SLAC TRANS - 102

HIGH-ENERGY INTERACTIONS OF GAMMA QUANTA AND ELECTRONS WITH NUCLEI

by

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PART I

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This year's winter seminar at the Institute of Engineering Physics is devoted to a wide range of problems in nuclear physics and the physics of elementary particles. Many of the lectures will be of interest to all the seminar participants. Much attention is devoted to the structure of the nucleus, the physics of fission, meson physics, and nuclear reactions.

The first portion of the materials is being published in time for the start of the seminar. The lectures appear in the order in which the editor received them.

We wish to thank the authors for their help in preparing the written versions of their lectures, also those members of the Institute staff who assisted with the publication of this collection.

Seminar Management

HIGH-ENERGY INTERACTIONS OF GAMMA QUANTA AND ELECTRONS WITH NUCLEI

V.N. Gribov

In a study by B.L. Ioffe, I.Ya. Pomeranchuk and this writer[1] the question was raised as to the possibility of determining experimentally what the important distances are in strong interactions at high energies. It was shown that, if the amplitude of the scattering of a particle a on a certain target b (Fig. 1) is substantially dependent on the square of the four momenta P_a^2 (the mass), then the distances important in the interaction will be the longitudinal ones, which increase with an energy of the order of $P/\mu^2 (\hbar = c = 1)$, where P is the momentum of the incident particle in the laboratory system, μ a certain characteristic mass.

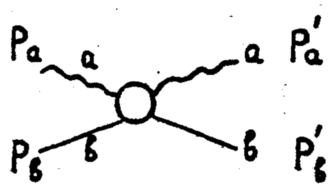

Figure 1

Unfortunately, it was found that the method of experimental investigation of the amplitude dependence on the particle "mass" through analysis of the bremsstrahlung, as proposed in ref. 1, cannot answer the question as to the important longitudinal distances because of the contractions due to the charge preservation[2]. In this paper we wish to turn our attention to the fact that studying the interaction of gamma quanta and electrons with nuclei makes it possible to ascertain experimentally what longitudinal distances are important in the electromagnetic interactions of hadrons.

Bell has reported an interesting phenomenon[3] whereby, if an interaction of gamma quanta with nucleons is dominated by vector mesons, the neutrinos by pi-mesons, in the interaction of gamma quanta and neutrinos with nuclei surface effects appear, i.e., the amplitudes contain, along with the volume terms proportional to the number of nucleons in the nucleus A, also surface terms proportional to $A^{2/3}$. At a high energy the surface terms have been found to be the governing ones, and this has been regarded as a specific property ρ or π of the dominant model.

In this paper we shall show that the character of the interaction of gamma quanta and neutrinos with nuclei and the development of the surface effects at high energies have no connection with ρ-meson or π-meson dominance but are determined solely by which distances are significant in those interactions. We shall show that, if the large longitudinal distances of the order of $\delta = P/\mu^2$, then the total cross-section, e.g., of gamma quanta with heavy nuclei, which includes only the hadron processes, will be

$$\sigma_\gamma = 2\pi R^2(1 - Z_3), \tag{1}$$

where R is the radius of the nucleus, Z_3 the charge-renormalization constant due to the hadrons. $1 - Z_3$ can be expressed in terms of the hadron part of the Lehmann density of the Green's function of the photon, or in terms of the cross-section of annihilation of electron-positron pairs on hadrons,

$$1 - Z_3 = \frac{e^2}{\pi}\int \rho(\varkappa^2)\frac{d\varkappa^2}{\varkappa^2} \quad . \tag{2}$$

Formula (1) has a simple physical meaning: $2\pi R^2$ is the total cross-section of interaction of hadrons with the nucleus, $1 - Z_3$ the length of time that a gamma quantum spends in the hadron state.

The assumption that the large distances are important in the interaction is equivalent to assuming the convergence of the integrals (2). The condition for the applicability of (1) is $\delta^2 \gg R\ell$, where ℓ is the mean path length of the hadrons in the nucleus. If the characteristic mass μ is of the order of the meson mass ρ, and the path length is of the order of $1/m_\pi$ (m_π being the mass of the pi-meson), the surface effect should develop at an energy exceeding 10 GeV.

The arising of the surface effects and formula (1) can be understood almost without calculations, as follows. Let us imagine that a gamma quantum interacts with the nucleons of the nucleus in the following manner: first it virtually decays into hadrons, the hadrons then interacting with the nucleons of the nucleus. Let us assume that this fluctuation lasts for a length of time δ . Then the total cross-section of the interaction of gamma quanta with the nucleus will be determined by the probability of a gamma quantum's hitting the nucleus, πR^2 , the probability that the fluctuation will develop inside the nucleus, $R/137\delta$, and the probability that the hadrons forming will have time to complete an interaction with any nucleon of the nucleus, δ/ℓ . Hence σ_γ will be of the order of $\pi R^2 \cdot R/137\delta \cdot \delta/\ell \sim 1/137\ \pi R^2 \cdot R/\ell \sim A \cdot \sigma_{\gamma N}$. This reasoning is valid, however, only if $\delta \lesssim \ell$. If in the coordinate system in which the quantum has a low energy the duration of the fluctution is of the order of $1/\mu$, then in the laboratory system, in which the quantum has the momentum P , the duration of the fluctuation will be $\delta \simeq P/\mu^2$, i.e., increases with the energy of the quantum. If $R > \delta > \ell$, the cross-section of the quantum's interaction will be of the order of $\pi R^2 \cdot R/137\delta$, i.e., will decrease as the energy increases. Actually, as Bell in essence pointed out[3], the possibility of development of a fluctuation having the dimension δ greater than the free path ℓ is depressed by quantum-mechanical interference by a factor of δ/ℓ , and the cross-section will be of the order of $\pi R^2 \frac{R\ell}{137\delta^2}$, i.e., decreases with increasing energy still faster. Under these conditions one can no longer neglect the probability that a fluctuation may develop outside the nucleus. When δ becomes greater than the radius of the nucleus, all fluctuations will mainly develop outside the nucleus, and the hadrons that formed in one of the 137 cases will collide with the nucleus with a cross-section πR^2, i.e., the cross-section of the quantum's interaction will be of the order of $1/137 \cdot \pi R^2$. In this way we arrive at a cross-section of type (1).

The presence in (1) of the factor $1 - Z_3$ is also easy to explain if the amplitude of the gamma quantum's elastic forward scattering on the nucleus, which determines the total cross-section, is visualized with the diagram in Figure 2.

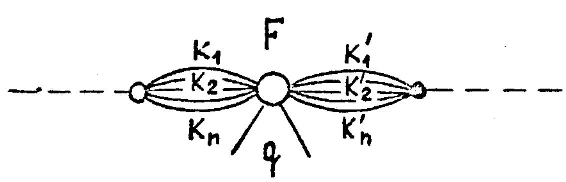

Figure 2

The amplitude F of the scattering of a beam of hadrons on a nucleus of radius R varies appreciably with variation of the transverse momenta of the particles by a quantity of the order of $1/R$, rapidly decreasing when the momenta vary greatly. Since $1/R$ is much smaller than the scale of momenta of significance in the diagram of figure 2, the momenta of the particles K_i' differ almost not at all from K_i. The usual amplitude of the elastic scattering of a single particle can be written under analogous conditions in the form $i2\pi R^2\delta(q)$. The corresponding amplitude of the scattering of a group of particles is proportional to $i\cdot 2\pi R^2 \Pi\delta(K_i - K_i')$. As a result, the diagram in figure 2 is equivalent to the diagram in figure 3 multiplied by $i\cdot 2\pi R^2$. The diagram in figure 3 defines the charge renormalization.

