## SLAC TRANS - 64

INVESTIGATION OF ( $\mathrm{e}^{+} \mathrm{e}^{-}$) PAIR PRODUCTION BY ELECTRONS WITH $\overline{\mathrm{E}}=2.4 \mathrm{GEV}$

## J. Bohm et al.

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Direct production of ( $e^{+} e^{-}$)-pairs with Ey $=10 \mathrm{Mev}$ by 1.24-4 Gev electrons was investigated with the help of 24-Ifter propane bubble chamber of the High Energy Laboratory of the Ioint Institute for Nuclear Research. The obtained cross section $\sigma \mathrm{tr}_{\mathrm{r}}=55+24 \mathrm{mb}$ is in agreement with the theoretical value obtained by Bhabha. The method proposed admits a successful measuring of the $\sigma$ tr by means of a hydrogen bubble chamber.

## Par. 1. Introduction.

The process of the production of electron-positron pairs by cherged particles has been studied in a number of theoretical studies/1-8/. The first one to calculate the cross section of the process

$$
\begin{equation*}
e^{-}+z \rightarrow e^{-}+e^{+}+e^{-}+z \tag{1}
\end{equation*}
$$

to the Vaitszeker*-Williams approximation was Bhabha/1/. In studies $/ 4,5 /$, the cross section of this reaction was calculated with the help of Feinman's* diagram. In this calculation only two diagrams out of the eight possible ones were taken into consideration (Figure lc, ld) and the contribution of the remaining ones was evaluated approximately. In the same studies, the Vaitszeker\%-Williams approximation analysis was carried out and the authors came to a conclusion that the cross sections obtained by Bhabha/l/ can be compared with the experimental ones when the energy of the primary electrons ( $E_{0}$ ) is greater than 10 Gev.
[Trenslator's Note: *-- transliterated from Russian7

Study/7/ gives calculations of differential cross sections of the reaction (1) taking into consideration all eight diagrams (Figure 1) with the help of an electronic computer for $E_{0} \approx 6$ Gev and for great momentum transfers to ( $e^{+} e^{-}$)-pair.

The obtained results differ from those calculated by the Bhabha's formulas by two-three times. From this we make a conclusion that all of the diagrans (in Figure l) should be taken into consideration if $E_{0}=1-10 \mathrm{Gev}$.

Figure 2 shows total cross sections for $E_{0}=0.01-1$ 100 Gev calculated by the formulas obtained in studies/1-6/ for a molecule of propane ( $\mathrm{C}_{3} \mathrm{H} 8$ ). It can be seen from Figure 2 that the theoretical cross sections differ from one another by $\approx 30 \%$. Therefore it is interesting to calculate the cross sections of the process (1) with the help of electronic computer taking all of the feinman's diagrems correct to $\approx 5 \%$.
2. The production of the ( $e^{+} e^{-}$)-pairs by the electrons has been studied experimentally in many works (for example, see studies/9-23/). Almost all experimental results were compared with the Bhabha's theory/1/. The basic difficulty in the experimental determination of the cross section of the process (1) is the isolation of the background events occurring as a result of the conversion of the breaking $\gamma$-quanta in the material (pseudotridents) because the cross section of the breaking radiation is approximately 100 times greater than the cross section of process (1). The background of pseudotridents depended on the conditions of the experiments and was $80 \%$ of all registered ( $\mathrm{e}^{+} \mathrm{e}^{-}$)-pairs.

The first series of experinents was of a qualitative nature and was carried out with the help of diffusion chambers. the source of electrons was $\beta$-active nuclei (see, for example , survey work/9/).

The second series of studies was connected with the methods of nuclear emulsions. In most of the experiments, nuclear emulsions were exposed to cosmic rays (for example /10-16/). In the experiments of /17-18/, electron beams from accelerators ( $E=380$ and 560 Mev) were used. The work/10/ gives the calculations of the dependence of the number of pseudotridents on the distance of the vertex of the ( $e^{+} e^{-}$)-pair from the track of the primary electron in the emulsion plane. These results have been used by many researchers for isolating the tridents from the background. Figure 3 shows the mean free paths of the electrons in the flford $G-5$ emulsion for process (1) obtained in works/10-187. It can be seen from

Figure 3 that for $E_{0}>1$ Gev, the experinental values of the cross section are $2-4$ times greater than predicted by the Bhabha's theory.

The authors of study/19/ have analysed the method of measurement of reaction (I) in nuclear emulsions and consider that the disagreement betwe en the ory and experimental results was established. on the other hand, article/20/ points out the very low statistics in all of the se studies ( 60 tridents and pseudotridents), a high level of the background ( $\approx=80 \%$ ), greet inaccuracies in measuring the energies of primary electrons, due to which the experinent al results become questionable.

