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TITLE: PERTURBATIONS TO THE HORIZONTAL OFF-ENERGY FUNCTIONS IN THE ARCS*

The perturbation of off-energy functions in the arcs affects the SLC performance in two ways. First of all, it indroduces additional emittance blow-up in the arcs through synchrotron radiation loss. Secondly, if the perturbation is too large, the chromatic correction in the final focus cannot completely suppress the eta at IP resulting in a larger beam size. Both effects reduce the luminosity. In this report an analysis is made of the disturbances to the horizontal eta-function generated by imperfections in the arcs and their effects are estimated.

1. THE DISTURBED OFF-ENERGY FUNCTION.

The real arcs will have an off-energy function $\eta(s)$ which is different from the ideal $\eta_o(s)$ calculated for the design arcs. We make here some estimates of the disturbances to the horizontal $\eta(s)$ that will arise from various kinds of imperfections in the arcs. A similar analysis of the vertical eta function will be reported in a later report. The main assumption we make is that the arcs are sensibly linear so that the effect of any one imperfection is independent of the others, and that the various effects can be added linearly.

Quite generally, $\eta(s)$ satisfied the differential equation[1]

$$\eta'' + K(s)\eta = G(s) \tag{1}$$

where $G(s)$ is the curvature function ($= 1/\rho$) and $K(s)$ is the focussing function ($=B'/B\rho$)– both taken along the actual, on-energy trajectory. The design

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function $\eta_o(s)$ is the solution of Eq.(1) for which G and K have their nominal values $G_o(s)$ *and* $K_o(s)$, and which is also a periodic function in the achromats.

Clearly, η_o satisfies

$$\eta_o'' + K_o\,\eta_o = G_o \tag{2}$$

Consider now the on-energy trajectory for the imperfect machine. Along this disturbed trajectory the G and K will differ from G_o and K_o. We write

$$G = G_o + \delta\overline{G}; \quad K = K_o + \delta\overline{K} \tag{3}$$

The bar above $\delta\overline{G}$ and $\delta\overline{K}$ is to emphasize that they are to be taken along the disturbed orbit - - and, thereby, to include the effects of orbit displacements as well as possible 'intrinsic' errors in G and K along the design path.

Now, let's take $\overline{\eta}$ for the perturbation to η_o, namely,

$$\eta = \eta_o + \overline{\eta} \tag{4}$$

Using Eqs. (1) and (2), we get the differential equation for $\overline{\eta}$

$$\overline{\eta}'' + K(s)\overline{\eta} = f(s) = \delta\overline{G} - \eta_o\delta\overline{K} \tag{5}$$

The perturbation $\overline{\eta}$ executes a betatron oscillation with a driving term $f(s)$. Notice that $K(s)$ in Eq. (5) is the disturbed K; so that a free oscillation of $\overline{\eta}$ in the disturbed machine is just like a free transverse oscillation. Eq. (5) was obtained without any assumption about the size of the distrubances.

The perturbation $\overline{\eta}$ is driven both by field 'errors' $\delta\overline{G}$ and by gradient 'errors' $\delta\overline{K}$, and the latter is proportional to the value of the unperturbed η_o at the disturbance. The perturbation $\overline{\eta}$ is not yet completely defined; we must still specify some 'initial' conditions. For this study, we assume the off-energy functions are matched at the entrance of the arc. In other words, we consider those solutions which satisfy $\overline{\eta}(0) = \overline{\eta'}(0) = 0$ at the beginning of the arc. The different possible solutions are, however, closely connected; they will differ only by a solution of the homogeneous part of Eq.(5) - - namely, by some free oscillation.

We now look at the $\overline{\eta}$ that may result from various imperfections in the arcs. We consider in this report only the perturbations to the horizontal eta function - - so $\overline{\eta}$ should be read everywhere as $\overline{\eta}_x$.

2. INTRINSIC GRADIENT ERRORS

The arc AG magnets are designed to give the required field distribution,

$$B_y(x,y) = B_o + gx + \frac{1}{2}S(x^2 - y^2) \tag{6}$$

$$B_x(x,y) = gy + Sxy \tag{7}$$

Where x and y are the horizontal and vertical displacement from the field center line, and

$$g = \frac{dB_y}{dx}, \quad S = \frac{d^2B_y}{dx^2} \tag{8}$$

are the quadrupole and sextupole components. At the design energy of 50 GeV, the field values are $B_o = 5.97KG, g = \pm 7.02KG/cm \; and \; S = 1.63KG/cm^2$ for focus and $-2.70KG/cm^2$ for defocus magnets.