Figure 3

Such a picture of the interaction is obtained by assuming that a low-energy quantum virtually decays into small masses of the order of μ. But it is possible that relatively frequently a quantum decays into very large masses. This corresponds to a divergence of the integral for $1 - Z_3$. The existence of such fluctuations involving large masses cannot possibly occur with a low quantum energy if the path length ℓ for the states with the large masses is great. But as the energy increases to where the length of the fluctuation δ, even for very large masses, becomes comparable with the path length ℓ in the nucleus, such masses start to participate in the interaction with a cross-section of the order of πR^2. Only masses for which the length of the fluctuation δ at any energy is less than the path length can, at any energy, make a contribution to the cross-section that is proportional to the volume of the nucleus. In a case in which the mass integral for $1 - Z_3$ diverges, the cross-section of the interaction of a gamma quantum with a nucleus can be written in the form:

$$\sigma_\gamma = 2\pi R^2 [1 - Z_3(\varkappa_0^2)] + \sigma_\gamma^V \quad , \tag{3}$$

$$1 - Z_3(\varkappa_0^2) = \frac{e^2}{\pi} \int_0^{\varkappa_0^2} \rho(\varkappa^2) \frac{d\varkappa^2}{\varkappa^2} \quad . \tag{4}$$

The mass x_o at which the integral (4) is cut off is defined by the condition

$$\delta(\varkappa^2) = \frac{2P}{\varkappa_0^2} \approx \ell(\varkappa_0^2, P), \tag{5}$$

where $\ell(x_o^2,P)$ is the path length of a group of particles having the total mass x^2 and momentum P. A proper definition of $\ell(x^2,P)$ will be given in the text.

If the integral (2) diverges logarithmically, i.e., the cross-section of annihilation of e^+, e^- on hadrons has the same order of magnitude as the cross-section of annihilation into leptons, then

$$1 - Z_3(\varkappa_0^2) \approx \frac{e^2}{\pi} \rho(\infty) \ln \frac{\varkappa_0^2}{\mu^2} \quad , \tag{6}$$

where μ is a certain constant. The second term in (3) is proportional to the volume of the nucleus and has the order

$$\sigma_\gamma^V \sim e^2 \pi R^2 \frac{R}{\ell(\varkappa_0^2, P)} \quad . \tag{7}$$

If $\ell(x_o^2,P)$ is not dependent on the energy, which is possible if $\ell(x^2,P) = \ell(x^2/P^2)$, then $x_o^2 \sim P$ and σ_γ^V do not depend on the energy, and the surface term increases logarithmically. If $\ell(x_o^2,P)$ increases with increasing energy, condition (5) will occur only up to energies at which $\ell(x_o^2,P) > R$. When $\ell(x_o^2,P) > R$, the cutoff of x_o^2 is determined by the condition

$$\ell(x_o^2,P) = R . \tag{8}$$

Here the volume term has the order $e^2\pi R^2$, while the surface term, which is the main one, either winds up a constant, if $\ell(x_o^2,P)$ is not dependent on P, or continues its logarithmic increase.

By studying experimentally the dependence of σ_γ on the energy and on A, we can isolate the two terms and find the dependence of x_o^2 on P. The dependence of x_o^2 on P reflects the energy dependence of the longitudinal distances $\delta = 2P/x^2$, important in the interaction of the gamma quanta with the nucleons. If x_o^2 increases with increasing P, but more slowly than P $(x_o^2/2P \to 0)$, then important will be the large longitudinal distances that increase with the energy but are smaller than with a finite $1 - Z_3$. If $x_o^2 \sim P\mu$, then the important ones are the longitudinal distances up to $1/\mu$. The increase of σ_γ with the energy can continue up to those energies at which $e^2/\pi \cdot \ell n\, x_o^2/\mu \sim 1$, and the perturbation theory for electromagnetic interactions becomes inapplicable.

If the integral (2) should diverge faster than logarithmically, the cross-section would increase exponentially.* The perturbation theory in electrodynamics would become inapplicable at energies much lower than usually assumed. We are not considering this possibility.

For a given P^2 , P_o the cross-section of the interaction of electrons with nuclei, described by the diagram in figure 4, will have the same properties. The only difference is that, instead of by $1 - Z_3$, it will be determined by the magnitude of the polarization operator (Fig. 3) when $P^2 \neq 0$.

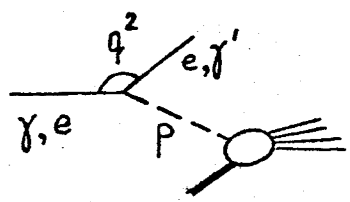

Figure 4

We shall show that

$$d\sigma^s = e^4 \cdot 2\pi R^2 \cdot \frac{1}{P^4}\left\{\left[4m^2 + p^2 + \frac{p^2}{P_0^2}(K_0 + K_0')^2\right]\cdot \Pi_1 + \right. \tag{9}$$

$$\left. + p^2\left[4m^2 + 2p^2\right]\cdot \Pi_2\right\} \cdot \frac{P_0}{2K_0K_0'}\,\frac{d^3K'}{(2\pi)^3} \quad ,$$

$$d\sigma_\gamma = d\sigma^s + d\sigma^v \quad , \tag{10}$$

where

$$\Pi_1(p^2, \varkappa_0^2) = \int^{\varkappa_0^2} \frac{\rho(\varkappa^2)\,d\ {}^2}{\varkappa^2 - p^2} \quad , \quad \Pi_2(p^2) = \int^{\varkappa^2} \frac{\rho(\varkappa^2)\,d\varkappa^2}{(\varkappa^2 - p^2)^2} \quad . \tag{11}$$

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* Translator's Note: Literally the Russian here says "powerwise" or "in a power manner."

It is interesting to note that in the case in which the important ones are the distances less than $2P/\mu^2$, i.e., in which the integral for Π_1 diverges, $\Pi_1(P^2,x_o^2)$ is not dependent on P^2 , whence the surface term in the cross-section does not depend on P^2 when $P^2<<x_o^2$.

The above results are obtained on the assumption that the interaction of fast hadrons with a nucleus can be regarded as the result of successive interactions with the nucleons of the nucleus, and the interaction of the nucleons of the nucleus can be described with the aid of pair correlations. This latter assumption is apparently not fundamental; dispensing with it would merely complicate the analysis.

1. *Interaction of Hadrons with a Nucleus at High Energies*

As pointed out above, the interaction of a gamma quantum with a nucleus at high energies occurs in such a way that the gamma quantum first converts to hadrons, and the hadrons then interact with the nucleus. So before turning our attention to the interaction of the gamma quantum with the nucleus, we shall discuss how the description of the interaction of the hadrons with the nucleus changes as one moves toward high energies as compared with the description at low energies. The total cross-sections and elastic interaction of hadrons with a nucleus at not very high energies is usually described either with an optical model or with the aid of the Glauber theory of successive collisions. These two approaches are similar if we take into account only the pair correlations of the nucleons in the nucleus, and they boil down to considering Feynman diagrams of the type in figure 5, which describe successive elastic scatterings on the nucleons of the nucleus. If we assume that the mean momenta of the nucleons in the nucleus are much smaller than the momenta that figure in the interaction of the hadrons, for the low energies we can consider only the elastic scatterings, since the inelastic processes require large transfers of momentum, which lead to a breakup of the nucleus.