There are also three studies/21-23/ in which the cross section was determined with the help of electronics for $\mathrm{E}_{0}=25,230$ and 31.5 Hev , respectively. The results of stu-dies/EI-22/ are in agreement with the the ory of Bhabha. The differential cross sections at $E_{0}=31.5 \mathrm{Mev} / \varepsilon 3 /$ are one-third of the theoretically calculated cross sections of reaction (I)/I/ and are within the limits of theoretical uncertainty of cross section calculation 47. The background of pseudotridents was determined in these studies with the help of measuring the cross section of the production of ( $e^{+} e^{-}$)-pairs in targets of various thicknesses.

Thus, at the present time the process of the production of ( $e^{+} e^{-}$)-pairs by electrons has not yet been sufficiently studied experimentaliy or theoretically. Therefore, it is interesting to study this process with the help of electron accelerators by the method of bubble or spark chambers with calculations of theo retical values of the cross sections of (1) on electronic computers.

Par. 2. Experimental Methods
The production of ( $e^{+} e^{-}$)-pairs by electrons was studied with the help of a 24-Ifter propane bubble chember of the High Energy Laboratory of the Joint Institute of Nuclear Research placed in a magnetic field of 14.3 kg [kilogauss7. The chamber was irradiated with a bean of $\pi-, \mu$-mesons and electrons with $\mathrm{pc}=4.00+0.06 \mathrm{Gev} / 24 \%$. The electron content in the primary beam was determined experimentally and was found to be equal to $(2.0 \pm 0.6 \%) / 25,26 /$ and $(1.3 \pm 0.13) \% / 27 \%$.

The events were selected according to the following scanning rules:

1. The ( $\left.e^{+} e^{-}\right)-p a i x$ or a pair consisting of a positive and negative relativistic rays lies on the particle track from the beam (without clearance between them).
2. At the point of pair production, no fracture at the primary particle track is noticeable ( $\theta \leqq 1^{\circ}$ ).
3. The events are within the effective volume of the chamber (see appendix I).

A total of 43 thousand frames were scanned and 203 events were found. Iwelve thousend frames were scanned twice. The effeciencies of a single and a double scanning were found to be equal to $\epsilon_{1}=(87 \pm 3) \%$ and $\epsilon_{2}=98 \%$, respectively. one event detected in the scanned material corresponded to a cross section of ( $1.2 \pm 0.12$ ) mb .

In $80 \%$ of events, the ( $e^{+} e^{-}$)-pair was identified by the ionization, energy throw-off, $\delta$-electrons and by the rangeenergy ratio (a negative track with a lower energy was considered to belong to the pair). only one event with a secondary interaction of the primary particle was found.

The analysis of the background events connected with $c \pi_{-m e-}^{-}$ sons indicated that their contribution to the selected events did not exceed $4 \%$ (see Appendix II).

The methods of measuring electron energ in a propane bubbie chamber taking into account the fonization and radiation losses were described in the work/28/。 Lrrors in the determination of electron energy were (20-25) \%.

The cross sections of the bremsstrehlung and the production of (e+e-)-pairs by $y$-quanta in a matter are well known/c9-33/ This fact, in principle, makes it poseible to isolate quantitatively the process of a direct production of ( $e^{+} e^{-}$)-pairs

$$
\begin{equation*}
e^{-}+z \rightarrow e^{-}+e^{+}+e^{-}+z \tag{1}
\end{equation*}
$$

against the background of the bremsstrahlung accompanied by a conversion of $\gamma$-quanta into the ( $e^{+} e^{-}$)-pairs:

$$
\begin{align*}
& \mathrm{e}^{-}+\mathrm{Z}+\gamma+\mathrm{e}^{-}+z  \tag{2}\\
& \gamma+z^{\prime} \rightarrow \mathrm{e}^{+}+\mathrm{e}^{-}+Z^{\prime} . \tag{3}
\end{align*}
$$

For an experimental determination of the cross section of process (I) in a propane bubble chamber we used a method similar to the one used in nuclear emulsion/10\%.

The diagram of the background process (2) and (3) in the chamber plane ( $x, y$ ), perpendicular to the direction of the magnetic field, is show in Figure 4. A is the point of orgination of the braking $\gamma$-quantum and $B$ is the point of its conversion into an (ete-)-pair, while AB' is the trajectory of the motion of the electron in the chamber. When the conversion lengths are small ( $\ell=\overrightarrow{A B}<9 \mathrm{~cm}$ ), it is possible to ignore the angular distribution of the breaking f-cuanta, multiple scattering of the electron in propane and its energy losses at $\ell \leq 9 \mathrm{~cm}$ (see Appendix III). In this approximation, the electron, ofter its emission, moves along a circle with $R=K(E-E \gamma)$, where $K=0.233 \mathrm{~cm} / \mathrm{Mev}$ for $\mathrm{H}=14.3 \mathrm{kG}$.