We define the axis of a magnet to be the place where the field strength B is the design value. Then there are no 'intrinsic' errors of G. The term 'intrinsic' is used to denote field errors not related to orbital and energy errors. At this magnetic axis, the field gradient K may have an intrinsic error $\delta K (= K - K_o)$; and these errors can produce a perturbation $\overline{\eta}$.

If we imagine a magnet of length $\triangle S$ which is ideal except for a gradient error δK, it will induce a perturbation to $\overline{\eta}$, which, from Eq. (5), will be given a 'kick' $\triangle\overline{\eta}'$ at the magnet (assumed to be short) of the magnitude

$$\triangle\overline{\eta}' = -\eta_o\,\delta K \triangle S \tag{9}$$

thereafter, $\overline{\eta}$ executes a free betatron oscillation whose amplitude $\triangle A_1$ is given by

$$\triangle A_1 = |\triangle\overline{\eta}'|\ \sqrt{\beta_1\beta_2}\ sin\phi_{12}$$

$$= \sqrt{\beta_1\beta_2}\ \eta_o\ K_o \triangle S (\frac{\delta K}{K_o}) sin\phi_{12} \tag{10}$$

where 1 denotes the source point and 2 the end of arc which is in the middle of a focus magnet, and ϕ_{12} is the phase advance between points 1 and 2.

Let's define ς such that

$$\triangle A_1 = \varsigma \frac{\delta K}{K_o}\ sin\phi_{12} \tag{11}$$

To see the effects of gradient error on the η-function, we summarize the design values of relevant parameters for the arc magnets at 50 GeV in Table I [2].

TABLE I

	Focus	**Defocus**
$B_o(KG)$	5.97	5.97
$G = (m^{-1})$	0.00358	0.00358
$K_o(m^{-2})$	0.421	-0.421
$\alpha(m^{-1})$	23.2	38.48
$\eta_{o,ave}\ (m)$	0.043	0.026
$\beta_{ave}\ (m)$	7.08	1.96

For easy reference, the beta-function and eta-function in one cell of the arc lattice are shown in Fig. 1 and Fig. 2 respectively.

For the two kinds of magnets we get (approximately) that

$$Focus : \varsigma_f = 0.32m$$

$$Defocus : \varsigma_d = 0.10m$$

Clearly, gradient errors in the focussing magnets are more important. A gradient error $(\delta K/K_o)$ of, say, 2×10^{-3} in one focussing magnet gives an oscillating $\overline{\eta}$ with an amplitude $\triangle A_1 \approx 0.64mm$.

If there is a systematic gradient error of the same size in all focussing magnets, the induced $\overline{\eta}$ is of the same general size, $\triangle A_f$. Because of the destructive interference that occurs across an achromat, to be explained in Sect.4, the effect can be expected to be small.

If there are random gradient errors, then there can be a growth of $\overline{\eta}$ through the arcs. We can estimate the r.m.s. expected value of $\overline{\eta}$ at the end of the arc as

$$(\overline{\eta})_{rms} = \sqrt{\frac{N}{2}(\triangle A_f^2 + \triangle A_d^2)} \tag{12}$$

where $N = 230$ is the number of each type of magnet in one arc. Random errors of $\delta K/K_o$ are expected to be about 2×10^{-3} (Ref. 3). Such a value gives $(\overline{\eta})_{rms} \approx 7.2mm$, a perturbation of about 17% of the maximum value of η_o.

Note that this eta error depends only on the intrinsic gradient errors and is independent of the scale of the magnet survey errors.

