PEP-90
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Energy Loss to Parasitic Modes of the Accelerating Cavities

Introduction

At maximum stored current, each circulating beam in PEP will consist of three bunches, each about 10 cm long and containing 1.5×10^{12} particles. The large electric charge carried by such a bunch (2.5×10^{-7} coulomb) will, because of its short length, give rise to a large transient excitation of hundreds of parasitic modes in the accelerating cavities. The energy loss of the stored beam to the cavities from this process may be comparable to the loss to synchrotron radiation, and may, therefore, require a significant increase in power from the accelerating rf system.

In this note I consider three aspects of this effect. In Section I an attempt is made to estimate the magnitude of the energy loss of a bunch in a single passage through the accelerating cavities. In Section II I consider the effects of the periodic passages of the bunches in a single stored beam. And in Section III I look at the consequences of storing two counter-rotating beams. The general conclusions are that the magnitude energy loss to the parasitic modes is serious, though probably not disastrous; and that, in general, the separate stored bunches will act incoherently.

The excitation of parasitic modes can have consequences beyond the mere subtraction of energy from the stored beam. In particular, there will be a modification of the restoring force for the energy oscillations, bunch lengthening effects, and the possible anti-damping of coherent energy oscillations. These effects are not analysed here.

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I. Single Pass Energy Loss

In connection with studies of electron ring accelerators, Keil[1] has calculated the energy loss of a relativistic bunch travelling down an idealized periodic cylindrical structure. I will use Keil's results to estimate the energy loss of a stored electron bunch in PEP to the rf accelerating cavities. The major part of Keil's work is a numerical solution for the electromagnetic fields subject to appropriate boundary conditions at the structure walls and at the bunch. He considers only an infinitely long structure, so that all fields are strictly periodic (in the direction of the beam axis). The consequence of this, and other, approximations made in the calculations -- especially when applied to a small group of cavities are not clear to me. Also, the PEP cavity structure differs markedly from the idealized form considered by Keil. The results obtained here can, clearly, be considered only an estimate.

Most significantly for our purposes, Keil shows that his numerical results for ultra-relativistic particle energies agree in some detail with an algebraic expression for the energy loss obtained from an "optical resonator model" developed by Sessler and Vainshtein for configurations in which the latter should be valid. We can, therefore, use this algebraic formula for our estimate.

The structure considered is a sequence of identical "cavities" which have cylindrical symmetry about the axis of the beam, as shown in Fig. 1. The significant dimensions are the cavity length (structural period) $\underline{b}$, the hole radius $\underline{a}$, and the bunch length $\underline{\ell}$. The optical resonator model presumes that the disc thickness is much less than the structure period

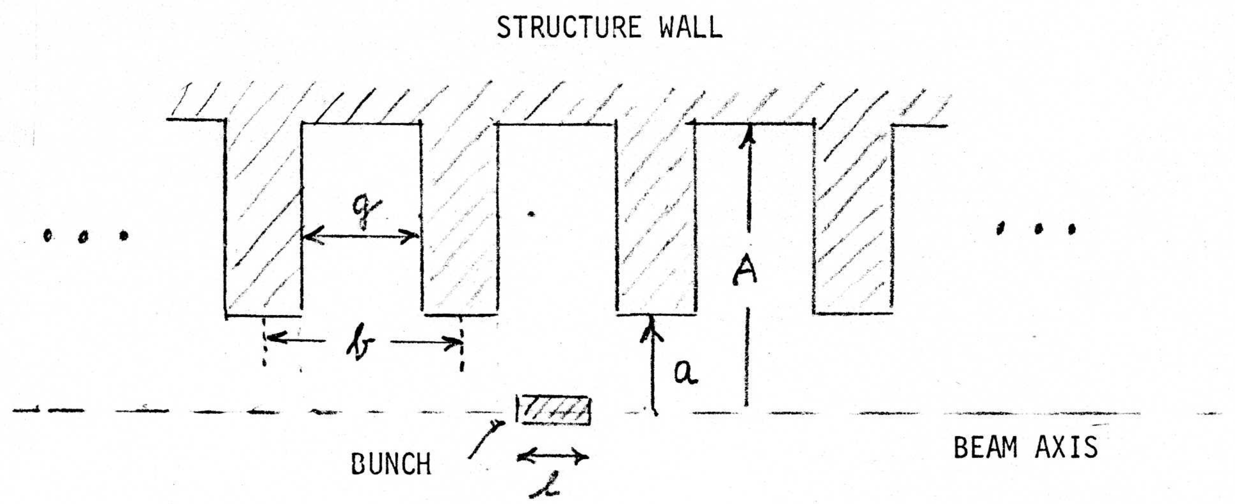

Fig. 1. Section of Periodic Structure.

b so that the gap length g is nearly the same as b, and that the radius of the bunch is much less than the hole diameter a. The model is not sensitive to the major cavity radius A, assuming only that it is sensibly larger than a. The numerical results are also not sensitive to A for ultra-relativistic particles.

Keil (following a suggestion of Lawson) believes that the optical resonator model should be valid for energies for which $\gamma = E/mc^2$ is much larger than a critical γ_c. For $a < b$ and $(b - g) \ll b$,

$$\gamma_c \simeq 50\ b/a. \tag{1}$$

For PEP, γ is certainly well above γ_c.

I consider a bunch containing N particles (each with electronic charge) and let u_0 be the average energy loss per particle due to the passage of the bunch through a sequence of M cavities. The formula given by Keil -- his Eq. (37) -- can conveniently be written as

$$u_0 = U \cdot F(\alpha,\lambda)$$

where U is the "basic" energy loss factor for the structure and $F(\alpha,\lambda)$ is a "form factor" for the structure and beam. The basic energy loss is

$$U = mc^2 r_0 N \frac{Mb}{a^2} \qquad (2)$$

where:

mc^2 = rest energy of an electron = 0.511 MeV

r_0 = classical electron radius = 2.82×10^{-13} cm.

N = number of particles in the bunch

M = number of cavities

b = cavity length (period of the structure)

a = hole diameter

$mc^2 r_0 N$ = 0.216 MeV/cm for PEP.