The distance between point $B^{\prime}$ and $B$ is designated by $\xi$ 。 Let us assume that $\xi>0$ if B lies outside of the circle ( $R$ ), and $\varepsilon_{0}<0$ if $B$ is located within ( $R$ ). It is evident that magnitude $\xi$ has a wide distribution for the background process $(\xi \geq 0)$, while for process (I) $\xi=0$ if we ignore measurement errors.

Taking into account the approximations performed above, we obtain that

$$
\begin{equation*}
\xi=\frac{\mathrm{l}^{2}}{2 R} . \tag{4}
\end{equation*}
$$

The magnitude $\varepsilon$ was measured on a microscope MBI-9 LBiological Immersion Microscope 7 with an eyepiece micrometer (15 X 6.3). One division of the micrometer corresponded to 1.82 microns in the frame. Hereafter all values of the magnitude $\xi$ are given in the photoeraphs in the number of divisions.

The first method for measuring $\xi$ consisted in the following (see Figure 5). The coordinates $X$ of points $X_{l}$, $X_{2}, X_{3}$, and $X_{4}$ were determined with the help of an eyepiece micrometer. The rules of the event selection ensured that $\mathrm{X}_{2}<\mathrm{X}_{3}$ and $\mathrm{X}_{4}>\mathrm{X}_{1}$. The value of $\xi_{0}$, was calculated in the following way: $\xi_{1}=x_{4}-x_{3}$, if $x_{2} \geq x_{1}$ and $x_{4} \geq x_{3} ; \xi_{1}=x_{2}-x_{1}$, if $X_{4} \leq X_{3}$ and $X_{2} \leq x_{1}$. In all other instances $\varepsilon_{1}=1\left(X_{4}+X_{2}\right)$ $/ 2-\left(X_{3}+X_{1}\right) / 2$.

The second method cons isted in measuring the distance between the axes of the electron tracks and the (e+e-)-pair ( $\xi_{0}$ ). The distribution histogram of $\Delta \xi_{0}=\xi_{1}-\xi_{2}$ is shown
in figure 6. It was obtained from this that $\Delta \xi=0.15$ divisions and $\sigma(\Delta \xi)=1.9$ divisions. Since these two methods for measuring $\xi$ yielded identical results, hereafter we shall not differentiate between them。

In order to obtain quantitative results, it is necessary to determine the error in measuring the value of $\xi$. The following messurements were conducted for this purpose.

On the primery $\pi^{\prime}$-meson tracks ( $p c=4$ Gev), the value was measured by the same method as it was done by us for the events with the (e+e-)-pairs. From this we obtain the minimum possible values of the Rus /root-mean-square 7 error of $\sigma_{E_{0}}=2$ divisions (see Figure 7 ).

The value of $\xi_{0}$ was al so measured for 150 instances of occurrences of scattering of $\pi^{-m e s o n s ~(~} \mathrm{pc}=4 \mathrm{Gev}$ ) on the electrons ( $E \delta=200-680 \mathrm{Mev}$ ) in the propane bubbic chamber. The meen angle of escape of $\delta$-electrons ( $\hat{\theta}_{5}=10-2$ radian) by the order of magnitude is equal to the mean angle between the electron and the positron in the ( $\left.e^{+} e^{-}\right)$-pair. For the
 distribution histogram of events according to $\xi_{0}$ is shown in Figure 8. Here $\sum_{0}=-0.7$ divisions and $\sigma_{2}=3.7$ divisions. Due to the fact that the point of the $\pi \theta-$ - Sattering is not determined as well as the point of production of the ( $e^{+} e-$-pair (only double ionization and two secondary tracks), we consider that $\sigma_{\alpha_{0}}=3.7$ divisions is the maximum value for $\sigma_{z}$ in the case of ( $\left.e^{+} e^{-}\right)$-pairs.

The magnitude $\xi_{\text {for }} 63$ ( $\mathrm{e}^{+} e^{-}$)-pairs was also measured by two physicists. In order to make these measurements independent, another area of tracks removed by $3-4$ bubbles from the apex of the pair was usea during the second measurement of $\xi$. It was determined that the error in the difference of the two measurements, i. $\theta . \xi_{12}-\xi \delta$, is equal to $\sigma=3$ divisions. A precise detemination of the value of full ermor in the magnitude $\xi$ presents great difficulties. Hence it was assumed in this study that 2.5 divisions $\leq \sigma_{\varepsilon} \leq 3.5$ divisions, i. $\theta$. the values are included between the minimum and maximum pos. sible values. The mean wiath of the electron track was also measured. It was found to be equal to $(30 \pm 8)$ divisions.

The distribution histogram of 203 ( $\left.\mathrm{e}^{+e^{-}}\right)$-pairs according to the value of $\zeta$ is shown in Figure 9. Most of the events ( $90 \%$ ) have the value of $\zeta \geq-3$ divisions. A discussion of the effects which may influerice this aistribution is given in appendices II and III.

