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How Can a Heavy Higgs Boson be Consistent with the Precision Electroweak Measurements?

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ABSTRACT

The fit of precision electroweak data to the Minimal Standard Model currently gives an upper limit on the Higgs boson mass of 170 GeV at 95% confidence. Nevertheless, it is often said that the Higgs boson could be much heavier in more general models. In this paper, we critically review models that have been proposed in the literature that allow a heavy Higgs boson consistent with the precision electroweak constraints. All have unusual features, and all can be distinguished from the Minimal Standard Model either by improved precision measurements or by other signatures accessible to next-generation colliders.

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1 Introduction

Is there a Higgs boson? What is its mass? These are among the most pressing questions of contemporary elementary particle physics. The final months of experiments at the LEP e^+e^- collider at CERN showed tantalizing hints of the appearance of the Higgs boson. But with the LEP run now ended, we will not see further experimental evidence to confirm or refute these suggestions for many years. Thus, it is important to revisit the indirect constraints on the Higgs boson mass and to understand their power as well as possible.

The most important indirect information on the Higgs boson comes from precision measurements of the weak interactions [1]. The Minimal Standard Model (MSM)—defined as the $SU(3) \times SU(2) \times U(1)$ gauge theory of quarks and leptons with a single elementary Higgs field to break the electroweak symmetry—provides a good fit to the corpus of precision electroweak data. The fit presented at the most recent International Conference on High-Energy Physics predicts the mass of the Higgs boson to be less than 170 GeV at 95% confidence [2]. Though this limit may be weakened slightly by improved measurements of the renormalization of α [3], it remains true that the MSM with a Higgs boson of mass above 250 GeV is strongly inconsistent with the current data.

On the other hand, it is likely that the correct picture of electroweak symmetry breaking requires ingredients beyond the MSM. In principle, these new ingredients could affect the electroweak fit and weaken the upper limit on the Higgs boson mass. Specific models have been presented in which there is no significant upper limit. In this paper, we will review and catalogue models with new physics beyond the MSM which allow a heavy Higgs boson to be consistent with the precision electroweak measurements. We will give strategies for producing such models, and we will investigate what properties these models share.

The predictions of the MSM depend on the Higgs boson mass through loop diagrams which contain the Higgs boson as a virtual particle. A general model of electroweak symmetry breaking might not contain a Higgs boson as a light, narrow resonance. However, any such model must contain an $SU(2) \times U(1)$ gauge theory and some new particles or fields which spontaneously break its symmetry. These new fields must couple to the W and Z bosons and thus contribute to electroweak radiative corrections. The constraint of the precision electroweak fit is that these corrections should be of the same size as those produced by a light elementary Higgs boson. Models in which the Higgs boson is composite or the symmetry-breaking sector is strongly interacting typically contain larger corrections, comparable to those of a heavy elementary Higgs boson. The precision electroweak constraint then requires that other radiative corrections in these models cancel this contribution down to the

small value produced by a light Higgs boson.

About ten years ago, at the beginning of the era of precision electroweak measurements, several groups studied these corrections in the simplest technicolor models of a strongly interacting Higgs sector. They found that the new contributions typically add to the heavy Higgs effect rather than cancelling it, giving an even stronger disagreement with the precision data [4–6]. To build models with a heavy Higgs boson that are compatible with the precision data, we need to find the counterexamples to this general trend.

One way to address this question is to represent the Higgs sector by the most general possible effective Lagrangian. Recently, a number of groups have shown that, by adding high-dimension operators to this effective Lagrangian, it is possible to compensate the effect of a heavy Higgs boson and relax the upper bound on the Higgs boson mass [7–10]. We believe that this line of argument, though correct, is incomplete. The effective Lagrangian description of the Higgs sector is obtained by starting with a complete theory of electroweak symmetry breaking and integrating out the high-energy degrees of freedom. The full theory predicts not only the particular operator coefficients relevant to the Higgs mass bound but also other effects, which might include interesting low-energy signatures [11]. Also, it could happen that a particular set of operator coefficients cannot be produced from any complete theory, or may require a full theory so contrived as to be unacceptable. To investigate these issues, we must go beyond the effective Lagrangian description and ask what ingredients are needed in the full theory to compensate the effect of a heavy Higgs boson.

In this paper, we will attempt to make general statements about the Higgs mass bound based on explicit models. It is not so easy to make statements that cover all possible models. However, as we will review in Section 2, the question of how to relax the constraint on the Higgs boson mass is related to other questions about the electroweak constraints that were raised just shortly after the inception of the precision electroweak program ten years ago. Considerable ingenuity has been applied to these questions, and a substantial literature has been generated. In this paper, we will review this literature and extract lessons from it.

We have noticed that all explicit models proposed in the literature to relax the bound on the Higgs boson mass use one of three specific mechanisms, which we will attempt to describe transparently. In Section 2, we will briefly review the present status of constraints on the Higgs boson mass from precision electroweak interactions, using the language of S and T variables [6]. Then, in Sections 3, 4 and 5, we will discuss the three mechanisms in turn, showing that each has a simple explanation in terms of the S, T formalism. For definiteness, we will focus on models of new physics that would make a Higgs boson of 500 GeV with Standard Model couplings consistent with the precision electroweak bounds. This is a less severe criterion than

that of allowing a model with no narrow Higgs resonances and true Higgs-sector strong interactions.

Before we begin, we have one more important introductory comment. In models in which electroweak symmetry breaking arises from an elementary Higgs boson, theoretical consistency often places a stringent upper bound on the mass of the Higgs boson which is independent of any requirement from the data. In particular, the postulate that all interactions in Nature are weakly-coupled up to a grand unification scale at 10^{16} GeV implies by itself a very strong constraint on the Higgs boson mass [12]. The general class of supersymmetric grand unified theories has been studied exhaustively and found to give an upper bound of 205 GeV [13]. In addition, because of decoupling, models which contain an elementary Higgs boson—even those which, like supersymmetry, contain a huge number of new particles—typically give only small additional contributions to electroweak radiative corrections beyond the effects present in the Standard Model. Thus, the most familiar examples of physics beyond the Standard Model are compatible only with a light Higgs boson. To allow the Higgs boson to be heavy, we must go further afield.

2 S–T analysis

The precision electroweak constraints are conveniently represented by fitting the data to the MSM augmented by two 'oblique' parameters, S and T, which represent the effects of new physics on the W and Z vacuum polarization amplitudes [6,14]. We describe the method briefly. The parameter S describes weak-isospin-symmetric and T describes weak isotriplet contributions to W and Z loop diagrams. Precision electroweak observables are linear functions of S and T. Thus, each measurement picks out an allowed band in the S–T plane, and measurements of several processes restrict one to a bounded region in this plane.