3. GEOMETRIC DISTURBANCES.

Suppose that the beam has the 'right' energy, and that the magnets have the 'right' fields, but that the beam trajectory has disturbances due to magnet placement errors. In any given piece of magnet the beam will pass at some horizontal displacement δ_x from the ideal axis of the magnet, we have for that path segment

$$\delta\overline{G} = K_o\ \delta_x\ + \frac{1}{2B\rho} S \delta_x^2, \quad \delta\overline{K} = \alpha\ K_o\ \delta_x, \tag{13}$$

where

$$K_o = \frac{g}{B\rho} \tag{14}$$

and

$\delta K = \alpha K_o \delta x$

$$\alpha = \frac{1}{K_o} \frac{dK_o}{dx} = \frac{S}{g} \tag{15}$$

which represents the strength of the sextupole component with respect to the gradient in the magnet. The driving term $f(s)$ in Eq.(5) then becomes

$$f(s) = K_o\ \delta_x(1 - \eta_o\ \alpha) + \frac{1}{2} \frac{1}{B\rho} S \delta_x^2$$

$$= \frac{\delta_x}{B\rho}(g - \eta_o S) + \frac{1}{2} \frac{1}{B\rho} S\ \delta_x^2 \tag{16}$$

To obtain an appreciation of the order of magnitude of geometric perturbations, we calculate the $\triangle A_f$ and $\triangle A_d$ amplitudes caused by each term in the driving force $f(s)$ shown in Eq. (16), assuming they are active and independent of each other. The results are summarized in Table 2.

TABLE 2

f(s)	$\triangle A_f$ (mm)	$\triangle A_d$ (mm)
$K_o \delta x$	1.12	-1.12
$-K_o \eta_o \alpha \delta x$	-1.12	1.12
$\frac{1}{2B\rho} S\ \delta x^2$	0.0020	-0.0065

In the estimate, an rms orbital error $\delta x = 150$ microns is assumed.

From Table 2, two conclusions are obvious. First, the contributions from the first two terms cancel each other. Secondly, the δ_x^2-dependent field error is much smaller than that introduced through δ_x-dependent term. If we use a criterion that the contribution to $\overline{\eta}$ at the end of the arc should not be larger than that from the intrinsic gradient error, then the limit on δ_x is 10 times larger than 150 microns. That means the δ_x^2 term is not important until δ_x approaches 1.5 mm.

We have shown that the average of the factor $(1-\eta_o\,\alpha)$ over a magnet is very close to zero. It is, essentially, a necessary design condition of the achromat, because the strength of sextupole component is chosen to cancel the gradient at off-momentum orbit. That means that so long as a magnet is somewhat shorter than β, the displacement of the orbit with respect to the magnet axis will not generate a significant η disturbance. (Because of the symmetry of η_o about a magnet center, this conclusion is true even when there is an angle between the orbit and magnet axis.) This effect was discovered when the moving magnet scheme was proposed[4] for orbital correction.

Besides the large displacement error mentioned above, there are two possible sources of incomplete cancellation of $(g - \eta_o S)$. The first is the fact that we neglect the contribution from $\overline{\eta}\delta K$ in Eq. (5). At the location where the perturbation $\overline{\eta}$ becomes large, the driving term $f(s)$ given by Eq. (16) should read,

$$f(s) = \frac{\delta x}{B\rho}(g - (\eta_o + \overline{\eta})S) = -\frac{\delta x}{B\rho}\overline{\eta}S \qquad (17)$$

which will become important when $\overline{\eta}/\eta$ is approaching 10-20%. The second source of incomplete cancellation is due to the deviation of S from design value at higher excitation, but which is only important when SLC is run at an energy higher than 60 GeV.

4. ENERGY OFF-SETS

Consider two cases: (a) off-set of the centered beam energy without re-steering and (b) an energy off-set with re-steering. Case (a) just tells us about the non-linear dependence of η_o on energy. Case (b) refers to arc operation with a mis-match to the Linac energy.

