SINGLE PASS COLLIDER MEMO CN- SLAC-TN-16-002 #310

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TITLE: MEAN LIFE OF KLYSTRONS*

1. INTRODUCTION

It would be useful to have the best possible estimate of this mean life–time of our new klystrons based on the most recent, available operating experience. I give here a simple formula for this best estimate, based on the maximum likelihood method. This method also provides an indication of the reliability of the estimated lifetime.

The results given here apply uniquely to a uniform klystron population for which we can assume that deaths occur randomly, and independently of the previous history (operating time) of any one klystron. That is, I assume that the probability ΔP that any given operating tube will fail in the next, small time interval Δt is given by

$$\Delta P = \frac{\Delta t}{\tau} \tag{1}$$

where τ is the mean life–time of the population.

If this assumption is not justified (from prejudice, or because it is clearly in violation of operating experience), the method given here would not be valid. In general, I would then suggest that the analysis proposed in SLAC–TN–68–2 (M. Sands, 1968) be used.

2. THE DATA

I assume that at any given moment in history – say "now" – we have the following information:

(*a*.) The accumulated ON – time $t_{\ell i}$ for each of the tubes that still survive.

(*b*.) The accumulated ON – time t_{di} until death for each of the n tubes that have died in service. The time t_{di} is, of course, the "actual" lifetime of each failed tube.)

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3. THE EXPECTED LIFETIME

We are assuming that the tube population has some mean life τ which I would like to estimate. The best estimate $\bar{\tau}$ of τ is given simply by

$$\bar{\tau} = \frac{T}{n}, \tag{2}$$

where

$$T = T_\ell + T_d = \Sigma_1^m t_{\ell i} + \Sigma_1^n t_{fi} \tag{3}$$

is just the total accumulated ON time of all tubes. As an example, suppose we had run 16 tubes, and 15 were still alive after a total (combined) operating time of 13,000 hours, with one which failed after 2,000 hours. The best estimate of the mean life would be $\bar{t}$ = 15,000 hours.

4. THE LIKELIHOOD FUNCTION

Clearly, the best estimate in the example just given would not be very reliable. We might guess that the actual lifetime τ could easily be anything between 10,000 and 30,000 hours. How can we quantify that intuition?

In probability theory a function L(τ; T, n) is defined which is proportional to the "likelihood" that the actual lifetime of the population has the value τ, if the total observed ON time is T, and there have been n failures. I will write this function simply as L(τ) – assuming that it refers to some particular T and n.

If n is at least 1, the function L(τ) falls to zero for $\tau = 0$ or $\tau = \infty$, and has a maximum in between. In fact the best estimate $\bar{\tau}$ is defined to be that value of τ which makes L(τ) take on its maximum value. ($\bar{\tau}$ is then said to be the "maximum likelihood estimate".) Note that L(τ) is a relative likelihood function. Generally speaking, its absolute magnitude is of no useful significance. But L(τ) is useful in comparing the relative likelihood that two different values of τ represent the true value.

For the present problem the likelihood function can be written as

$$L(\tau) = Ce^{-T/\tau}\tau^{-n}, \tag{4}$$

where C is some arbitrary constant. (This expression is derived in the Appendix.) Notice that for fixed T and n, L(τ) does indeed have its maximum value at $\tau = \bar{\tau}$.

In the figure below I have plotted L(τ) as a function of τ for several values of n from 1 to 10. I have for convenience chosen to plot the ratio of L(τ) to its maximum value L_o = L ($\bar{\tau}$). For each n this ratio takes on its maximum value of 1 at $\tau = \bar{\tau}$. It is evidently also convenient to chose $\frac{\tau}{\bar{\tau}}$ as the horizontal coordinate of the graphs.

From the figure you see that the likelihood curve for n = 1 is very broad. The likelihood is only down to 70% of its peak at $\frac{\tau}{\bar{\tau}}$ = 0.5 or at 2.5. The "best" estimate of $\tau = \bar{\tau}$ is not very good! But it is better than no estimate at all! By the time n = 10, the likelihood curve is beginning to narrow somewhat, and we can begin to have some confidence that our best estimate is correct to within about 30%. Indeed, for large n (say $\geq$10) the likelihood function becomes a gaussian error curve with a standard deviation about $1/\sqrt{n}$. We can then take that the "error" $\Delta\bar{\tau}$ in the estimate of τ is $\frac{\bar{\tau}}{\sqrt{n}}$.

Incidently, equations (2) and (4) are still correct for n = 0. In this case the "best" estimate of $\bar{\tau}$ is ∞! (After all no failures have been observed.) The likelihood function is $e^{\frac{-T}{\tau}}$, however, which tells us that an estimated τ of 2.8T is only 30% less likely than an infinite life.

I should emphasize that all of the foregoing presupposes that we have absolutely no other useful information related to the tube lifetime. If we have any a priori information (say, from previous experience with similar tubes) which would bias our expectations this information should be used and would modify our estimate of the lifetime.

Further, it should be kept in mind that if the operating conditions of some tubes are significantly different from others – or if they are changed after some period of use – it may be that the tubes under the new operating condition should be defined as members of a new "population", and a separate estimate of mean life should be made for the two populations.