The form factor $F(\alpha,\lambda)$ depends only on two dimensionless parameters characteristic of the cavity and beam dimensions:

$$\alpha = \frac{2\pi\sqrt{2}}{\zeta(\frac{1}{2})} \frac{a}{b} = 6.083 \frac{a}{b} , \qquad (3)$$

$$\lambda = (\ell/b)^{\frac{1}{2}} , \qquad (4)$$

$$F(\alpha,\lambda) = K\left\{\frac{\alpha}{(\alpha+1)^2+1} - \frac{\alpha\lambda}{(\alpha+\lambda)^2+\lambda^2} + \arctan\frac{\alpha(1-\lambda)}{(\alpha+1)(\alpha+\lambda)+\lambda}\right\}, \quad (5)$$

with $K = 2\ J_0^{-1}(0)\ ^2/\pi = 3.682.$

In order to get a qualitative feel for the nature of the form factor, I have used these expressions to compute $F(\alpha,\lambda)$ for various ratios b/a and ℓ/b. The results are shown in Fig. 2. The dependence of the energy loss on the ratio b/a is not very strong -- varying by only a factor of two, or so, over the range of practical interest. The decrease of $F(\alpha,\lambda)$ even for small bunch lengths is perhaps surprising. The energy loss at b/a $\simeq$ 6 drops to one-half the value for a point bunch when the bunch length reaches only about 1/3 of the hole radius. (Incidentally, the model gives zero energy loss for a bunch length equal to the cavity length; that is, for $\ell/b = 1$.)

I should say here that the bunch length terms in $F(\alpha,\lambda)$ were arrived at in a rather crude way by saying that a bunch of length $\underline{\ell}$ contains the same frequency components (in its current, for example) as a point bunch up to the frequency c/ℓ, and none for higher frequencies. For PEP, the bunch will presumably be more-or-less Gaussian. In keeping with the roughness of the bunch-length treatment in the energy loss, I think we will not go too far astray if we take $\ell = 2\sigma_y$, where σ_y is the root-mean-square longitudinal bunch spread.

Let's now look at the energy loss expected for PEP, and, at the same

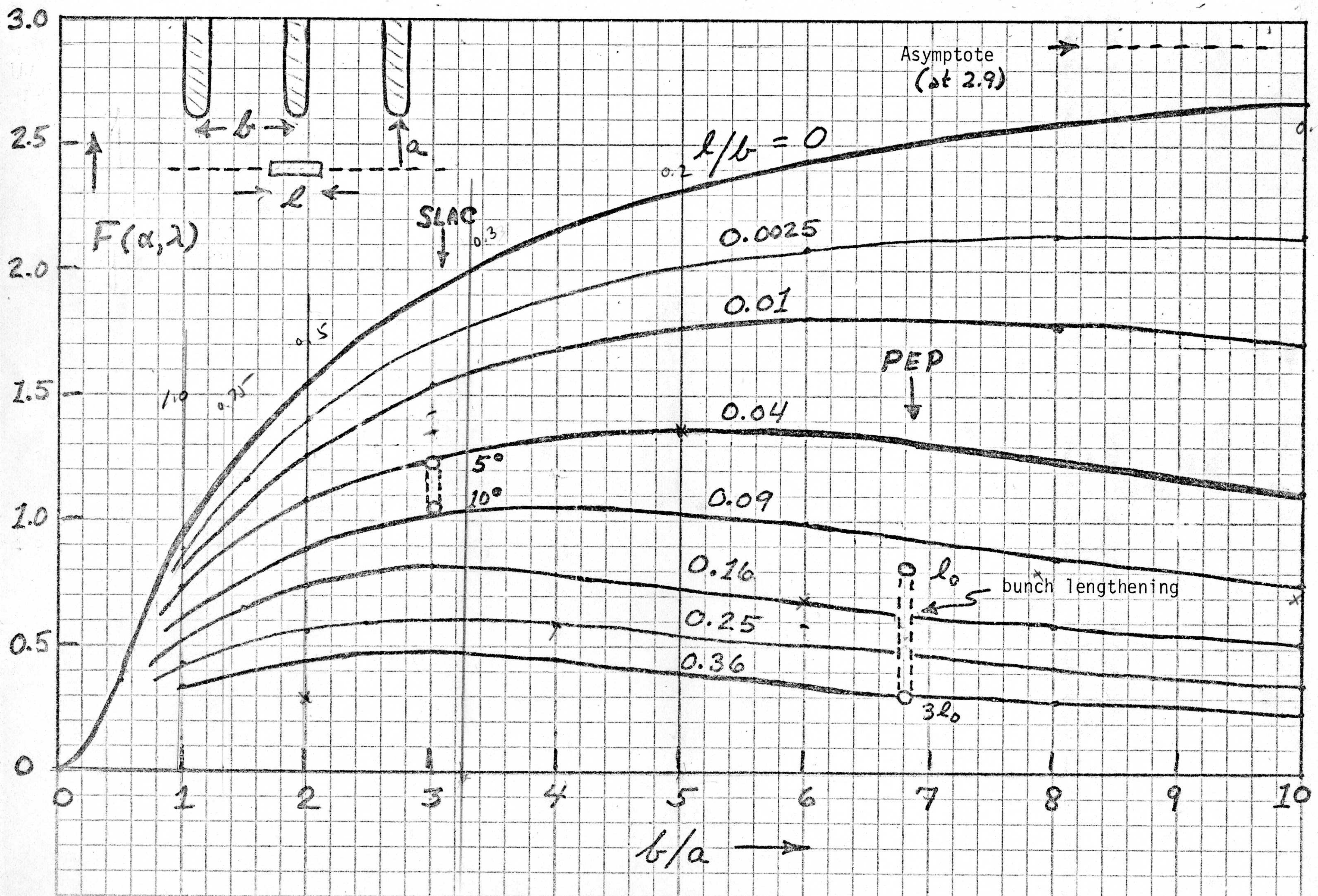

Fig. 2. Form Factor for the Energy Loss.

time, the loss expected for the SLAC accelerator structure, so that we can compare with the energy loss that has been measured there for a single bunch. Table 1 gives the appropriate parameters and the energy loss U computed from Eq. (2).

Table 1. The basic energy loss for PEP and SLAC

Quantity	PEP	SLAC
N*	1.5×10^{12}	1.0×10^{9}
M	1.2×10^{2}	0.86×10^{5}
a	6.0 cm	1.15 cm
b	41 cm	3.5 cm
b/a	6.83	3.04
U	29.5 MeV	32.8 MeV

*For PEP, I consider only a single bunch (electrons or positrons) with a beam current of 100 mA for a 3-bunch beam.

It is a little more difficult to know what to take for the bunch length in both PEP and SLAC. For PEP, the bunch length for small currents is expected to be about 5 cm. Most electron storage rings have, however, anomalous bunch lengthening, of up to a factor of 3 or more, that increases with increasing charge in the bunch. It will be surprising if there is not a comparable lengthening in PEP. (It may indeed be that at least a part of the bunch lengthening is due to the same energy loss mechanism we are considering here.) I choose to evaluate the form factor $F(\alpha,\lambda)$ for two lengths: the natural length ℓ_0 at 15 GeV, and three times ℓ_0.

