Algorithms To Automate LCLS Undulator Tuning

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Abstract

This note describes the operation of a computer program which was written to automate LCLS undulator tuning. The algorithms used to compute the shim sizes and locations are discussed.

1 Introduction[1]

Automation of the LCLS undulator tuning offers many advantages to the project. Automation can make a substantial reduction in the amount of time the tuning takes. Undulator tuning is fairly complex and automation can make the final tuning less dependent on the skill of the operator. Also, algorithms are fixed and can be scrutinized and reviewed, as opposed to an individual doing the tuning by hand. This note presents algorithms implemented in a computer program written for LCLS undulator tuning.

The LCLS undulators must meet the following specifications[2]. The maximum trajectory walkoff must be less than 5 μm over 10 m. The first field integral must be below 40×10^{-6} Tm. The second field integral must be below 50×10^{-6} Tm^2. The phase error between the electron motion and the radiation field must be less than 10 degrees in an undulator. The K parameter must have the value of 3.5000 ± 0.0005. The phase matching from the break regions into the undulator must be accurate to better than 10 degrees. A phase change of $113 \times 2\pi$ must take place over a distance of 3.656 m centered on the undulator. Achieving these requirements is the goal of the tuning process.

Most of the tuning is done with Hall probe measurements. The field integrals are checked using long coil measurements. An analysis program written in Matlab takes the Hall probe measurements and computes the trajectories, phase errors, K value, etc. The analysis program and its calculation techniques were described in a previous note[3]. In this note, a second Matlab program containing tuning algorithms is described. The algorithms to determine the required number and placement of the shims are discussed in detail.

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[2]H. D. Nuhn et al., "General Undulator System Requirements", LCLS Physics Requirements Document 1.4-001.
[3]Z. Wolf, "Introduction to LCLS Undulator Tuning", LCLS-TN-04-07, June, 2004.

2 Trajectory Tuning

This section describes the algorithm to straighten the x and y trajectories. The specification requires that the trajectory must remain straight to 5 μm over 10 m. Our tuning goal is to make the RMS trajectory walkoff less than 2 μm in a 3.4 m long undulator section. Implicit is the assumption that the trajectory is not changing rapidly causing large phase errors. The field integral specifications, which refer to the trajectory exit position and slope, will automatically be satisfied by the algorithm.

The following subsections describe how the trajectory is first fit by a collection of line segments. The slope changes at the beginning of each segment required to make the segments form a straight line at the zero position are calculated. Each slope change corresponds to a shim which is applied to the undulator. The shims are then combined to reduce their number. The shim dimensions which give a certain slope change are determined empirically and input to the program. The pole at which the shim is applied must be chosen carefully and the selection process is presented in detail. Knowing the slope changes, the required integrated field strength of each shim is calculated. The integrated field strengths let us add the shim fields to the measured fields in order to predict the effect of the shims. The field integrals are ideally made zero by this procedure. Long coil measurements will be used to remove small errors.

2.1 Segmented Trajectory

The x and y trajectories of an electron beam of given energy are calculated from the undulator magnetic field measurements. An example of an x trajectory is shown in figure 1. The heavy line shows the trajectory averaged over one undulator period, which we call the average trajectory. Trajectory shims must be added to change the slope at certain places in order to make the average trajectory straight.
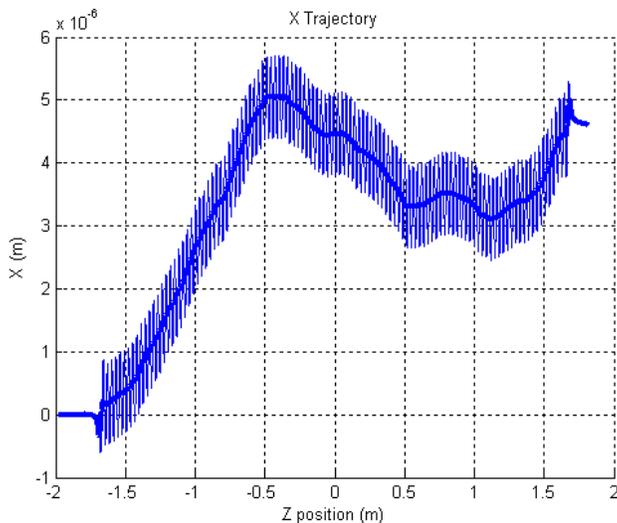
Figure 1: Calculated x trajectory from the measured fields.

The trajectory shimming algorithm begins by fitting a set of line segments to the average trajectory. Both x and y trajectories are treated in the same manner. The line segments are equally spaced along the undulator and joined to form the "segmented trajectory". An example of a segmented trajectory with 20 segments is shown in figure 2.
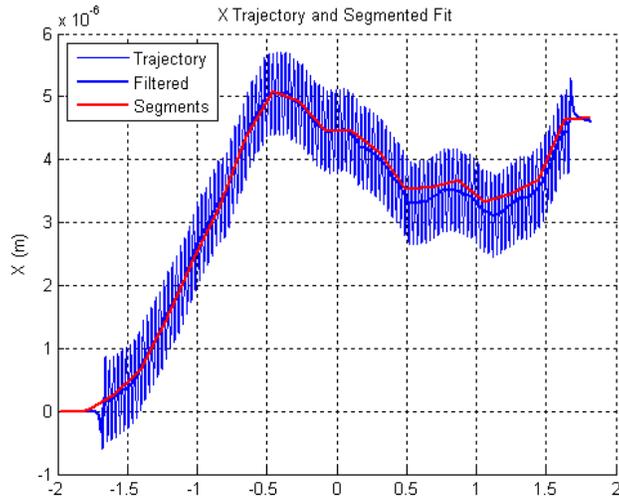
Figure 2: Line segments are fit to the trajectory and joined, making the segmented trajectory.

The segmented trajectory offers several advantages. The linear fits provide noise filtering. The segments turn the problem into a discrete problem instead of a continuous problem. The behavior of the segments is thoroughly understood. A set of shims can be calculated which make the segmented trajectory mathematically perfect. This known solution acts as a starting point for practical solutions which reduce the number of shims.

2.2 Required Segment Slope Changes

The segmented trajectory is formed by equally dividing the measured trajectory into N segments along z, the undulator axis. A line is fit to each segment. The slope of the i'th line m_i is determined from the fit. The change in trajectory position in the i'th segment is

$$\Delta x_i = m_i \Delta z_i \tag{1}$$

The trajectory starts at zero. The changes Δx_i are added together to get the trajectory position at the end of the n'th segment

$$x_n = \sum_{i=1}^{n} \Delta x_i \tag{2}$$

Or,

$$x_n = \sum_{i=1}^{n} m_i \Delta z_i \tag{3}$$

3

Suppose we add a shim at the beginning of the j'th segment. The shim causes a slope change Δm_j to the j'th segment and all succeeding segments. Conversely, after shimming, the slope of the j'th segment is the original slope plus the sum of the slope changes of the j'th segment and all the previous segments. Let M_i be the slope after shimming,

$$M_i = m_i + \sum_{j=1}^{i} \Delta m_j \tag{4}$$

Let X_n be the trajectory position after shimming,

$$X_n = \sum_{i=1}^{n} M_i \Delta z_i \tag{5}$$

Inserting the shimmed slope, we get

$$X_n = \sum_{i=1}^{n} (m_i + \sum_{j=1}^{i} \Delta m_j) \Delta z_i \tag{6}$$

Using equation 3 for the original trajectory position x_n, we find

$$X_n = x_n + \sum_{i=1}^{n} \sum_{j=1}^{i} \Delta m_j \Delta z_i \tag{7}$$

The goal of shimming is to determine the Δm_j such that $X_n = 0$ for all n. Setting $X_n = 0$, we get

$$\sum_{i=1}^{n} \sum_{j=1}^{i} \Delta m_j \Delta z_i = - x_n \tag{8}$$

We rewrite the sum on the left as (note the index changes)

$$\sum_{j=1}^{n} \sum_{i=j}^{n} \Delta z_i \Delta m_j = - x_n \tag{9}$$

We form the matrix

$$Z_{nj} = \sum_{i=j}^{n} \Delta z_i \tag{10}$$

With the convention that $Z_{nj} = 0$ for $j > n$, equation 9 can be written as

$$\sum_{j=1}^{N} Z_{nj} \Delta m_j = - x_n \tag{11}$$

where N is the number of segments, and $n = 1 \ldots N$. This has the form of a matrix equation

$$\mathbf{Z} \, \mathbf{\Delta m} = - \mathbf{x} \tag{12}$$

4

We solve for $\mathbf{\Delta m}$ by multiplying by the inverse of the $\mathbf{Z}$ matrix

$$\mathbf{\Delta m} = -\mathbf{Z}^{-1}\mathbf{x} \tag{13}$$

This equation is our result. It tells us the trajectory slope change required from the shim of each segment in order to make the segmented trajectory a perfect line at zero.

Figure 3 demonstrates the algorithm. The upper line in the top plot shows the segmented
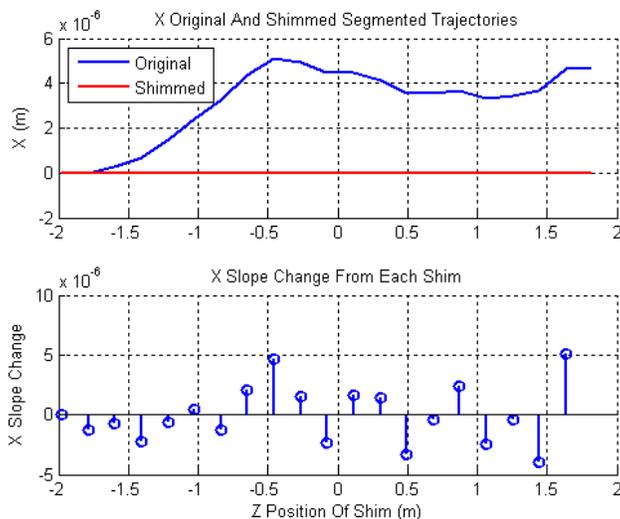
Figure 3: One shim per segment is calculated to make the segmented trajectory flat. The required slope changes are shown.

linear fit to the original trajectory. Using the algorithm described in this section, a shim with the calculated slope change was added to each segment resulting in the bottom line at zero. The corrected trajectory is a perfect line along the beam axis. The calculated shim slope changes are shown in the bottom of the figure. After shimming, the corrected trajectory would be along the beam axis with small deviations due to the residuals of the linear fits and due to errors from placing the shims at the beginning of the segments instead of at the exact problem locations. Both of these trajectory deviation sources are reduced by using a large number of segments.