By convention, the point $S = T = 0$ corresponds to the prediction of the MSM for fixed 'reference' values of the top quark and Higgs boson mass. In this paper, we will take the reference values to be: $m_t = 174.3$ GeV (the current central value from the Tevatron experiments [15]) and $m_h = 100$ GeV. Shifts in these reference values can be compensated by shifts in S and T. In Fig. 1, we show the 68% confidence contour (1.51 σ) for a current S–T fit.

A fit with different reference values of m_t and m_h has confidence contours of the same shape but with a different center. The diferences from the S and T values at the original reference masses indicate the shifts in S and T that best compensate the change in the contributions from the top quark and Higgs boson masses. Following Takeuchi, we translate the ellipse for a given (m_t, m_h) back to the position for a

parameter	current value	α effect
m_W (GeV)	80.434 ± 0.037	± 0.003
$\sin^2\theta_w^{\rm eff}$	0.23147 ± 0.00017	± 0.00006
Γ_ℓ (MeV)	83.984 ± 0.086	± 0.028

Table 1: Current values of the three best measured electroweak parameters, from [2]. In the fit described in the text, the uncertainty in the last column [17], due to the uncertainty in $\alpha(m_Z^2)$, is added in quadrature.

light Higgs boson, and then consider the translation to represent the (S,T) position associated with the new top quark and Higgs masses. With this definition, we obtain the (S,T) values shown in the figure as a banana-shaped grid to represent the MSM with $m_t = 174.3 \pm 5.1$ GeV [15] and m_h running from 100 GeV to 1000 GeV. The condition $S = T = 0$ means 'no new physics'. From the figure, the fit strongly favors a light Higgs boson and excludes a 500 GeV Higgs at the 5σ level.

The fit in Fig. 1 uses only the values of the three best measured electroweak observables, m_W, $\sin^2\theta_w^{\rm eff}$ (the value of the weak mixing angle which appears in Z^0 decay asymmetries), and Γ_ℓ (the leptonic width of the Z^0). Accurate analytic expressions for the Standard Model predictions for these quantities have been given in [16,17]. The current values of these quantities are displayed in Table 1. A fit with this data only gives

$$S = 0.05 \pm 0.10 \ , \qquad T = 0.07 \pm 0.11 \ . \tag{1}$$

This restricted data set actually carries most of the information in a complete fit to the corpus of weak interaction data. A rather sophisticated fit presented by Swartz at the 1999 Lepton-Photon Conference [18] gave the same errors as in (1) with a central value $(S,T) = (-0.04, -0.06)$. Our fit to three data points with the values used by Swartz gives again the same errors and a central value $(S,T) = (0.02, -0.02)$. The compact procedure used here makes little difference for most of the paper, but it will considerably simplify the analysis of Section 5.

The earliest fits to S and T had central values which were substantially negative compared to the Standard Model prediction. It was pointed out in [6] that a negative value of S is especially problematic; since S is the zeroth moment of a distribution whose first momemt is positive and whose second moment is zero, S will be positive in any simple model. This led to a number of papers on mechanisms which generated negative S. These mechanisms are directly relevant to our present concern. If there is a heavy Higgs boson, it is clear from Fig. 1 that we must add additional ingredients to the theory to compensate the effect of this Higgs boson on S and T. Since it is difficult to generate a negative shift in S, the list of helpful additions is severely restricted.

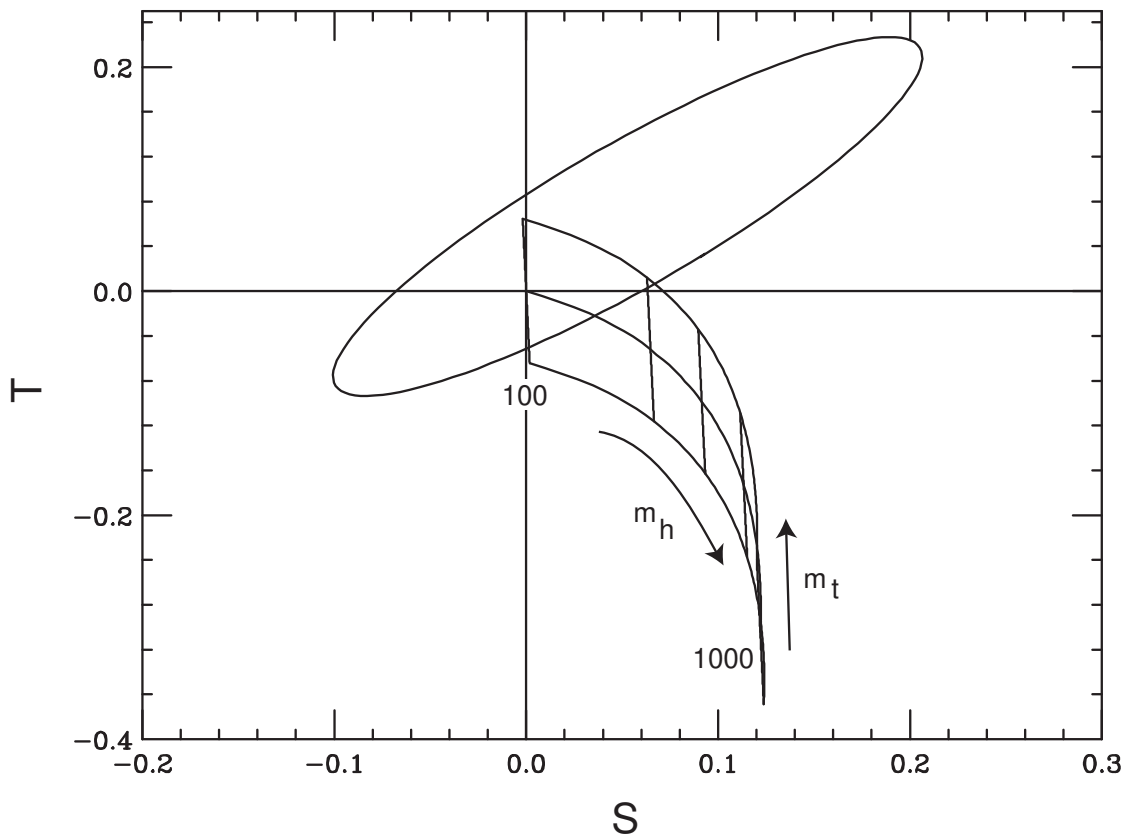

Figure 1: Fit of the precision electroweak data to the MSM plus the S, T parameters described in the text. The fit is based on the values of m_W, $\sin^2\theta_w^{\rm eff}$, and Γ_ℓ shown in Table 1. The ellipse shows the 68% two-dimensional confidence region (1.5 σ). The banana-shaped figure shows the central value of a fit to the MSM for $m_t = 174.3 \pm 5.1$ GeV and m_h varying from 100 to 1000 GeV, with $m_h = 200$, 300, 500 GeV marked with vertical bands. An active version of this figure can be obtained by downloading the additional files deposited with the eprint.