Par. 3. Experimental Kesults
The distribution of the backeround events (processes $(2)$ and (3) in relation to the value of $\xi$ was calculated (see Appendix IV, equation (4.IV)). When accurate to $\approx 3 \%$, the form of distribution does not depend on the enereies Ey and E. In the calculation, it was assuned that measurement errors for all values of 5 were distributea according to Gauss with a co dispersion. the histograms for the background processes ( $(\mathbb{C})$ and ( 3 ) are shown in rigure 9 by broken lines. Normalization was done according to the number of events in an interval of $\xi=3-15$ divisions.

It was found from the difference of the background and experimental histograns for on $=3$ divisions that the cross section of ( $\theta^{+} \theta^{-}$)-pair production in propane (C3H8) was on tr $=$ $=55 \pm 4 \mathrm{mb}$. The error mentioned here takes into consideration statistical fluctuations of the number of primary electrons/27/, the number of detected tridents and the number of events in the intervai of $\xi=3-15$ divisions, by which the background distribution was nomalized. If we also consider the indefiniteness in $\sigma_{\zeta}\left(2.5-3.5\right.$ division) as a systematic error, then $\sigma_{t r}=55 \pm 24 \mathrm{mb}$

Figure 10 shows a distribution histogran of the selectec events in relation to the energy of the ( $\mathrm{e}^{+} \mathrm{e}^{-}$)-pairs for Er $\geq 10 \mathrm{Mev}$ The broken lines represent a theoretically calcurated distribution of events by the energies of the ( $\mathrm{e}^{+} \Theta^{-}$)--pairs for processes (1), and (2), (3). The distribution for tridents (equation 7,IV)) was normalized for 48 events and the background curve (equation 6,IV) was obtained with the assurnption that the number of electrons in the primary bean is equal to ( $1.3 \pm 0.13$ ) $\% / 57 /$. The error shown in Figure 10 in the first energy interval ( $10-60 \mathrm{Mev}$ ) is connected with the errors in calculating the background curve ( $\approx 12^{\circ}$ ) and the obtained number of tridents ( $\approx 45 \%$ ). The calculated histogran has an error of $\approx 12 \%$ for the energy of $E_{\gamma} \geq 150 \mathrm{MeV}$. It can be seen from Figure 10 that the theoretical and the experimental histograms are well in agreement for Ey $\geq 60$ ihev. The experimental histogram for $E y=(20-60)$, Lev lies lower than the theoretical one but agrees with it within the limits of a double error.

Figure 11 shows a distribution histogram of the detected events by the energies of the primary electrons ( $\mathrm{E} \geq 1.25$ Gev). Broken lines show the thecretical histogram only for the background processes normalized for the total area. The theoretical aistribution is in a qualitative agreement with the experiment.

Pable 1 provides celculated values of the number of background events for three values of $\xi \max$ (equation (E,IV)). The cited error is connected with the indefiniteness of the lower limits of enereies $E \gamma$ min $=10 \pm 3$ Hev and
$E_{\text {min }}=1.25+0.25$ Gev and the number of electrons in the beam. It can be obtained from here that the number of tridents for $\xi_{\text {max }}=9$ divisions is equal to $14 \pm 19$, which is in agreement with the number of tridents determined by the distribution of events in relation to $\varepsilon_{0}$.

The cross section of the production process of ( $e^{+} e^{-}$)pairs by the electrons in propane was calculated according to the formulae given in the article/l/. It was found to be equal to $0_{\mathrm{tr}}=(60+5) \mathrm{mb}$ (equation $(3, I V)$ ). The shielding of the nucleus by the atomic electronswas ignored in the calculations and it was assumed that $E \gamma \min =10 \pm 3 \mathrm{Mev}$ and $\mathrm{Emin}_{\mathrm{m}}=$ $=1.25 \pm 0.25 \mathrm{Gev}$. If the shielaing is teken into account, then the cross section (1) becomes lower by about 15\%, but on the other hand, we did not consider the contribution to the cross section by the production process of the $\left(\mathrm{e}^{+} \mathrm{e}^{-}\right)$pairs in the ficla of atomic electrons which makes the cross section of the trident production $\approx 18 \%$ higher. Thus, withIn the limits of the abovementioned crrors, we obtained an agreement between the theoretical and the experimental values of the cross section for process (1).

During scanning, we found only one event with a secondary nuclear interaction which was not connected with a direct generation of an ( $e^{+} e^{-}$)-pair by a $\pi$-meson ( $\varepsilon=-26$ divisions). Hence, it is possible to conclude that the cross section of the process

$$
\begin{equation*}
\pi^{-}+C_{0}^{12} \rightarrow \pi^{-}+e^{-}+e^{+}+C_{0}^{12} \tag{5}
\end{equation*}
$$

is less than $\approx 20 \mathrm{microbar}$. A rough evaluetion of the expected cross section of the diffraction production of the ( $e^{+} e^{-}$)-pairs by $\pi$-mesons on a carbon nucleus results in a magnitude of the order of several microbarns/34\%.