Similarly, there is some uncertainty about what to take for the bunch length in SLAC. According to Greg Loew, the distribution of current with rf phase, when averaged over many pulses, is roughly uniform over an interval of about 10° of rf phase. Roger Miller points out, however, that some of this spread may be due to pulse-to-pulse jitter between the injector and the accelerator, and that a single bunch may be as short as 5° , the value he has measured for the injector on a test stand. The appropriate length here is, of course, the single pulse current distribution. Facing this uncertainty, I choose to calculate the energy loss for both 5° and 10°. These correspond to lengths of 0.15 cm and 0.29 cm respectively.

The energy losses obtained for these assumptions are shown in Table 2.

Table 2. The energy loss including effect of bunch length.

Quantity	PEP	SLAC
ℓ	5 cm (= ℓ_0)	0.15 cm (5°)
ℓ/b	0.122	0.043
$F(\alpha,\lambda)$	0.82	1.22
u_0	24 MeV	40 MeV
ℓ	15 cm (= $3\ell_0$)	0.29 cm (10°)
ℓ/b	0.37	0.083
$F(\alpha,\lambda)$	0.30	1.05
u_0	9 MeV	34 MeV

The loss for PEP is estimated to lie between 10 MeV and 20 MeV, and for

SLAC, between 35 MeV and 40 MeV.

How reliable are these estimates? I find it difficult to guess. First, let's look at the SLAC result. It has been reported earlier by Koontz, Loew and Miller that the energy loss for a single bunch of 10^9 electrons traversing SLAC is measured to be about 38 MeV. Recently, however, a re-analysis of the data indicates that this was an apparent maximum energy loss, while the mean energy loss in the bunch is probably nearer to about 20 MeV. It would appear that the theoretical estimate may be nearly a factor of two too large.

One of my main worries about the analytical model is that it is derived by taking an integral over a spectral density -- which integral is rather arbitrarily truncated at the low frequency end at the frequency c/b. Unfortunately, the spectral density is large and changing rapidly at this low-frequency limit, so the integral is quite sensitive to the choice of the cut-off frequency. What surprises me, therefore, is that Keil's numerical calculation -- which considers all low-frequency modes individually, and uses the analytical model only to account for the highest-frequency modes that are too numerous to treat individually -- gives the same answer within a few percent.

The SLAC waveguide has a geometry that is very close to the idealized structure taken for the calculations. How can we account for the discrepancy? A couple of factors come to mind. First, the electron bunch in SLAC has a radius that is not small in comparison with the hole radius a (as assumed in the calculations). It is my guess that this would decrease the energy loss to higher modes, but I have not yet made an estimate of

Finally, I mention one other concern I have about the theoretical estimates. The model (and Keil's numerical work) considers the energy loss in an infinite periodic structure. The SLAC structure should probably be reasonably approximated by such a model since each guide section contains 100 cavities. The PEP structure, on the other hand, consists of groups of only five cavities. There may be some significant differences from a strictly periodic structure.

Because of all the uncertainties in the computed energy loss, it seems likely to me that the results could be in error by perhaps a factor of two (either way). Since the estimated loss for PEP is comparable to the radiation loss of 26 MeV, a factor of two could have serious consequences for the PEP design. It would be important to have an experimental measurement of the energy loss in the PEP cavities as soon as possible.

II. Energy Loss with Periodic Beam Passes

The energy loss considered in the preceding Section was for a single passage of a bunch through the cavities. Perry Wilson[2] has considered a modification of this energy loss due to the periodic passages through the cavities of the bunches of a stored beam. I will now consider this problem -- for which I obtain a significantly different result.

The energy Nu_0 deposited in a cavity by the single passage of a bunch of N particles appears as energy stored in the electromagnetic field in the cavity. It is convenient to consider this field as the sum of the fields of each of the normal modes of the structure. Since the total energy is the sum of the energy in each mode, we can write the energy loss u_0 as

$$u_0 = \Sigma u_{on} , \tag{6a}$$

where u_{on} is the energy lost by each particle to the n^{th} mode -- which energy is found in the mode immediately after the passage of the bunch. The total energy deposited in the n^{th} mode by the whole bunch is Nu_{on} (I choose to call mode "zero" the "fundamental" -- that is, the lowest frequency that is used for the accelerating field.)

When the energy Nu_{on} is deposited in the n^{th} mode, the fields will afterwards ring down with an oscillation frequency ω_n and a decay constant α_n. A particle that traverses the cavity at the time t after the passage of the bunch will feel an effective accelerating voltage $v_n(t)$ from that mode that is given by

$$v_n(t) = V_{on} e^{-\alpha_n t} \cos \omega_n t , \tag{7}$$

where

$$V_{on} = -\frac{2u_0}{e} , \tag{8}$$

and e is the charge of the electron. I may also note for future reference that u_{on} is, for a point bunch, related to the shunt impedence R_n and the quality factor Q_n through *

$$u_{on} = N e^2 \frac{\omega_n R_n}{4Q_n} . \tag{9}$$

Now consider what happens when bunches all containing the same number of particles traverse the cavity periodically with a time separation T. On each passage through the cavity, a bunch leaves behind a field -- in the mode n -- that can be written as an infinite sum of terms like Eq. (7). In fact, in the steady state, the effective voltage seen by a bunch as it crosses a cavity is

$$V_n = v_n(T) + v_n(2T) + v_n(3T) + \ldots \tag{10}$$

where $v_n(T)$ is the expression of Eq. (7) evaluated at t = T.

When a bunch traverses the cavity each particle will gain the energy $e\,V_n$ from the n^{th} mode, and it will still lose the energy u_{on}. The net loss in energy in the state u_n is, then,

$$u_n = u_{on} - e\,V_n . \tag{11}$$

The sum of Eq. (10) is easily evaluated (see, for example, Ref. 2) and we get for u_n

* I think this R_n is really the SLAC R^*
"Real" $R_{II} = R^*/2$

$$u_n = u_{on} f(\delta_n, \theta_n) , \tag{12}$$

where

$$f(\delta,\theta) = \frac{1 - e^{-2\delta}}{1 - 2e^{-\delta} \cos\theta + e^{-2\delta}} , \tag{13}$$

with

$$\begin{aligned} \delta_n &= \alpha_n T , \\ \theta_n &= |\omega_n T - 2\pi j_n| , \end{aligned} \tag{14}$$

with j_n an integer chosen to make $|\theta_n| \leq \pi$.