3 Method A: Negative S

As we have indicated in the introduction, we have exhaustively surveyed explicit models of electroweak symmetry breaking which produce shifts in the S and T parameters. It turns out that all such models use one of three mechanisms to move S and T from the region predicted by a heavy Higgs boson to the region preferred by the S–T fit to data. In this and the next two sections, we will discuss these mechanisms in turn. For definiteness, we will consider models which contain a 500 GeV Higgs particle and additional content associated with dynamical electroweak symmetry breaking. We refer to the contributions from this additional content as ΔS, ΔT.

The first method for reconciling a heavy Higgs boson with the precision electroweak fits is to add particles whose vacuum polarization integral shifts S in the negative direction. Typically, new heavy particles give a positive shift in S. However, several specific multiplets have been found which can give negative contributions to S. In this section, we will review models of this type.

Georgi [19] and Dugan and Randall [20] considered a scalar field which transforms according to a definite representation of $SU(2) \times SU(2)$, where the first factor is the weak interaction gauge group and the second factor is the 'custodial $SU(2)$' required to preserve the small value of the ρ parameter [21]. We denote the representation by (j_L, j_R) according to the spin under each $SU(2)$ group. When electroweak symmetry is spontaneously broken, the diagonal $SU(2)$ is preserved, and the large multiplet breaks up into smaller multiplets of definite spin J under this symmetry. The smallest possible value of J is $j_- = |j_L - j_R|$. It turns out that, if the particles with smallest J are the lightest, the multiplet produces negative ΔS. As long as the $SU(2)$ symmetry is exact, the contribution to ΔT is zero.

It is interesting to ask how large a value of ΔS can be produced in this model. To make a simple estimate, assume that the particle with $J = j_-$ has the lowest mass m, and all other particles in the multiplet have a common mass M. Then ΔS contains a logarithm of the mass ratio,

$$\Delta S \sim \frac{1}{3\pi} X \log \frac{M^2}{m^2} \tag{2}$$

with

$$X = -\left[\left(\frac{(j_+ + 1)}{(j_- + 1)} \right)^2 - 1 \right] \frac{j_-(j_- + 1)(2j_- + 1)}{12} \ , \tag{3}$$

and $j_+ = (j_L + j_R)$. We give a more complete expression for ΔS in the Appendix. The simplest example, $(j_L, j_R) = (1, \frac{1}{2})$, yields a puny coefficient $X/3\pi = -0.024$. Larger values can be obtained by using multiplets with larger weak isospin. It is important

to note that the logarithm cannot be large. Since the mass splitting between M and m violates weak isospin, this splitting must be generated by electroweak symmetry breaking and so cannot be greater than about 100 GeV.

In Fig. 2, we plot the contributions to ΔS from some representative multiplets as a function of the light mass m, assuming a mass splitting of 100 GeV. The values shown should be compared to the contribution $\Delta S = +0.11$ from a 500 GeV Higgs boson. Since the scalars involved in this mechanism couple to the weak interactions, they will certainly be found at an e^+e^- collider that can reach their pair-production threshold. From the figure, we see that it is possible that the required particles might escape detection at 500 GeV e^+e^- collider, but that this requires large multiplets of new particles and isospins $J > 2$.

We are aware of only one paper that makes use of this mechanism within a fully developed model of electroweak symmetry breaking. Luty and Sundrum [22] devised a set of technicolor models in which the pseudo-Goldstone bosons contribute a negative ΔS. However, to obtain $\Delta S \simeq -0.1$ from this source, they needed technifermions with $j_L = 2$ and pseudo-Goldstone bosons as light as 200 GeV. Larger values of $|\Delta S|$ could be obtained from larger isospin multiplets. However, these large multiplets gave compensatory positive contributions to ΔS from the technicolor dynamics. Other problematical aspects of these models are also pointed out in [22].

Gates and Terning [23] noticed that one can obtain negative S by introducing electroweak-singlet Majorana fermions which also have Dirac mass terms with isodoublet particles. In this mechanism, the contribution to S again contains a logarithm of the mass ratio of the particles split in mass by electroweak symmetry breaking. The largest negative contributions to ΔS that can be obtained by this method are of the size

$$\Delta S \sim -\frac{1}{6\pi} \log \frac{M_1^2}{M_2^2} \tag{4}$$

where M_1 and M_2 are mass eigenvalues split by electroweak symmetry breaking. This formula briefly led to excitement that the negative S values found in electroweak fits could be explained by a chargino of mass 60-80 GeV [24]. Unfortunately, for charginos heavier than m_Z, the contribution decouples and vanishes as m_Z^2/M_2^2.

Thus, to explicitly obtain negative ΔS by introducing new particles, one must either introduce very large multiplets or require that some of these particles remain light. Light particles associated with this mechanism necessarily have electroweak charge and can be found an an e^+e^- collider.

