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SINGLE PASS COLLIDER MEMO CN-356

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TITLE: BETATRON OSCILLATION FROM A MAGNET MOVER*

Introduction

The Magnet Movers of the Arcs are provided primarily for steering the beam along the design trajectory. They can also be used to excite deliberately a betatron oscillation in order to investigate the optical performance of the guide field. We give in this note an analytic analysis of the betatron motion expected from the displacement of a magnet mover in an ideal Achromat.

The assumption is made that all trajectory distortions and betatron amplitudes are small enough that a linear approximation can be used for the Arc guide fields. This means also that the betatron oscillation excited by the motion of a magnet will occur relative to any actual Central Trajectory that exists at the moment.

We will derive here the amplitude and phase of the betatron oscillation launched in the particular Achromat that contains the moved magnet and using the coordinate system appropriate to the Achromat. The propagation of the oscillation into the following downstream Achromats - - including coupling into the other transverse coordinate has been described in an earlier Note (CN-355).

In view of the linearity assumption we have made, the betatron motion in an imperfect machine will be the same as we would get by displacing a magnet in an otherwise ideal achromat, with a beam that was initially proceeding along the design trajectory. So, we consider only such a situation.

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Magnet Motion

The Magnet Mover mechanism displaces transversely the upstream end of a magnet while the position of its downstream end is held fixed, so that the magnet rotates about its downstream end as a rigid bar. The Mover and the downstream pivot plane are located close to the effective "magnetic" ends of the magnet. Within each Achromat all magnets have the same median plane - - which we use to define the local $x-$ and $y-$ coordinates. An $x-$ mover displaces "horizontally" the end of a focussing magnet, and a $y-$ mover displaces "vertically" the end of a defocussing magnet, where, "horizontal" and "vertical" refer to the local $x-$ and $y-$ directions in the Achromat.

Effect on the Trajectory

Let's look first at the $x-$motion introduced by a horizontal mover. Let me use x and $x' = dx/ds$ to represent the horizontal displacement and slope of an electron's path with respect to the design trajectory, and take $\bar{x}$ and $\bar{x}'$ (with bars above) to be the displacement and slope with respect to axes attached to the moved magnet. See Fig. 1.

Now take an electron moving along the trajectory so that it arrives at the entrance to a focussing magnet (call it point "a") with $x_a = x'_a = 0$. Next, say that the magnet end has been displaced by the amount X, equal to the mover displacement which will also cause a rotation about the downstream end by the angle $\theta = X/\ell$, where ℓ is the length of the magnet. The displacement and slope at point "a", measured with respect to the magnet, will then be (see Fig. 1).

$$\bar{x}_a = -X \; ; \; \bar{x}'_a = \theta = X/\ell \tag{1}$$

At the end of the magnet (call it point "b") the displacement and slope $\bar{x}_b$ and

$\bar{x}'_b$ are related to $\bar{x}_a$ and $\bar{x}'_a$ by the usual matrix product:

$$\begin{pmatrix} \bar{x}_b \\ \bar{x}'_b \end{pmatrix} = M \begin{pmatrix} \bar{x}_a \\ \bar{x}'_a \end{pmatrix}, \qquad (2)$$

where, for a magnet of uniform focussing strength K and length ℓ,

$$M = \begin{pmatrix} \cos\mu & \frac{1}{\sqrt{K}} \sin\mu \\ -\sqrt{K} \sin\mu & \cos\mu \end{pmatrix}, \qquad (3)$$

with $\mu = \sqrt{K}\ell$.

At point "b" we can return to the Achromat frame by taking

$$x_b = \bar{x}_b \, ; \; x'_b = \bar{x}'_b - \theta = \bar{x}'_b - X/\ell \qquad (4)$$

The effect of the Magnet Mover is, then, to produce

$$\begin{pmatrix} x_b \\ x'_b \end{pmatrix} = M \cdot \begin{pmatrix} -X \\ X/\ell \end{pmatrix} - \begin{pmatrix} 0 \\ X/\ell \end{pmatrix} \qquad (5)$$

Or, factoring out X/ℓ

$$\begin{pmatrix} x_b \\ x'_b \end{pmatrix} = \frac{X}{\ell} \left[M \begin{pmatrix} -\ell \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right] \qquad (6)$$

In the Arcs a focussing magnet has K = 0.42 m^{-2} and ℓ = 2.5 m, so that, numerically,

$$\begin{pmatrix} x_b \\ x'_b \end{pmatrix} = X \begin{pmatrix} 0.666 \\ 0.228 m^{-1} \end{pmatrix} \qquad (7)$$

For a Mover desplacement of X, we get at the end of the magnet a beam displacement of 0.666 X, with a slope of 0.228 (X/1 meter).

