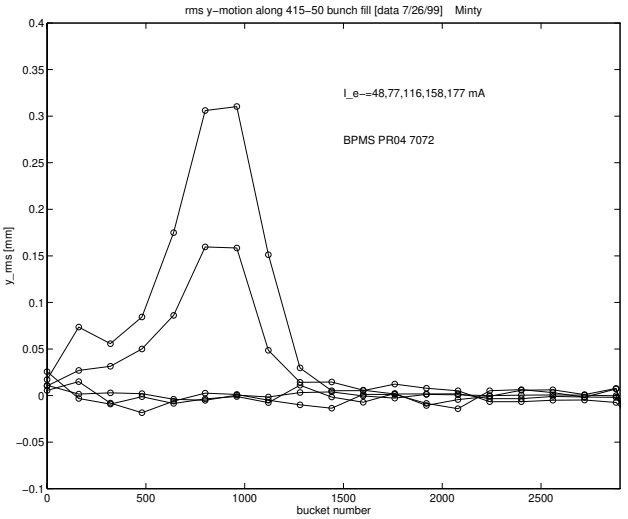

Figure 5: Vertical rms oscillation amplitude measured in the HER at the indicated total beam currents with 415 bunches minus a 50 bunch gap and a closed x'-bump at the IP.

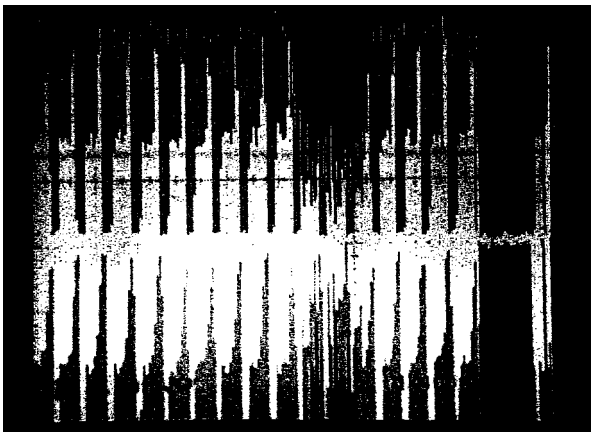

Figure 6: Bunch intensity monitor data taken in the HER taken during beam injection. The horizontal full scale is 10 μs. At this time it was not possible to accumulate more charge at roughly the $\frac{2}{3}$ point along the fill pattern.

bunches. In this measurement the beam was first injected to 1 mA per bunch in a 50 bunch train with feedback on. The vertical feedback loop was then opened. The data acquisition was synchronized to acquire data while opening the horizontal feedback loop. To improve the probability of time-overlap between these events, the BPMs were sampled every 100th or 200th turn. These data are shown in Fig. 8. The first column shows the measurements with the BPMs gate centered on a low-current bunch. The second column shows measurements gated on a bunch for which there was more current in the final state. While the horizontal motion is significantly larger in amplitude, when the data are normalized to the beam size, the vertical motion was observed to dominate.

With a 100-bunch train and design bunch spacing, the

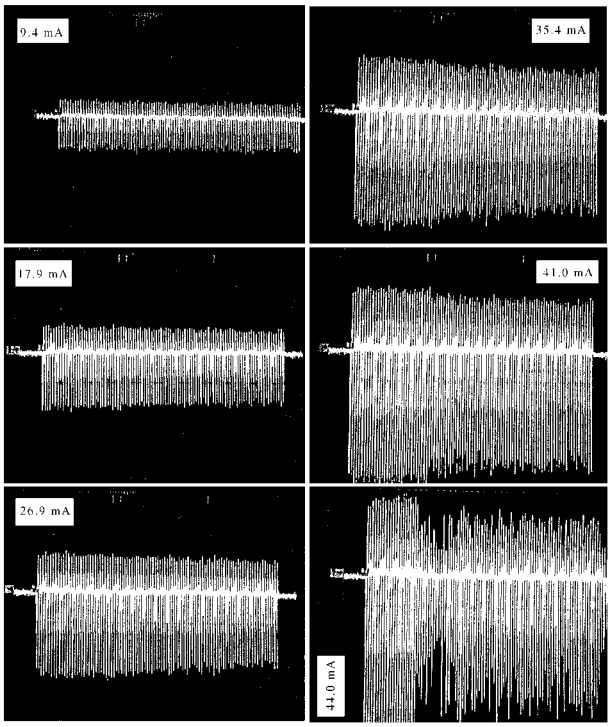

Figure 7: Bunch-by-bunch current monitor data from the HER with a 100-bunch train with twice the design bunch spacing without transverse multibunch feedback.