Evidently, $e^{-\delta}$ represents the decay of the fields between bunch passages, and θ is the phase slippage of the mode from one bunch passage to the next.

The function $f(\delta,\theta)$ thus represents the correction to u_{on} that must be made because of the "memory" in the mode of past traversals. I shall call $f(\delta,\theta)$ the "resonance factor" for each mode. The nature of this resonance factor is illustrated in Figs. 3, 4 and 5.

The essential features of the resonance factor are as follows:

(a) If $\delta \gtrsim 1$, there is little memory from one passage to the next and $f(\delta,\theta)$ is near 1 for any θ, and the energy loss is near u_{on} for the mode.

(b) If $\delta < 1$, there is significant memory and the modification of the energy loss depends strongly on θ.

(c) If $\delta \ll 1$, the resonance factor is <u>greater</u> than 1 for $\theta < \sqrt{2\delta}$, and <u>less</u> than 1 for $\theta > \sqrt{2\delta}$.

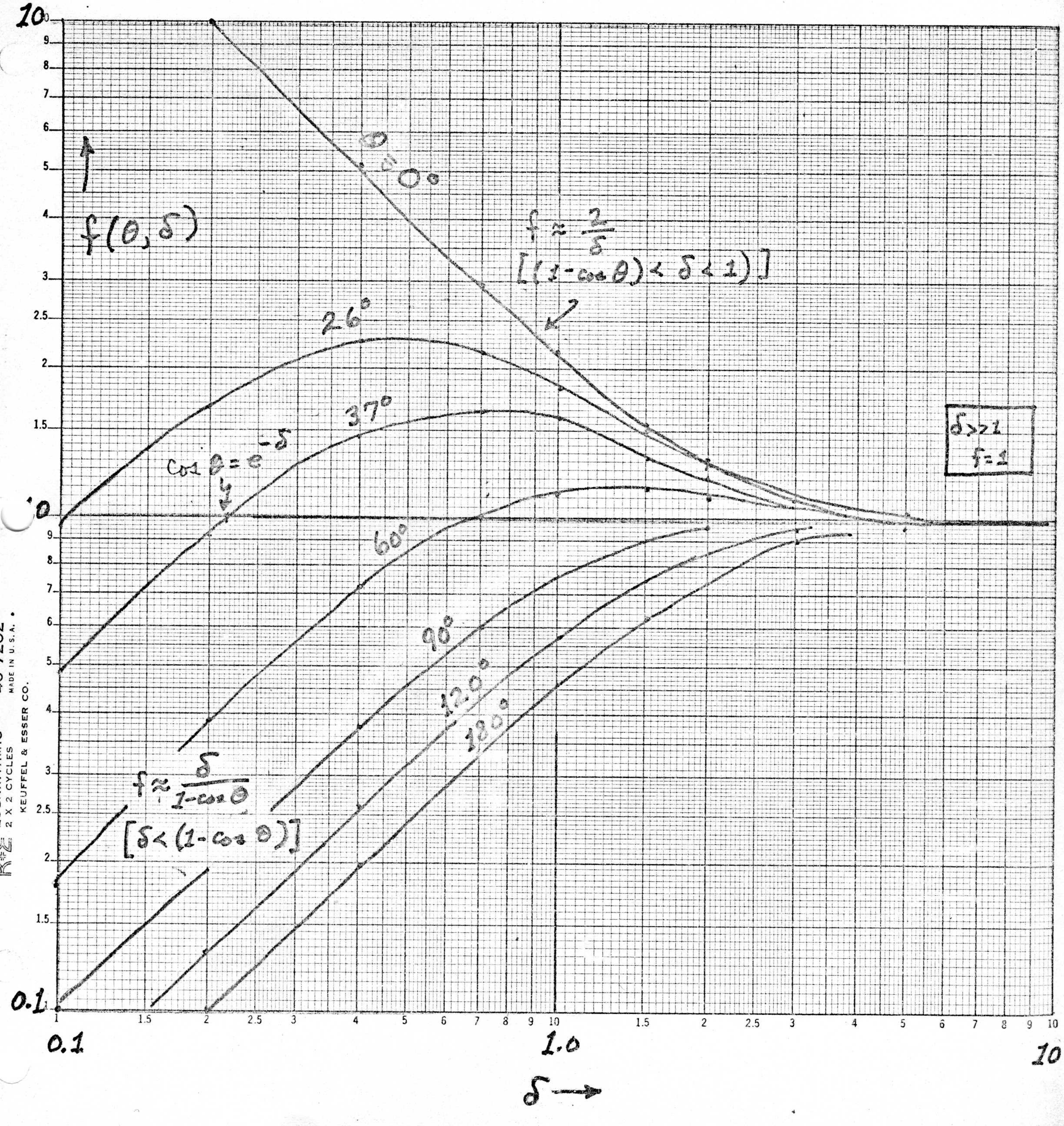

Fig. 3. The resonance factor $f(\delta,\theta)$ plotted against δ.