The Disturbed Trajectory

Knowing x and x' at the end of the magnet, we can find their values downstream with the Transport matrix from that point onward. Alternatively, it may be convenient to think of the disturbed trajectory as a free betatron oscillation, which we can write, generally, as

$$x(s) \;=\; a\sqrt{\beta(s)}\,cos\,\{\phi(s)\;+\;\phi_o\} \tag{8}$$

where $\beta(s)$ and $\phi(s)$ are the known parameters of the betatron oscillations, and a and ϕ_o are the invariant amplitude and phase constant of any particular solution. We can find them from the values of x and x' at any point using (see SLAC-121).

$$a^2 \;=\; \frac{x^2}{\beta} \;+\; \beta(x' - \frac{\beta'}{2\beta}x)^2 \tag{9}$$

$$tan(\phi + \phi_o) \;=\; -\frac{\beta x'}{x} + \frac{\beta'}{2} \tag{10}$$

Since the horizontal motion can only be observed at the horizontal Beam Position Monitor (BPM's) which are attached to the upstream (moving) end of the focussing magnets, it will be convenient to chose a reference point for the longitudinal coordinate s at one of them. Let us chose here the BPM attached to the moved magnet as our reference point, at which we let $s = 0$, and, so also, $\phi(s) \;=\; 0$.

The values of β, β' and $\phi(s)$ for the reference point and for the downstream end of the magnet can be obtained from a Transport printout and are given in Table I. (Recall that $\beta' = \; -2\alpha$.) Using the values for the downstream end in Eqs. (9) and (10) together with the values in Eq. (7), we find that

$$\begin{aligned} a &= 1.39X \\ \phi_o &= -\,98.3deg. \end{aligned} \tag{11}$$

TABLE I. Optic Parameters for x-Motion

Place	s(m)	β(m)	β'	ϕ (deg)
X-BPM	0	4.055	5.302	0
Downstream end of foc. mag.	2.54	4.267	-5.458	21.7
Next Y-BPM	5.19	4.055	5.302	108

Displacements at X-BPMs

Let us call x_n the displacement of the disturbed trajectory at the n-th BPM after the moved magnet (counting the reference BPM as "zero"). Using Eq. (8), we can write this

$$x_n = A\ cos\ (n\phi_1 + \phi_o) \tag{12}$$

in which

$$\begin{aligned} A &= \sqrt{\beta}_{BPM}\ a = 2.80X \\ \phi_1 &= 108 deg.\ ;\ \phi_o = -\ 98.3 deg. \end{aligned} \tag{13}$$

Since ϕ_o is so close to 90 deg., it is perhaps more convenient to express x_n as

$$x_n = A\ sin\ (n\ \phi_1\ -\ \phi_o^*) \tag{14}$$

with $\phi_o^* = 8.3$ deg.

The disturbed orbit - - as seen by the BPM's - - looks like a sine curve which received its initial "kick" from the moved magnet at 8.3 deg.phase advance after the BPM. This is, at a point which is rather near (geometrically) to the center of the moved magnet.

The above results are expected to apply in the presence of small misalignment errors; but will not be correct if there are any errors in the focussing strengths of the Arc magnets. Indeed, observed departures from the predicted behavior can be useful in diagnosing those errors.

I remind you that the results obtained here apply only to the disturbed trajectory in the same Achromat, or, at least, until the next roll is encountered. At which time, the results of this note must be combined with those of CN-355 to get the disturbed trajectory after the roll.

Vertical Motion

The vertical and horizontal lattices of the Arc are almost identical, and the Y-BPM is attached to the defocussing magnet (vertically focussing) in the same way as the X-BPM is, to its magnet. So the disturbed trajectory in y relative to its reference Y-BPM is the same as we found above for x. With this change in reference point, Eqs.(11) through (14) - - with all x's changed to y's - - apply also for the y-motion.

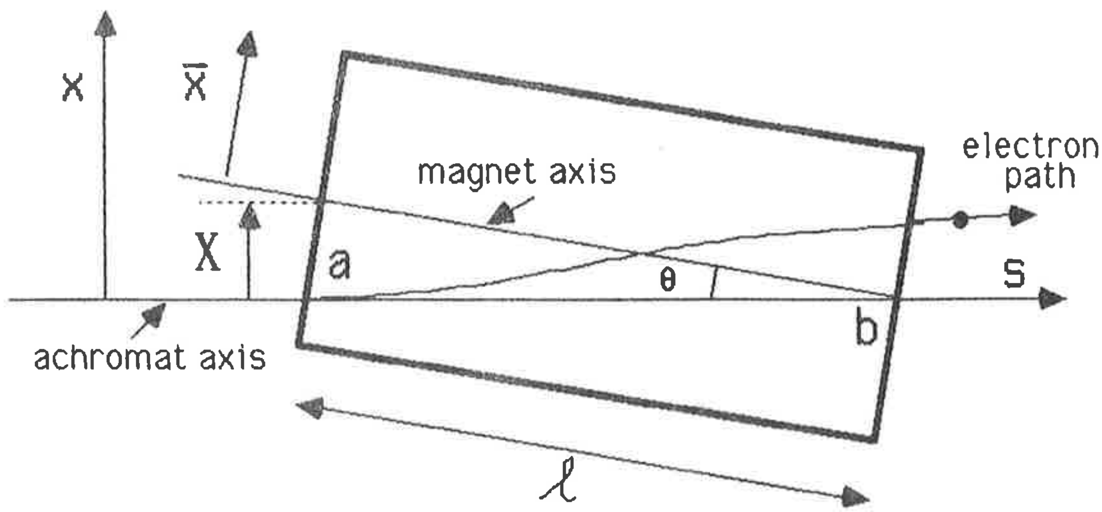

Fig. 1. Geometry of a moved magnet.

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