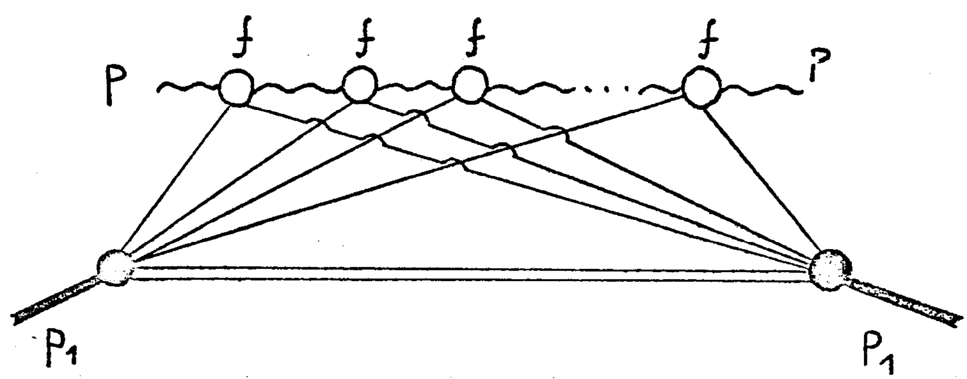

Figure 5

It is shown that as the energy increases, when the transferred momenta necessary for the production of particles decrease and become of the order of the momenta of the nucleons in the nucleus, the inelastic processes and diagrams in figure 6 do have to be taken into account.

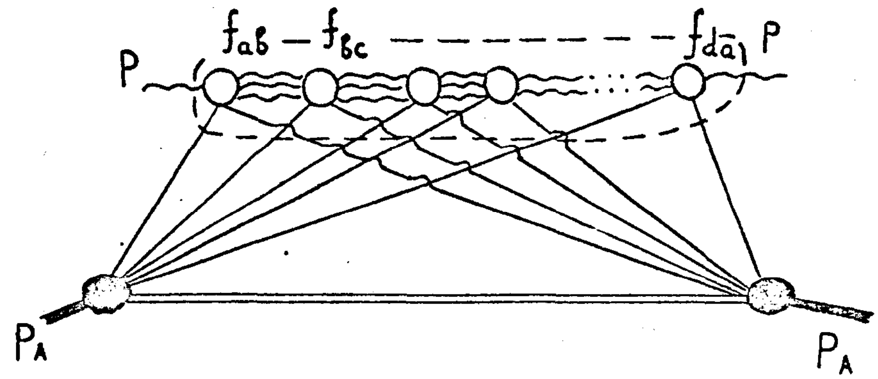

Figure 6

Before turning our attention to the diagrams in figure 6 and their influence on the character of the total cross-sections, we shall look briefly at how one calculates the total cross-sections for high energies, but energies such that the inelastic processes are still unimportant. Everywhere in what follows we shall disregard the contraction of the diffraction cone in the hadron processes. Calculation of the diagram in figure 5 for these energies yields the following. The amplitude of the forward scattering F_n corresponding to the nth rescattering is (see the appendix, for instance):

$$F^{(n)} = \left(\frac{iN}{4pmV}\right)^{n-1} \cdot \frac{N^2}{V} \int d^2\rho_1 \, dz_1 \ldots dz_n \, f \cdot æ(z_1 - z_2) \ldots æ(z_{n-1} - z_n) f, \tag{12}$$

where N is the number of nucleons in the nucleus; V the volume of the nucleus; P the momentum of the incident particle; ρ_i , Z_i the coordinates of the nucleons, Z_i in the direction of the momentum of the incident particle, ρ_i perpendicularly to P ; $x(Z_i - Z_{i+1})$ the correlation function of two nucleons in the nucleus $x(\infty) = 1$; f the scattering amplitude; m the nucleon mass; $Z_1 > Z_2 \ldots > Z_n$.

If the amplitude F , equal to

$$F = \sum_n F^{(n)} , \tag{13}$$

is written in the form

$$F = \frac{N^2}{V} \int F(\rho, z)\, dV , \tag{14}$$

we can then be sure that F(ρ,Z) satisfies the equation

$$F(\rho, z) = f + \frac{iN}{4pmV} \int_{-z_0(\rho)}^{z} dz' f \, \ae(z - z')\, F(\rho, z') , \tag{15}$$

$$z_0 = \sqrt{R^2 - \rho^2} ,$$

which is the analogue of the equations for the optical model with the scattering amplitude f , which plays the role of a potential. If we neglect the correlations, i.e., set x = 1 , from (15) we get the trivial result

$$F(\rho, z) = f e^{-\frac{1}{l}[z + z_0(\rho)]} , \tag{16}$$

$$\frac{1}{l} = -\frac{iN}{4pmV} f \tag{17}$$

and

$$F = \frac{N^2}{V} \int F(\rho, z)\, dV = 2pmN \cdot 2\pi R^2 \cdot i , \tag{18}$$

$$\sigma_t = 2\pi R^2 .$$

The idea behind our further analysis is that an equation of type (15) will remain valid with a small variation if by the amplitudes f and $F(\rho, Z)$ we mean the amplitudes of the interaction of groups of hadrons with a nucleon, with a transition to other groups of hadrons, which enter the diagrams in figure 6.

In the appendix we calculate the diagram in figure 6 on the assumption that the nucleons in the nucleus are nonrelativistic, their momenta much smaller than the transferred momenta that figure in the strong interactions at high energies. The latter is equivalent to assuming that the path length of the hadrons in the nucleus is greater than the distance between the nucleons. It is assumed that under these conditions we may confine ourselves to considering only the correlations between the nucleons that participate in two successive collisions. For the amplitude of a process that includes interaction with n nucleons the result can be written in the form

$$F_{aa}^{(n)} = \left(\frac{iN}{4pmV}\right)^{n-1} \frac{N^2}{V} \sum_{b,c,d\ldots} \int d^2\rho_1 dz_1 \ldots dz_n f_{ab} \cdot e^{-iq_z^b (z_1 - z_2)} \cdot \varkappa(z_1 - z_2) f_{bc} \cdot e^{-iq_z^c (z_2 - z_3)} \ldots f_{da}, \tag{19}$$

f_{bc} being the amplitude of a process corresponding to the diagram in figure 7. We sum over the real intermediate states

$$q_z^b = \frac{m_b^2 - \mu^2}{2p} , \tag{20}$$

where m_g is the mass of the intermediate state, μ the mass of the incident particle.

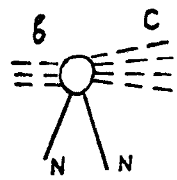

Figure 7

We introduce the operator $F(\rho,Z)$, whose matrix elements between any states are defined by the equality

$$F_{ab}(\rho,z)=\sum_n\left(\frac{iN}{4pmV}\right)^{n-1}\sum_{c,d,e..}\int dz_2\ldots dz_n f_{ac}\, e^{-iq_z^c(z_2-z_3)}\times$$
$$\times\,\text{\ae}(z_2-z_3)f_{ed}\ldots e^{-iq_z^e(z_{n-1}-z_n)}\cdot\text{\ae}(z_{n-1}-z_n)f_{eb}\ . \tag{21}$$

The operator $F(\rho,Z)$ satisfies the equation

$$F(\rho,z)=f+i\zeta\int_{-z_0(\rho)}^{z}dz' f\cdot e^{-iq_z(z-z')}\,\text{\ae}(z-z')F(\rho,z')\ , \tag{22}$$

$$\zeta=\frac{N}{4pmV}\ ,\qquad (q_z)_{cd}=\delta_{cd}\cdot\frac{m_c^2-\mu^2}{2p}\ .$$