Thus, the following results have been obtained in this study.

1. The cross section of the ( $e^{+} e^{-}$)-pairs production by electrons with $E=1.25-4 \mathrm{Gev}$ in propane $\left(\mathrm{C}_{3} \mathrm{H}_{8}\right)$ has been studied. The obtained value of $\sigma$ tr $=(5 b+24) \mathrm{mb}$ agrees with the theoretical $\sigma$ tr $=(60 \pm 5) \mathrm{mb} / 1 /$. Due $\bar{t}_{0}$ this fact, a aisagreement of 2-4 times between the theory and the experiment (Par.1) appears to be scarcely probable for propane.
2. The methods used in this experiment permit a successful study of the production process of ( $\theta^{*} \theta^{-}$)-pairs by electrons with the help of a hydrogen bubble chamber beceuse the background caused by pseudotridents in this case will be
about 10 times smeller $\left(I_{\text {rad }}(H)=10 \mathrm{M}, \mathrm{I}_{\mathrm{rad}}\left(\mathrm{C}_{3} \mathrm{H}_{8}\right)=1 \mathrm{~m}\right)$. In order to determine the cross section of Otr accurate to $\approx 7 \%$, 1t is necessary to have about 70 thousand frames (10 electrons per 1 freme) from a bo-cn hydrogen bubble chamber.

The authors are grateful for profitable discussions to I. M. Gramenitskiy, M. Iresh, M. I. Podgoretskiy, V. N. Strel'tsov and members of the scaming and measurement groups.

## Appendix I.-

Events of the (2) and (3)-type were selected for $\varepsilon_{\max } \approx 20$ divisions. Therefore the beginning of the effec$t$ ve maxea is 10 cm from the inlet of the chamber ( $Y=-15 \mathrm{~cm}$ ). The thickness of the inlet port of the chamber was 0.32 of the radiation unit of length. At the end of the chambor the scanning efficiency becomes lower. In view of this, the effective volume was set as follows:

$$
3 \leq X \leq 27 \mathrm{~cm}, \quad-15 \leq Y \leq 12 \mathrm{~cm}, \quad 2 \leq Z \leq 9 \mathrm{~cm} .
$$

Out of 849 registered evonts, 221 were located in this area. In 18 cases it was impossible to measure the value of $\varepsilon$ and in 21 ceses we could not determine the pulses and the direction cosines of the electrons.

The distributions of the points of ( $e^{+} e^{-}$)-pair production along the coordinates $X, Y, Z$ are shown in Figure $\eta$.

The distributions of the direction cosines $\ell, m, n$ of the prinary tracks at the points of ( $e^{+} e^{-}$)pair production are shown in Figure $\Pi \mathbb{R}_{0}$

The distributions in Figure 71 and $\cap 2$ are well in agreement with the distributions obtained in the work/27/.

Appendix II
Background Processes

1. In order to determine the number of accidental overlaps of (e+e-)-pairs caused by conversion of $\gamma$-quanta forming in the port of the chamber, 7 thousand frames were scanned on the tracks of the $\pi$-mesons. We discovered 737 ( $e^{+} e^{-}$)-pairs directed along the flight of the prinary particles ( $\pm 5^{\circ}$ ). Hence, it was determined that there should be
several occurrences of accidental overlaps on our material.
2. The cross sections of braking radiation and a direct production of the ( $e^{+} e^{-}$)-pairs by $\mu^{-}$and $\pi^{-}-$mesons are many times smaller then in the case of electrons/34/. Therefore the contribution of these processes to the $\sigma$ tr cross section was not taken into consideration.

## Appendix III

1. The mean solid angle of the braking $\gamma$-quantum escape in relation to the primary election is equal to $\theta_{1_{\gamma}} \approx \frac{m c^{2}}{E} / 29 /$ ( $E$ is the energy of primary electron, mas its mass) and almost does not depend on the energy of $\gamma$-quantum EY: The braking $\gamma$-quanta in $90 \%$ of cases escape at an angle of ${ }^{\theta} 1 \gamma \leqslant \frac{4 m c^{2}}{F}$.

The distribution of the angle a (projection of the solid angle $G y$ on the plane ( $x, y)$ ) is approximately a normail distrjbution/10/ with $\bar{a}=0$ and $\sigma_{x} \approx \frac{m c^{2}}{F}$.
2. The displacement dispersion of coordinate $x$ of the electron trajectory caused by multiple scattering in propane is equal to $x^{2} \approx 10-7 \ell^{3} \mathrm{~cm}^{2}$.
3. Between points $A$ and $B^{\prime}$ (see Figure 4), the eleatron loses its energy basically due to the braking radiation. Mean energy losses at a length of $A B^{\prime} \leq 9 \mathrm{~cm}$ are equal to $\approx 6 \%$. In view of this, the calculated total number of background events is increased (see Appendix IV)。
4. The optical system of the bubble chamber consists of two carreras (focus $f=6.1 \mathrm{~cm}$ ) mounted on a base $B=30 \mathrm{~cm}$ at a height of $H=64 \mathrm{~cm}$ above the chamber. In connection with the distribution of the discovered events along $Z$ (see Figure MIc), average increase was found to be equal to (10 10.4 ).

