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Matt Sands
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Parasitic Cavity Losses in SPEAR-2

I. Introduction

In PEP the large number of particles in a bunch, together with the small bunch length, may cause grievous energy loss from the beam to parasitic modes in the accelerating cavities. The same energy loss mechanism may also influence significantly the operation of SPEAR-2. Unfortunately, there is some quantitative uncertainty in the theory of these losses -- in part, because the consequences of the approximations used are not well understood. As it stands, however, the theory does give an estimate of the energy loss that, if correct, would require a notable increase in the required rf power of high-luminosity storage rings.

I have recently tried to estimate the parasitic cavity in PEP[1], based on a paper of Keil, and I have obtained the result that the loss to parasitic modes will be about 10 MeV per particle per revolution for a bunch length of about 10 cm. I do not have, however, high confidence in this estimate. I am afraid that the loss might be higher by a factor of 2; although I am inclined to guess that the loss may be lower by a factor of 2, or more.

Since the normal loss to synchrotron radiation in PEP will be 26 MeV at 15 GeV, an additional energy loss of 10 or 20 MeV would have important implications for the design of the rf system -- and might have other grave consequences for the performance of the ring as well.

It seems urgent, therefore, to obtain a more reliable value of the loss as soon as possible. And perhaps the best way to do so would be an experimental measurement of the loss in SPEAR-2. In this note, I bring together

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some of the considerations that might bear on an experimental investigation of the loss using SPEAR-2.

II. Energy Loss

It is convenient to treat the energy loss to the cavities as consisting of two parts: (a) the loss to the fundamental accelerating mode, which is well understood (and which must be treated as part of the rf engineering design), and (b) the parasitic loss -- by which I mean the energy lost by the beam in the transient excitation of all cavity modes other than the fundamental accelerating mode. It is only this latter loss about whose magnitude there is great uncertainty. Also, it is only the parasitic loss that must be supplied directly from the rf source, since it combines linearly with the energy loss to synchrotron radiation.

It is convenient to define the parasitic energy loss S as the energy lost by each particle of the stored beam in making one complete revolution around the ring. In Ref. 1 I have shown that this loss to the rf cavities alone can be written

$$S = C_1 M\, N_B\, g(\ell) , \qquad (1)$$

where

- C_1 is a coefficient that depends only on the geometry of a single cavity,
- M is the number of such cavities in the ring,
- N_B is the number of particles stored in each circulating bunch,
- ℓ is the length of the bunch,

$g(\ell)$ is a bunch length factor that depends on the bunch length and the geometry of the cavities.

You will notice that Eq. (1) does not depend on the number of stored bunches, nor on the revolution period of the ring -- despite the fact that the parasitic cavity modes may have energy storage times that are long in comparison with the time between the passage of successive bunches through a cavity. It is shown in Ref. 1 that the energy (and fields) stored in the parasitic modes does not change the expected value of the energy loss. Although the energy loss to any one mode may, in the steady state, be increased or decreased over the transient loss due to a single passage through the cavities, the increases and decreases compensate, on the average, leaving the total loss unchanged. This statement is clearly a statistical inference, so deviations from the expected energy loss should be expected in any particular circumstance. I emphasize, therefore, that the parasitic loss S of Eq. (1) is only the expected value, from which some departure will occur. I shall say more about this later.

In Ref. 1, I have obtained estimates for C_1 and $g(\ell)$ for the cavities to be used in PEP and SPEAR-2. These estimates are, however, uncertain by a factor of 2 (or more), so I shall for the present purposes take a nominal value for C_1, which is, it should be remembered, the important parameter we would hope might be determined by experiment. The other constants written below as various C_i are directly proportional to C_1. The nominal value is:

$$C_1(\text{nominal}) = 5.55 \times 10^{-8} \text{ eV} \quad . \tag{2}$$

I have defined the length factor $g(\ell)$ to be 1 for $\ell \simeq 10$ cm; and according to the same theory that gave the nominal C_1, it varies with ℓ as shown in Fig. 7 of Ref. 1.

It is convenient to give an expression for S also in terms of the beam current. We can write

$$S = C_3\, g(\ell)\, I \quad , \tag{3}$$

where I is the average circulating current in one beam, and

$$C_3 = \frac{C_1 M T_0}{eB} \quad , \tag{4}$$

where

T_0 is the revolution time of ring,

B is the number of circulating bunches in one beam,

e is the electronic charge.