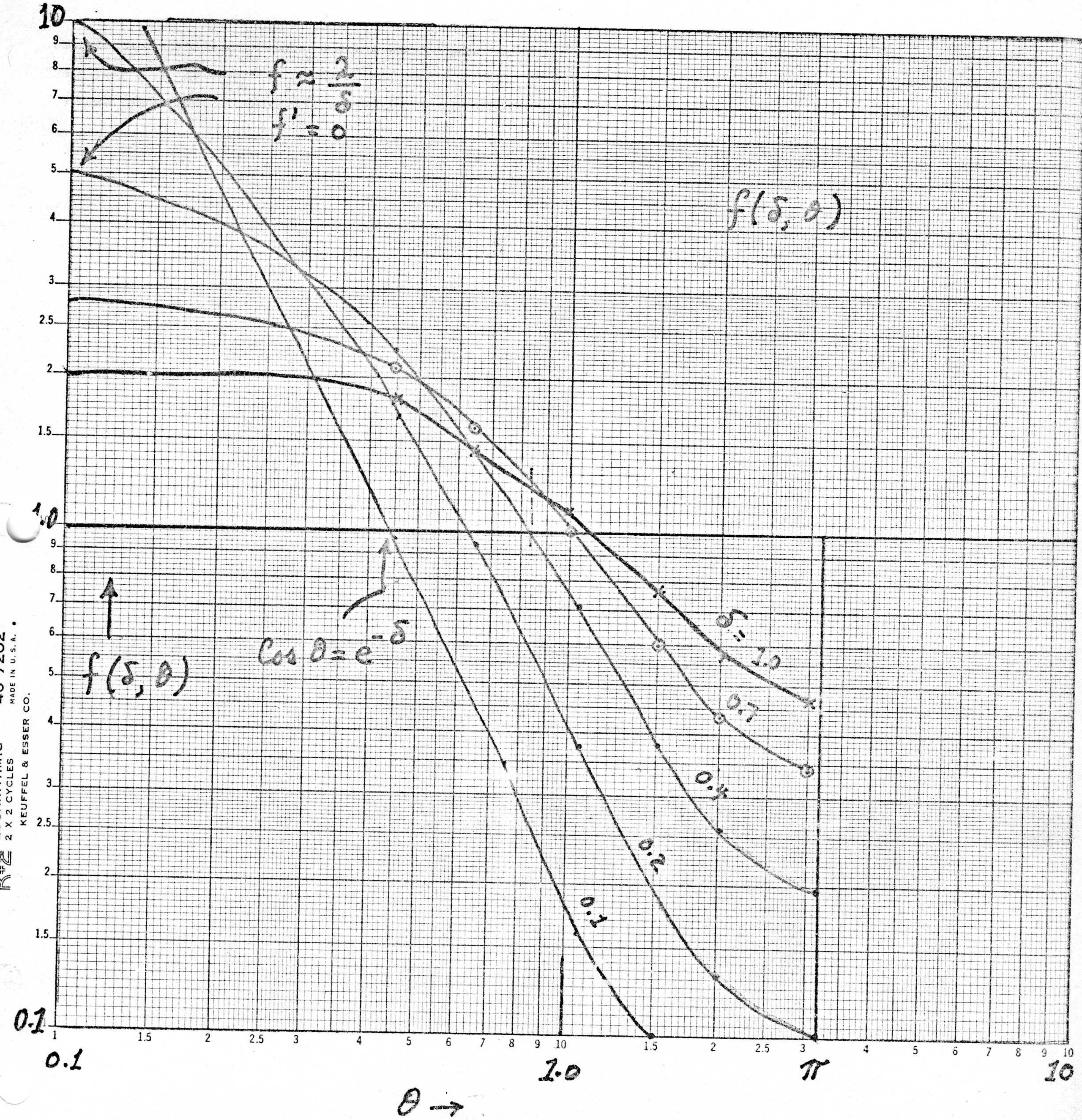

Fig. 4. The resonance factor $f(\delta,\theta)$ plotted against δ (log scales).

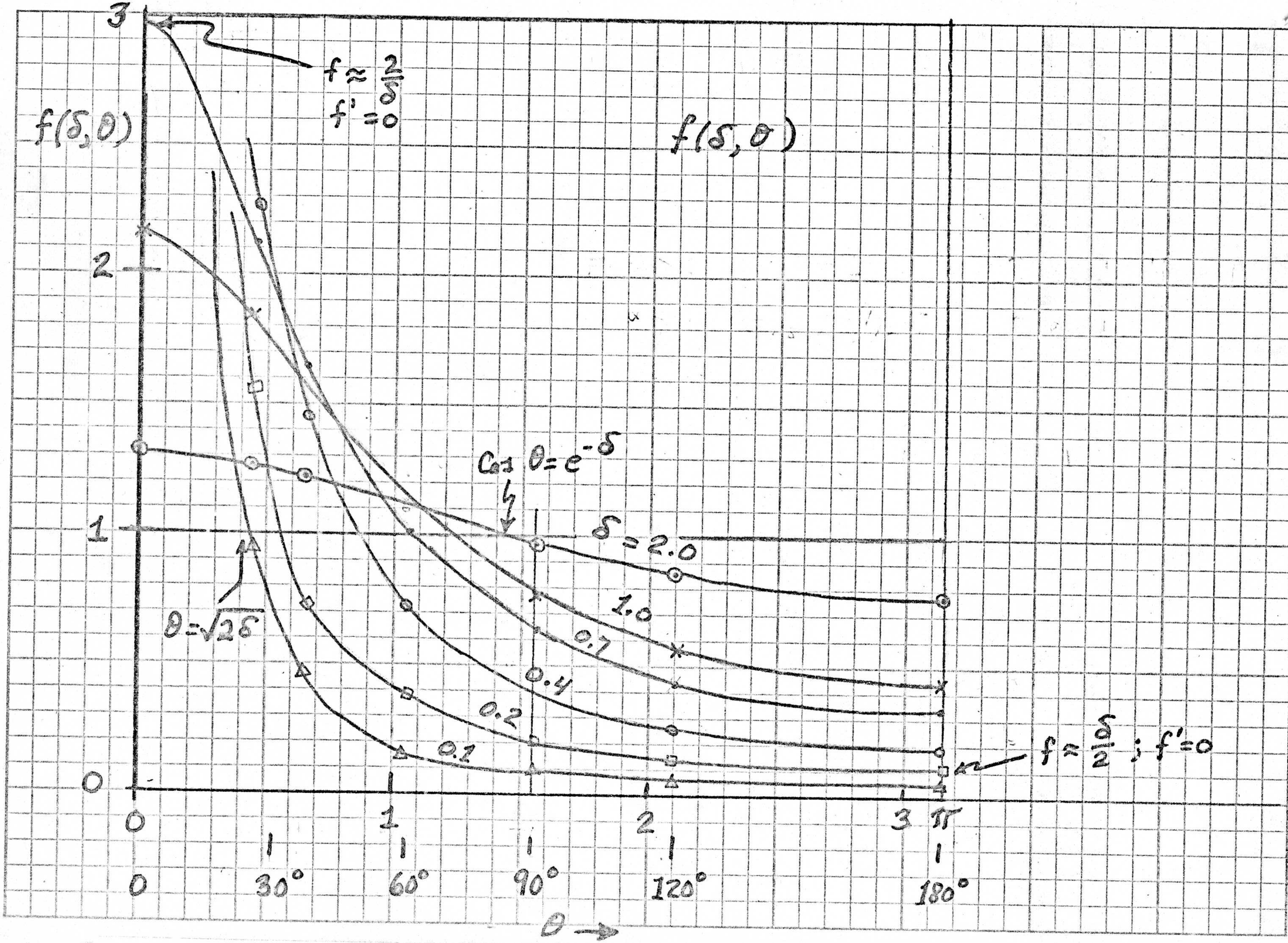

Fig. 5. The resonance factor $f(\delta,\theta)$ plotted against θ (linear scales).

(d) If $\theta \lesssim \delta$ and $\delta < 1$, the mode is "resonant" and $f(\delta,\theta) \simeq 2/\delta$, so that there is an increase of the energy loss in inverse proportion to the decay term δ.