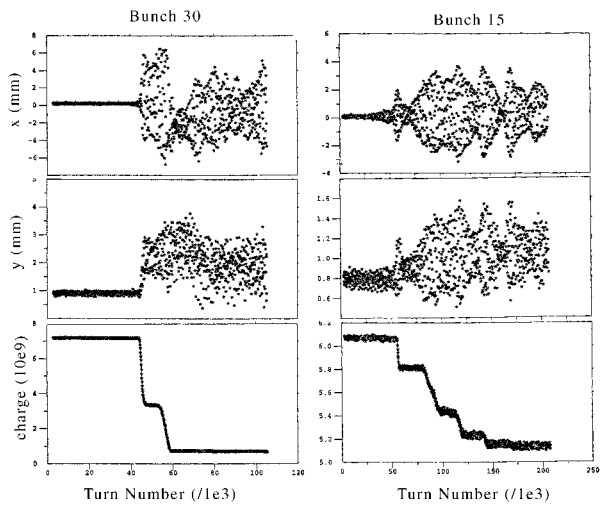

Figure 8: Transverse motion in the HER of selected bunches in a 50 bunch train with the design bunch spacing (4.2 ns) recorded as transverse feedback was turned off. Plotted are the measured horizontal (top) and vertical (middle) beam centroid positions, and the beam intensity (bottom).

transverse position rms along the train was measured for different beam currents as shown in Fig. 9. The time evolution of the vertical motion measured with a 150-bunch train with the design bunch spacing during the first 20 ms after transverse feedback was turned off is shown in reference [13]. These data from the HER show clearly the self-excitation of the beam moving towards the front of the train as the beam current was increased. These data support previous results indicating that the excitations were preceeded by motion in the horizontal plane. Multibunch beam dynamics with bunch trains in the LER evidenced similar features of beam loss along the fill pattern [12]. An example is given in Fig. 10 which shows the measured current distribution for various train lengths, spacing between trains, and total beam currents.

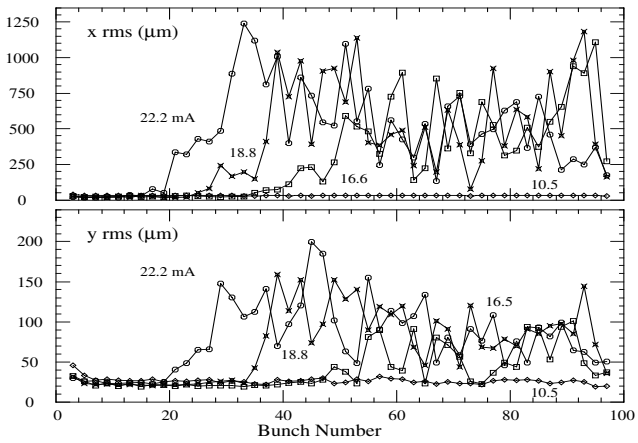

Figure 9: Growth of horizontal (top) and vertical (bottom) bunch centroid motion in the HER along a 100-bunch train with the design bunch spacing.

6 BEAM STABILITY DURING COLLISIONS

During early commissioning with high-current beams in collision it was found that the required gains of the transverse feedback system could be substantially reduced. Two experiments were performed to better quantify this effect. For these measurements the current distribution consisted of 786 bunches spaced at twice the nominal bunch separation including a 10% gap in the fill pattern.

In the first measurement the transverse feedback gain required to damp the measured 0-mode excitation of the HER beam was measured with a spectrum analyzer as function of electron beam current. The data are shown in Fig. 11. The single-beam measurements show that with 150 mA electrons about 15 dB of gain was needed to damp the horizontal centroid motion to the -120 dB noise floor of the spectrum analyzer. In the vertical plane, with a maximum relative gain of 30 dB, above 150 mA there was insufficient gain to fully damp the coherent motion.