Within the chamber, the value $\xi$ is a vector $\vec{\xi}=\left[\xi_{x}, \xi_{y}, \xi_{i}\right]$

 lion factors, then

$$
\begin{gathered}
\vec{\xi}_{\pi}-\vec{\xi}_{\pi} \equiv\left[\left(\xi_{x \pi}-\xi_{x \pi}\right), 0\right] \quad \text { и } \epsilon \equiv \xi_{x \pi}-\xi_{x \pi}=1 \frac{B-2 x}{(H+z)^{2}} \xi_{z} . \\
-10
\end{gathered}
$$

(Point E' in Figure 4 has the coordinates $B^{\prime}=[\bar{X}, Y, Z]$ in the chamber ). In view of the prescnce of multiple scattering and the escape angle of $\Theta_{1 \gamma}, \epsilon_{\max }<1$ division and $\bar{\xi}=0$ 。

Approximately for $30 \%$ of the detected events, the value of $\xi$ was measured on the left and the right frames. The histogram of the value of $\epsilon=\left(\varepsilon_{e}^{-}-\xi_{r}\right)$ is shown in Figure $\Pi 3$. From this we obtained $\epsilon=0.15 \pm 2.2$ divisions.
5. An electron between the points $A B^{\prime}$ (see Figure 4) can be scattered by atomic electrons. Approximately in $3 \%$ of events the electron scatters at an angle of $\theta_{\delta} \geqslant 6^{\prime}\left(\Phi_{\delta} \geq 3\right.$ Mev), which results in an adaitional displacenent $\varepsilon_{\delta}>\sigma_{\mathcal{E}}$ ( $\sigma_{\xi} \equiv 3$ divisions). Thus, the above effects wich can influence the $\xi$, distribution of the detected events are unimportant.

Appendix IV
Let us introduce the following designations.
$N_{\text {A }}-$ Avogadro's number, A -- the weight of a propane gram-nolecule, p-- density of propane, $t$-- thickness of the target penetrated by the electron (in the midale of the chamber $t=0.64 \mathrm{rad}$. of length), $\Delta Y-$ widh of the interval along axis $Y$ in the chamber to the center of which the thickness is equal to $t, \mathrm{E}_{0}=4 \mathrm{Gev}$-- energy of the electrons in front of the chamber (for $t=0$ ), $\mathrm{E}-\mathrm{-}$ energy of primary electrons within the chamber, $\mu(E, Y)$-- conversion probability of a $\gamma$-quanta with energy of $E \gamma$ on one radiation length $L$ in propane/32/, No -- number of electrons felline onto the charaber, w(E,t, EO)dE -- probability that on electron with initial energy $\mathrm{E}_{\mathrm{O}}$ after passing the target of $T$ thickness will have an energy in the interval of ElE+dE/33/. O (E $Y$, E)dEY designates the cross section of the formation of a braking $\gamma$-quantum with an energy in the intervel $E \gamma, E_{Y}+d E_{y}$ by an electron with energy $E$ in propane taking into consideration the scanning and the radiation in the field of atomic electrons /31\% O $\left.\quad \sigma_{Y}, E\right)$ dEY is the cross section of a direct production of $\left(\theta^{+} \theta^{-}\right)$-pairs in propane with an energy in the interval of Ey, Ey+dEY by an glectron with energy E without taking scanning into account/l/.

$$
\begin{equation*}
M=N_{0} \frac{N_{A}}{A} \rho . \tag{I.IV}
\end{equation*}
$$

1. Total number of pseudotridents in propane is determined by the expression:

$$
\begin{align*}
& E_{0} E-m \sqrt{2 K\left(E-E_{y}\right\} \xi_{m a x}} \\
& N=M \Delta Y \sum_{t} \int_{E_{m!n}}^{0} d E E_{\gamma \ldots m i n} d E_{\gamma} \int_{0}\left(E, t, E_{0}\right) \sigma\left(E_{\gamma}, E\right) \times  \tag{2.IV}\\
& \mu\left(E_{\gamma}\right) \exp \left[-\mu\left(E_{\gamma}\right) P\right] d \mathcal{C} \text {. }
\end{align*}
$$