2.3 Combine Shims

The method of shimming the segmented trajectory gives a mathematically perfect shimmed segmented trajectory. There are certain trade-offs, however. The more segments that are used, the closer a shim is placed to the actual problem in the undulator, giving smaller resulting errors. Also, the residuals to the linear fits are smaller. The disadvantage is that a large number of shims are required. As a compromise, we use the segmented trajectory in order to determine an ideal set of shims, and then we combine shims in order to reduce their number to a manageable level. We do not wish to remove a shim because then the

resulting trajectory would have a net slope at all points after the removed shim. Rather, we move a shim to its neighboring segment and combine shims if the effect is small enough.

Moving a shim to a neighboring segment is the same as putting a negative cancelling shim in the segment and a positive shim of the same size in the neighbor. These two shims of equal and opposite strength result in a translation of the trajectory, but with no net slope change. We start at the first segment. If moving the shim to the second segment causes a translation below a threshold value, we move it and combine the shims into the second segment. Then we consider moving the new second segment shim to the third segment. If the resulting translation is below the threshold, we perform the move. If the translation is above the threshold, we leave the shim and move on to the next segment. This process is continued through all the segments.

The procedure of moving and combining shims if the resulting trajectory translation is small enough can lead to significant trajectory excursions if the small translations accumulate. To deal with this possibility, another threshold is applied to the above procedure. Once the trajectory translations accumulate to the threshold value, the shim is left in place instead of moved.

At this point, a second pass is applied. After the shims are combined, the new trajectory can be made to go to zero at all the final shim locations after the first shim by adjusting the shims slightly. Consider a shim after combining shims. Its strength is adjusted so that the trajectory at the next combined shim location goes to zero. The shim at the next location is adjusted to cancel the slope change from the first shim. This process is repeated down the undulator. This procedure helps keep the trajectory excursions small.

If we take the shims of figure 3 and apply this algorithm with a threshold of $\frac{1}{2}$ μm for the allowed translation and 1 μm for the allowed translation accumulation, then we get the new set of shims shown in the bottom of figure 4. The top of figure 4 shows the
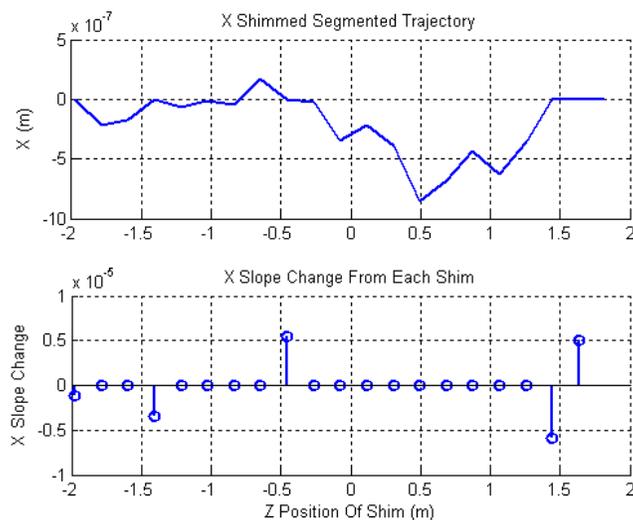
Figure 4: Shims are combined leaving an imperfect but acceptable trajectory. The combinations result in a great reduction in the number of shims.

resulting segmented trajectory. It is no longer ideal, but it is within the specifications. The number of shims has gone from 20 to 5, making the application of the shims much more manageable. The trajectory translation threshold and the trajectory shift accumulation threshold are parameters that will have to be tuned. Preliminary studies show that a value of $\frac{1}{2}$ μm for the trajectory translation threshold and 1 μm for the trajectory shift accumulation threshold give a large reduction in the number of shims with an acceptable resulting trajectory if 20 segments are used.

2.4 Shim Dimensions

The trajectory is made straight and the field integrals are reduced to acceptable values by making slope changes along the trajectory. The slope is changed by adjusting the magnetic field of a pole in the undulator. This is done by applying steel rods, called shims, near the appropriate poles. The trajectory shims are shown in the top left in figure 5[4]. The

Figure 5: Trajectory shims are shown in the upper left. Phase shims are shown in the lower right.

shims weaken the field of a pole by shunting flux out of the gap. The strength of the shim is determined by the distance from the undulator pole to the rod. The slope change as a function of distance is determined experimentally by many measurements at different distances. The data is fit and the fit parameters are input to the program. Once the desired slope change is known, the distance between the shim and the pole is calculated from the fit. Fit results using both one rod and two rods are illustrated in figure 6[5].

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[4]R. Dejus (Editor), "LCLS Prototype Undulator Report", ANL/APS/TB-48, January, 2004.

[5]Y. Levashov, private communication.
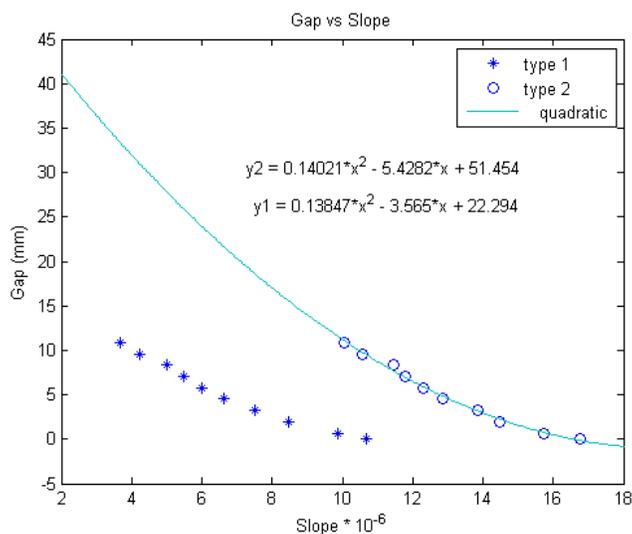
Figure 6: The distance between the shim and a pole is calculated using a fit to measured distance vs slope change data.

2.5 Shim Placement

It is helpful to think of the shim as adding a superposed field in the opposite direction to the pole field. It is easy to see how this superposed field changes the slope of the trajectory. If a pole bends the beam to the right, an added shim will bend the beam to the left, and visa versa. This makes placing the shim on the proper pole important.

Shims for the x trajectory must be placed on both the upper and lower poles to cancel any horizontal fields. This is illustrated in figure 7.
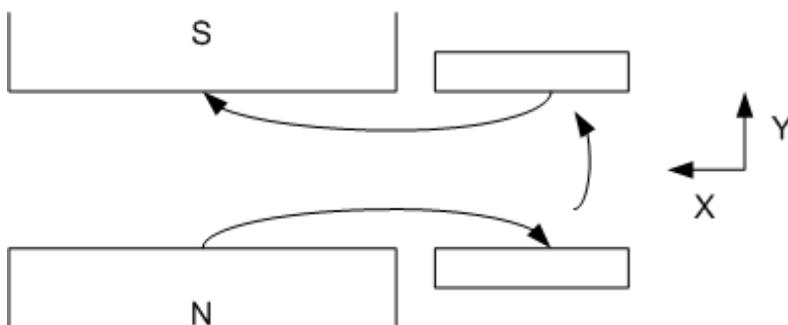
Figure 7: The x trajectory shims are placed on both the upper and lower poles to cancell the horizontal field.

The algorithm to place a shim for the x trajectory is as follows. The nearest pole location to the beginning of a segment is found. If the required integrated field strength of the shim has the opposite sign as the sign of the field from the poles, the shim stays at this

8

pole location. If the required integrated strength of the shim has the same sign as the pole field, the shim is moved to the next set of poles and placed on both the upper and lower poles.

For the y trajectory, a horizontal field is desirable. If one shim is placed at an upper south pole, for example, the other shim is placed on the next lower south pole. In this way, a net horizontal field is generated to kick the beam vertically, but no net vertical field is generated which would kick the beam horizontally. This is illustrated in figure 8.
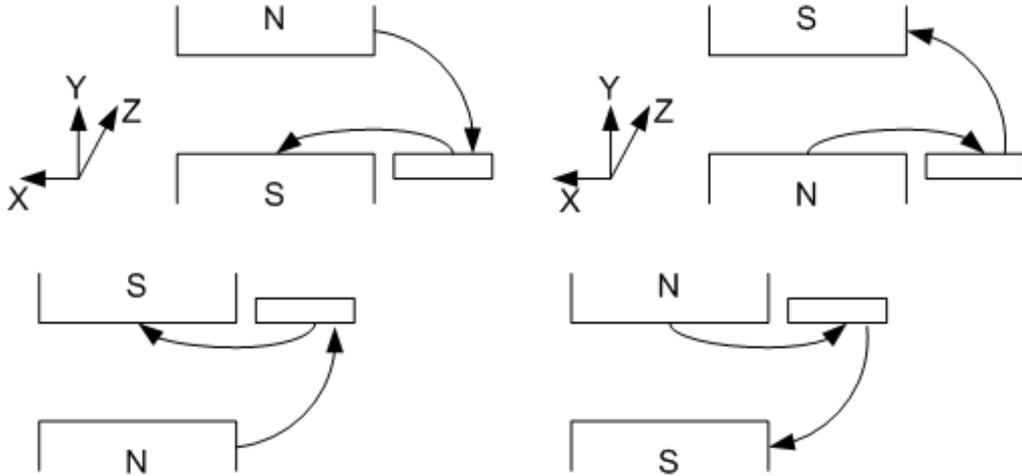
Figure 8: Shims for the y trajectory are placed on succeeding sets of poles.

The algorithm to place a shim for the y trajectory is slightly more complicated than for the x trajectory. A shim placed on a north pole causes a horizontal field in the $-x$ direction, as shown in figure 8. This deflects the electron beam in the $+y$ direction. A shim placed on a south pole causes a horizontal field in the $+x$ direction. This deflects the beam in the $-y$ direction. For a segment that needs a shim deflecting the beam in the vertical direction, the pole pair nearest the the start of the segment is found. If the beam requires a slope change in the $+y$ direction, a shim is placed on the north pole of the pair and the north pole of the following pair. If the beam requires a slope change in the $-y$ direction, a shim is placed on the south pole of the pair and the south pole of the following pair. The algorithm uses the measured field direction to determine whether to place the shim on an upper or lower pole. If the field direction is $+y$, the pole pair has a north on the bottom and a south on top. If the field direction is $-y$, the pole pair has a north on the top and a south on bottom.