This equation describes all possible transformations in the beam of hadrons on interacting with the nucleons of the nucleus. It converts to (15) if only one intermediate state with a mass m , equal to the mass of the incident particle, is possible. A symbolic solution of this equation is easily found if $F(\rho,Z)$ is written in the form:

$$F(\rho,z)=\frac{1}{2i\pi}\int_{\alpha-i\infty}^{\alpha+i\infty}d\xi\cdot e^{\xi(z+z_0)}\,F(\xi)\quad, \tag{23}$$

$$F(\xi)=\frac{1}{1-i\zeta\cdot f\cdot \varkappa(\xi+iq_z)}\cdot\frac{f}{\xi}\quad, \tag{24}$$

$$\varkappa(\xi)=\int_0^\infty e^{-\xi z}\ \varkappa(z)\,dz\quad . \tag{25}$$

The scattering amplitude is

$$F=\frac{N^2}{V}\int F(\rho,z)\,dV=\frac{N^2}{V}\int d^2\rho\int\frac{d\xi}{2\pi i}\cdot\frac{e^{+2z_0\xi}-1}{\xi}\,F(\xi)\quad . \tag{26}$$

We shall write (24) in the form

$$F(\xi)=\frac{1}{1-i\zeta f\varkappa(iq)}\cdot\frac{f}{\xi}+\frac{1}{1-i\zeta f\varkappa(iq)}\cdot i\zeta f\varkappa'(iq)\frac{1}{1-i\zeta f\varkappa(iq)}f+\tilde{F}(\xi), \tag{27}$$

$$\tilde{F}(0)=0\ .$$

Substituting (27) into (26) and integrating, we get

$$F=N^2\frac{f}{1-i\zeta f\varkappa(iq)}+\frac{N^2}{V}\pi R^2\frac{1}{1-i\zeta f\varkappa(iq)}\cdot i\zeta f\varkappa'(iq)\times$$
$$\times\frac{1}{1-i\zeta f\varkappa(iq)}f+\tilde{F}\quad, \tag{28}$$

$$\tilde{F}=\frac{N^2}{V}\int d^2\rho\int_{-\alpha-i\infty}^{-\alpha+i\infty}\frac{d\xi}{2\pi i}\ \frac{e^{2z_0\xi}}{\xi^2}\cdot\frac{1}{1-i\zeta f\varkappa(\xi+iq)}f\quad . \tag{29}$$

The first term, proportional to the number of nucleons in the nucleus, actually equals zero when applied to a real state having a mass μ^2, for then $q = 0$

$$[x(iq)\sim\frac{1}{iq}\ ,\ q\to 0]\ .$$

For the same reason the second term equals $2PmN \cdot i2\pi R^2$.

The last term in (28) $\tilde{F}$ is defined by the poles of the integrand in (29) . These poles are placed for negative $\xi = -\bar{\xi} \equiv -\frac{1}{\ell}$ and determine the damping $F(Z,\rho)$. Here $\tilde{F}$ is of the order $\frac{N^2}{V} f\ell$ where $\ell = \frac{1}{\xi}$ is the path length. Therefore

$$\begin{aligned} F &= 2pmN\left[2\pi R^2 \cdot i + O\left(\frac{N}{V}\sigma\ell^3\right)\right] , \\ f &\sim 2pm \cdot i\sigma , \end{aligned} \tag{30}$$

Hence the amplitude of the scattering of a group of particles on a sufficiently large nucleus is a diagonal operator, and the total cross-section, as at lower energies, is $2\pi R^2$.

We stress in conclusion that the volume absorption equals zero only for a real state of the incident particle, i.e., for the amplitude of scattering on a mass surface. If P^2 of the incident particle does not coincide with μ^2 of the intermediate state, then $q_Z \neq 0$, and we get a volume absorption proportional to $\left(\frac{P^2 - \mu^2}{2P}\ell\right)^2 N$. This means that for low energies the scattering amplitude of a virtual particle is substantially different from the amplitude of scattering on a mass surface.

Translator's Note:

The author's hand-written notation seems occasionally ambiguous. He appears, for instance, to switch back and forth between P and p .

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2. Interaction of Gamma Quanta with Nuclei - The Large Distances

In this section we shall give a derivation of formula (1) in which the concept of the distances at which interaction occurs will explicitly figure. For this we shall write the amplitude of the virtual forward Compton effect $F_{\nu\nu}(s,x^2)$ in the form of an integral of the time-ordered product of the electromagnetic currents.

$$F_{\nu\nu}(s,\varkappa^2) = ie^2\int e^{ipx}\langle A|Tj_\nu(-x_1)j_\nu(x_2)|A\rangle d^4x_1 d^4x_2, \tag{31}$$

where $x^2 = P^2$ is the quantum "mass".* As was discussed in ref. 1, the amplitude $F_{\nu\nu}(s,x^2)$ is dependent on x^2 at high energies s only in the case in which large longitudinal distances of the order P/μ^2 are important in the integral (31), μ being a certain characteristic mass, P the quantum momentum in the laboratory system $S \sim 2PM$, M the mass of the nucleus.

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* Translator's Note:
Again the author's handwriting is troublesome. His flowerlike symbol $\varkappa$ has been guessed by the translator to denote a lower-case x, but in equation (31) we find it mixed in with a more conventional-looking x. Are both symbols suppose to be x? And again, the line that the author draws (if any) between lower-case and upper-case Roman letters is extremely unclear.

Actually, by writing the index of the exponential in (31) as

$$px = p_0 t - pz \approx p_0(t-z) + \frac{\varkappa^2}{2p_0} z,$$

we see that important in (31) are the $t - z \sim \frac{1}{P_o}$, and $F_{\nu\nu}(s,x^2)$ depends on x^2 only if $z \sim t \sim \frac{P}{\mu^2}$ are significant. Assuming this to be the case and making use of the reduction formulae, we write $\langle A|Tj_\nu(x_1)j_\nu(x_2)|A\rangle$ in the form:

$$\langle A|Tj_\nu(x_1)j_\nu(x_2)|A\rangle = \\ = i\int e^{-ip_A(y-y')}d^4y\,d^4y'\langle 0|Tj_\nu(x_1)j_\nu(x_2)U_A(y')\bar{U}_A(y)|0\rangle\,, \tag{32}$$

where $U_A(y')$, $\bar{U}_A(y)$ are operational sources of the nuclear field. Substituting (32)into (31) and changing the variables, we get:

$$F_{\nu\nu}(s,\varkappa^2) = -e^2\int e^{-ip(x_1-x_2)+ip_A\xi} \times \\ \times \langle 0|Tj_\nu(x_1)U(0)\bar{U}(\xi)j_\nu(x_2)|0\rangle d^4x_1\,d^4x_2\,d^4\xi \quad . \tag{33}$$

Remembering that $X_{10} - X_{20} \to \pm\infty$ and $\xi_o \sim \frac{1}{M}$, we may consider that in (33) the points 0, $\xi_o \sim \frac{1}{M}$ are between the points X_{10} and X_{20}, and in place of (33) we can write

$$F_{\nu\nu}(s,\varkappa^2) = -e^2\int e^{ip(x_2-x_1)+ip_A\xi} \times \\ \times \langle 0|j_\nu(x_2)T(U(0)\bar{U}(\xi))j_\nu(x_1)|0\rangle d^4x_1\,d^4x_2\,d^4\xi \\ + x_1 \to x_2\;, \tag{34}$$

or, expanding the product of operators with respect to the intermediate states, we get

$$F_{\nu\nu}(s,\varkappa^2) = e^2\sum_{\substack{n,m \\ p_n=p_m=p}}\frac{\langle 0|j_\nu|n\rangle}{p_{no}-p_o}\langle n|\int e^{ip_A\xi}TU(0)\bar{U}(\xi)d^4\xi|m\rangle\frac{\langle m|j_\nu|0\rangle}{p_{mo}-p_o} \\ + p \to -p \quad . \tag{35}$$