With the beams nominally colliding head-on, the measurement was repeated as indicated using crosses in Fig. 11. With these beam currents, it was possible to turn off entirely the horizontal multibunch feedback loop. In the vertical plane, the beam-beam interaction damped the residual motion by 30 dB. The apparent increase in gain required at high beam currents may have resulted from a small separation of the beam positions at the IP.

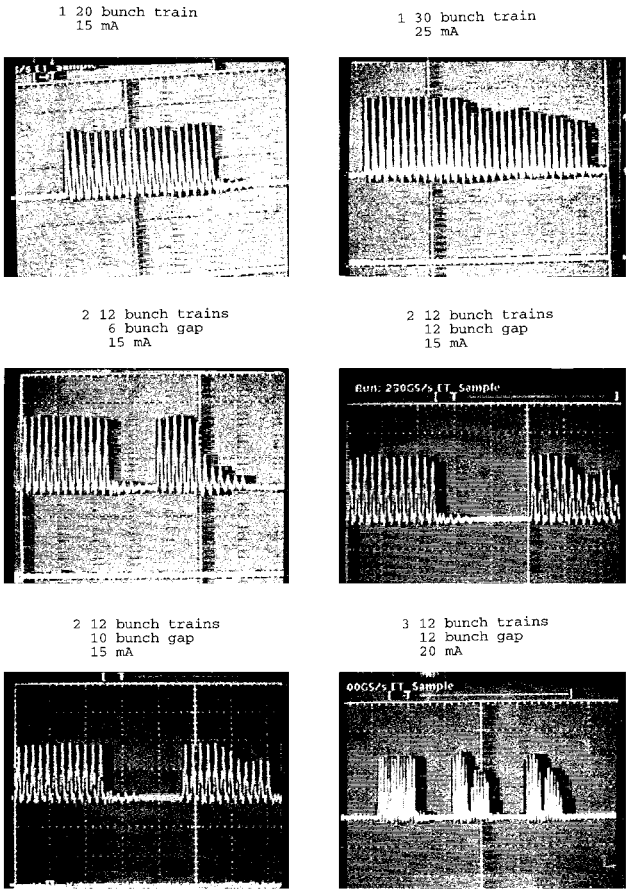

Figure 10: Bunch current monitor data in the LER showing measured current distributions as indicated.

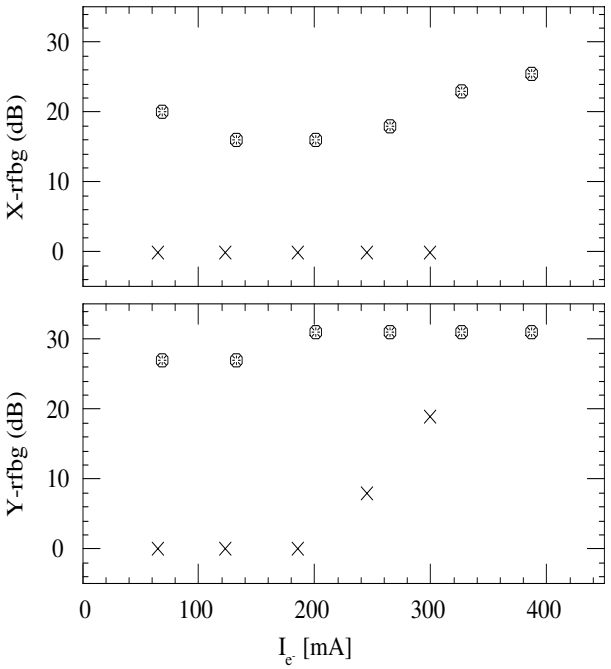

Figure 11: Required feedback gain (rfbg) versus electron beam current with beams in collision (crosses) and with the HER beam only (circles) in the horizontal (top) and vertical (bottom) planes with 786 bunches in both cases. With collisions the total LER beam current was fixed at 0.5 A.

In a separate measurement, the 0-mode instability amplitude was measured as a function of the vertical separation between the beams as shown in Fig. 12 (top) with transverse feedback off. Under these conditions with relative separations of up to about 5 Σ_y the horizontal motion of the beam remained fully damped. Comparing with the simultaneously measured luminosity (bottom) reveals that the residual motion was smallest with the beams best centered vertically.