2. Total number of direct ( $\left.e^{+} e^{-}\right)$-pairs:

$$
N=M \Delta Y \underset{t}{\Sigma} \int_{E_{m / n}}^{E_{0}} d E E_{\gamma_{m i n}}^{E-m}\left(E, t E_{0}\right) \sigma_{B}\left(E_{y}, E\right) d E_{\gamma}
$$

3. Distribution of pseudotridents depending on $\mathcal{E}$ is determined by the expression:

$$
\begin{aligned}
& \frac{d N}{d \xi_{0 \times D}}=M \Delta Y \sum \sum_{i}^{E_{0}} d E \quad \int_{\min }^{E-m} d E_{\gamma_{\min }} \int_{0}^{\xi_{\max }} w\left(E, t, E_{0}\right) \sigma\left(E_{\gamma}, E\right) \mu\left(E_{y}\right) \times \\
& \times \exp \left[-\mu\left(E_{\gamma}\right) \sqrt{\left.2 \mathrm{~K}\left(E-E_{\gamma}\right) \xi\right]} \frac{\sqrt{2 \mathrm{R}(\mathrm{E}-\tilde{E} \gamma)}}{2 \sqrt{\xi}} \frac{1}{\sqrt{2 \pi} \sigma} \exp \left[-\frac{1}{2 \sigma^{2}}(\xi-\xi \exp )^{2}\right] \mathrm{d} \xi .\right.
\end{aligned}
$$

4. Distribution of primary energy E for pseudotridents

$$
\begin{aligned}
& \times \exp \left[-\mu\left(E_{\gamma}\right) \ell\right] \frac{1}{\sqrt{2 \pi} \beta E} \exp \left[-\frac{1}{2 \beta^{2}}\left(1-\frac{E}{E}\right)^{2}\right] d \ell .
\end{aligned}
$$

The calculations were done with $\mathrm{E}_{\mathrm{min}}=800 \mathrm{Mev}$ and $\beta=0.2$ 。 Here $\sigma=\beta \mathrm{E}$ is a RMS error of the measurement of energy E.
5. Distribution of energies of the ( $e^{+} e^{-}$)-pairs for pseudotridents

$$
\begin{aligned}
& \times \mu\left(E_{\gamma}\right) \exp \left[-\mu\left(E_{\gamma}\right) \ell\right] d \ell .
\end{aligned}
$$

6. Distribution of energies of the $\left(e^{+} e^{-}\right)$-pairs for tridents

$$
\frac{d N}{d E_{\gamma}}=M \Delta Y \sum_{i} \int_{E_{m i n}}^{E_{0}} w\left(E, t, E_{0}\right) \sigma_{B}\left(E_{\gamma}, E\right) d E .
$$

It was assumed in all calculations that $\Delta y=3 \mathrm{~cm} . \Sigma$ designates, for pseudotridents, sumation along the area of $Y=(-16.5 ; 7.5) \mathrm{cm}$ which corresponds, on the average, to the braking $\gamma$-quantum production area and for tridents $\mathrm{y}=(-13.5$; $10.5) \mathrm{cm}$.

Table 1

| $\xi_{\max }$ div. | N theory | N experiment |
| :---: | :---: | :---: |
| 20 | $204 \pm 23$ | $203 \pm 14$ |
| 15 | $176 \pm 19$ | $182 \pm 13$ |
| 9 | $137 \pm 15$ | $151 \pm 12$ |

## Interature

1. H.J. Bhabha. Proc. Roy. Soc. (London) A 152, 559 (1935).
2. G. Racah. Nuovo Cimento, 14, 93 (1937), ; 4, 112 (1937).
3. M.M. Block, D.T. King, W.T. Wada. Phys. Rev. 96, 1627 (1954).
4. T. Murota, A. Ueda, H. Tanaka. Prog. Theoret. Phys. 16, 482 (1956).
5. T. Murota, A. Ueda. Prog. Theoret. Phys. 16, 497 (1956).
6. F. F. Ternovskiy, ZnETF (Journal of Experinental and Theoretical Physics) 37, 793 (1959)。
7. E. G. Johnson. Phys. Rev. 140, 4B, 1005 (1965).
8. J. Bjorken, S. Drell, S. Frautsche, Phys. Rev. 112, 1409 (1958).
9. H. Crane, J. Halpern, Phys. Rev. 55, 838 (1938).
10. M. Koshiba, M.F. Kaplon. Phys. Rev. 97, 193 (1955).
11. J. E. Naugle, P.S. Freier. Phys. Rev. 92, 1086 (1953).
12. M. Koshiba, M.F. Kaplon. Phys. Rev. 100, 327 (1955).
13. M. Gailloud, R. Weill. Ph. Rosselet. Helv. Phys. Acta 30. 281 (1957).
14. M.M. Block, D.T. King, W.W. Wada. Phys. Rev. 96, 1627 (1954).
15.J. F. Naugle. P.S. Freier. Phys. Rev. 104, 804 (1956).
15. P.K. Aditya. Nuovo Cimento 11, 546 (1959).