For SPEAR-2 ($M = 20$, $B = 1$, $T_0 = 0.78 \times 10^{-6}$ sec),

$$C_3 = 5.4 \text{ MeV/amp} \quad . \tag{5}$$

So with an 0.1 ampere beam, the parasitic energy loss is expected to be 0.54 MeV per turn for a bunch length of 10 cm. This is to be compared with 1.78 MeV of loss to synchrotron radiation at 4.0 GeV or 0.56 MeV at 3.0 GeV. The expected loss is not negligible.

Since the parasitic cavity loss is comparable with the radiation loss, we might hope to measure S by observing the quantum lifetime of the stored beam as a function of peak rf voltage. Such a measurement will give an important result, although it may not measure S directly. First, the beam energy distribution may be different from the assumed one, and, second, there is likely to be also some significant parasitic losses into other "accidental" cavities of the ring (such as the vacuum boxes). Although the practical consequences of the two losses are the same, it would be useful to measure each separately in SPEAR-2 so that the results could be applied to PEP where the proportions of the two will be different. (It might also be useful if it turns out to be necessary to take corrective measures in SPEAR-2.)

Another measurement of the parasitic losses to the cavities can be made by measuring the power dissipated in the cavity walls of a detuned cavity. Expressions for this power are given in the next Section.

III. Power Dissipated in the Cavities

The average rate at which energy is deposited by one beam in the parasitic modes only of a physically-connected block of individual cavities is

$$P_{cs} = \frac{N_B\, B}{T_0} \frac{m}{N} S \ , \tag{6}$$

where

B is the number of bunches in the beam,

m is the number of cavities in the block.

This can be written in terms of the stored current I as

$$P_{cs} = C_5\, I^2 \ , \tag{7}$$

with

$$C_5 = \frac{C_1}{e^2} \frac{T_0 m}{B} . \tag{8}$$

For SPEAR-2, taking m = 5,

$$C_5 = 1.35 \text{ Megawatts/amp}^2\text{/block} . \tag{9}$$

For a single stored beam of 0.1 amp, the parasitic power to a block of five cavities is 13.5 kilowatts. It should be possible to measure this power by measuring the heat delivered to the cooling water of a cavity block.

This method also suffers from some difficulties. In addition to the difficulty of measuring the heat loss (heat leakage, etc.), there is also an rf power "leakage" from the cavities, in that the energy lost to a cavity does not appear solely in the cavity walls. Some of the energy in the parasitic modes may be coupled out to the load (which, incidentally, could be reduced by placing a resonant filter at the coupling loop), and an uncertain amount of the energy will pass out through the cavity openings into the beam tube to be deposited as heat in other parts of the ring vacuum chamber.

The parasitic power P_s dissipated in the cavity walls is, of course, in addition to wall losses from the fundamental mode. If the fundamental mode is excited to a steady oscillation with a peak rf voltage $\hat{V}$, the wall dissipation is

$$P_{CF} = \frac{\hat{V}^2}{R_s} , \tag{10}$$

where R_s, the shunt resistance, and $\hat{V}$ refer to a block of cavities. When the cavity is excited by both the rf generator and the beam, the fundamental mode does not have a precisely steady level of oscillation, although, generally, the variations of rf level between beam pulses will be small and Eq. (10) can be used -- taking for $\hat{V}$, the measured peak voltage of the mode.

For a block of five SPEAR-2 cavities, R_S = 40 Megohms, so with a peak voltage (for five cavities) of, say, 500 kV, the wall loss is about 6.3 kW, which is comparable with, and in addition to, the parasitic loss.

The most precise measurement of the parasitic losses could be made by disconnecting a block of cavities and detuning the fundamental mode well off resonance. Then the losses to the fundamental mode are not only reduced from the resonant value, but are suppressed well below the energy loss that would occur with an impulse excitation. Since the loss to the fundamental is, however, quite sensitive to the tuning, I shall give here an expression for the power loss in the fundamental mode for a cavity that is not excited by an external generator, but is, instead, coupled to a passive load.

Let the coupling to the load be such that the quality factor Q_L of the cavities is reduced by the factor ρ below its natural value Q_0. That is,

$$Q_L = \rho Q_0 \quad . \tag{11}$$

(The factor ρ is sometimes written as $1/(1 + \beta)$, with β the "coupling coefficient".) Next, I define a "tuning angle" θ of the fundamental mode by

$$\theta = \frac{(\omega - \omega_0)\, T_0}{B} = \Delta\omega T_B \quad , \tag{12}$$

where ω is the resonant frequency of the cavity, ω_0 is the frequency when resonant with a harmonic of the beam frequency, T_B is the time between bunches and $\Delta\omega$ is the "detuning" $(\omega - \omega_0)$. Finally, I define a decay parameter δ by

$$\delta = \alpha_L T_B = \frac{\omega_0 T_B}{2Q_L} = \frac{\omega_0 T_B}{2\rho Q_0} = \frac{k}{2B \rho Q_0} , \tag{13}$$

where α_L is the decay constant of the loaded cavity and $k = \omega_0 T_0 / 2\pi$ is the "harmonic number" of the rf system.