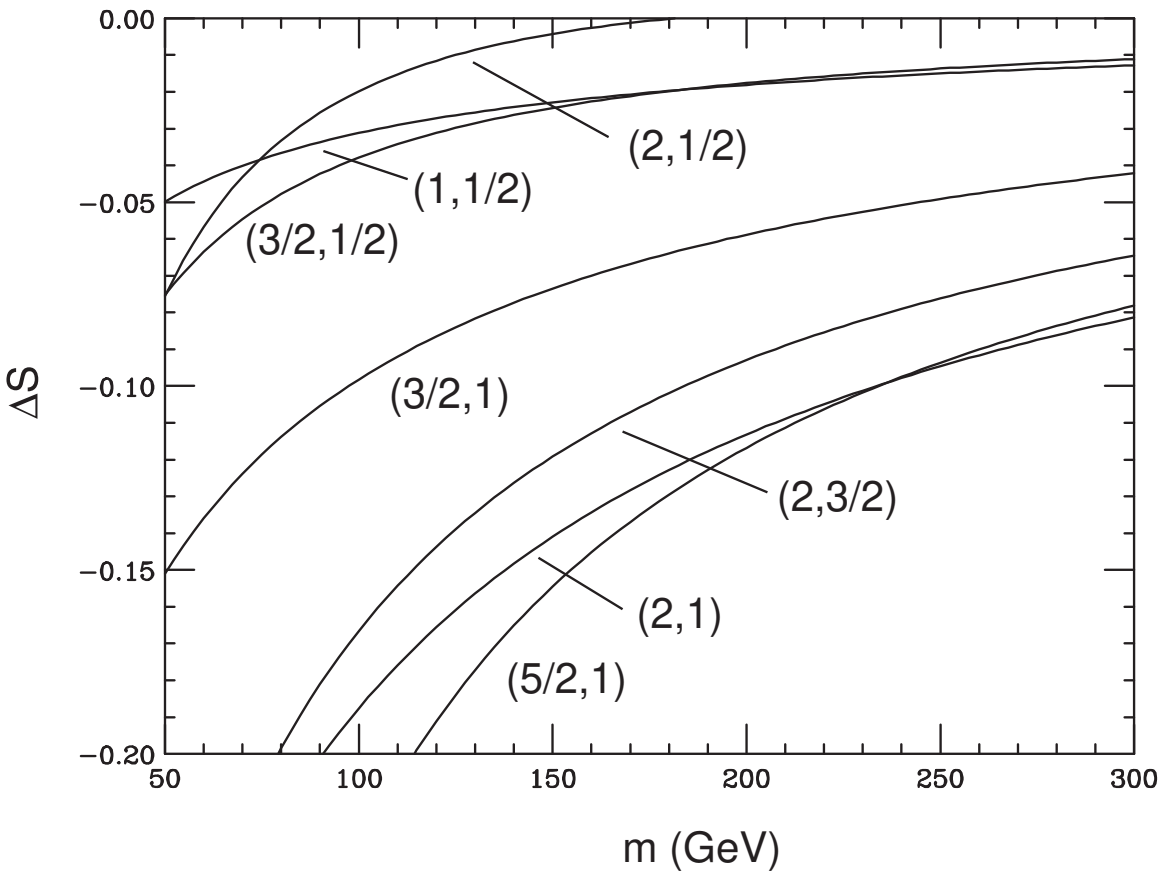

Figure 2: Shift in S induced by the vacuum polarization of various multiplets in the Dugan-Randall scenerio described in the text. The curves assume that the lightest state in the multiplet is split in mass from the other states by 100 GeV. The various Dugan-Randall multiplets are labeled by (j_L, j_R), and the corresponding shifts in S are plotted against the mass of the lightest state.

4 Method B: New vectors

The second method for reconciling a heavy Higgs boson with the precision electroweak fits is to change the weak-interaction gauge group, adding heavy $Z^{0\prime}$ vector bosons. The effects of such new bosons on the precision electroweak fits were studied by a number of groups in the early 90's [25–28]. Rizzo explicitly studied their effect as a method for obtaining negative ΔS [29].

In all of these papers, the effects of the $Z^{0\prime}$ is studied by mapping it to a shift of three variables representing the oblique electroweak corrections, S, T, and U. We find it more instructive to use a slightly different strategy which ignores U. We will compute the shifts in our three well-measured electroweak parameters due to the $Z^{0\prime}$, fit the data to S and T taking these shifts into account, and see if the resulting effect on S and T can compensate the effect of a heavy Higgs boson.

For the pattern of shifts induced by a $Z^{0\prime}$ boson, we find the following: Consider a $Z^{0\prime}$ boson whose mixing with the standard Z^0 is represented by the mass matrix

$$m^2 = \begin{pmatrix} m^2 & \gamma m_Z^2 \\ \gamma m_Z^2 & M^2 \end{pmatrix} , \tag{5}$$

where γ is a parameter of order 1. It is natural that the off-diagonal terms are of the same order of magnitude as m_Z and much less than M^2, since in typical models the heavy mass M^2 results from an $SU(2) \times U(1)$-singlet expectation value, while both the Z^0 mass and the off-diagonal terms result from the expectation values of standard Higgs fields. The observed Z^0 mass is given by the lower eigenvalue of this matrix, $m_Z^2 = m^2(1-\delta)$, and the physical Z^0 contains an admixture ξ of the original $Z^{0\prime}$, where

$$\delta = \gamma^2 \frac{m_Z^2}{M^2} , \qquad \xi = \gamma \frac{m_Z^2}{M^2} , \tag{6}$$

to leading order in (m_Z^2/M^2). Let the current coupling the $Z^{0\prime}$ to leptons ℓ^- have the form

$$\Delta\mathcal{L} = g' Z_\mu^{0\prime} \left\{ \overline{\ell}_L \gamma^\mu q_L \ell_L + \overline{\ell}_R \gamma^\mu q_R \ell_R \right\} . \tag{7}$$

Then the $Z^{0\prime}$ induces the shifts

$$\begin{aligned} \Delta m_W &= 57.\,\delta \quad (\text{GeV}) \\ \Delta \sin^2\theta_w^{\text{eff}} &= -0.33\,\delta + 0.22\,q_L\xi + 0.26\,q_R\xi \\ \Delta\Gamma_\ell &= 100\,\delta - 170\,q_L\xi + 150\,q_R\xi \quad (\text{MeV}) . \end{aligned} \tag{8}$$

Symbolic versions of these expressions are given in the Appendix.

To demonstrate the effect of these shifts, consider first the simple case $q_L = q_R = 0$, and take $\gamma = 1$. (This last choice is conservative, since typically γ is of order $\sin^2\theta_w$.)

Now set the Higgs boson mass to 500 GeV, add to the MSM prediction the shifts shown in (8), and fit for S and T. The result is the set of contours shown in Fig. 3. We see almost complete compensation of the heavy Higgs boson effect for $M \sim 2$ TeV. The figure shows how we would plot this compensation as a translation of the center of the fit, in the same way that we plotted the (S, T) contributions from shifts in m_t and m_h within the MSM.

In principle, the fit might have become worse as the $Z^{0\prime}$ pulls the three variables in directions that are not possible within the MSM. To show that this does not happen significantly, we have plotted the various new ellipses at the same χ^2 value as the reference ellipse copied from Fig. 1. If a third variable U were required, the contours would become smaller for low $Z^{0\prime}$ masses, but this clearly does not happen. In this special case, the effect of the $Z^{0\prime}$ is actually completely described by a shift of T, $\Delta T = \delta/\alpha$. However, it is true in the other examples we have studied that the main effect of the $Z^{0\prime}$ is to shift the center of the (S, T) fit while maintaining a fit with reasonable χ^2.