For case (a), there are two contributions to $\delta\overline{G}$ and $\delta\overline{K}$. One comes from the energy change δE and one from the orbit change δ_{xE} due to the energy change. We already know from Sect. 2 that the orbit change will not generate an $\overline{\eta}_x$, so we look only at the energy part. Then

$$\delta G = -G_o\,\frac{\delta E}{E}\;;\quad \delta K = -K_o\,\frac{\delta E}{E} \tag{18}$$

so that

$$f(s) = (-G_o + \eta_o\,K_o)\,\frac{\delta E}{E} \tag{19}$$

Now let us look at the effect of energy error on the off-energy function produced by a single magnet. Following Eq. (10), the contribution to the perturbation of η-function is

$$\triangle A_1 \;=\; (\eta_o K_o - G)\,\triangle S\sqrt{\beta_1\beta_2}\,\frac{\delta E}{E}\,sin\phi_{12}$$

$$= \xi\,\frac{\delta E}{E}\,sin\phi_{12} \tag{20}$$

From Table 1 and Eq. (20) we get for the two kinds of magnets,

$$Focus\;:\;\xi_f\;= 0.26m$$

$$Defocus\;:\;\xi_d\;= -0.14m$$

For the whole arc, this individual perturbation will not build up because of the destructive interference from the same type of magnets at different phases.

Since the beam energy error is the same for every magnet, the total contribution from all the focus magnets at the end of the arc is given by

$$\overline{\eta} = \sum_{i=1}^{230} \triangle A_f \ sin\phi_i$$

$$= \triangle A_f \sum_{i=1}^{230} sin\phi_i = 0 \qquad (21)$$

It sums up to zero due to the fact that the contribution from each focus magnet is the same, and the phase sum is zero since the phases are evenly distributed over many wave lengths. Same cancellation occurs for contributions from defucs magnets. This points out the fact that a constant energy off-set, as long as it is not rich in first harmonic of betatron frequency, is not as harmful as random change from magnet to magnet. Because in the latter case the perturbation tends to build up.

It is interesting here to introduce another way of summing Eq. (19) by showing that the factor in parentheses is necessarily zero when averaged over one magnet cell. This fact can be seen as follows. rearrange the defining equation for η_o, Eq. (2), and take the average of both sides to get

$$\langle \eta_o'' \rangle = \langle G_o - K_o \ \eta_o \rangle \qquad (22)$$

Now the average of η_o'' over a magnet cell, which is a pair of focusing and defocusing magnets, is proportional to the integral of η_o'', which is just the change in the <u>first</u> <u>derivative</u> $\triangle\eta_o'$ across the cell. But the ideal η_o is periodic with the period of a cell, so that $\triangle\eta_o'$ is zero. It follows then that the right-hand-side of Eq. (22) is zero, provided that the average is taken over a whole cell. This can also be confirmed by putting numerical values from Table 1 into Eq. (22). Where the F-D pair is not complete, as there are several locations in the arcs, there are residual $\overline{\eta}$ contributions at the end of the arc, but they are very small.

There is a small residual high frequency component to $\overline{\eta}$ which should, however, be quite small. The driving terms for $\overline{\eta}$ contributed be each magnet in a cell are equal and opposite, and the magnets are about 50^o apart in betatron phase, so there is some small residual $\overline{\eta}$ from one cell. Because the phase shift is 108^o per cell, however, the growth of $\overline{\eta}$ within an achromat is small and is precisely zero for one whole achromat. There is a 'destructive interference' of the individual contribution.

We show that an energy shift by itself - - Case (a) - - does not produce significant η-disturbance. If we now re-steer the beam - - Case (b) - - there will still be no generation of any η-disturbance, because we are merely adding on a geometric disturbance, which were found in Sect. 3 to have no effect.

5. SUMMARY OF HORIZONTAL ETA EFFECTS.

We conclude that the horizontal η is quite insensitive to nearly all arc perturbations - - geometric errors, energy errors and systematic gradient errors. The only significant effect we have found is from random gradient errors which may, we estimate, produce an η-error at the end of the arcs whose r.m.s. value is about 7.2 mm. One effect we have not included is the perturbation of the horizontal η due to coupling from vertical dispersion - - a coupling that would come about from magnet roll. We will consider this effect in another note where perturbations to vertical η will be treated.

A corollary of our results is that it is almost impossible to correct any η-errors within the arcs themselves. No pattern of magnet motions will help. Some trim quads would do nicely, but, so far, no plans exist for their implementation. A proper pattern of back-leg trim coils would help but they also are not in the present plan.

Fortunately, we do not expect the horizontal η-error to be very large - - and this even if the geometric errors (survey, BPM, etc.) are larger than expected. Unless, of course, the geometric errors become large enough to generate large second-order effects, or coupling from vertical dispersion turns out to be important.