2.6 Magnetic Fields From The Shims

Once the slope change required from each shim is known, the integrated field strength can be calculated. This will allow us to model the effect of the shims. The Lorentz force law for a particle of charge q travelling in the z direction with speed v_z through a shim with

fields B_x and B_y is

$$\gamma m \frac{d^2 x}{dt^2} = -qv_z B_y \tag{14}$$

$$\gamma m \frac{d^2 y}{dt^2} = qv_z B_x \tag{15}$$

The time and z position are related by $z = v_z t$. Making z the independent variable instead of t gives $\frac{d}{dt} = v_z \frac{d}{dz}$, yielding

$$\frac{d^2 x}{dz^2} = -\frac{q}{\gamma m v_z} B_y \tag{16}$$

$$\frac{d^2 y}{dz^2} = \frac{q}{\gamma m v_z} B_x \tag{17}$$

Integrating through the shim and using $\Delta m_x = \frac{dx}{dz}\big|_{after\ shim} - \frac{dx}{dz}\big|_{before\ shim}$ and a similar expression for Δm_y, we find

$$\Delta m_x = -\frac{q}{\gamma m v_z} \int B_y\,dz \tag{18}$$

$$\Delta m_y = \frac{q}{\gamma m v_z} \int B_x\,dz \tag{19}$$

Knowing the required slope change of the shim, the integrated field strength of the shim is easily calculated

$$\int B_y\,dz = -\frac{\gamma m v_z}{q} \Delta m_x \tag{20}$$

$$\int B_x\,dz = \frac{\gamma m v_z}{q} \Delta m_y \tag{21}$$

2.7   Predicted Trajectory After Shimming

Once we know the integrated field strength of the shims, we can model their effect. We distribute a shim field over a pole of length $\lambda_u/2$ such that the integrated strength is preserved. We then add the shim field to the measured fields. The resulting shimmed trajectory can be predicted by sending the shimmed field through the analysis software. The result is shown in figure 9. It looks similar to the shimmed segmented trajectory. It results in a trajectory within the specifications.

2.8   Undulator Field Integrals

As noted in the introduction, the field integrals have tight specifications. The field integrals are related to the exit slope and position of the beam. The exit slope is given by

$$x' = -\frac{q}{\gamma m v_z} I_1 \tag{22}$$

where q is the electron charge (negative), γ is the electron Lorentz factor, m is the electron mass, v_z is the electron velocity in the forward direction (this can be set to c, the speed of
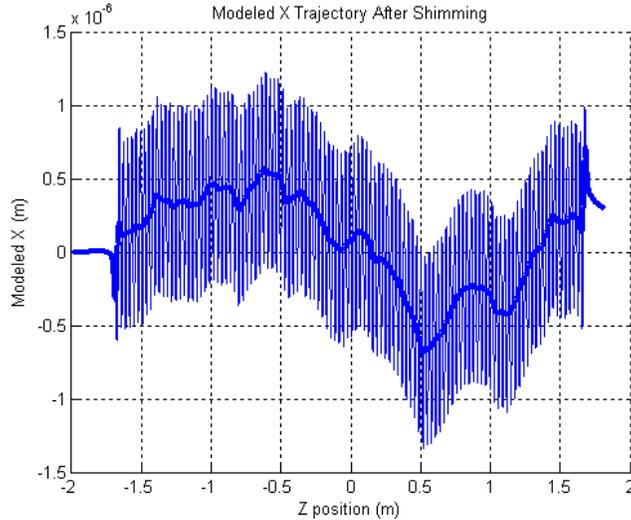
Figure 9: The required shim fields are added to the measured fields and the resulting trajectory is calculated giving the predicted shimmed trajectory.

light, with small error). I_1 is the first field integral which is specified. Using $I_1 = 40 \times 10^{-6}$ Tm and $\gamma = 26693$, we find $x'_{\max} = 8.80 \times 10^{-7}$ at the undulator exit.

The second field integral I_2 determines the position at which the beam exits the undulator relative to the entrance position. The formula is

$$x = -\frac{q}{\gamma m v_z} I_2 \tag{23}$$

Using the LCLS parameters and $I_2 = 50 \times 10^{-6}$ Tm^2, the maximum position shift is $x_{\max} = 1.10 \times 10^{-6}$ m.

The algorithm of using the segmented trajectories given above guarantees that the last segment has zero slope and zero offset. It is built into the procedure. To correct for small residual errors, a small shim will be added at the entrance of the undulator and at the exit to adjust the field integrals within specifications. A long coil will make an accurate measurement of I_1 and I_2. The shim at the undulator entrance is used to adjust the exit position, or I_2. The shim at the exit will adjust the exit slope, or I_1. The gap between the shim and the pole required to make a given field integral change will be determined from fits to experimental data, as discussed above.

3 Phase Tuning Overview

This section gives an overview of phase tuning. It begins with a general discussion of slippage and phase in which these quantities are defined and their method of calculation is presented. This leads to a definition of phase errors and a description of their calculation. Once all terms are defined, an overview of the steps for phase tuning is presented. Later sections will discuss these steps in detail.

3.1   Slippage, Phase, Phase Errors

Consider a radiation wave in the undulator along with the electrons. An electron can not keep up with the radiation, which is travelling at the speed of light. Also, the large, alternating undulator magnetic field B_y causes the x trajectory to oscillate. This oscillation adds path length to the trajectory. The added path length, coupled with the fact that the electron is travelling slower than the speed of light, causes the electron to lag behind the radiation wave. The distance between a point on the radiation wave and the electron is called slippage. In spite of the slippage, the electron motion must keep a proper phase relationship with the radiation if the electron is to add to the radiation field in such a way as to maximize the intensity. The relative motion between the radiation wave and the electron will have important implications for the performance of the undulator.

The slippage at the magnetic field peaks can be used to calculate the radiation wavelength. Consider an electron at the positive peak of its trajectory corresponding to the peak field of an undulator period as shown in the top of figure 10. The electron adds
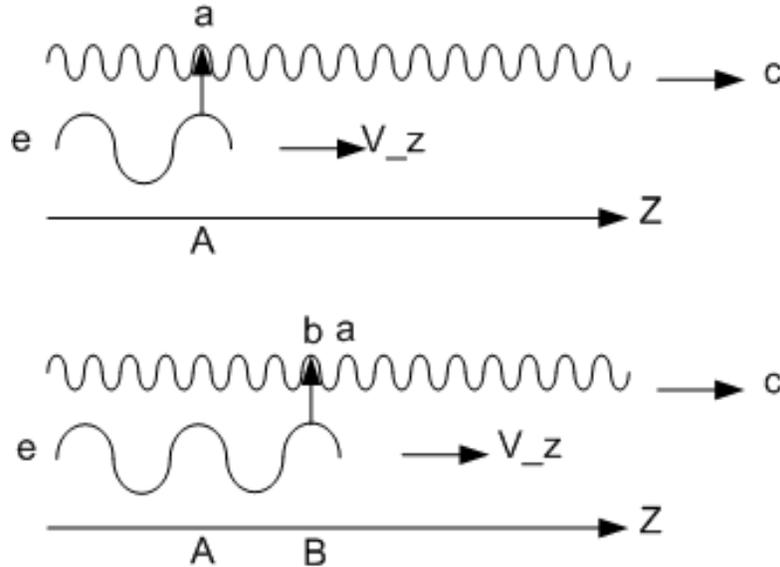
Figure 10: The upper figure shows an electron at position "A" adding photon "a" to the radiation wave. The lower figure shows the situation after the electron has moved one undulator period to position "B". The electron adds photon "b", which is behind "a" because "a" is moving forward faster than the electron.

radiation to the radiation wave. After the electron has moved one undulator period, it adds more radiation as shown in the lower part of the figure. Suppose this radiation is added in such a way as to maximize the intensity of the radiation wave. The new radiation can not add to the radiation from the previous positive peak because of the forward speed difference between the radiation and the electron. Rather, it adds to the radiation wave one wavelength back. The slippage changes by one radiation wavelength in one undulator period.

12

The radiation in the undulator moves with the speed of light c in the z direction and the electron moves with speed v_z. In time dt, the slippage, S, changes by

$$dS = (c - v_z)dt \tag{24}$$

We now wish to change variables. Instead of using time, we use the z position of a point on the radiation wave as our independent variable. With this change of variables, we have $dz = cdt$. Then,

$$dS = (1 - \frac{v_z}{c})dz \tag{25}$$

With z giving the position of a point on the radiation wave, the position of the electron is z minus the slippage distance. The velocity in the forward direction is given by

$$v_z \simeq c(1 - \frac{1}{2}\frac{1}{\gamma^2} - \frac{1}{2}x'^2 - \frac{1}{2}y'^2) \tag{26}$$

The slippage differential can now be written as

$$dS = (\frac{1}{2}\frac{1}{\gamma^2} + \frac{1}{2}x'^2 + \frac{1}{2}y'^2)dz \tag{27}$$

Integrating from initial position z_0 to z, we find the change in slippage

$$\Delta S = \int_{z_0}^{z}(\frac{1}{2}\frac{1}{\gamma^2} + \frac{1}{2}x'^2 + \frac{1}{2}y'^2)dz_1 \tag{28}$$

If we define the initial position to have zero slippage, $S(z_0) = 0$, we can define the slippage as

$$S(z) = \int_{z_0}^{z}(\frac{1}{2}\frac{1}{\gamma^2} + \frac{1}{2}x'^2 + \frac{1}{2}y'^2)dz_1 \tag{29}$$

The first term arises because the electron is moving slower than the speed of light. The second and third terms come from the extra path length of the trajectory when the electron is moving transverse to the undulator. Outside the undulator, where $x' = y' = 0$, the slippage varies linearly with z as $S(z) = \frac{1}{2}\frac{1}{\gamma^2}z + const$. Inside the undulator, the slope terms contribute and the slippage varies more rapidly. This is illustrated in figure 11.

If we fit a line to the slippage vs z curve in the undulator, the average slope is proportional to the wavelength the undulator is radiating at. The average slope is one radiation wavelength per undulator period, as noted above. The residuals from the fitted line indicate errors, or lack of synchronization. The slippage residuals at the poles are shown in figure 12. Let $S(z_n)$ be the slippage at pole n. Then, when we do the linear fit to the slippage at the poles, we have

$$S(z_n) = S(z_0) + \frac{\lambda_r}{\lambda_u}(z_n - z_0) + S_{resid}(z_n) \tag{30}$$

where the slope of the fitted line, λ_r/λ_u, has been inserted.