Using this formalism, we can investigate the region of parameters for any $Z^{0\prime}$ model in which the shifts due to the $Z^{0\prime}$ compensate those of a heavy Higgs boson. In Fig. 4, we show the results for the fit centers as a function of the parameters of the $Z^{0\prime}$, for the model described in the previous paragraph and for several other models from the literature. A commonly discussed class of $Z^{0\prime}$ models are the rank-1 E_6 models, obtained by considering the $Z^{0\prime}$ to be an arbitrary linear combination of the two $U(1)$ bosons in E_6 that are orthogonal to the bosons of $SU(3) \times SU(2) \times U(1)$. The $SO(10)$ $Z^{0\prime}$ and the 'superstring-inspired' $Z^{0\prime}$ are particular cases of these models. The predictions of these models for precision electroweak parameters depend through ξ on the quantum numbers of the Higgs field responsible for Z–Z' mixing. This Higgs field could be either an H_u, with $I = \frac{1}{2}$, $Y = \frac{1}{2}$, or an H_d, with $I = \frac{1}{2}$, $Y = -\frac{1}{2}$, or the mixing could receive contributions from Higgs fields of both types. Explicit formulae for the various contributions from these models are given in the Appendix. In Fig. 4, we plot the compensation as a function of the E_6 mixing angle for each of the two extreme cases, for fixed values of M. In these models, partial compensation is possible only for relatively low values of M, below 1.5 TeV.

The final model shown in Fig. 4 is a model involving extra space dimensions [30]. This is an example of a number of models presented recently in which gauge fields live in a higher-dimensional space that is compactified to our ordinary $3 + 1$ dimensions [31–34]. In these models, the new vectors are the Kaluza-Klein excitations of the MSM gauge bosons.

These models with extra dimensions are interesting to this study because one of the simplest of these theories—with gauge fields in 5-dimensions and the Higgs boson and fermions in 4-dimensions—exhibits compensation between the effects of the new

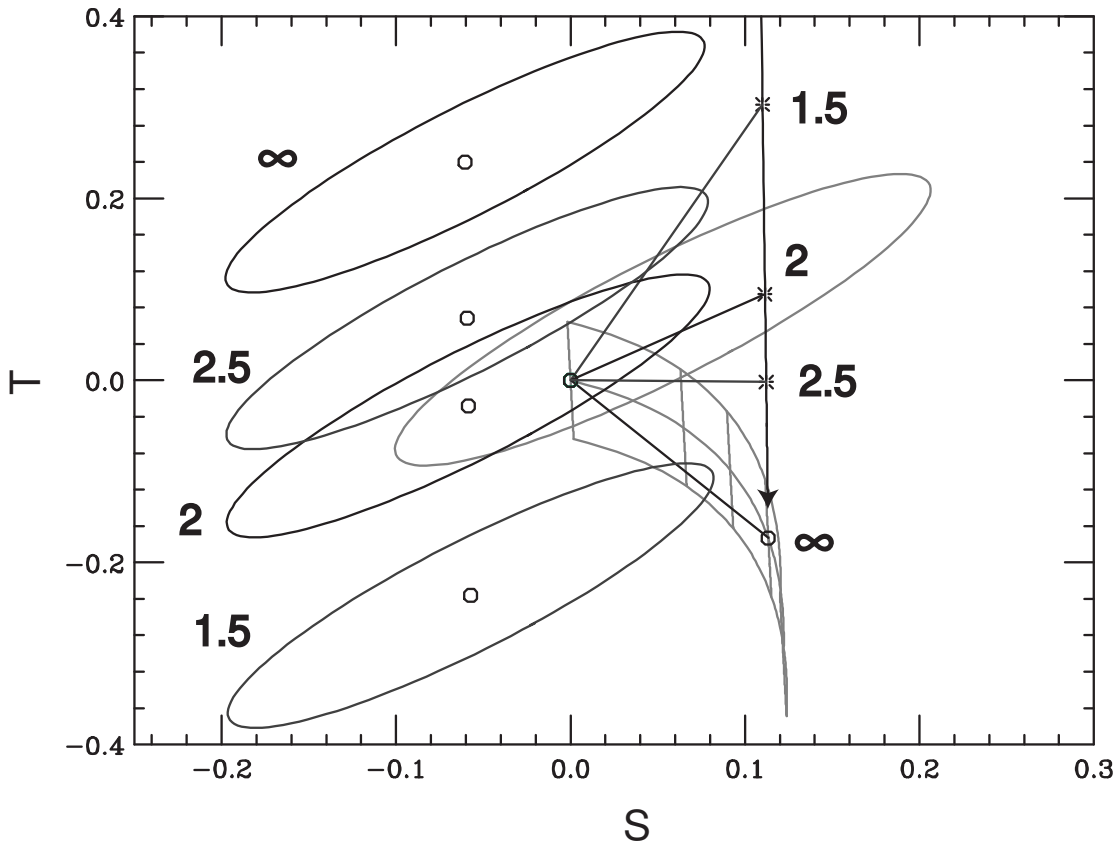

Figure 3: Fit of the precision electroweak data to the MSM with $m_h = 500$ GeV and shifts of the electroweak parameters due to a $Z^{0\prime}$, plus the effects of the S, T parameters. The four darker ellipses correspond to fits with $M = 1.5$, 2.0, 2.5 TeV and ∞. The lighter ellipse and grid are those plotted in Fig. 1. This diagram shows how the centers of the various fits with different values of M can be plotted as a line in the (S, T) plane representing the combined contribution of the $Z^{0\prime}$ and the heavy Higgs boson.