5. APPENDIX

According to probability theory (see Reference 1) if we observe M tubes that last t_{li} without failure (i= 1 to M) and n tubes that last t_{fi} until failure (i = 1 to n) then the likelihood function that the mean life of the population is τ is given by

$$L(\tau) = K(P_1 \cdot P_2 \cdot P_3 \cdots P_m) \cdot (Q_1 \cdot Q_2 \cdot Q_3 \cdots Q_n) \qquad \text{(A1)}$$

where P_i is the probability that any one tube will last the time t_{li} (assuming a lifetime τ) and Q_i is the probability that any one tube will live for the time t_{fi}, and then fail in some small time Δt afterward, and K is some constant. (If Δt were set equal to O, then all Q_i would be zero, so let's leave Δt small, finite and fixed.) For our assumed population,

$$P_i = e^{-t_{li}/\tau} \qquad (5)$$

and

$$Q_i = e^{-t_{fi}/\tau} \cdot (\Delta t/\tau)$$

Using these formulas in Eq. (A1), we get that

$$L(\tau) = Ke^{-T/\tau} \cdot (\Delta\tau/\tau)^n,$$

which is the same as Eq. (4) if we identify $C = K(\Delta t)^n$, which is quite arbitrary.

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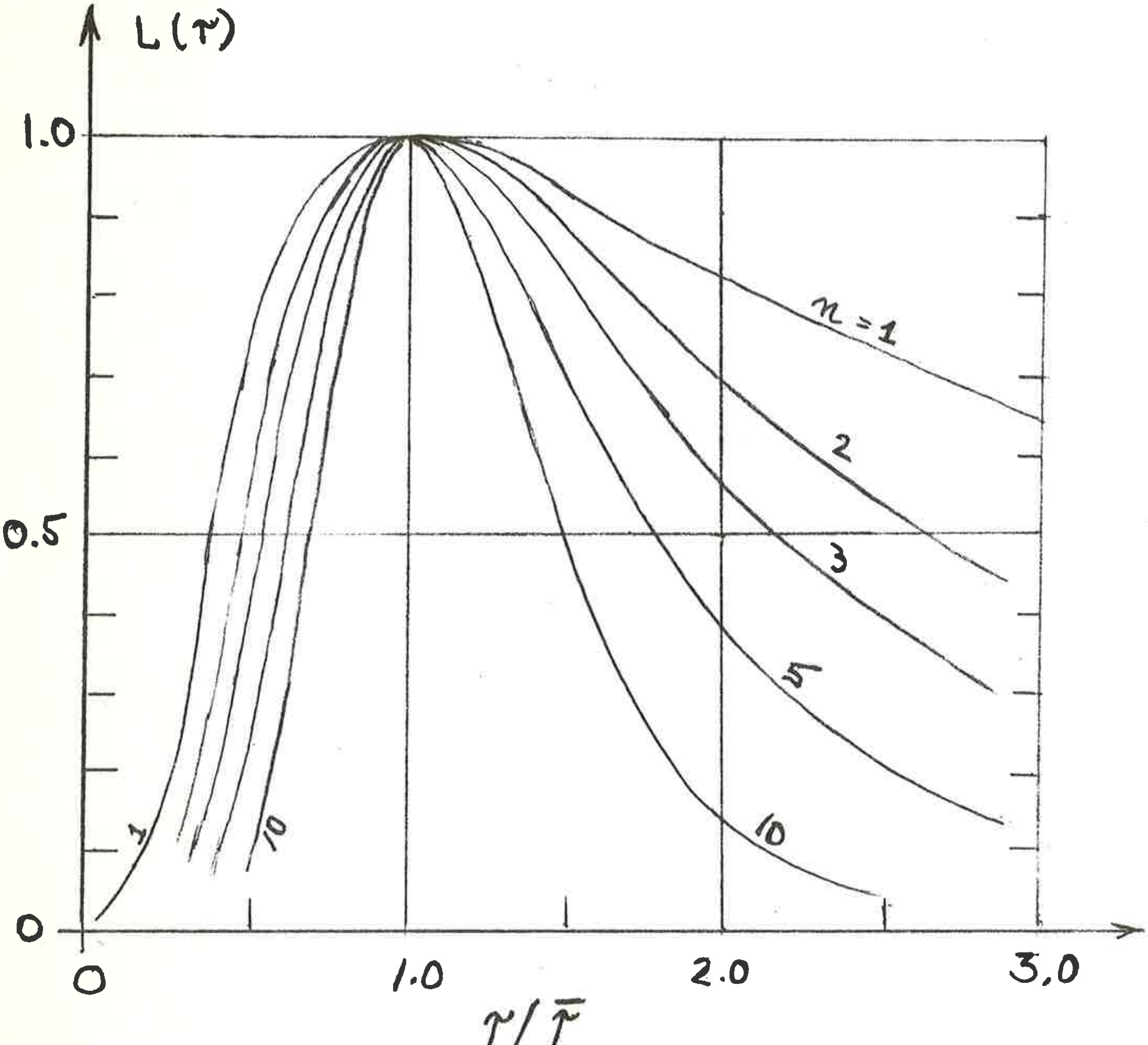

RELATIVE LIKELIHOOD THAT τ DIFFERS FROM $\bar{\tau}$ FOR n FAILURES.

FIGURE 1

REFERENCES

1. W. Eadie, et. al., Statistical Methods in Experimental Physics, North – Holland (1971).