We define phase by dividing the slippage by λ_r and multiplying by 2π. Let P be the phase,

$$P(z) = \frac{2\pi}{\lambda_r}S(z) \tag{31}$$

13
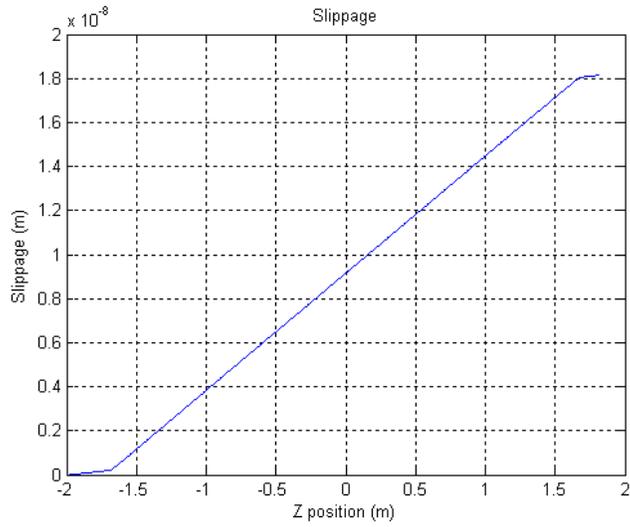
Figure 11: The slippage change is small outside the undulator and large in the undulator where the slope terms contribute.
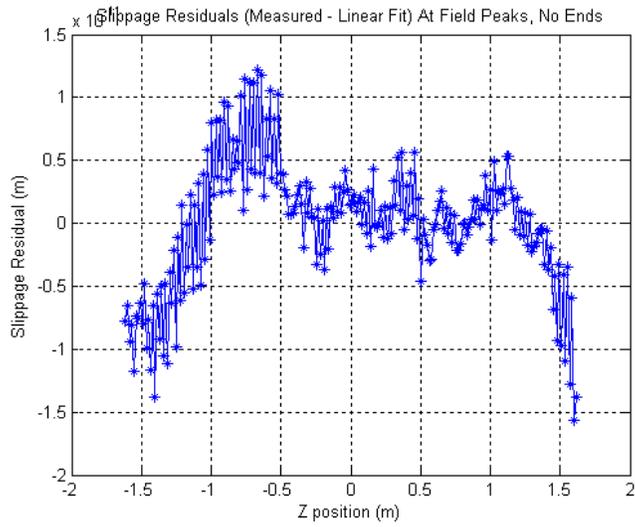
Figure 12: Residuals from a linear fit to the slippage in the undulator.

14

The phase at pole n is given by

$$P(z_n) = \frac{2\pi}{\lambda_r}S(z_0) + \frac{2\pi}{\lambda_u}(z_n - z_0) + \frac{2\pi}{\lambda_r}S_{resid}(z_n) \qquad (32)$$

If the residuals are zero, the phase changes by 2π when z changes by λ_u. Deviations from this case are called phase errors. We define the phase error to be

$$P_{err}(z_n) = \frac{2\pi}{\lambda_r}S_{resid}(z_n) \qquad (33)$$

A plot of phase errors using degrees instead of radians is shown in figure 13.
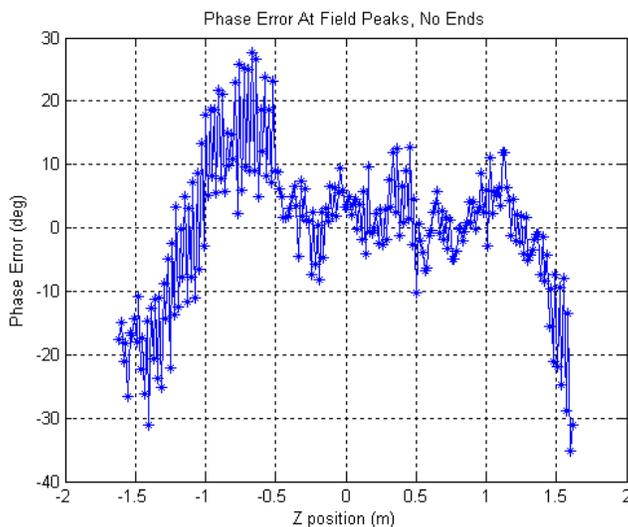
Figure 13: Plot of phase errors at the undulator poles.

Note that the phase is calculated using the λ_r determined for the undulator, not the ideal λ_r. Phase errors measure how consistently the undulator radiates at λ_r. It is an entirely separate matter to set λ_r to the proper value.

3.2 Phase Tuning Basic Steps

The basic steps for phase tuning are illustrated in figure 14. The different steps are labelled a through i. The slippage for each step is along the vertical axis. The z position along the undulator is along the horizontal axis. The outermost set of vertical lines show the cell boundary. A cell includes the undulator and an equal drift distance at the entrance and exit. The phase change through the cell is specified as a tuning requirement. The middle set of vertical lines marks the end of the undulator. The innermost set of vertical lines marks the interior of the undulator, the region away from the ends where the fields are periodic and uniform. The tuning steps shown in each part of the figure will now be described.

15
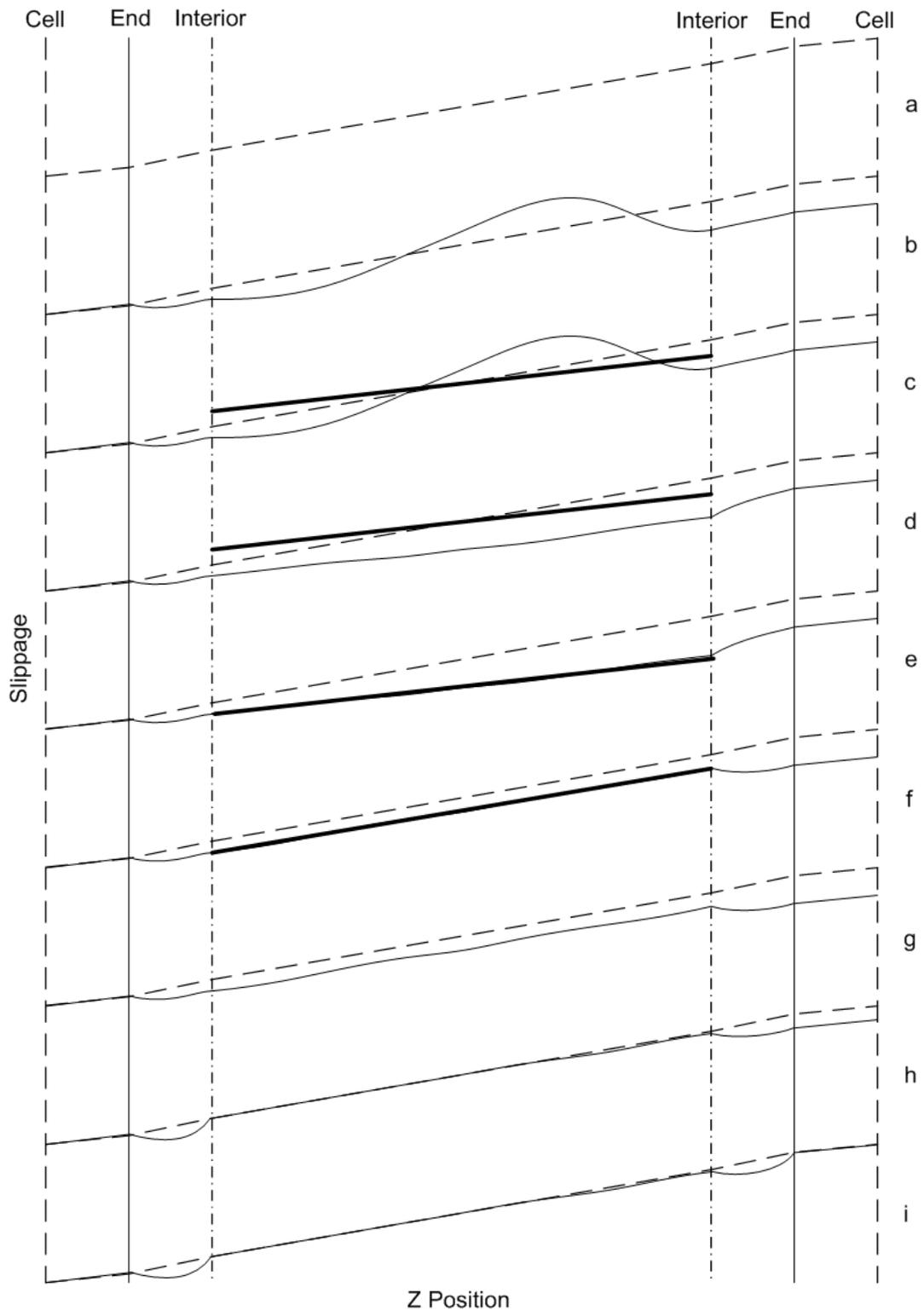
Figure 14: Illustration of the steps in phase tuning.

a) The ideal slippage through the cell is shown as a dotted line. This is repeated in all the other drawings. The ideal slippage is the goal that the tuning tries to achieve.

b) The actual slippage through the cell is shown as a solid line. All differences between the solid line and the dotted line need to be corrected.

c) A linear fit is made to the slippage in the interior of the undulator. The fitted line is the heavy line in the drawing. The slope is used to calculate the radiation wavelength. Deviations from the fitted line are the slippage residuals.

d) **Phase Error Tuning** In this step, the phase errors are tuned to zero. There is a technical issue, however, which must be discussed. In the LCLS undulators, there is no way to rapidly increase the slippage, only to rapidly decrease it. This means that the slippage residuals at the undulator entrance can gradually be made small, but a region of several centimeters with large slippage errors will remain. This is unacceptable and could cause large phase matching problems. Instead of making the slippage residuals zero, we make the residuals all through the undulator equal to the residuals at the entrance. After tuning, the slippage vs z curve in the undulator is a straight line parallel to the fitted line.

e) If the undulator was re-measured and the slippage inside the undulator was fit again, the heavy line would result. The slope of the fitted line remains the same as in step d), it is only shifted. The slippage is nice and uniform with z, but the slope is not equal to the ideal slope.

f) **K Value Tuning** In this step we change the field uniformly in the undulator to adjust the slope of the line to the ideal value. This is known as K value tuning, and will be discussed in a future section.

g) At this point the phase errors and K value have been tuned. The drawing shows the resulting slippage vs z curve. The slippage, however, does not have the ideal value in the undulator.

h) **Phase Match At Entrance** A phase matching shim is added to the undulator entrance region to make the slippage equal to the ideal value all through the undulator. This shim adds a constant offset to the slippage at all points after the shim. Everything is correct except in the exit region.

i) **Phase Match At Exit** A phase matching shim is added to the undulator exit region. This ensures the phase is correct at the exit cell boundary.

4   Phase Error Tuning

This section gives a description of step d) above, phase error tuning. An overview of the process is as follows. The algorithm to correct phase errors uses straight line fits to the phase errors, making the "segmented phase error". Two types of shims, phase shims and gap shims, are used to adjust the segments to minimize the phase errors. The physical dimensions of the phase shims are determined empirically. The gap change required of a gap shim, however, is calculated analytically. The magnetic effects of the phase and gap shims are calculated from the required phase changes. The magnetic effects are added to the measured fields to predict the phase errors after shimming. We now discuss these steps in detail.