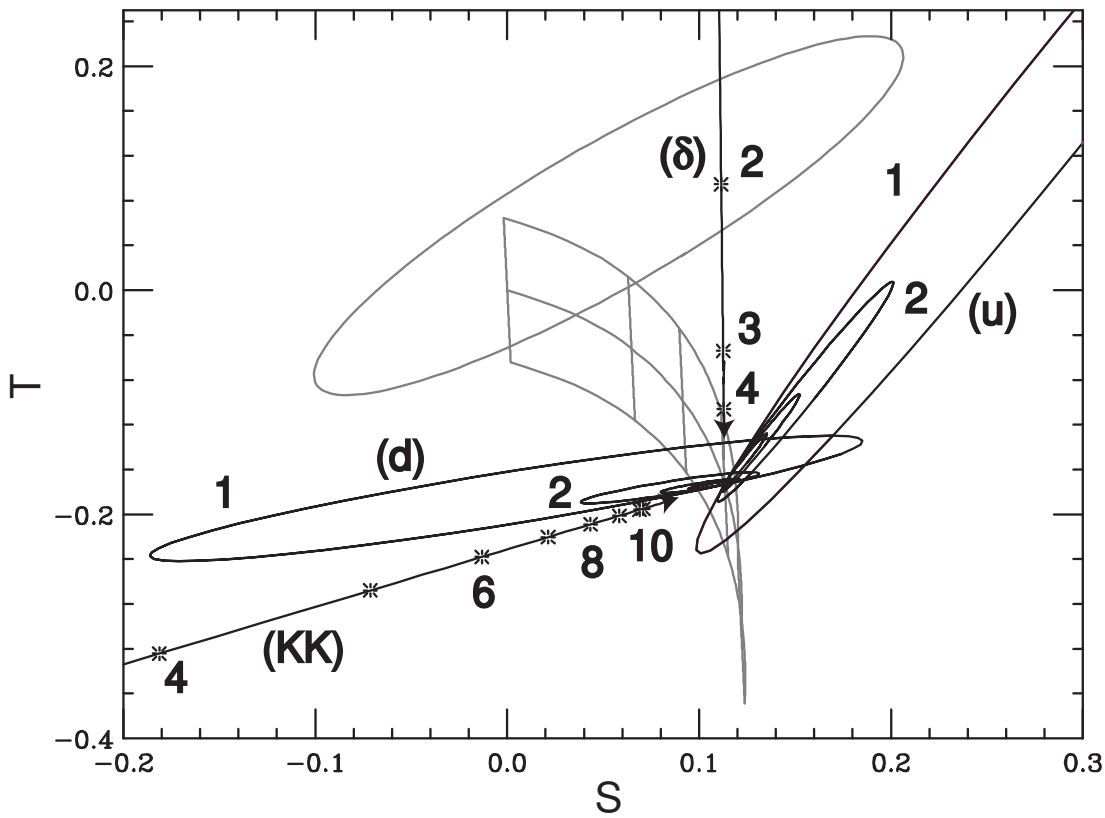

Figure 4: Contributions to S and T from a Higgs boson with $m_h = 500$ GeV, plus a heavy $Z^{0\prime}$. The contributions are computed and displayed as indicated in Fig. 3. Four different models are considered: (δ): model of Fig. 3, with $\gamma = 1$, $q_{L,R} = 0$; (u): rank-1 E_6 models with mixing due to a Higgs field H_u; (d): rank-1 E_6 models with mixing due to a Higgs field H_u; (KK): extra-dimension model of ref. [30]. The numbers indicate the values of the $Z^{0\prime}$ mass M, always in TeV. All of the $Z^{0\prime}$ predictions tend to the 500 GeV MSM point as $M \to \infty$.

vector bosons and the heavy Higgs boson. If we denote the mass of the first KK excited state of the gauge bosons by M_{KK}, the electroweak observables are altered in this model by

$$\begin{aligned} \Delta m_W &= 87.0 \frac{m_Z^2}{M_{KK}^2} \quad \text{(GeV)} \\ \Delta \sin^2 \theta_w^{\text{eff}} &= -1.09 \frac{m_Z^2}{M_{KK}^2} \\ \Delta \Gamma_\ell &= -220 \frac{m_Z^2}{M_{KK}^2} \quad \text{(MeV)} \ . \end{aligned} \tag{9}$$

The effect of these changes is to move the (S, T) value to a region of the plane where it is poorly constrained. A detailed global fit analysis [30] indicates that the 95% C.L. of the Higgs boson could reach as high as 300–500 GeV for M_{KK} in the range 3–5 TeV. This compensation of heavy Higgs and light KK states is seen in our three-observable fit, although with the latest experimental numbers we find that the Higgs mass cannot extend beyond 300 GeV.

Another interesting property of this model is that the coupling of the new sector is large. By this we mean both that a large number of new states participate and that the couplings of individual states are larger by a factor $\sqrt{2}$ than those of the corresponding MSM bosons. These features allow compensation for a mass of the lightest new vector about twice as high as the mass of the single $Z^{0\prime}$ boson in the E_6 models considered above.

The common feature of these four models, and of other $Z^{0\prime}$ models we have studied, is that the values of the $Z^{0\prime}$ mass needed to compensate the effect of a heavy Higgs boson is well within the reach of next-generation colliders. The LHC should be able to find a $Z^{0\prime}$ as a narrow resonance for masses up to 4 TeV. A 500 GeV e^+e^- linear collider can see the effect of the $Z^{0\prime}$ as a perturbation of the cross section for $e^+e^- \to f\overline{f}$, with similar sensitivity in the $Z^{0\prime}$ mass [35]. The information from proton and electron colliders is complementary, and a complete picture of the $Z^{0\prime}$ is obtained by combining the two sets of measurements. In the case of the extra-dimension model, the mass of the first new vector excitation is predicted to be higher. However, in this model, the larger couplings give enhanced sensitivity, up to 6 TeV for the LHC and above 10 TeV for a 500 GeV e^+e^- linear collider, so the general conclusion applies to this model as well.

5 Method C: Positive T

In both methods of compensating a heavy Higgs boson that we have discussed so far, the compensation leads to new physics signatures that should be observed at next-generation pp and e^+e^- colliders. However, there is one further compensation strategy that can evade this requirement. Looking again at Fig. 1, we see that it is possible to bring a model with a heavy Higgs boson back into reasonable agreement with the precision electroweak fit without changing S at all, by adding new particles that lead to positive ΔT. For example, the shift $\Delta S = 0$, $\Delta T = 0.3$ due to new physics brings a model with a 500 GeV Higgs boson within 1 sigma of the central value.

Most models with new physics produce a nonzero, positive ΔT [36]. In fact, the contribution to ΔT can easily be of order 1. Particles with mass much larger than 1 TeV can contribute to ΔT if their masses have an up-down flavor asymmetry. The contribution is of the order of

$$\Delta T \sim \frac{m_U^2 - m_D^2}{m_U^2 + m_D^2} \ . \tag{10}$$

Even though $m_U - m_D$ can be at most of order 100 GeV because it must arise from electroweak symmetry breaking, this contribution can easily be large enough to compensate the effect of a heavy Higgs boson for values of the U and D masses that are inaccessible to any collider.