(e) If $\theta \gtrsim 1$ and $\delta < 1$, the mode is "anti-resonant" and $f(\delta,\theta) \simeq \delta/2$, and the energy loss is depressed in proportion to the decay term δ.

The total energy loss per revolution of each particle in a bunch is the sum of the energy losses to each mode:

$$u = \Sigma\, u_n = \sum_{n=0}^{\infty} u_{on}\, f(\delta_n,\theta_n) \quad . \tag{15}$$

If we knew the resonance frequencies ω_n and the decay constants α_n, as well as the single-pass energy loss u_{on}, for all modes, we could, in principle, compute the total loss u for the steady state. There are, however, so many significant modes that the determination of the parameters of each is not feasible -- either experimentally or theoretically.

Wilson[2] has made an estimate of u based on two assumptions: (a) that θ_n can be placed "off resonance" for all modes expect for mode zero, and (b)* that all modes have the same decay parameter δ. I believe that this estimate is optimistic on two counts. First, it is difficult (impossible?) to guarantee that all relevant modes will be off-resonance, and second, we should expect that δ_n should decrease with ω_n as $\omega_n^{-1/2}$, so that the reduction in u_{on} even for ω_n off resonance (which goes as $1/\delta_n$) will for the higher

*This is equivalent to the assumption made by Wilson that his function $f(\alpha_0)$ follows the form $(1 + K\alpha_0^2)$ for values of α_0 that are comparable to 1.

modes be less than in Wilson's estimate.

I would like to suggest that an estimate of u as given in Eq. (15) be made in the following way. First, let's separate out the fundamental mode n = 0 from the sum and write

$$u = u_0 f(\delta_0, \theta_0) + S \tag{16}$$

with

$$S = \sum_{n=1}^{\infty} u_{on} f(\delta_n, \theta_n) . \tag{17}$$

It will also be convenient to let

$$S_0 = \sum_{n=1}^{\infty} u_{on} . \tag{18}$$

The energy loss to the fundamental mode requires a special treatment, anyway, because the fields of this mode add coherently with the rf accelerating fields of the drive system of the cavity, modifying the total cavity loss at the fundamental frequency. This problem has been handled routinely in accelerator and storage ring design, and is also covered in detail in Wilson's report.

In PEP, the bunch period T is much larger than the period, $2\pi/\omega_0$, of the fundamental mode and is, therefore, even larger than the periods of the higher modes. This fact (together with the odd-shaped cavity structure of PEP) leads me to propose the following assumption:

That the phase shifts θ_n for $n \geq 1$ will be distributed randomly from 0 to π with uniform probability. We can then ask: What is $\langle S \rangle$, the expectation value of S, under this assumption? From Eq. (17),

$$\left\langle S \right\rangle = \sum_{n=1}^{\infty} u_{on} \left\langle f(\delta_n, \theta_n) \right\rangle_\theta \tag{19}$$

where

$$\left\langle f(\delta,\theta) \right\rangle_\theta = \frac{1}{\pi} \int_0^\pi f(\delta,\theta) d\theta \ . \tag{20}$$

The integrand of Eq. (20) is of the form $a/(1 - b \cos \theta)$. Evaluating the definite integral (leaning on a table of integrals), I get the convenient result that

$$\left\langle f(\delta,\theta) \right\rangle_\theta = 1 \ , \tag{21}$$

so that

$$\left\langle S \right\rangle = \sum_{n=1}^{\infty} u_{on} = S_0 \ . \tag{22}$$

I can interpret this result in the following way: Although the loss S to the higher modes will, in general, differ from S_0 for any given cavity, we cannot say a priori whether it will be greater than or less than S_0 ; and, in fact, the resonant increase of the energy loss in some modes will, in general, be compensated for by the off-resonance decrease in other modes. So our best guess should be that $S = \left\langle S \right\rangle = S_0$.

Physically, this result means that although a mode that lands "on resonance" will have its energy loss increased by the factor $2/\delta$, the chance of its doing so is only $\sim \delta/2$. So that if there are many modes with the same δ and the same excitation, the decreased energy loss of the off-resonance modes would, on the average, just cancel the increased loss of the on-resonance modes.

Unfortunately, in a PEP cavity, the many parasitic modes will have widely differing decay constants δ_n and excitations u_n. It is, therefore, not unlikely that a nasty fluctuation could place one (or more) of the dominating modes on resonance with no chance of compensation by other off-resonant modes. In particular, the first several modes will have significantly different excitations and decay constants, and we should be prepared for significant departures from the expected S in any particular cavity design. We may expect, however, that for the large number of modes with frequencies greater than a few times ω_0, there will always be many modes with comparable excitations and decay constants so that the averaging to 1 for these modes should be fairly reliable.

With respect to the lower-lying modes, it should be possible to provide the cavities with a subsidiary tuning device for shifting the frequencies of the modes; and, with only moderate luck, it should be possible to adjust any given cavity so that $S \lesssim S_0$. Alternatively, if all cavities were constructed with somewhat different dimensions, the averaging would occur among the cavities and the chance of any large fluctuation from the expected S would be much reduced.

It is clear that the distribution of the likelihood of any particular value of S is <u>not</u> normally distributed. It might, in fact, be interesting and useful to try to derive this distribution -- with a reasonable assumption about the mode distribution -- to get some feeling for what the expected deviations from the "expected" mean may be like.

The energy loss S_0 to the parasitic modes can be obtained by subtracting the single pass energy loss to the fundamental mode from the total single pass loss.

$$S_0 = u_0 - u_{00} .$$

To get u_{00}, we can use Eq. (9), taking the experimentally observed values for the various factors, namely:

$$\omega_0 = 2.25 \times 10^9 \text{ sec}^{-1}$$
$$R_{s0} = 9.5 \times 10^8 \text{ ohms}$$
$$Q_0 = 2.8 \times 10^4 .$$

For a bunch of 1.5×10^{12} particles,

$$u_{00} = 4.6 \text{ MeV} . \qquad (23)$$

Taking the two possible values of u_0 for PEP from Table 2, we get:

$$\ell = 5 \text{ cm} \; ; \; S_0 \simeq 19 \text{ MeV}$$
$$\ell = 15 \text{ cm} \; ; \; S_0 \simeq 4.3 \text{ MeV} \qquad (24)$$

This, then, is our best estimate of energy loss S of the stored particles to higher frequency modes of the cavities. The rf system must provide this energy in addition to the radiation loss U_γ -- which, for comparison, is 26 MeV at 15 GeV.