4.1 Segmented Phase Error

The phase error shim analysis is done in a manner similar to that used for the trajectory shims. The phase error is fit with a number of line segments. The line segments are joined to smooth the curve. To join the segments, the point is found halfway between the end of one segment and the beginning of the next. These midpoints are then joined to form the segmented phase error. Figure 15 shows a typical phase error curve and its segmented fit.
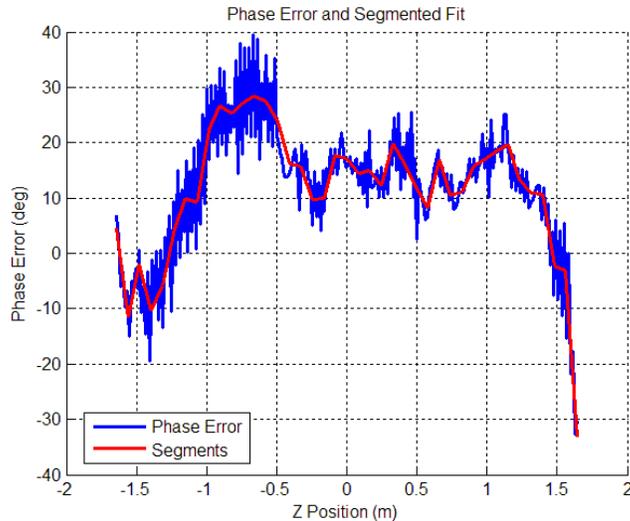
Figure 15: The phase error is fit with line segments giving the segmented phase error.

4.2 Phase Error Shims

In the shimming process, we use two types of shims to make the phase errors as small as possible. The first type of shim, phase shims, are strips of steel that shunt flux out of the gap and weaken two poles. By weakening two poles equally, no net transverse force is applied to the beam, so the trajectory does not get a net slope. The resulting trajectory in the weakened poles is straighter, however, allowing the electrons to catch up a little to the radiation. The slippage decreases, resulting in a decrease in phase over the two poles. Phase shims were shown in the lower right of figure 5. Their operation is shown in figure 16.

In the segmented phase error, a phase shim at the beginning of a segment causes a negative translation of that segment and all succeeding segments. The slopes of all segments remain the same. The effect of a phase shim on a segment is illustrated in figure 17. Phase shims are easy to apply making them the shim of choice when it is appropriate to use them.

The second type of shim, gap shims, sit above and below the pole assemblies. They are used to make local gap changes which either increase or decrease the field in the gap. The operation of a gap shim is shown in figure 18. The magnetic field change causes the path length of the electron motion to change, resulting in a phase change.

18
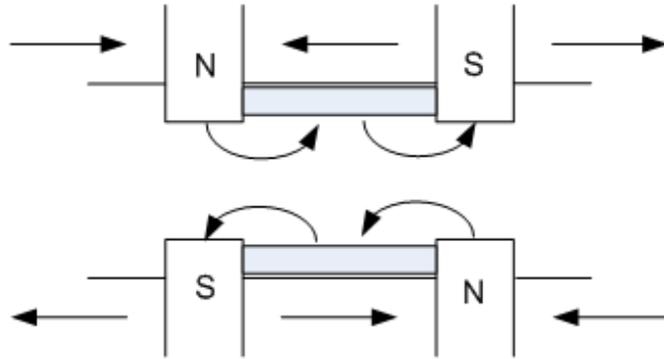
Figure 16: Phase shims shunt flux out of the gap and weaken two poles.
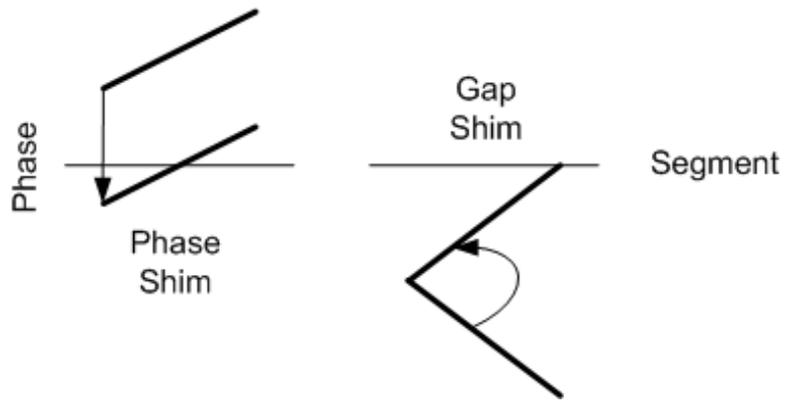
Figure 17: Phase shims cause a rapid phase decrease moving the segment downward. Gap shims change the slope of a segment.
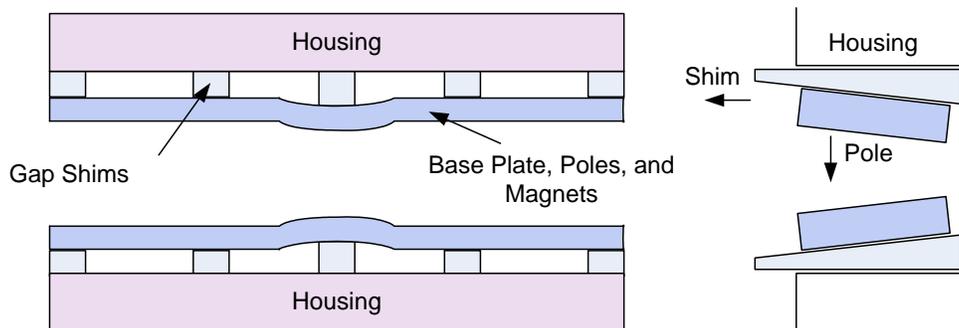
Figure 18: Gap shims are used to increase or decrease the gap. The magnetic field in the gap changes, which in turn changes the phase.

Gap shims change the slope of a segment because the phase change keeps accumulating along the segment. The slope change is illustrated in figure 17. Gap shims can not suddenly increase the phase of a segment, so if a segment starts with a negative phase, it can only slowly be brought up to zero phase. When a gap shim is applied, the net phase change in the segment is added to all succeeding segments. Practically, gap shims are difficult to apply. Bolts must be loosened and shims adjusted between the pole assemblies and the housing before the bolts are tightened again. Gap adjustments at the few micron level are required. The length over which this is done must be accurately determined. Gap shims are used when no alternative exists.

It has been mentioned that we have no mechanism to rapidly increase phase. Gap shims reduce the gap over many centimeters, increasing the phase slowly. Because of this, we are not able to correct a negative phase error at the start of the undulator. Instead, we tune the phase errors to the constant initial phase error. This is the same as calling the initial phase error zero, as was done in figure 15. The phase errors are then tuned to this zero.

4.3   Phase Error Shim Selection Algorithm

The algorithm to determine the phase error shims maximizes the use of phase shims since they are much easier to apply than gap shims. A first pass is made in which phase shims are placed at the beginning of all segments where the mean phase error increases above a threshold value, typically 2 degrees, compared to the previous segment. The strength of the phase shim is chosen to bring the mean phase error to the same value as the previous segment. After applying the phase shims, the segmented phase error of figure 19 results. The upper segments in the upper plot show the original phase error. The lower segments in the upper plot show the phase error after the phase shims have been added. The lower plot shows the field changes required from the phase shims.

The next step is to use gap shims to increase the phase and bring the phase errors to zero. A gap shim is added whenever the phase error goes below a threshold value, typically 6 degrees. The gap shim acts over a large distance, on the order of 20 cm. It is applied so the end of the gap shim is at the location where the phase error crosses the threshold value. When the gap shims are applied, the phase errors of figure 20 result. In this example, the gap shims extend over two segments, so each gap shim is shown as two boxes next to each other in the lower plot of the figure.

4.4   Phase Error Shim Dimensions

The thickness of a steel phase shim is not calculated analytically in the computer program. Rather different thicknesses of the shim material are placed on a magnet in the undulator and the resulting phase change is measured. A fit is made to the phase change vs shim thickness measurements. The computer program uses the fit to determine the proper shim thickness for the given desired phase change.

The amount the gap should be increased or decreased in a gap shim can be calculated analytically. The equations are derived in the subsection titled "Gap Shim Magnetic Strength". The computer program uses these equations to calculate the gap shim dimensions.
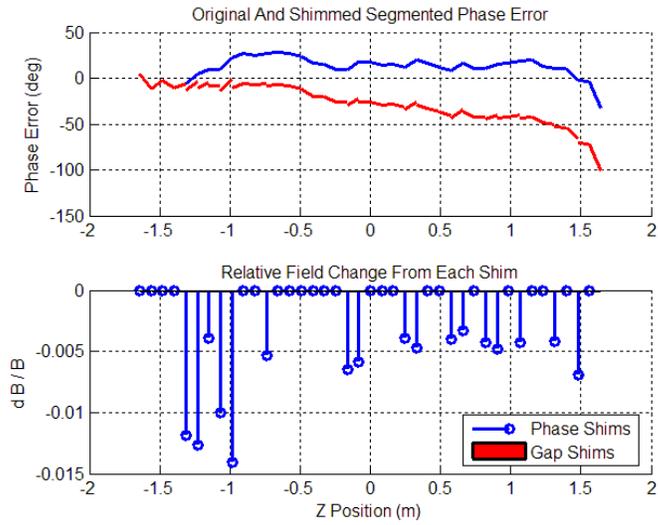
Figure 19: Phase shims are applied in all segments where the phase error increases above a threshold value.
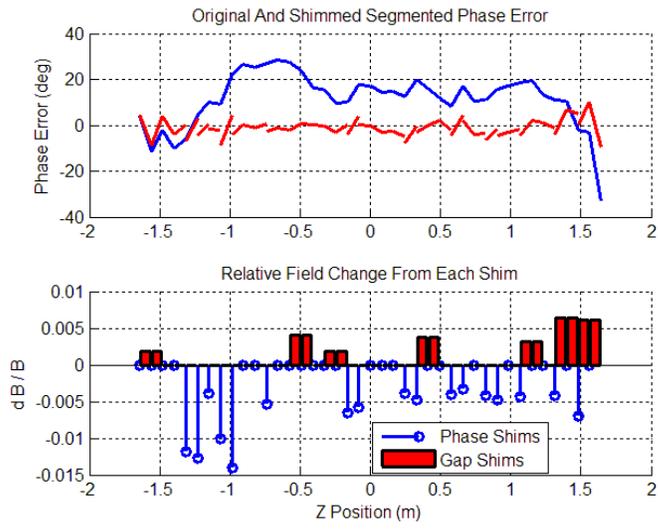
Figure 20: Gap shims are applied to bring the phase errors to zero.

4.5   Phase Shim Magnetic Strength

The required phase change from a phase shim is calculated using the segmented phase error. Knowing the required phase change, we can determine the required magnetic field change. This allows us to model the effect of the phase shims and predict the phase errors after shimming. In these calculations, we approximate the undulator field by a pure sinusoidal field.