Two recently proposed models allow a heavy Higgs boson to make use of this mechanism. The first is the 'topcolor seesaw' of Dobrescu and Hill [37]. In this model, the new physics needed to break electroweak symmetry arises from a heavy, weak-$SU(2)$-singlet fermion χ. In the simplest topcolor seesaw model, one finds [38]

$$\alpha \Delta T = \frac{3}{16\pi^2} \frac{g_{tc}^4}{4} \frac{v^2}{m_\chi^2} \left[1 + 2 \frac{\lambda_t^2}{g_{tc}^2} \log \frac{m_\chi^2}{m_t^2} \right] \ , \tag{11}$$

where $g_{tc} \sim 3$ is the topcolor coupling, $\lambda_t = 1$ is the top quark Yukawa coupling, $v = 246$ GeV is the weak interaction scale, and m_χ is the mass of a new heavy fermion. For $m_\chi = 1$ TeV, this expression gives $\Delta T = 7.2$. However, it is permissible in this model to raise m_χ arbitrarily, at the expense of fine-tuning. For $m_\chi = 5$ TeV, $\Delta T = 0.3$ and we obtain a reasonable fit to the precision electroweak data with heavy Higgs boson. A mapping of the ellipse in Fig. 1 into the m_χ, m_h plane gives the interesting contour seen in Fig. 7 of [38].

A recent paper by Chankowski *et al.* [39] argues that the two-Higgs-doublet model can be made consistent with the electroweak fits for a Higgs boson mass of 500 GeV.

The strategy of this paper is to adjust the Higgs spectrum to give the required positive contribution to the ρ parameter ($\Delta\rho = \alpha\Delta T$); the model gives only a tiny shift in S.

Technicolor models can also be made consistent with the precision electroweak fits through this strategy. Most technicolor models lead to values of S larger than the value for a 1000 GeV Higgs boson, $S > 0.12$ [6]. In typical cases, the values of S and T are positive and of order 1. Models have been proposed in which the technicolor enhancements to S and T are of order 0.1 or smaller [40–42]. But in all models that have been studied, except for [22] cited above, the lower bound for S still applies. Still, it is possible to construct a technicolor model that is consistent with the electroweak data in spite of this bound, by including enough weak isospin breaking give a small positive correction to T. Such a model would, for example, have $S \sim 0.15$, $T \sim 0.2$. Models of this type would not have a visible Higgs boson and might not have any new particles below the first techi-rho resonance at about 2 TeV [43].

Though the models which use this strategy predict little or no new physics at the next generation of colliders, they are clearly distinguishable from the MSM by improved precision electroweak measurements. Foreseeable improvements in the precision electroweak fit are shown in Fig. 5. The larger contour shows the effect of a measurement of m_W to 15 MeV, as might be expected from the LHC [44]. The smaller contour shows the effect of the precision measurements expected from a high-luminosity e^+e^- linear collider run at the Z^0 and at the W^+W^- threshold: m_W to 6 MeV, $\sin^2\theta_w^{\rm eff}$ to 0.00002, Γ_ℓ to 0.04 MeV [45–47]. With this latter set of measurements, the point in the (S, T) space favored by this strategy is separated from prediction of the MSM with a light Higgs boson by more than 5 sigma. Thus, these measurements would clearly prove the presence of new physics and would indicate the route by which today's precision electroweak constraint is evaded.

6 Conclusions

The precision electroweak data are consistent with the Minimal Standard Model only if the Higgs boson mass is very low, $m_h < 170$ GeV at 95% C.L. This result would predict that the colliders of the next generation, possibly including the upgraded Tevatron, will be able to discover and study the Higgs boson. However, if the Minimal Standard Model is not correct, there are scenarios in which new physics contributions conspire with a heavier Higgs boson to allow agreement with the precision electroweak data.

In this paper, we have argued that, despite this, one cannot freely assume that the Higgs boson is heavy in the face of the precision electroweak constraint. The

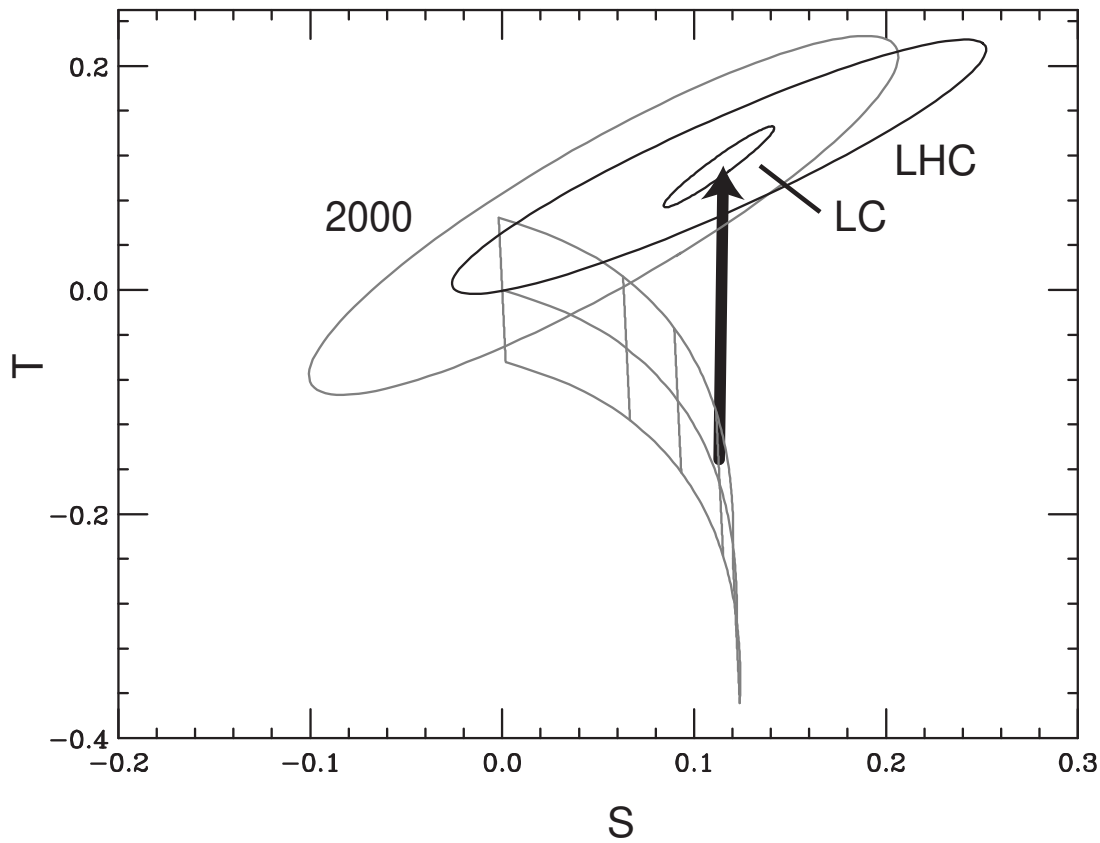

Figure 5: Future improvements in the determination of precision electroweak parameters. The lighter ellipse and grid are those plotted in Fig. 1. The heavier ellipses, both centered at $(S, T) = (0.11, 0.11)$, correspond to an improved W mass measurement with an error of 15 MeV, as would be expected from the LHC, and measurements of m_W, $\sin^2\theta_w^{\rm eff}$, and Γ_ℓ with errors of 6 MeV, 0.00002, and 0.04 MeV, respectively, as would be expected from the precision electroweak program at an e^+e^- linear collider [45–47].

particular new physics that compensates the effect of a heavy Higgs boson has a price, and, to evade the MSM constraint on the Higgs boson mass, one must be prepared to pay it.