The power loss P_{CF} to the fundamental can now be written as

$$P_{CF} = R_S \rho^2 I^2 h(\delta,\theta) , \tag{14}$$

where the resonance function $h(\delta,\theta)$ is defined by

$$h(\delta,\theta) = \frac{\delta(1 - e^{-2\delta})}{2(1 - 2e^{-\delta} \cos \theta + e^{-2\delta})} . \tag{15}$$

Incidentally, the cavity voltage can be obtained from Eq. (14), using the fact that $\hat{V}^2 = P_{CF} R_S$. One further remark: Eq. (15) assumes that the stored bunch is short in comparison with the wavelength of the fundamental mode, namely that $\omega \ell / c << 1$. For long bunch lengths, an additional correction factor dependent on the square of the bunch length should be introduced.

For the SPEAR-2 cavities, ρ is about 1/3 and Q_0 is about 28,000, so $\delta \approx 0.092$. So long as $\delta << 1$, formula (15) is well approximated by

$$h(\delta,\theta) = \frac{\delta^2}{2(1 - \cos \theta) + \delta^2 (2 - \cos \theta)} . \tag{16}$$

If the undriven cavity were to be tuned to resonance, $\theta = 0$ and $h(\delta,\theta) = 1$, so that $P_{CF} = R_S \rho^2 I^2$. For a circulating current of 0.1 ma, P_{CF} would be 440 kilowatts for a block of five cavities. Note, however, that, for the fundamental mode, the fields of two circulating beams add coherently, and two 0.1 ma beams would produce four times the power loss -- for a power of 1.7 Megawatts per block. These powers are, clearly, much larger than the power available in SPEAR-2, so beam currents of 0.1 ampere cannot be stored with a passive cavity block on resonance.

For small tuning angles, $\theta \approx \delta$, Eq. (16) is well approximated by

$$h(\delta,\theta) = \frac{\delta^2}{\theta^2 + \delta^2} , \qquad (17)$$

which has the form of a standard resonance formula. The factor $h(\delta,\theta)$ will be significantly less than 1 only when θ becomes larger than δ. For any $\theta > \delta$, Eq. (16) becomes

$$h(\delta,\theta) = \frac{\delta^2}{2(1 - \cos\theta)} , \qquad (18)$$

which, if $\theta < 1$, is approximately δ^2/θ^2 in agreement with Eq. (17).

The minimum of $h(\delta,\theta)$ will occur if the cavity can be detuned to the anti-resonant condition $\theta = \pi$. Then, $h(\delta,\theta) = \delta^2/4$. With this condition for SPEAR-2, P_{cF} for a block of five cavities is about 2.2×10^{-3} of the resonant value; or, with a 0.1 amp stored beam, $P_{cF} \approx 1.0$ kilowatt. This power is about 7% of the nominal parasitic loss P_s, and can easily be corrected for.

IV. Experimental Program

It would be useful to measure both the energy loss S and the power loss P_s due to the SPEAR-2 cavities to obtain an experimental number for the basic parameter C_1. It is important, however, that the measurements be made in such a way that there can be some confidence that the measurement is near the "expected" value, and not significantly larger than "normal" due to an accidental fluctuation in the influence of a single parasitic mode. This possibility can be checked on in a number of ways: (a) By changing the frequency of the accelerating rf drive. (To be meaningful, such changes should be about 10^{-4} of the frequency.) (b) By storing more than one bunch and testing with one or more different spacings between bunches. (c) By varying the tuning of the test cavity. (It is likely that the frequency of any dominant mode will be strongly influenced by any tuning plug.)

As mentioned earlier, it would be most useful if both S and P_S could be measured independently to determine whether there are significant contributions to the measured S from energy losses to other structures in the vacuum chamber of the ring.

Finally, it would be useful to have some check on the dependence (at least the slope with ℓ) of the bunch length factor $g(\ell)$, as might be determined by varying the rf accelerating voltage, or -- should there be a current-dependence of the length -- by measuring the current-dependence of S or P_S.

Reference

1. Matt Sands, PEP Note 90, July 1974.