For a pure sinusoidal field,

$$B_y = B_0 \cos(\frac{2\pi z}{\lambda_u}) \tag{34}$$

The trajectory slope is given by

$$x'(z) = -\frac{q}{\gamma m v_z}(\frac{\lambda_u}{2\pi})B_0 \sin(\frac{2\pi z}{\lambda_u}) \tag{35}$$

We define K for a sinusoidal field such that

$$x' = \frac{K}{\gamma} \sin(\frac{2\pi z}{\lambda_u}) \tag{36}$$

So, for a sinusoidal field,

$$K = -\frac{q}{m v_z}(\frac{\lambda_u}{2\pi})B_0 \tag{37}$$

Using equation 36 to calculate the slippage and evaluating the slippage change between poles, we get

$$\Delta S_p = \frac{1}{2}\frac{1}{\gamma^2}(1 + \frac{1}{2}K^2)\Delta z_p \tag{38}$$

The slope of the slippage at the poles vs z curve in a sinusoidal field is

$$\frac{\Delta S_p}{\Delta z_p} = \frac{1}{2}\frac{1}{\gamma^2}(1 + \frac{1}{2}K^2) \tag{39}$$

The slope is equal to λ_r/λ_u, so,

$$\lambda_r = \frac{1}{2}\frac{1}{\gamma^2}(1 + \frac{1}{2}K^2)\lambda_u \tag{40}$$

If we change K over two poles using a phase shim, the resulting slippage change is

$$\delta S_p = \frac{1}{2}\frac{1}{\gamma^2}K\,\delta K\,\lambda_u \tag{41}$$

The phase change is

$$\delta P_p = \frac{2\pi}{\lambda_r}\,\delta S_p \tag{42}$$

Inserting the expression for λ_r and δS_p, we get

$$\delta P_p = 2\pi \frac{K}{(1 + \frac{1}{2}K^2)}\delta K \tag{43}$$

22

Since K depends linearly on the field amplitude,

$$\frac{\delta K}{K} = \frac{\delta B_0}{B_0} \qquad (44)$$

The phase change over the two poles in terms of the magnetic field change is given by

$$\delta P_p = 2\pi \frac{K^2}{(1 + \frac{1}{2}K^2)} \frac{\delta B_0}{B_0} \qquad (45)$$

We now solve this expression for the required field change over the two poles which gives the desired phase change. This is our result.

$$\frac{\delta B_0}{B_0} = (\frac{1}{2\pi}) \frac{(1 + \frac{1}{2}K^2)}{K^2} \delta P_p \qquad (46)$$

As an example, in the LCLS the K value is 3.5. For a $10°$ phase change, the relative field change from a phase shim must be $\delta B_0/B_0 = 1.6 \times 10^{-2}$.

4.6 Gap Shim Magnetic Strength

Gap shims change the undulator gap in order to change the magnetic field strength over their length. This changes the phase over the length of the gap shim. Both the required magnetic field strength change and the required gap dimension change can be calculated analytically.

The magnetic field strength change is calculated in a manner similar to the calculation for phase shims. The slippage change over a gap shim of length L_s between poles is given by

$$\Delta S_s = \frac{1}{2} \frac{1}{\gamma^2} (1 + \frac{1}{2}K^2) L_s \qquad (47)$$

When we change the K value, the slippage changes by

$$\delta S_s = \frac{1}{2} \frac{1}{\gamma^2} K \, \delta K \, L_s \qquad (48)$$

Over the segment, the phase changes by

$$\delta P_s = 2\pi \frac{K}{(1 + \frac{1}{2}K^2)} \frac{L_s}{\lambda_u} \delta K \qquad (49)$$

Using the linear relation between K and B_0 and solving for the required field change in the segment, we get

$$\frac{\delta B_0}{B_0} = (\frac{1}{2\pi}) \frac{(1 + \frac{1}{2}K^2)}{K^2} \frac{\lambda_u}{L_s} \delta P_s \qquad (50)$$

Note that $\delta P_s/L_s$ is the desired phase error slope change in the segment from the gap shim. This result lets us model the shimmed field after we know the desired phase changes from the segmented phase error. Knowing the change in the fields from the shims lets us predict the phase errors after the gap shims are applied.

23
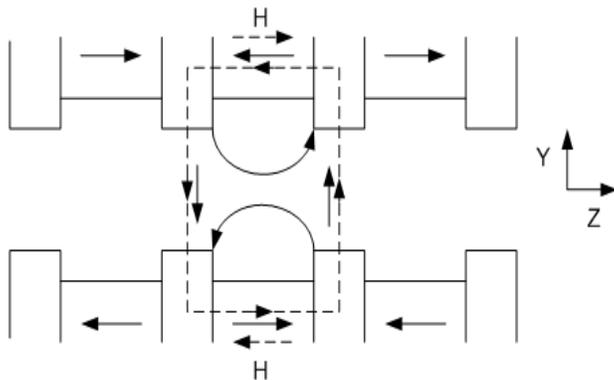
Figure 21: The dashed line shows the path used in the calculation of Ampere's law. The solid arrows show B. The two dashed arrows show H in the permanent magnet blocks.

In order to tune the undulator, we must know how much to change the gap in order to get the required field change. This calculation is done using Ampere's law with the magnetic circuit shown in figure 21. The fields in the undulator are assumed to vary sinusoidally in z with minimal additional harmonic content. This means that the fields vary as the hyperbolic sine or cosine in y with minimal additional terms. Under these conditions, the fields in the undulator are given by

$$B_y(y,z) = B_0 \cosh(\frac{2\pi y}{\lambda_u}) \sin(\frac{2\pi z}{\lambda_u}) \tag{51}$$

$$B_z(y,z) = B_0 \sinh(\frac{2\pi y}{\lambda_u}) \cos(\frac{2\pi z}{\lambda_u}) \tag{52}$$

Applying Ampere's law around the dashed line in figure 21 gives

$$2\int_{-g/2}^{g/2} \frac{B_0}{\mu_0} \cosh(\frac{2\pi y}{\lambda_u}) dy - 2H_m L_m = 0 \tag{53}$$

In this expression, g is the total gap height of the undulator, B_0 is the strength of the peak vertical field on the midplane, H_m is the magnetic field intensity in the permanent magnet blocks, and L_m is the length of the permanent magnet blocks. The poles are assumed to have infinite permeability. Note that H in the block is in the opposite direction of B, hence the minus sign. Performing the integral, we get

$$2\frac{B_0}{\mu_0} \sinh(\frac{\pi g}{\lambda_u})(\frac{\lambda_u}{2\pi})2 - 2H_m L_m = 0 \tag{54}$$

Simplifying this expression, we find that the field on the midplane is related to the gap height by

$$B_0 \sinh(\frac{\pi g}{\lambda_u}) = \frac{\pi}{\lambda_u} \mu_0 H_m L_m \tag{55}$$

We now let the gap change, but we assume that H in the magnetic material is a constant source and does not change as we change the gap. As g goes to $g + \Delta g$, the field on the

24

midplane B_0 goes to $B_0 + \Delta B_0$. These changes are related by

$$\Delta B_0 \sinh(\frac{\pi g}{\lambda_u}) + B_0 \cosh(\frac{\pi g}{\lambda_u})\frac{\pi}{\lambda_u}\Delta g = 0 \tag{56}$$

Rewriting this expression gives

$$\frac{\Delta B_0}{B_0} = -\coth(\frac{\pi g}{\lambda_u})\frac{\pi}{\lambda_u}\Delta g \tag{57}$$

We can now solve for the gap change required to give the desired phase change over a segment. Using equations 50 and 57, we get

$$\Delta g = -\frac{\lambda_u}{\pi \coth(\frac{\pi g}{\lambda_u})}\left(\frac{1}{2\pi}\right)\frac{(1+\frac{1}{2}K^2)}{K^2}\frac{\lambda_u}{L_s}\delta P_s \tag{58}$$

The program uses this formula to compute the gap changes whenever a gap shim is required.

4.7   Predicted Phase Error After Shimming

The magnetic effect of the phase error shims is calculated using the formulas given above. If the resulting shim fields are superimposed on the measured fields, the predicted phase errors after shimming can be calculated. When this is done, the results shown in figure 22 are obtained. The predicted RMS phase error has gone from 11.3 degrees to 5.88 degrees.
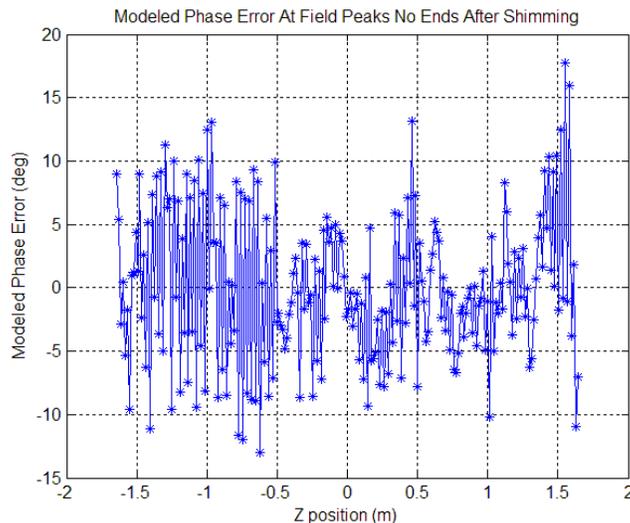
Figure 22: The magnetic effect of the phase error shims are calculated and superimposed on the measured field. The phase errors are then calculated, resulting in this prediction of the phase errors after shimming.

In this example, however, the trajectory was not straightened first. The phase error shims are correcting for extra phase errors coming from the trajectory slope. In practice, the trajectory is straightened before the phase error shimming begins. This will significantly reduce the RMS phase error further.

5 K Value Tuning

After the tuning described so far, the trajectory in the undulator is straight and the phase errors are small. We must now ensure that the undulator is radiating at the proper wavelength. To adjust the wavelength, we adjust the slope of the slippage vs z curve through the entire undulator. This is known as K value tuning.

In order to discuss K value tuning, we must revisit our definition of K. In the section on phase tuning, we defined K for sinusoidal magnetic fields in terms of the slope of the beam trajectory. We now wish to extend the definition of K to apply to actual fields, not just pure sinusoidal fields. To do this, we define K in terms of the slope of the curve of slippage at the poles vs position along the undulator. Let $(\Delta S_p/\Delta z_p)_{fit}$ be the slope of the line fitted to the slippage at the poles. We define K such that

$$\left(\frac{\Delta S_p}{\Delta z_p}\right)_{fit} = \frac{1}{2}\frac{1}{\gamma^2}(1 + \frac{1}{2}K^2) \tag{59}$$

This definition preserves the value of K for pure sinusoidal fields. It applies, however, to all other fields. This K value is often called the effective K, or K_{eff}. We will continue to denote it by K, remembering its definition.