Many popular models of physics beyond the Standard Model do not allow a heavy Higgs boson at any price. Supersymmetric grand unified theories are an example. Among models that allow a heavy Higgs boson in principle, any successful model must introduce new physics that can perturb the precision electroweak observables in the correct direction by a sufficiently large amount. In this paper, we have made that statement precise, using the S, T formalism, and we have reviewed the various strategies suggested in the literature. In fact, the entire literature to date is exhausted by only three strategies, which we have described in detail.

Two of these strategies, method A, which gives new contributions of negative ΔS, and method B, which introduces new vector bosons, have distinctive signatures that should be observed at the next e^+e^- linear collider. Method A requires new light bosons or fermions with electroweak charge. Method B requires that the new vector bosons be sufficiently light to create large perturbations of the cross sections for e^+e^- annihilation to fermion pairs. In fact, models using this strategy create a very interesting physics scenario, in which the new vector particles are also directly observable as resonances at the LHC.

The third strategy, method C, is less dramatic, and specific models exist in which no new particles beyond the MSM are observable either at an e^+e^- linear collider or at the LHC. However, this strategy leads to a prediction for the precision electroweak parameters that is distinctive, and that can be distinguished from the MSM prediction with a high level of confidence by the improved level of precision in the electroweak parameters that a next-generation e^+e^- linear collider should achieve.

We cannot close off the idea that the Higgs boson is very heavy on purely theoretical grounds. But we have emphasized in this paper that models in which the Higgs boson is heavy have specific properties which must be taken into account in any discussion of future experimental prospects. In particular, these models generally lead in their own way to an interesting experimental program for the next-generation colliders. A 500 GeV e^+e^- linear collider has an important—and, in some cases, crucial—role in untangling the new physics that these models present.

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A Appendix

In this appendix, we present various explicit formulae that are used in the text.

In Section 3, we analyzed the Dugan-Randall models [20] in the most favorable case in which the scalar particle with smallest J has a low mass m while the other particles in the multiplet have a large mass M. In (2), we quoted only the leading logarithm in the formula for ΔS. The complete formula for this case is

$$\Delta S = \frac{1}{3\pi}\left[X \log\frac{M^2}{m^2} + 2X' B(m,M)\right] \ , \tag{12}$$

where

$$\begin{aligned} X &= \left[1 - \left(\frac{(j_+ + 1)}{(j_- + 1)}\right)^2\right] \frac{j_-(j_- + 1)(2j_- + 1)}{12} \\ X' &= \left[1 + \left(\frac{(j_+ + 1)}{(j_- + 1)}\right)^2 - \frac{j_+(j_+ + 2) + j_-^2}{j_-(j_- + 1)}\right] \frac{j_-(j_- + 1)(2j_- + 1)}{12} \\ B(m,M) &= -\frac{m^4(m^2 - 3M^2)}{(M^2 - m^2)^3} \log\frac{M^2}{m^2} + \frac{5M^4 - 22M^2m^2 + 5m^4}{6(M^2 - m^2)^2} \ . \end{aligned} \tag{13}$$

In Section 4, we analyzed the effect of a $Z^{0\prime}$ boson on the best-measured precision electroweak observables. In (8), we quoted numerical formulae for the shifts in m_W, $\sin^2\theta_w^{\rm eff}$, and Γ_ℓ induced by a $Z^{0\prime}$ in terms of the parameters δ, ξ, $q_{L,R}$ defined in (6), (7). The corresponding analytic formulae are

$$\begin{aligned} \Delta m_W &= \frac{1}{2}\frac{c^2}{c^2 - s^2} m_W \delta \\ \Delta \sin^2\theta_w^{\rm eff} &= -\frac{s^2c^2}{c^2 - s^2}\delta + s\xi(q_R(1 - 2s^2) + 2s^2 q_L) \\ \Delta\Gamma_\ell &= \Gamma_\ell \Bigg\{ \left(\frac{1 - 2s^2}{s^2c^2} + \frac{4(1 - 4s^2)}{1 - 4s^2 + 8s^4}\right)\frac{s^2c^2}{c^2 - s^2}\delta \\ &\qquad - \frac{4}{1 - 4s^2 + 8s^4} s\xi(q_L(1 - 2s^2) - 2s^2 q_R)\Bigg\} \ , \end{aligned} \tag{14}$$

where $s = \sin\theta_w$, $c = \cos\theta_w$. In the rank-1 E_6 models [25],

$$q_L = \cos\theta \frac{3}{2\sqrt{6}} + \sin\theta \frac{1}{6}\sqrt{\frac{5}{2}}$$
$$q_R = \cos\theta \frac{1}{2\sqrt{6}} - \sin\theta \frac{1}{6}\sqrt{\frac{5}{2}} \ , \tag{15}$$

where θ is the mixing angle between the two $U(1)$ bosons, defined so that $\theta = 0$ corresponds to the $SO(10)$ boson χ and $\theta = \pi/2$ to the E_6 boson ψ. The expressions for δ and ξ require a parameter γ, which depends on the quantum numbers of the Higgs boson responsible for $SU(2) \times U(1)$ breaking and Z^0–$Z^{0\prime}$ mixing. In general, we would expect both Higgs fields H_u and H_d to obtain vacuum expectation values, which are conventionally written

$$\left\langle H_u^0 \right\rangle = \frac{v}{\sqrt{2}} \sin\beta \qquad \left\langle H_d^0 \right\rangle = \frac{v}{\sqrt{2}} \cos\beta \ , \tag{16}$$

with $v = 246$ GeV. Then

$$\gamma = 2s\sin^2\beta(\cos\theta\frac{1}{\sqrt{6}} - \sin\theta\sqrt{\frac{5}{18}}) + 2s\cos^2\beta(\cos\theta\frac{1}{\sqrt{6}} + \sin\theta\sqrt{\frac{5}{18}}) \ . \tag{17}$$

The cases (d) and (u) shown in Fig. 4 correspond to the cases $\beta = 0$ and $\beta = \pi/2$, respectively.

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