It may be worth pointing out also that the energy loss of (24) will for the most part appear in the walls of the cavities. The cavity cooling must take care of this loss as well as the wall losses in the fundamental (accelerating) mode. To get the magnitude of this loss, we can multiply the energy loss per electron of (24) by the total number of circulating charges (in both beams). We get

$$P_s \simeq 0.8 \text{ to } 3.8 \text{ Megawatts.} \tag{25}$$

This power is a significant addition to the cavity losses in the fundamental mode, which are about 2 Megawatts.

III. Cavity Losses with Two Stored Beams

The preceding Section considered the energy loss of the particles in a single stored beam of several equal bunches with the equal time separation T -- as would occur with either electrons, or positrons, only, in the ring. I consider now what will be the effects when beams of electrons and positrons are stored together. When two beams are stored, each cavity will be excited by two trains of bunches, each train will have a bunch separation T, but the two trains will be interleaved with a time displacement, or "stagger", of T_1. See Fig. 6.

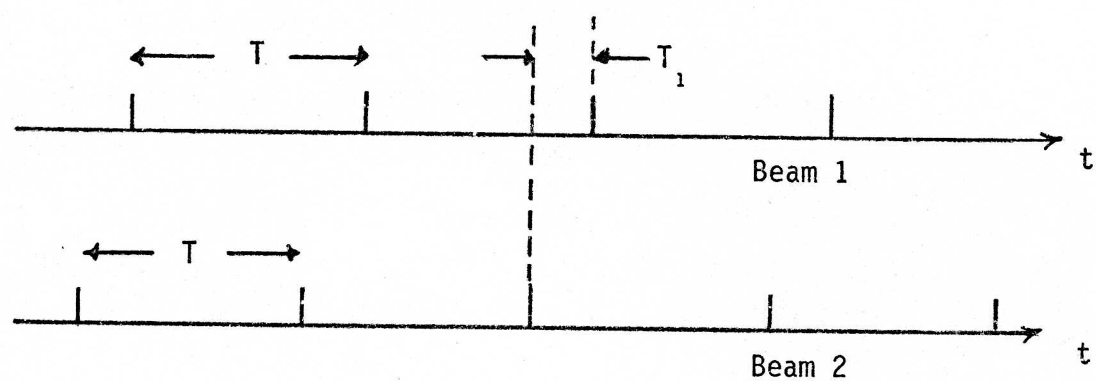

Fig. 6. Interleaving of two beams at a cavity.

We could, of course, analyse the fields (and energy losses) associated with each cavity mode when it is subjected to the sum of the two staggered bunch trains. The following qualitative arguments show, however, that such an analysis is not necessary.

First, I note that the total field produced in each cavity is just the sum of the fields produced by each train acting separately. Each particle will experience the fields produced by its own beam -- as considered in Section II -- and an additional cross-coupling field V_n^* produced by the other beam. If the stagger time T_1 were zero at any cavity, the two fields would add coherently, and the energy loss per particle would be doubled. Fortunately, however, the PEP design places the accelerating cavities at a distance of some tens of meters from a beam interaction point so that the stagger time T_1 is much larger than the period of the fundamental accelerating field (whose wavelength is 0.83 meters). The cavities are, of necessity, placed so that the time T_1 is precisely an integral multiple of the fundamental period $2\pi/\omega_0$. For all of the other cavity modes, however, such synchronism would be purely accidental.

The effective accelerating field of any one cavity mode <u>due to one train alone</u> can be written as

$$v_n(t) = A_n \cos(\omega_n t - \psi_n) , \tag{26}$$

with t measured from the time of passage of any one bunch, and $A_n \cos \psi_n$ is equal to the V_{on} of Eq. (7). Let's define ϕ_n by

$$\phi_n = |\omega_n - 2\pi j_n| \tag{27}$$

with j_n an integer chosen to make $\phi_n \leq \pi$. Then the cross-coupling field V_n^* becomes

$$V_n^* = A_n \cos(\phi_n - \psi_n) \tag{28}$$

Now the fact that $\omega_n T_1 >> 2\pi$ leads us to expect that the phase slippages ϕ_n will be distributed more-or-less randomly over the interval 0 to π. In addition, the starting phase, ψ_n, of each mode will also be randomly distributed. We may reasonably expect, therefore, that on the average over many modes, the cross-coupling energy loss will be zero.

Some of the same qualifications made in Section II about accidental resonances should be made again here. It is always possible that one of the dominant lower-frequency modes might have $(\phi_n - \psi_n)$ near zero, so that there would be an enhanced energy loss to that mode without complete compensation by other modes. The effect would only be significant, however, if that particular mode also happened to be on resonance for a single beam. Even such coincidences should not concern us much, however, because of another mitigating circumstance. The stagger time T_1, and, therefore, the phase slippage ϕ_n will be different for each of the 120 separate accelerating cavities. And although the various values of T_1 are integral multiples of the fundamental period $2\pi/\omega_0$, they will, in general, not be neatly related to the periods of the parasitic modes. This additional randomness should lead to a rather smooth distribution of the phases ϕ_n, and an average cross-coupling energy loss that is much, much less than the cavity losses of a single bunch or of a single beam.

In summary, I believe that the two stored beams will act incoherently in their excitation of the higher cavity modes. The energy loss of each particle in a bunch will be just equal to the energy loss in single beam operation, and, as we have seen, this loss (to the higher modes) is expected to be about equal to the single pass energy loss. Similarly, the power deposited in the cavities -- as evaluated at the end of Section II -- is just the sum of the powers delivered by the two beams acting alone. The interaction of the two beams with the fundamental cavity mode is, as remarked earlier, intentionally coherent, and must be treated differently.

IV. Dependence of Parasitic Loss on Ring Parameters

The energy loss figures given above were obtained using the nominal parameters of the PEP design. It may be useful to record here some general formulas that relate the energy loss to the stored current and other parameters that might be varied -- all assuming a given rf cavity structure. I do this only for the parasitic loss S -- that is, only to the loss to the parasitic modes of the cavities. (The loss due to the fundamental mode is complicated by the interaction with the accelerating voltage, and has been treated in Ref. 2.)

For the present purposes, I shall assume a nominal value for the parasitic loss S, which I shall take to be

$$S_{nom} = 10 \text{ MeV at } \ell = 10 \text{ cm} . \tag{29}$$

When more reliable values are obtained for this figure, all of the following formulas will need to be corrected by the same factor: S_{actual}/S_{nom}.