It remains true that there is one radiation wavelength of slippage in one undulator period, so the slope of the slippage curve is λ_r/λ_u. The radiation wavelength is given by

$$\lambda_r = \frac{1}{2}\frac{1}{\gamma^2}(1 + \frac{1}{2}K^2)\lambda_u \tag{60}$$

We want the undulator to radiate at the desired wavelength, so we must set K to the proper value.

The slope of the slippage curve, or equivalently the K value, is changed by adjusting the magnetic field through the entire undulator. A larger field causes larger transverse motion of the beam, which increases slippage. The reverse is true for a lower field.

Changing the K value is relatively easy in the LCLS undulators. The poles are canted so the undulator gap changes with x position. This is illustrated in figure 23. The magnetic field strength changes with x, which means that the K value changes with x. To set the K value, K is measured as a function of x. A fit is made and the x position giving the desired K value is determined. A final measurement at this x position is used to confirm the K value. The undulator is fiducialized so the beam location is at this x position.

Using the analysis of how the field changes with undulator gap given above, we can estimate how the K value changes with x position in the undulator. The analysis assumes a pure sinusoidal field, so its accuracy is greatest if the field has low harmonic content.

We start with equation 57 for the field change as a function of gap change.

$$\frac{\Delta B_0}{B_0} = -\coth(\frac{\pi g}{\lambda_u})\frac{\pi}{\lambda_u}\Delta g \tag{61}$$

Since K and B_0 are linearly related for a sinusoidal field, we have

$$\frac{\Delta K}{K} = -\coth(\frac{\pi g}{\lambda_u})\frac{\pi}{\lambda_u}\Delta g \tag{62}$$
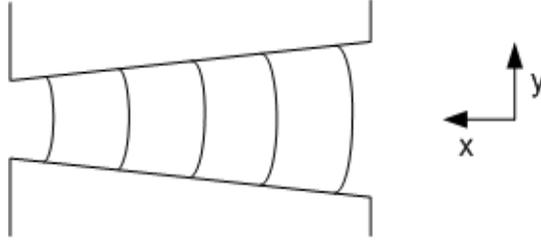
Figure 23: The poles of the LCLS undulators are canted to produce a field which varies with x.

This expression tells us the gap change required for a given K change. The gap change is related to the x position change through the cant angle according to $\Delta g = \frac{dg}{dx}\Delta x$. The total cant angle between LCLS undulator poles is 4.5 mrad, so $\left|\frac{dg}{dx}\right| = 4.5 \times 10^{-3}$. We now incorporate the fact that the gap change is due to the canted poles,

$$\frac{\Delta K}{K} = -\coth(\frac{\pi g}{\lambda_u})\frac{\pi}{\lambda_u}\frac{dg}{dx}\Delta x \tag{63}$$

For the LCLS, $g = 6.8 \times 10^{-3}$ m, $\lambda_u = 3.0 \times 10^{-2}$ m, $\coth(\frac{\pi g}{\lambda_u}) = 1.634$, and $\frac{dg}{dx} = -4.5 \times 10^{-3}$. Inserting these values, we find

$$\frac{\Delta K}{K} = (0.77 \ \frac{1}{\text{m}})\Delta x \tag{64}$$

For a K change of $\frac{\Delta K}{K} = 1.5 \times 10^{-4}$, a horizontal translation of $\Delta x = 1.95 \times 10^{-4}$ m is required.

6 Phase Matching

6.1 General Requirements

In a previous section we noted how the electron motion must stay in phase with the radiation wave in the undulator to maximize output power. We considered small imperfections in the undulator field causing phase errors which we must correct. There is a more global problem, however. We need to adjust each of the individual undulator sections so that the radiation produced in each section is in phase with the overall radiation wave.

The entire LCLS structure is broken into individual undulators with drift regions in between. We associate a certain drift distance with an undulator in the center and call the combination a cell. The phase change in a cell is required to be a multiple of 2π. The phase change in any remaining drift distance outside a cell is also required to be a multiple of 2π. In this way, the global phase relationship can be maintained.

We require the proper phase change from the beginning of the cell into the body of the undulator. Consider figure 24. It shows the x trajectory of an electron in an undulator with an even number of poles, as in the LCLS. The cell boundaries are marked with dotted lines.

Reference poles, which determine the region of the center of the undulator, are marked with arrows. The reference poles are the third pole in from each end in this example. Notice that with the reference poles an equal distance in from the ends, there is an extra half oscillation cycle between reference poles. The phase change between reference poles is an integer times 2π plus π for the extra half cycle. We write this as $\Delta P_{und\_ctr} = (2n+1)\pi$, or $\Delta P_{und\_ctr} = \pi$ modulo 2π. If the cell is required to have a phase change of an integer times 2π, the phase change at the entrance and exit of the undulator must add to π modulo 2π. We divide this evenly between the ends. Thus we require the phase change from the cell entrance to the first reference pole to be $\Delta P_{enter} = \pm\frac{\pi}{2}$ modulo 2π, and the phase change from the second reference pole to the far cell boundary $\Delta P_{exit} = \pm\frac{\pi}{2}$ modulo 2π. The signs must be the same for entrance and exit.
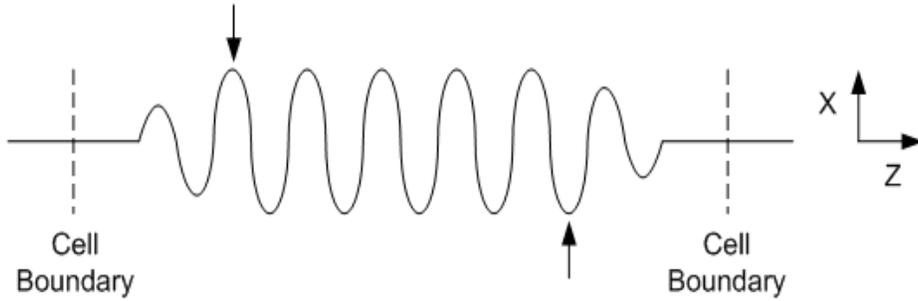
Figure 24: Electron trajectory with the reference poles marked with arrows. The phase from the cell boundary to the first reference pole, the phase between reference poles, and the phase from the second reference pole to the far cell boundary must all be set to required values.

Let ΔP_{enter} be the phase change from the entrance cell boundary at z_{c1} to a given reference pole at z_{r1}.

$$\Delta P_{enter} = \frac{2\pi}{\lambda_r} \int_{z_{c1}}^{z_{r1}} (\frac{1}{2}\frac{1}{\gamma^2} + \frac{1}{2}x'^2)dz \tag{65}$$

We determine ΔP_{enter} from our measurements and perform adjustments until $\Delta P_{enter} = 2\pi n \pm \frac{\pi}{2}$, where n is an integer.

Similarly, let ΔP_{exit} be the phase change from the second reference pole near the end of the undulator at z_{r2} to the cell boundary at the exit at z_{c2}.

$$\Delta P_{exit} = \frac{2\pi}{\lambda_r} \int_{z_{r2}}^{z_{c2}} (\frac{1}{2}\frac{1}{\gamma^2} + \frac{1}{2}x'^2)dz \tag{66}$$

We determine ΔP_{exit} from our measurements and perform adjustments until $\Delta P_{exit} = 2\pi n \pm \frac{\pi}{2}$. The sign of the $\frac{\pi}{2}$ must be the same as in ΔP_{enter}.

Using this scheme for all undulators, the electron motion will be in phase with the radiation in the body of all the undulators. The cells independently preserve the phase relationship.

6.2 Phase Matching Shim Dimensions

The phase matching shim calculation is done in a manner similar to phase error shimming. Phase shim dimensions are determined empirically by measuring the phase change as a function of shim thickness. Once the desired phase change is known, the computer program uses the experimental data to determine the proper shim thickness. If the phase needs to be increased, either a preset phase shim can be removed, or the gap can be reduced.

6.3 Delta Function Model

In order to understand phase matching in more detail, it is very useful to make a model of the undulator ends that lets us calculate the phases at the poles. A model with sinusoidal fields turns out to involve much algebra. A simpler model, which gives much insight into the undulator ends, involves replacing the field from each pole by a delta function with the same integrated field strength as the pole. This model will give a clear explanation of the end pole strengths, namely in the ratio 1/4, 3/4, 1. It will also lead to simple expressions for the phase change through the undulator and through the cell. The derived expressions give fairly good agreement with the numerical calculations of the undulator field. The model will explain why the total phase change in an LCLS cell is $113 \times 360°$, as given in the specification, and where the contributions to the phase come from.

Consider the effect of an electron going through the field of a single pole. From equation 18, the slope of the trajectory will change. If the trajectory slope change is $\Delta x' = m$, we replace the pole in our model by a very narrow pole with integrated field strength also giving $\Delta x' = m$. This is illustrated in the upper part of figure 25. Replacing all the poles in the undulator by the very narrow poles gives the situation shown in the lower part of figure 25. The integrated field strength of each pole is represented by a vertical line. The trajectory is a series of straight lines between the narrow poles. The poles cause the slope of the trajectory to change. From the cell boundary to the undulator magnet end, the trajectory has $x' = 0$. From the start of the undulator to the first pole, a distance of $\frac{\lambda_u}{4}$, the slope is also $x' = 0$. The first pole has one quarter strength and gives the trajectory a slope of $x' = \frac{1}{4}m$. The amplitude of the trajectory at the second pole is $x = x'\Delta z = \frac{m}{4}\frac{\lambda_u}{2}$. The second pole is $-3/4$ strength and changes the slope by $\Delta x' = -\frac{3}{4}m$. The slope after the second pole is $x' = \frac{1}{4}m - \frac{3}{4}m = -\frac{1}{2}m$. The amplitude at the third pole is $x = x_{init} + x'\Delta z = \frac{m}{4}\frac{\lambda_u}{2} - \frac{m}{2}\frac{\lambda_u}{2} = -\frac{m}{4}\frac{\lambda_u}{2}$. This is the negative of the amplitude at the second pole. The third pole is full strength and changes the slope to $x' = -\frac{1}{2}m + m = \frac{1}{2}m$. At the fourth pole, the amplitude of the trajectory is back to $x = \frac{m}{4}\frac{\lambda_u}{2}$. This pattern continues through the undulator. Notice how the 1/4, 3/4, 1 pole ratios results in a trajectory with no net slope and no net offset. At the far end of the undulator, this process is reversed as shown in figure 25.

The undulator model with impulsive forces results in a trajectory consisting of straight line segments. This makes it very easy to calculate the slippage in the cell. Each straight line segment adds a slippage change given by equation 28, which for constant slope becomes $\Delta S = (\frac{1}{2}\frac{1}{\gamma^2} + \frac{1}{2}x'^2)\Delta z$. The slippage changes for all the straight line sections of the model can be calculated.