We start the analysis assuming that the dispersion function is well-matched both at the entrance from Linac to Arc and from reverse bend to Arc. If the dispersion function is mismatched, it will execute free betatron oscillation through the Arc. Therefore, any large dispersion error in the early achromats indicates the mismatch from Linac to Arc or RB to Arc and should be corrected before proceeding. On the other hand, if a specific η value at the end of the Arc is to be desired, controlled mismatch of dispersion functions from RB to Arc may be created to achieve that goal as long as the required mis-match is not excessive.

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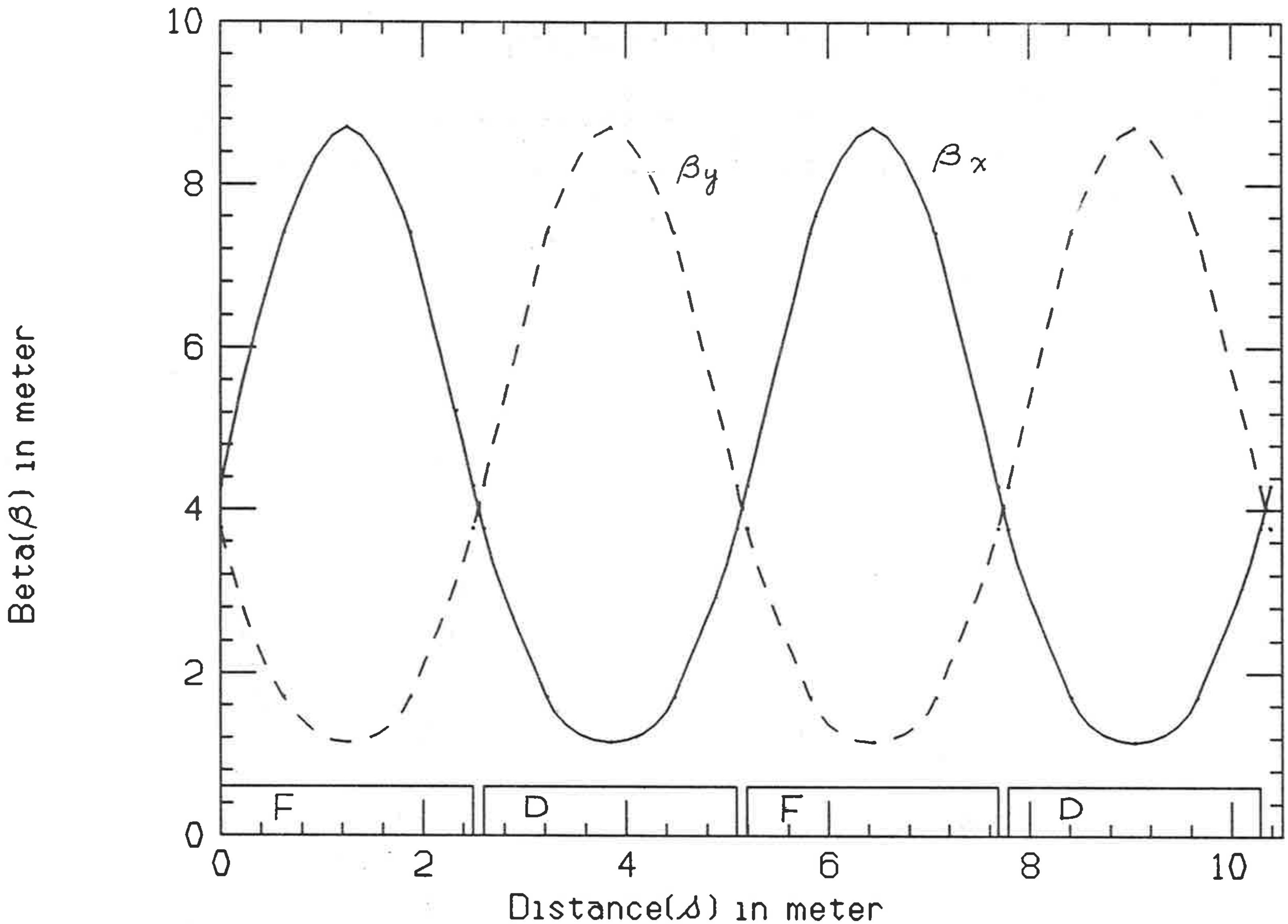