17, L.L. Stanley, Bull. Amer, Phys. Soc. 2, 167 (1956).
18. F.J. Loeffler. Phys. Rev. 108, 1058 (1957).
19. R. Weill, Helv. Phys. Acta 31, 641 (1958).
20.P.K.Aditya, Nuovo Cimento 13, 1013 (1959).
21. N.S. Shiren, R.F. Post. Phys. Rev. 86, 617 (1952).
22. M. Camac. Phys. Rev. 88, 745 (1952).
23. L. Criegee, Z. Physik, 158, 433 (1960).
24. Kin Khi In, A. A. Kuznetsov, V. V. Miller. Preprint of the OIVaI (Joint Institute for Nuclear Research) C09c, Dubna, 1965.
25. V. G. Grishin, E. P. Kıstenev, Mu Chun. YaF (Nuclear Ehysics), 2, 888 (1965).
26. V. S. Pantuyev. Preprint of the OIYaI. 2100,

Dubna, 1965.
27. Ya.Böhn V. G. Grishin, V. D. Ryabtsov. Letters of ZnETF, Vol. IV, Issue 3, 106 (1966). 28. V. G. Grishin et al. Preprint of the OXYaI P-2277, Dubna, 1965.
29. H. W. Koch, J. W.Motz. Rev, Mod. Pbys., , 31, 920 (1959).
30. H. A. Bethe, W. Heitler. Proc. Roy. Soc. London. A146, 83 (1934).
31. J. A. Wheeler, W. E. Lamb. Phys. Rev. 55,858 (1939); Phys. Rev. 101,1836 (1956).
32. Ya. Bön; V. G. Grishin. Preprint of the Oryal

H-c606, Dubna, 1966.
35. V. Gaitler. Guantum Theory of Radiation.

GITHL Press, Moscow, 1956.
34. A. I. Akiyezer, I. Ya. Pomeranchuk, UFN (Progress in Physical Sciences), Vol. LXV, issue 4, 593 (1958).

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Figure 1. Feinman's diagrems for the production


Figure M1. Distribution of space coordinates of the points of production of ( $e^{+} e^{-}$)-pairs in the chamber for $20 Z$ events. a) along $X$, b) along $Y$, c) along $Z$.




Figure Ma. Distribution of primary electronic in processes (1), (2), and (3).
a) Along $\left.l=\cos \theta_{1 z}, b\right)$ Along $N=\cos O_{l y}$, c) Along $n=\cos \theta_{l z}$, where $\theta 1 x, \bar{\Theta}_{l y}, \theta_{z}$ are the angles between the direction of fiction of primary electrons at the point of production of the ( $e^{+} e^{-}$)-pair and the axes $X, Y$, and 2, respectively, which are connected with the chamber.


Figure 3. Mean free path $\ell$ cm for process (I) in nuciear emulsions IIford $\mathrm{G}-5$ obtained in various exporimente. The solid curve was calcule tea according to theory $/ \bar{I}$.

* Not legible.


Figure $\Pi 3$. Distribution inistogram for the nagnitude $\Delta \xi=\xi_{l}-\xi_{r}$
$\xi_{i}$ were measured in the left and the right frames, respectively.


Figure 4. Diagram of the background process (2), (3) In the plane of the chamber ( $x, y$ ) perpendicular to the direction of the magnetic field. A is the origination point of the braking $\gamma$-quantum. $B$ is the point of its conversion into a ( $e^{+} e^{-}$)-pair; l-- primary electron; R -- radius of the circumference along which electron 2 moves.


Figure b. Weasurement diagram of $\xi$ with the sid of an eyepiece micrometer. Straight lines show the averaging of the $X$ coordinates over 3-4 bubbles.


Figure 6. Distribution histogram for the magnitude $\Delta \xi=\xi_{1}-\xi_{2}$.


Figure 7. Distribution histogram for the magnitude ${ }_{5}^{5}$ measured on the primary $\pi^{-}$-meson tracks.


Figure 8. Distribution histogram of the magnitude $\xi$ for the process of scattering of $\pi$-nesons on electrons.


Figure 9. Distribution histogram of the magnitude $\stackrel{1}{5}$ for the detected events. Broken Ines show the histogrems of the calculated distributions of the magnitude $\xi$ for the background processes (2), (3) with three values of measurement error $\left(\sigma_{\xi}=2.5 ; 3.0\right.$ and 3.5 division).


Figure 10. Histogram of the distribution of events according to the energies of the $\left(e^{+} e^{-}\right)$--pairs for the process (1) and the background. Broken lines show a histogram of the total of calculated distributions for tridents and for the background (absclute number).


Figure 11. Histogrem of the distribution of events accordine to the energy of primary electrons. Broken lines show the calculated histogram for the background processes alone with consideration of measurement errors for the energies of the electrons (20\%) 。

- END -
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