The expression $\langle n | \int e^{ip_A \xi} T\, U(0) \bar{U}(\xi) d^4\xi | m \rangle = F_{nm}$. F_{nm} is the amplitude, discussed in the preceding section, of the forward scattering of a group of particles with momentum P on a nucleus. Keeping in mind that $p_o \approx p + \frac{x^2}{2p}$, $p_{no} \approx p + \frac{M_n^2}{2p}$, $p_{mo} = p + \frac{M_m^2}{2p}$, we get

$$F_{\nu\nu}(s, x^2) = e^2 \sum_{n,m} \frac{\langle 0 | j_\nu | n \rangle}{M_n^2 - x^2} F_{nm} \frac{\langle m | j_\nu | 0 \rangle}{M_m^2 - x^2} (2p)^2 \ . \tag{36}$$

The second term in (35) , corresponding to the change $p \to -p$, is small since the denominator $p_n + p_o \approx 2p$ instead of $(M_n^2 - x^2)/2p$. Small, too, for an analogous reason are the contributions of the other regions in (33) with a time ratio in contrast to (34) if the mass integrals of type (34) converge. As has been shown,

$$F_{nm} = 2\pi R^2 i \cdot 2pM \cdot \delta(n-m) \ . \tag{37}$$

Substituting (37) into (36) , we get:

$$F_{\nu\nu}(s, x^2) = 2\pi R^2 i \cdot 2pM \cdot e^2 \int \frac{dM_n^2}{(M_n^2 - x^2)^2} \rho_{\nu\nu}(M_n^2) \ , \tag{38}$$

$$\begin{aligned} \rho_{\alpha\beta}(p_n) &= \sum \langle 0 | j_\alpha(0) | n \rangle \langle n | j_\beta(0) | 0 \rangle (2\pi)^4 \delta(\sum K_i - p_n) = \\ &= (-\delta_{\alpha\beta} p_n^2 - p_{n\alpha} p_{n\beta}) \rho(p_n^2) \ , \end{aligned} \tag{39}$$

$$\rho_{\nu\nu} = e_\alpha^\nu e_\beta^\nu \rho_{\alpha\beta}(p_n) = p_n^2 \rho(p_n^2) \ ,$$

where e_α^ν is the quantum-polarization vector, $\rho(M_n^2)/M_n^2$ the spectral density of the Green's function of the photon. Hence the total cross-section of the interaction of a real photon with a nucleus is:

$$\sigma_t = 2\pi R^2 (Z_3^{-1} - 1) \approx 2\pi R^2 (1 - Z_3) \tag{40}$$

$$1 - Z_3 = e^2 \int \frac{dM^2}{M^2} \rho(M^2) \quad . \tag{40a}$$

In deriving (40) we assumed that the integral for $1 - Z_3$ converges. To estimate with what degree of accuracy (40) is valid, and to consider the case in which the integral for $1 - Z_3$ diverges, it is found more convenient first to express the amplitude of the scattering of the quantum on the nucleus in terms of the amplitudes of the interaction of the quantum and hadrons with the nucleons of the nucleus, then to use formulae of type (33) or dispersion relations with respect to the mass, but now for the amplitudes of the interaction of the quantum with the nucleon. We do this in the next section.

3. *Interaction of a Gamma Quantum with a Nucleus - The Not Very Great Distances*

Under the assumptions formulated in section 1 the amplitude of the forward scattering of a gamma quantum on a nucleus $F_{\gamma\gamma}(s)$ can be depicted in the form of a set of diagrams like those in figure 6. $F_{\gamma\gamma}$ can be written as (19) , the only difference being that the state (a) is the gamma quantum, and f_{ab} and f_{da} are replaced by $f_{\gamma b}$, $f_{d\gamma}$. The other amplitudes are, as before, amplitudes of the hadron processes.

The parameters q_z for a real quantum are equal to $m_b^2/2p$. By analogy with (21) we introduce the amplitudes $F_{\gamma\gamma}(\rho,z)$ and $F_{\gamma\alpha}(\rho,z)$, where a is the hadron state. These amplitudes satisfy equations similar to (22):

$$F_{\gamma\gamma}(\rho,z) = f_{\gamma\gamma} + i\zeta \int_{-z_0(\rho)}^{z} dz' f_\gamma e^{-iq_z(z-z')} \varkappa(z-z') F_\gamma(z',\rho), \tag{41a}$$

$$F_\gamma(\rho,z) = f_\gamma + i\zeta \int_{-z_0(\rho)}^{z} dz' f e^{-iq_z(z-z')} \varkappa(z-z') F_\gamma(\rho,z'), \tag{41b}$$

where $f_{\gamma\gamma}$ is the amplitude of the Compton effect on the nucleon, f_γ the amplitude of photoproduction of the hadron state. Equations (41) are written in operator form with respect to the hadron states. Solving equations (41) through recourse to the momentum expression (24), we get

$$F_\gamma(\xi) = \frac{1}{1 - i\zeta f \varkappa(\xi + iq_z)} f_\gamma , \tag{42a}$$

$$F_{\gamma\gamma}(\xi) = \frac{f_{\gamma\gamma}}{\xi} + i\zeta f_\gamma \varkappa(\xi + iq_z) \frac{1}{1 - i\zeta f \varkappa(\xi + iq_z)} \frac{f_\gamma}{\xi} . \tag{42b}$$

Hence, by calculating $F_{\gamma\gamma} = \frac{N^2}{V}\int F_{\gamma\gamma}(\rho,z)\,dV$, , instead of (28) we can write

$$F_{\gamma\gamma} = N^2 \left[f_{\gamma\gamma} + i\zeta f_\gamma \varkappa(iq_z) \frac{1}{1 - i\zeta f \varkappa(iq_z)} f_\gamma \right] + \tag{43}$$

$$+ \frac{N^2}{V} \pi R^2 \cdot i\zeta f_\gamma \varkappa(iq_z) \frac{1}{1 - i\zeta f \varkappa(iq_z)} \frac{\varkappa'(iq)}{\varkappa(iq)} \frac{1}{1 - i\zeta f \varkappa(iq_z)} f_\gamma + \tilde{F}_{\gamma\gamma},$$

$$(q_z)_{a\beta} = \delta_{a\beta} \frac{m_\beta^2}{2p} .$$

The first term in (42) is the volume interaction $F^V_{\gamma\gamma}$, the second being the surface one $F^S_{\gamma\gamma}$, while the third is small like (30) . Unlike (30) , the volume term does not equal zero and the surface term does not equal $2\pi R^2 \cdot i2pmN$, since f_γ is different from f and $q_z \neq 0$ in application to a real state. We shall estimate the order of magnitude of the individual terms in (42). We note that in order of magnitude

$$-i\zeta f \sim -i \frac{N}{V \cdot 4pm} \cdot i 2pm\sigma = \frac{1}{2} \frac{N}{V} \sigma \sim \frac{1}{2\ell} ,$$

where σ is the cross-section of the hadron process, ℓ the path length. The correlation function $x(iq_z)$ depends on iq_z and on the mean distance between particles r_o . When $q_z r_o << 1$, $x(iq_z) = 1/iq_z$, and when $q_z r_o > 1$, $x(iq)$ decreases. The characteristic denominators determining the dependence of $F^V_{\gamma\gamma}$ and $F^S_{\gamma\gamma}$ on the quantity q_z in the intermediate states have the form $2\ell/x(iq_z) + 1$, and for small q_z are equal to $2i\ell q_z + 1$. These denominators are of