Being able to turn off the horizontal feedback loop with beams in collision indicates that the tune spread generated by the beam-beam interaction was large compared to the instability growth rate. Taking as an approximate measure of the Landau damping rate ξf_{rev} and the larger of the electron and positron vertical tune shifts for the data of Fig. 12, the imperfect damping of multibunch motion with head-on collisions suggests an instability growth time less than $(\xi f_{rev})^{-1} \approx 0.5$ ms with $\xi_{y,e^+} = 0.015$. With multibunch feedback designed [5] to damp up to three times the predicted resitive wall instability growth rate of 0.3 ms^{-1}, it has yet to be determined whether any residual motion can be fully suppressed. A more detailed analysis of these data may be found in reference [14]. In the future we hope to make similar measurements both with and without multibunch feedback and as a function of the number of bunches at fixed single-bunch current to better characterize beam stability with collisions.

7 HIGH-CURRENT EFFECTS

At the design beam currents of 2.14 A in the LER and 750 mA in the HER, the total synchrotron radiation power is 1.29 and 2.64 MW respectively. In the LER, a potential electron-cloud instability might arise due to electrons emitted from the chamber walls (from either the primary photon flux or via secondary emission) which congregate in the electric potential of the positron beam. With transverse feedback on, up to 1.7 A was stably stored in single-beam mode suggesting that this instability mechanism, if present, was relatively weak. However, in certain bunch fill patterns, a nonlinear pressure increase with beam current has been observed in the LER which is attributed to multipacting electrons [7]. Fortunately, this effect seems to subside quickly as the ring vacuum improves. Whether or not this affects beam stability has not been studied.

The luminosity measured along the bunch fill pattern with 1 A in the LER and 650 mA in ther HER was constant within the measurement resolution of about 10%. This suggests that phase variations due to the ion clearing/abort kicker timing gap and/or instability processes which can deplete a portion of the fill in single-beam operation have not been relevant with beams in collision.

While the luminosity per bunch L_{sb} with 830 bunches is about 1.7×10^{30} in standard units (su) of cm^{-2} s^{-1} mA^{-2},

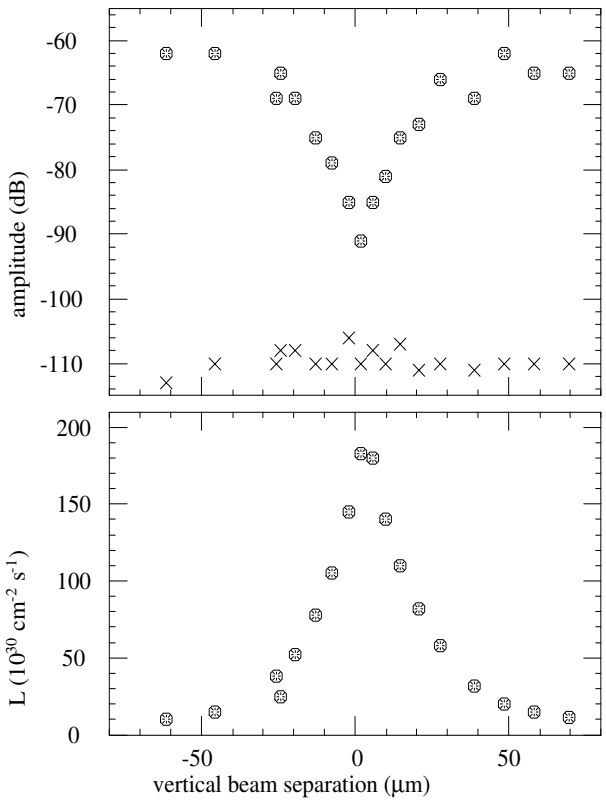

Figure 12: Measured horizontal (crosses) and vertical (circles) instability amplitude (top), and luminosity (bottom) versus relative separation of the colliding electron and positron beams. In this measurement the total currents of the 786 colliding electron and positron beams was respectively 315 mA and 450 mA.

there have been instances where L_{sb} was as high as 2.4 su (which is well above the design of 1.8 su) with 430 colliding bunches. The latter measurement was taken at a time when the HER current was vacuum-limited to about 300 mA. Measurements have yet to be made to determine whether this is due to multibunch motion, phase variations across the fill due to gap transients (the measured phase difference between the beams along the fill was 3° with 830 bunches at 890/630 mA in the LER/HER, respectively, and 1° with 415 bunches with 800/300 mA), or to some other mechanism.