At the entrance, let L_d be the drift length from the cell boundary to the magnet end.
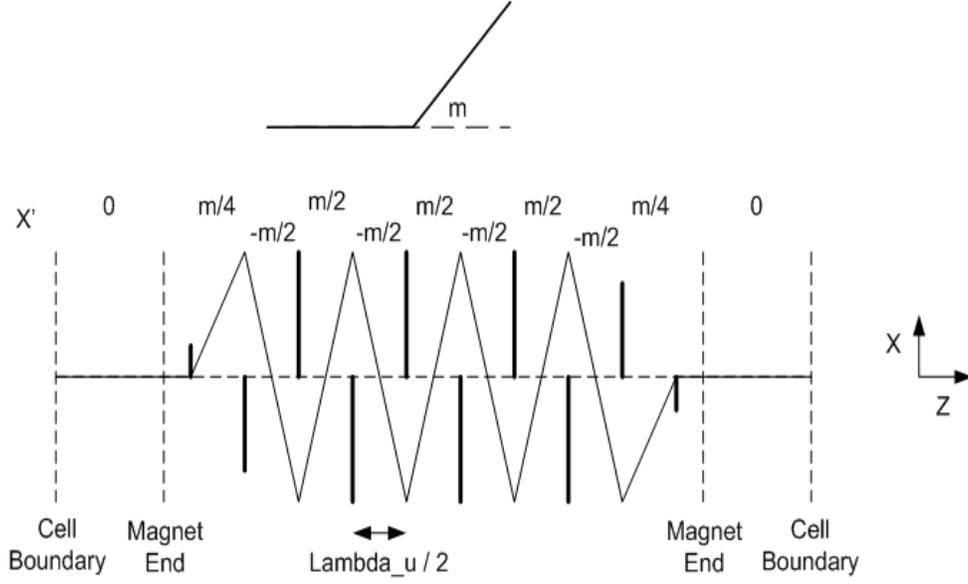
Figure 25: The undulator field is replaced by impulsive forces. The upper part of the figure shows the effect of a short pole giving the trajectory a slope change equal to that of a full strength pole. The lower part of the figure shows the model of the undulator field replaced by the impulsive forces. The trajectory is shown and the trajectory slopes are given.

The slippage in the drift region is $\Delta S_d = (\frac{1}{2}\frac{1}{\gamma^2})L_d$.

In the region from the magnet end to the first pole, the slope is zero and the distance is $\frac{\lambda_u}{4}$. The slippage change in this region is $\Delta S_{01} = (\frac{1}{2}\frac{1}{\gamma^2})\frac{\lambda_u}{4}$.

From the first pole to the second pole the slope is $\frac{m}{4}$ and the distance is $\frac{\lambda_u}{2}$, giving $\Delta S_{12} = (\frac{1}{2}\frac{1}{\gamma^2} + \frac{1}{2}(\frac{m}{4})^2)\frac{\lambda_u}{2}$.

From the second pole to the third pole, the slope is $-\frac{m}{2}$. The distance is $\frac{\lambda_u}{2}$. The slippage is $\Delta S_{23} = (\frac{1}{2}\frac{1}{\gamma^2} + \frac{1}{2}(\frac{m}{2})^2)\frac{\lambda_u}{2}$. The slippage between the rest of the poles in the body of the undulator has this same value. We summarize the results in table 1. The slippage changes at the exit end behave in a similar manner.

A plot of the slippage vs z in this model in the body of the undulator would be a straight

Region	Slippage Change
Entrance drift	$\Delta S_d = (\frac{1}{2}\frac{1}{\gamma^2})L_d$
Magnet end to pole 1	$\Delta S_{01} = (\frac{1}{2}\frac{1}{\gamma^2})\frac{\lambda_u}{4}$
Pole 1 to pole 2	$\Delta S_{12} = (\frac{1}{2}\frac{1}{\gamma^2} + \frac{1}{2}(\frac{m}{4})^2)\frac{\lambda_u}{2}$
Pole 2 to pole 3	$\Delta S_{23} = (\frac{1}{2}\frac{1}{\gamma^2} + \frac{1}{2}(\frac{m}{2})^2)\frac{\lambda_u}{2}$
Pole 3 to pole 4	$\Delta S_{34} = (\frac{1}{2}\frac{1}{\gamma^2} + \frac{1}{2}(\frac{m}{2})^2)\frac{\lambda_u}{2}$
Pole n to pole n+1	$\Delta S_p = (\frac{1}{2}\frac{1}{\gamma^2} + \frac{1}{2}(\frac{m}{2})^2)\frac{\lambda_u}{2}$

Table 1: Slippage change in each region.

Region	Slippage Change
Entrance drift	$\Delta S_d = (\frac{1}{2}\frac{1}{\gamma^2})L_d$
Magnet end to pole 1	$\Delta S_{01} = (\frac{1}{2}\frac{1}{\gamma^2})\frac{\lambda_u}{4}$
Pole 1 to pole 2	$\Delta S_{12} = \frac{1}{2}\frac{1}{\gamma^2}(1+\frac{1}{8}K^2)\frac{\lambda_u}{2}$
Pole 2 to pole 3	$\Delta S_{23} = \frac{1}{2}\frac{1}{\gamma^2}(1+\frac{1}{2}K^2)\frac{\lambda_u}{2}$
Pole 3 to pole 4	$\Delta S_{34} = \frac{1}{2}\frac{1}{\gamma^2}(1+\frac{1}{2}K^2)\frac{\lambda_u}{2}$
Center pole n to n+1	$\Delta S_{ctr} = \frac{1}{2}\frac{1}{\gamma^2}(1+\frac{1}{2}K^2)\frac{\lambda_u}{2}$

Table 2: Slippage change in each region in terms of K.

line. The slope of the line is

$$\frac{\Delta S_p}{\Delta z_p} = (\frac{1}{2}\frac{1}{\gamma^2} + \frac{1}{2}(\frac{m}{2})^2) \tag{67}$$

We know from the discussion of phase that the slope of the line is equal to λ_r/λ_u. So,

$$\lambda_r = (\frac{1}{2}\frac{1}{\gamma^2} + \frac{1}{2}(\frac{m}{2})^2)\lambda_u \tag{68}$$

From the definition of K, we know that

$$\frac{\Delta S_p}{\Delta z_p} = \frac{1}{2}\frac{1}{\gamma^2}(1+\frac{1}{2}K^2) \tag{69}$$

Equating terms, we find that

$$(\frac{1}{2}\frac{1}{\gamma^2} + \frac{1}{2}(\frac{m}{2})^2) = \frac{1}{2}\frac{1}{\gamma^2}(1+\frac{1}{2}K^2) \tag{70}$$

and

$$(\frac{1}{2}\frac{1}{\gamma^2} + \frac{1}{2}(\frac{m}{4})^2) = \frac{1}{2}\frac{1}{\gamma^2}(1+\frac{1}{8}K^2) \tag{71}$$

This allows us to rewrite the table as shown in table 2.

We can now calculate the phase change in each region. We multiply the slippage change by $\frac{2\pi}{\lambda_r}$, or

$$\Delta P = \frac{2\pi}{\frac{1}{2}\frac{1}{\gamma^2}(1+\frac{1}{2}K^2)\lambda_u}\Delta S \tag{72}$$

Rewriting the table in terms of phase changes, we find the results in table 3. The phase changes at the exit end of the magnet have this same pattern.

Numerical estimates of the phase at the poles are easily made. For $K = 3.5$, we have the phase changes given in table 4 expressed in degrees. In the table, ΔP_d is the phase change of the drift region from the cell boundary to the undulator. The LCLS undulators have 226 poles with spacing 0.015 m. The distance $L_d = ((3.656 - 226 \times 0.015)/2)$ m $= 0.113$ m. The phase at pole 2 is $180° + 86.8°$. For proper phase matching, the phase on pole 2 should be $180° + 90°$. The impulse model predicts the phases at the poles very well. If the measured phase at a designated pole is different than the ideal value, a shim is applied. The total phase change in the cell is approximately $\Delta P_{cell} = 180° + 90° + 223 \times 180° + 90° + 180° = 113 \times 360°$, as required.

31

Region	Phase Change
Entrance drift	$\Delta P_d = 2\pi \frac{1}{(1+\frac{1}{2}K^2)}\frac{L_d}{\lambda_u}$
Magnet end to pole 1	$\Delta P_{01} = 2\pi \frac{1}{(1+\frac{1}{2}K^2)}\frac{1}{4}$
Pole 1 to pole 2	$\Delta P_{12} = 2\pi \frac{(1+\frac{1}{8}K^2)}{(1+\frac{1}{2}K^2)}\frac{1}{2}$
Pole 2 to pole 3	$\Delta P_{23} = \pi$
Pole 3 to pole 4	$\Delta P_{34} = \pi$
Center pole n to n+1	$\Delta P_{ctr} = \pi$

Table 3: Phase change in each region.

Region	Phase Change
Entrance drift	$\Delta P_d = 190.3°$
Magnet end to pole 1	$\Delta P_{01} = 12.6°$
Pole 1 to pole 2	$\Delta P_{12} = 63.9°$
Pole 2 to pole 3	$\Delta P_{23} = 180°$
Pole 3 to pole 4	$\Delta P_{34} = 180°$
Center pole n to n+1	$\Delta P_{ctr} = 180°$

Table 4: Phase change in each region using LCLS parameters.

6.4 Phase Match Shimming

In order to do the phase match shimming, the analysis program calculates the phase at each pole. A reference pole is chosen and the measured phase at several poles near the reference pole (for averaging) is compared to the desired phase. The difference is the phase matching error which we must correct. The effect of phase error shims and gap shims are known. The program tells the operator the appropriate shim to apply to correct the phase difference.

7   Conclusion

It is highly desirable to automate the LCLS undulator tuning. Algorithms which automate tuning were presented. The algorithms have been implemented in Matlab.

The x and y trajectories are tuned using the segmented trajectory by finding the slope changes which make the segmented trajectory straight. The required shim dimensions are determined empirically and put into the program. First and second field integrals are made zero by this procedure.

The phase errors are tuned by again using a segmented fit. Phase shim use is maximized. Phase shims are applied in places where the phase error increases. The mean phase errors in these regions are brought to a constant value. Gap shims are used to increase the phase and bring the phase errors to zero. The phase change from a phase shim is determined empirically and put into the program. The gap change for a given required phase change is calculated analytically.

The K parameter is measured and the deviation from the desired value is determined.

The gap change required to get the desired value is calculated analytically. The gap change is obtained by moving horizontally in the canted poles. A detailed map of K vs x will be used to fiducialize the undulator.

Phase matching is done with phase shims and gap shims. The phase change into the undulator and the phase change of the cell must be corrected.