the order of unity when $q_z < \frac{1}{\ell}$ and are large when $q_z >> \frac{1}{\ell}$.
The contribution to $F^V_{\gamma\gamma}$, $F^S_{\gamma\gamma}$ from the regions $q_z << \frac{1}{\ell}$ and $q_z >> \ell$ can be written, respectively, as

$$F^V_{\gamma\gamma} = N^2 \left[f_{\gamma\gamma} - f_\gamma \frac{1}{f} f_\gamma \right] , \tag{44}$$

$$F^S_{\gamma\gamma} = i \frac{N^2}{V} \pi R^2 f_\gamma \frac{1}{\zeta f^2} f_\gamma$$

when $q_z << \frac{1}{\ell}$;

$$F^V_{\gamma\gamma} = N^2 \left[f_{\gamma\gamma} + \zeta f_\gamma \frac{1}{q_z} \left(1 - \frac{1}{2iq_z \ell}\right) f_\gamma \right]$$

$$F^S_{\gamma\gamma} = \frac{N^2}{V} \pi R^2 i \zeta f_\gamma \frac{1}{q_z^2} f_\gamma \quad , \tag{45}$$

when $q_z >> \frac{1}{\ell}$,

where $\frac{1}{2\ell} \equiv -i \zeta f$. If, as happens in the ρ-dominant model[3], $f_\gamma = g_\gamma f$ and $f_{\gamma\gamma} = g_\gamma f g_\gamma$ and $q_z \to 0$, since $q_z = \frac{m_\rho^2}{2p}$, then in accordance with $F^V_{\gamma\gamma} = 0$ we have

$$F^S_{\gamma\gamma} = 2pMi \cdot 2\pi R^2 \cdot g_\gamma^2 \ . \tag{46}$$

In order to calculate $F_{\gamma\gamma}$ without the assumption of ρ-dominance, we can either use formulae of type (31), but for the amplitudes of the interaction of the gamma quantum with the nucleon, or the dispersion relation with respect to the quantum masses for the same amplitudes.

In this section we use the dispersion relations. We shall assume that for the amplitude of the forward scattering of a virtual quantum having a mass P_1^2 , which is transformed into a quantum having a mass P_2^2 , there is a nonsubtractive dispersion relation with respect to the quantum masses which has the form

$$f_\gamma(s, p_1^2, p_2^2) = \frac{1}{\pi^2} \int \frac{d\varkappa_1^2 \, d\varkappa_2^2 \, \tilde{f}_{\gamma\gamma}(s, \varkappa_1^2, \varkappa_2^2)}{(\varkappa_1^2 - p_1^2)(\varkappa_2^2 - p_2^2)} , \tag{47}$$

$$\tilde{f}_{\gamma\gamma}(s_1, p_1^2, p_2^2) = \sum_{n,m} \Gamma_\gamma(K_1 \ldots K_n) \delta^4(p_1 - \Sigma K_i) \times \tag{48}$$

$$\times f_{nm}(K_1 \ldots K_n, K_1' \ldots K_m', q) \delta^4(p_2 - \Sigma K_i') \Gamma_\gamma(K_1' \ldots K_m') ,$$

where $f_{n,m}(K_1 \ldots K_n, K_1' \ldots K_m', q)$ are the amplitudes of the hadron processes on a nucleon with the momentum transferred to the nucleon q , $q^2 = + q_z^2$, $q_\perp^2 - q_0^2 = 0$, $\Gamma\gamma(\kappa_1 \ldots \kappa_n)$ being the vertex part of the conversion of the photon into n hadrons. Analogously, with the aid of a single dispersion relation the amplitude $f_{\gamma a}(s, p_1^2, \kappa_1 \ldots \kappa_a)$ can be written in the form:

$$f_{\gamma a}(s, p_1^2 \ldots) = \frac{1}{\pi} \int \frac{d\varkappa_1^2}{\varkappa_1^2 - p_1^2} \tilde{f}_{\gamma a}(s, \varkappa_1^2 \ldots) , \tag{49}$$

$$\tilde{f}_{\gamma a}(s, p_1^2 \ldots) = \sum_n \Gamma_\gamma(K_1 \ldots K_n) \delta^4(p - \Sigma K_i) f_{na}(K_1 \ldots K_n, K_1' \ldots K_a', q). \tag{50}$$

As to the possibility of using these dispersion relations, two questions naturally arise. Are not the dispersion relations (47) and (49), especially (49), violated owing to the presence of combined characteristics more complex than the threshold ones? And are the nonsubtractive dispersion relations valid? The combined or composite characteristics, even if they do exist, are not important at high energies. One can verify this with, for example, formula (36), which is nothing other than a double dispersion relation with respect to the quantum masses, and which is obtained only by assuming the convergence of the integrals over the intermediate states. Investigation of the analytic properties of the Feynman diagrams has led to the same result.

The use of nonsubtractive dispersion relations obviously constitutes a hypothesis that cannot be proved. We must stress, however, that there is an important difference between the usually employed dispersion relations with respect to the energy and dispersion relations with respect to mass. The increase of invariant amplitudes at high energies s is a normal phenomenon in both strong and weak interactions. On the other hand, it is natural to suppose that with large masses, i.e., in the case of highly virtual processes, the amplitudes decrease owing to the cutoff, caused by the strong interactions, in any event at masses of $x^2 \gtrsim s$.

Assuming the dispersion relations (47) - (50) and substituting them into (44), we get:

$$\frac{1}{2pm} F^{s}_{\gamma\gamma} = i\,2\pi R^2 \frac{1}{\pi^2} \int \frac{d\varkappa_1^2\, d\varkappa_2^2}{\varkappa_1^2\, \varkappa_2^2} \times \tag{51}$$

$$\times \Gamma_\gamma \left[\frac{1}{2iq\ell+1} \frac{1}{q} \frac{1}{2iq\ell+1} q \right] \Gamma_\gamma \quad ,$$

$$F^{V}_{\gamma\gamma} = N \frac{1}{\pi^2} \int \frac{d\varkappa_1^2\, d\varkappa_2^2}{\varkappa_1^2\, \varkappa_2^2} \Gamma_\gamma \frac{2iq\ell}{2iq\ell+1} f \Gamma_\gamma \quad . \tag{52}$$

If the integrals over x_1^2 , x_2^2 converge at finite masses of the order of μ^2 (large distances), then $q\ell \to \frac{\mu^2}{2p}\,\ell \to 0$ and

$$\frac{1}{2pm} F^{s}_{\gamma\gamma} = i\,2\pi R^2 \frac{1}{\pi^2} \int \frac{d\varkappa_1^2\, d\varkappa_2^2}{\varkappa_1^2\, \varkappa_2^2} \Gamma_\gamma \left[1 - 2i(q\ell + \ell q) - \right.$$

$$\left. -4(q\ell q\ell + \ell q \ell q + q\ell\ell q) \right] \Gamma_\gamma \quad , \tag{53}$$

$$F^{V}_{\gamma\gamma} = N \frac{1}{\pi^2} \int \frac{d\varkappa_1^2\, d\varkappa_2^2}{\varkappa_1^2\, \varkappa_2^2} \Gamma_\gamma \left[-\frac{q}{\varsigma} + 4(q\ell)^2 f \right] \Gamma_\gamma \quad . \tag{54}$$

Hence, using the optical theorem, we get the total cross-sections in the form:

$$\sigma^{s}_{\gamma} = 2\pi R^2 \left[1 - O\left(\frac{\mu^4}{p^2} \ell^2 \right) \right] (1 - Z_3^{-1}) \quad , \tag{55}$$

$$\sigma^{V}_{\gamma} = \frac{N}{\pi^2} \int \frac{d\varkappa_1^2\, d\varkappa_2^2}{\varkappa_1^2\, \varkappa_2^2} \Gamma_\gamma 4(q\ell)^2 f \Gamma_\gamma \approx N \sigma^{N}_{\gamma}\, O\left(\frac{\mu^4 \ell^2}{p^2} \right) , \tag{56}$$

i.e., with a satisfactory precision of the order of

$$\sigma^{V}_{\gamma} / \sigma^{s}_{\gamma} \sim \frac{1}{2\pi R^2} \cdot V \cdot \frac{1}{2\ell} \frac{\mu^4 \ell^2}{p^2} \sim \frac{R\ell}{p^2} \mu^4 \quad , \text{ and}$$

condition (40) is realized.