8 SUMMARY

At PEP-II longitudinal multibunch motion has caused little, if any, operational concern as residual motion is successfully damped by feedback in the low-level rf system, cavity HOM absorbers, longitudinal bunch-by-bunch feedback, and the low-frequency woofer link. In the transverse planes, the LER beam is also stable with feedback up to the highest current tested of 1.7 A. In the HER, however, the single-beam stability threshold is unexpectedly low. The driving mechanism is suspected to be either an impedance source near the interaction region or beam-ion interactions. The possibility of another cause is not excluded.

With beams in collision, the transverse stability thresholds are considerably higher. The frequency spread generated by the beam-beam interaction contributes to increased Landau damping, which is apparently much stronger than the damping provided by the transverse multibunch feedback (as evidenced by being able to turn off feedback with beams in collision). As the tune spread increases with beam current, prospects for beam stability at currents approaching design are quite promising.

Acknowledgements

We gratefully thank SLAC operations and colleagues for their suggestions and help with acquiring the data including J. Byrd, J. Clendenin, F.J. Decker, M. Donald, S. Ecklund, T. Fieguth, A.S. Fisher, W. Kozanecki, Z. Kvitky, D. Li, D. Manley, T. Mattison, M. Placidi, I. Reichel, M. Ross, H.V. Smith, T. Smith, M. Stanek, C. Steier, and B. Zotter. In addition we thank A. Chao, M. Furman, A. Hofmann, F. Willeke, and B. Zotter for insightful discussions and advice.

9 REFERENCES

[1] J.T. Seeman, `Status of the PEP-II B Factory', these proceedings.

[2] see also A.S. Fisher *et al*, `Design and Initial Commissioning the Beam Diagnostics for the PEP-II B Factory', Proc. 1997 Part. Acc. Conf. (Vancouver, Canada, 1997).

[3] M.J. Chin and J.A. Hinkson, `PEP-II Bunch-by-Bunch Current Monitor', Proc. 1997 Part. Acc. Conf. (Vancouver, Canada, 1997).

[4] S.R. Smith *et al*, `Beam Position Monitor System for PEP-II', Proc. 1997 Part. Acc. Conf. (Vancouver, Canada, 1997).

[5] W. Barry *et al*, `Design of the PEP-II Transverse Coupled-Bunch Feedback System', Proc. 1995 Part. Acc. Conf. (Dallas, TX, 1995).

[6] G. Oxoby *et al*, `Bunch-by-Bunch Longitudinal Feedback System for PEP-II', Proc. Eur. Part. Acc. Conf. (London, England, 1994) 1616.

[7] A. Kulikov, S. Heifets, J. Seeman, `PEP-II LER Nonlinear Vacuum Pressure Increase with Current', these proceedings.

[8] S. Prabhakar *et al*, `Commissioning Experience from PEP-II HER Longitudinal Feedback', 8th Beam Instrumentation Workshop (Stanford, CA, 1998) 529.

[9] J. Fox *et al*, `Multibunch Instability Diagnostics via Digital Feedback Systems at PEP-II, DAϕNE, ALS and SPEAR', Proc. 1999 Part. Acc. Conf. (New York, NY, 1999).

[10] R.A. Rimmer *et al*, `Updated Impedance Estimate of the PEP-II RF Cavity', Proc. Eur. Part. Accel. Conf. (Sitges, Spain, 1996) 2035.

[11] P. Corredoura, `Architecture and Performance of the PEP-II Low Level RF System', Proc. 1999 Part. Acc. Conf. (New York, NY, 1999).

[12] Summarized in `Coupled-Bunch Instability Measurements at PEP-II (Oct-Dec, 1998)' by M.G. Minty, SLAC-AP-118 (1999).

[13] S. Prabhakar *et al*, `Phase Space Tracking of Coupled Bunch Instabilities', Phys. Rev. ST Accel. Beams 2:084401 (1999).

[14] M.G. Minty, `Recent Beam-Beam Experience with Multiple High Current Bunches in PEP-II', Proc. Workshop on Beam-Beam Effects in Large Hadron Colliders (Geneva, Switzerland, 1999) 20.