Fig.1 Beta-function in one cell of Arc lattice

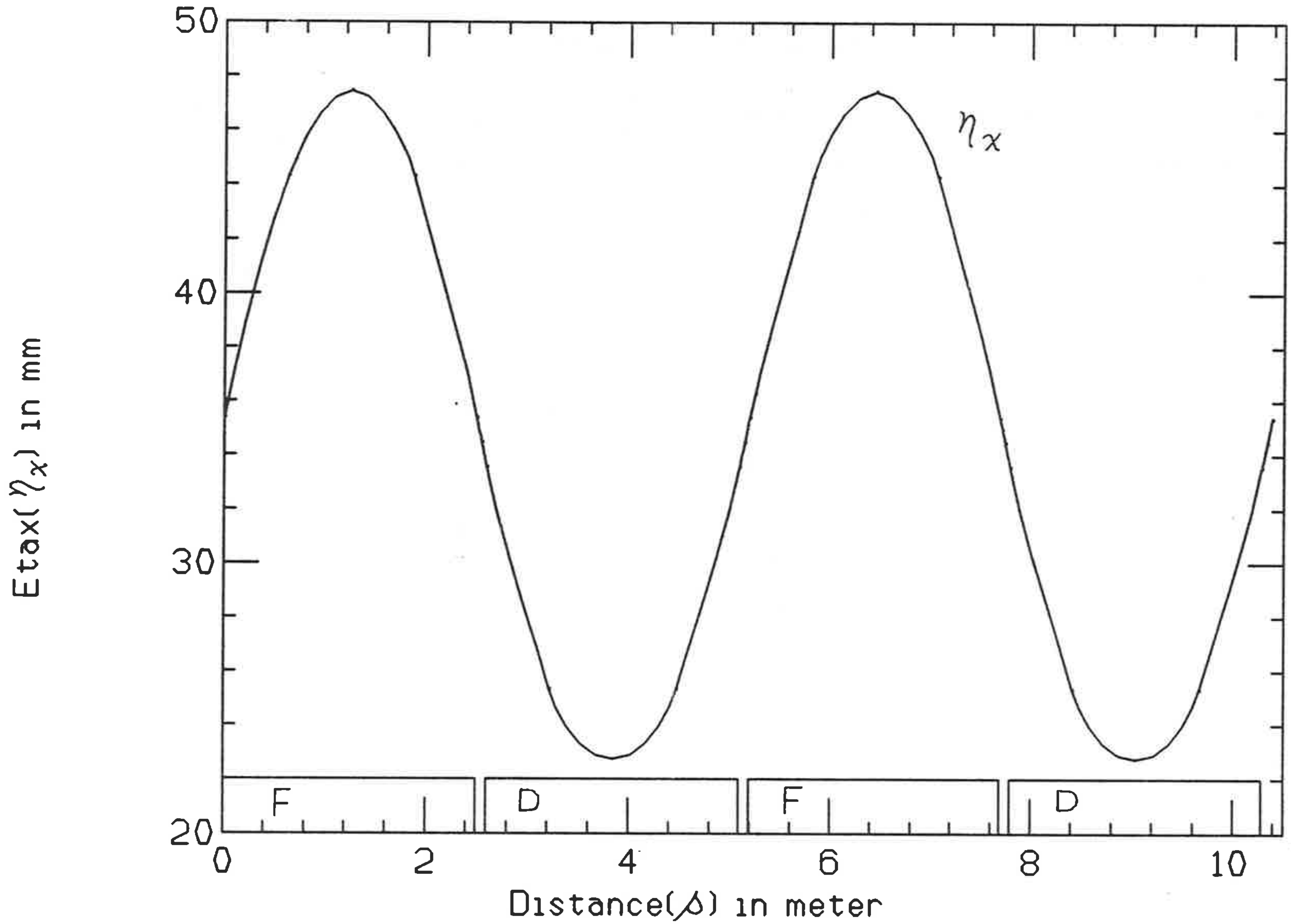

Fig.2 Eta-function in one cell of Arc lattice