If the integral (40a) for $1 - Z_3^{-1}$ diverges, i.e., if masses much greater than μ^2 are the important ones, then according to (51) $F^s_{\gamma\gamma}$ is determined by masses satisfying the condition $q\ell \lesssim 1$. The contribution of the large $q\ell$ is small owing to rapid decrease of the expression inside the square brackets. Here, if the integral (40a) over x^2 for $1 - Z_3^{-1}$ diverges logarithmically, then it is determined by the region $q\ell \ll 1$, and $\sigma^s_{\gamma\gamma}$ becomes

$$\sigma^s_{\gamma\gamma} = 2\pi R^2 \int^{\varkappa_0^2} \frac{d\varkappa^2}{\varkappa^2} \rho(\varkappa^2), \quad \varkappa_0^2 \sim \frac{2p}{\ell} \, . \tag{57}$$

The volume term, on the other hand, is determined by the region $q\ell > 1$ and in order of magnitude is:

$$\sigma^v_\gamma = N \sigma^N_\gamma (\varkappa_0^2) , \tag{58}$$

where $\sigma^N_\gamma (x_o^2)$ is the portion of the cross-section of the interaction of the gamma quantum with a nucleon attributable [from the standpoint of the dispersion integral (47)] to masses x_1^2 and x_2^2 larger than x_o^2. The question as to the dependence of x_o^2 on the energy was discussed in the Introduction. The assertions made there, from the standpoint of formulae (51) and (52), are self-evident.

4. *Interaction of Electrons with Nuclei*

The interaction of an electron with a nucleus leads to the interaction of a virtual gamma quantum with the nucleus (diagram in figure 4). The differential cross-section of the electron scattering with formation of an arbitrary number of hadrons can be written in the form:

$$d\sigma = \frac{e^2}{\kappa M}\frac{1}{p^4}\left[K_\mu K_\nu F_{\mu\nu} + p^2 F_{\mu\mu}\right]\frac{d^3\kappa'}{2\kappa_0'(2\pi)^3}, \tag{59}$$

where κ is the momentum of the incident electron in the laboratory system, $K_\mu = \kappa_\mu + \kappa'_\mu$, $p = \kappa'_\mu - \kappa_\mu$ the momentum of the virtual quantum, M the mass of the nucleus. The quantity $F_{\mu\nu}$ is the imaginary part of the forward-scattering amplitude for the virtual gamma quantum. In the preceding sections we calculated the quantity $e^\perp_\mu e^\perp_\nu F_{\mu\nu}$ for a real quantum, i.e., for $p^2 = 0$ and polarization vectors perpendicular to the quantum momentum. As seen from the foregoing, generalizing for the case $p^2 \neq 0$ presents no difficulties, for only in the final stage have we considered the photon mass equal to zero. A slight difficulty does arise only in calculating the contribution of longitudinally polarized quanta. This stems from the fact that expressions (38) and (39) are approximate, and from the fact that the longitudinal-polarization vector is dependent on the energy.

Expression (39) for $\rho_{\mu\nu}$ can be written in the laboratory system as:

$$\rho_{\mu\nu} = \left[-\delta_{\mu\nu} M_n^2 + p_\mu p_\nu + \alpha_\mu p_\nu - \alpha_\nu p_\mu + \alpha_\mu \alpha_\nu\right] \rho(M_n^2) \, ,$$
$$\alpha_\nu = \frac{p^2 - M_n^2}{2|p|} P_\mu^A \, , \tag{60}$$

where P_μ^A are the nuclear momenta. This expression is not gauge-invariant.At first glance it might appear that in this expression only the dominant term $p_\mu p_\nu$ has any meaning, while the other terms, of the order $\frac{1}{|P|^2}$ compared to it, should be discarded. But this term makes no contribution to any of the processes by virtue of preservation of the current, and we have therefore calculated an irrelevant quantity. In fact, if from the very beginning we consider $j_\lambda = j'_\mu e_\mu^{\perp\lambda}$, as was done above, these problems do not arise, and the dominant term becomes $\rho_{\lambda\lambda}^{\perp} = M_n^2$. The correction terms of the intermediate states of the other type $\sim 1/p$ compared to it, since no additional dependence on the energy can arise from $e_\mu^{\perp}$. The situation is different with $e_\mu^{\parallel}$. In this case, if $e_\mu^{\parallel} e_\mu^{\parallel} = -1$ enters the dominant term, $(e_\mu^{\parallel} P_\mu^A)^2 \frac{1}{p^2} \approx 1$ entering the correction term, we then get a contribution of the same order. This means that our calculation of the longitudinal polarization is not legitimate. To get around this difficulty, we write a general expression for $F_{\mu\nu}$ that satisfies the condition $p_\mu F_{\mu\nu} = 0$, $p'_\nu F_{\mu\nu} = 0$.

It has the form:

$$F_{\mu\nu} = -A\left[\delta_{\mu\nu} + \frac{p^2}{(p^A p)^2} p_\mu^A p_\nu^A - \frac{p_\mu^A p_\nu + p_\nu^A p_\mu}{p^A p}\right] + B\left(p_\mu p_\nu - p^2 \delta_{\mu\nu}\right). \tag{61}$$

On the other hand, by analogy with (38) we can write

$$F_{\mu\nu} = 2\pi R^2 i \cdot 2(p_a p^A) e^2 \int \frac{d M_n^2}{(M_n^2 - p^2)^2} \rho_{\mu\nu} \quad . \tag{62}$$

Comparing the dominant term on the right-hand side of (62) and (60), which is proportional to $p_\mu p_\nu$, with (61), we get:

$$B = 2\pi R^2 i \cdot 2 p_0 M \cdot e^2 \int \frac{d M_n^2}{(M_n^2 - p^2)^2} \rho \quad . \tag{63}$$

Calculating $F_{\mu\nu} e_\mu^\perp e_\nu^\perp$ with aid of (62) and (61), we get:

$$A + B p^2 = 2\pi R^2 i \cdot 2 p M \cdot e^2 \int \frac{M_n^2 \, d M_n^2}{(M_n^2 - p^2)^2} \quad . \tag{64}$$

Substituting (63) and (64) into (61) and (59), we obtain the formulae (9-11) given in the Introduction.

In conclusion I wish to express my profound gratitude to I.T. Dyatlov, B.L. Ioffe, L.B. Okun' and K.A. Ter-Martirosyan for their many helpful pointers.

APPENDIX

In this appendix we shall derive formula (21) for the amplitude of the scattering of a particle on a nucleus in terms of the amplitudes of the hadron processes on a nucleon. Formula (12), valid for low energies, is obtained from (21) as a particular case.

Looking at the diagram in figure 6, we shall regard the portion of it circumscribed by the broken line as the unitary amplitude of the scattering of a hadron on n nucleons (fig. 8) and shall designate it as $F_{aa}(p, p_1', p_1' + q_1', p_2', p_2' + q_2' \ldots)$.