I shall use the following symbols:

PEP

- S: energy loss per particle to all parasitic modes.
- P_{sm}: average power lost to a group of m cavities by one beam.
- N_B: number of particles per bunch.
- B: number of stored bunches in a beam.
- I: average current per beam.
- M: total number of accelerating cavities.
- m: number of cavities in a group.
- T_0: time for one revolution of the ring.
- ℓ: bunch length ($2\sigma_z$)

The energy loss calculated in Section I was for $N_B = 1.5 \times 10^{12}$, and for M = 120. In general,

$$S = C_1 g(\ell) M N_B , \tag{30}$$

with

$$C_1 = 5.55 \times 10^{-8} \text{ eV} , \tag{31}$$

and $g(\ell)$, a bunch length factor I choose to define to be 1 for a nominal bunch length of 10 cm. The "nominal" variation of $g(\ell)$ is shown in Fig. 7.

Using the fact that $N_B = IT_0/eB$, we can express S in terms of the stored current <u>in one beam</u> by

$$S = C_2 \frac{M T_0 I}{B} g(\ell) \tag{32}$$

with

$$C_2 = \frac{C_1}{e} = 3.47 \times 10^{11} \text{ eV/coul} . \tag{33}$$

For a given machine, M, B and T_0 are determined so that we can write

$$S = C_3 g(\ell) I \tag{34}$$

with

$$C_3 = \frac{C_2 M T_0}{B} . \tag{35}$$

For PEP,

$$C_3 = 100 \text{ MeV/amp.} \tag{36}$$

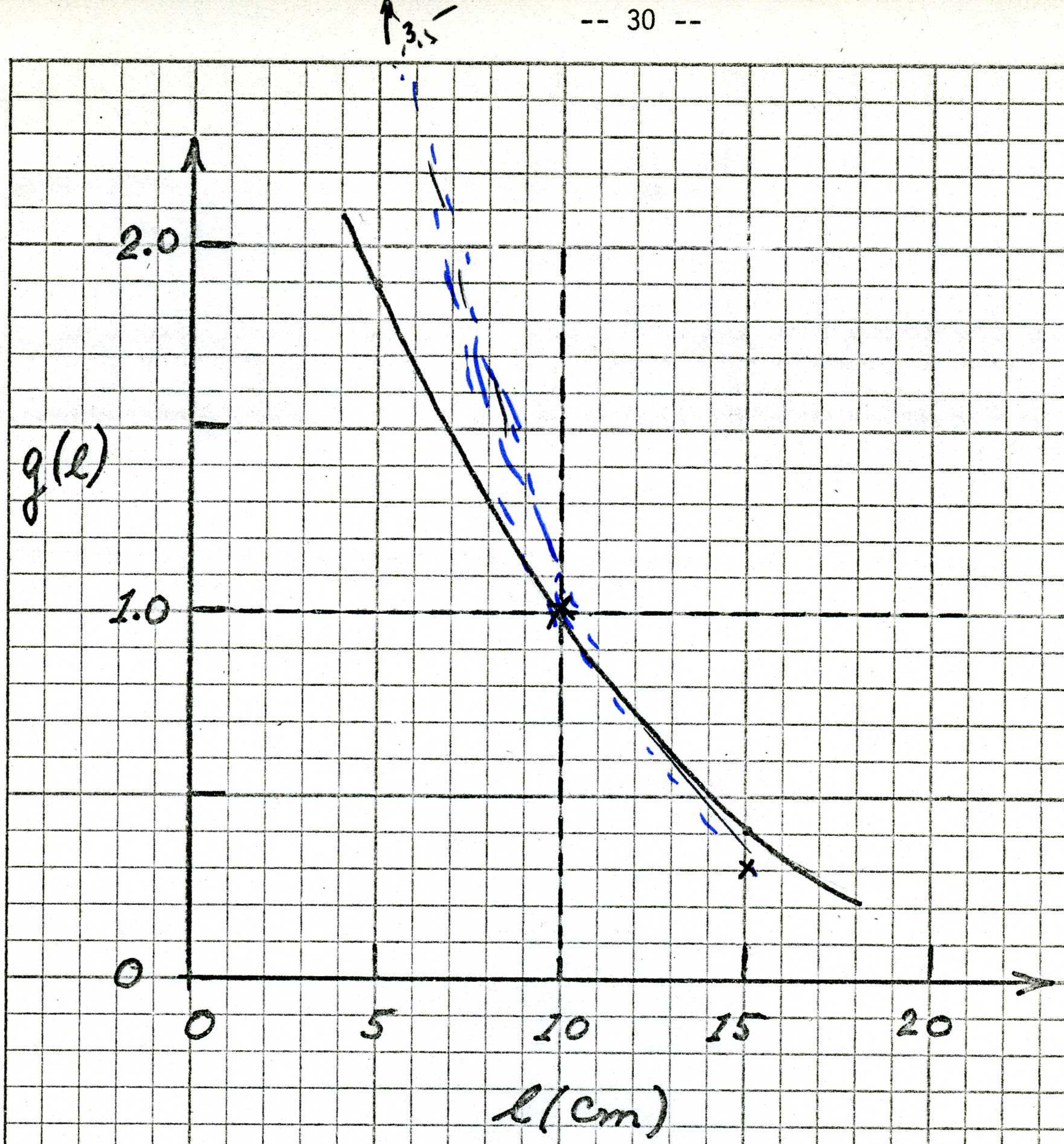

Fig. 7. Bunch length factor for S.

l	New	
5	14	3.5
10	4[illegible]	1
15	1.3	0.3

For engineering purposes, the average power delivered to the cavities is of interest. I give the relation for the power to a group of m cavities by <u>one</u> circulating beam. Multiply by 2 for two stored beams. The power P_{sm} is related to S by

$$P_{sm} = S \frac{m}{M} \frac{N_B B}{T_0} , \tag{37}$$

which can be written as

$$P_{sm} = C_4 g(\ell) \frac{m T_0}{B} I^2 \tag{38}$$

with

$$C_4 = \frac{C_1}{e^2} = 3.47 \times 10^{11} \text{ watts/amp}^2\text{-sec} . \tag{39}$$

For a particular machine, and particular m,

$$P_{sm} = C_5 I^2 , \tag{40}$$

with

$$C_5 = \frac{C_1}{e^2} \frac{T_0 m}{B} . \tag{41}$$

For PEP, taking B = 3 and m = 5,

$$C_5 = 4.18 \text{ Megawatts/amp}^2 . \tag{42}$$

References

1. E. Keil, "Diffraction Radiation of Charged Rings Moving in a Corrugated Cylindrical Pipe", Nucl. Inst. & Meth. 100, 419 (1972).

2. P. B. Wilson, "Beam Loading in High-Energy Storage Rings", PEP Note 37 (1973).