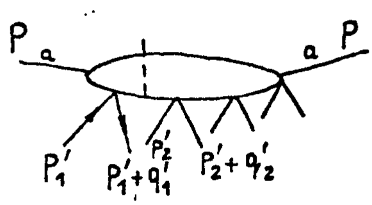

Figure 8

By assuming the nucleons of the nucleus to be nonrelativistic and by introducing the relative momenta of the nucleons $p_i' = \frac{p_1}{n} + \kappa_i'$, we readily note that $K_{i,0}' \sim q_{i,0}' \sim \frac{\bar{K}_i^2}{2m}$ are much smaller than the energies entering $F_{aa}(p, p_1', p_1' + q_1', \ldots)$. Here, neglecting the κ_{io}', q_{io}' in $F_{aa}(p, p_1', p_1' + q_1' \ldots)$, we find that

$$F_{aa}(p, p_1', p_1' + q_1', \ldots) = F_{aa}(p, \bar{q}_1', \bar{q}_2', \ldots \bar{q}_{n-1}', pp_A).$$

This enables us to integrate over κ_{io}', q_{io}' and to write the integral corresponding to the diagram in figure 6 in the nonrelativistic form

$$F_{aa}^{(n)} = \frac{1}{n!\,(N-n)!}\int \frac{\Gamma(\bar{K}_i')\,F_{aa}(p,\bar{q}_i',pp_A)\,\Gamma(K_i'+q_i')}{[N\Delta^2+(\sum_i \bar{K}_i')^2+\sum_i K_i'^2][N\Delta^2+(\sum(K_i'+q_i'))^2+\sum(K_i'+q_i')^2]}\times$$

$$\times\prod_{i=1}^{N-1}\frac{d^3K_i'}{(2\pi)^3 2m}\prod_{i=1}^{n-1}\frac{d^3q_i'}{(2\pi)^3 2m}\ , \qquad \text{(A-1)}$$

$$\Delta^2 = m^2 - \frac{M^2}{N^2}$$

or, by introducing the wave function of the nucleons in the nucleus in the conceived coordinate system, we get

$$\frac{\Gamma(K_i')}{N\Delta^2+(\sum K_i)^2+\sum K_i^2} = \sqrt{N!(2m)^{N-1}}\int e^{i\bar{K}_i\bar{r}_i}\,\psi(r_1\ldots r_{N-1})\,dV_1\ldots dV_{n-1}. \qquad \text{(A-2)}$$

With that choice of the factor in (A-2)

$$\int|\psi(r_1\ldots r_{n-1})|^2\,dV_1\ldots dV_{n-1} = N\ . \qquad \text{(A-2')}$$

Hence for $n \ll N$

$$F_{aa}^{(n)} = \frac{N^n}{n!}\frac{1}{(2m)^{n-1}}\int \chi(r_1\ldots r_n)\,e^{i\bar{q}_i'\bar{r}_i}\,F_{aa}(p,\bar{q}_i',pp_A)\prod_1^n dV_i\prod_i^{n-1}\frac{d^3q_i}{(2\pi)^3}\ , \qquad \text{(A-3)}$$

$$\chi(r_1\ldots r_n) = \int \psi^2(r_1\ldots r_{N-1})\,dV_{n+1}\ldots dV_{N-1}\ .$$

We represent the vector q_i' as $q_i' = q_{i\perp}' + q_{iZ}'$, where $q_{i\perp}'$ lies in a plane perpendicular to the momentum of the incident particle in the laboratory system and is analogous to $\bar{r}_i = \rho_{i\perp} + Z_i$. Then the integral $$\int e^{iq'_{i\perp}\rho_{i\perp}}\,\chi(r_1\ldots r_n)\,d^2\rho_1\ldots d^2\rho_{n-1}$$

will vary appreciably with variation of $q'_{i\perp}$ by an order of magnitude $1/R_n$, where R_n is the mean distance between the nucleons $r_1\ldots r_N$. We assume that the path length in the nucleus is greater than the transverse distances important in strong interactions at high energies. Since R_n is of the order of the path length, $1/R_N$ is much smaller than the transverse momenta important in the strong interactions and entering the amplitude $F_{aa}(p, q'_i, pp_A)$. This means that we can integrate over $q'_{i\perp}$ by setting $q'_{i\perp} = 0$ in $F_{aa}(p, q'_i, pp_A)$. Then the integration over d^2q_i gives $\delta(\rho_1 - \rho_2)\delta(\rho_2 - \rho_3)\ldots$ and

$$F^{(n)}_{aa} = \frac{N^n}{n!}\frac{1}{(2m)^{n-1}}\int d^2\rho_1\,dz_1\ldots dz_n\,\chi(\rho_1, z_1\ldots z_n)\,e^{iq'_{zi}z_i}\times F_{aa}(\rho, q'_{zi}, pp_A)\prod_{i=1}^{n-1}\frac{dq_{zi}}{2\pi}\;. \qquad \text{(A-4)}$$

We come to the most essential integration, over Z_i. Since $\chi(\rho, Z_1\ldots Z_n)$ is symmetrical with respect to $Z_1\ldots Z_n$ (allowing for the difference between neutrons and protons does not affect the result), we can omit the $1/n!$ in front of the integral in (A-4) and consider that

$$z_1 < z_2 < z_3 \ldots < z_{n-1}\;. \qquad \text{(A-5)}$$

The expression $\sum_i q'_{iz}Z_i$ can be written in the form

$$\sum_i q'_{iz} z_i = q_{1z}(z_1 - z_2) + q_{2z}(z_2 - z_3) + \dots + q_{n-1,z}(z_{n-1} - z_n), \qquad \text{(A-6)}$$

where $q_{1z} = q'_{1z}$, $q_{2z} = q'_{1z} + q'_{2z}$, $q_{3z} = q'_{1z} + q'_{2z} + q'_{3z}$.

We note that $S_1 = (p - q'_1)^2 = (p - q_1)^2 \approx p^2 + 2pq_{z1}$

equals the square of the mass of the intermediate state that occurs after the scattering on the first nucleon,

$S_2 = (p - q'_1 - q'_2)^2 = (p - q_2)^2 = p^2 + 2pq_{z2}$ the square of the mass of the intermediate state after scattering on two nucleons, etc. Hence integration over q_{1Z}, $q_{2Z} \dots$ constitutes integration over the masses of the intermediate states S_1, $S_2, \dots$ These integrations have the usual Feynman character (Figure 9). Since $\exp\,[iq_{Zi}(Z_i - Z_{i+1})]$

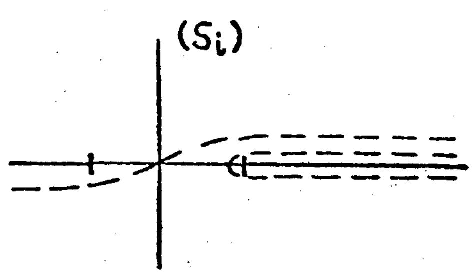

Figure 9

decreases in the lower half-plane, the contour of the integration over S_i can be closed in the lower half-plane, and the integral of $F_{aa}(\ )$ can be reduced to the integral of the absorption part, i.e., to integration over the real intermediate states, and we can substitute $q_{Zi} = \dfrac{S_i - p^2}{2p}$. As a result, $F_{aa}^{(n)}$ can be written as

$$F_{aa}^{(n)} = \frac{N^n i^{n-1}}{(4pm)^{n-1}} \int d^2\rho_1 dz_1 \ldots dz_{n-1} \sum_{b,c,d\ldots} f_{ab} e^{+iq_z^b(z_1-z_2)} \times$$
$$\times f_{bc} e^{iq_z^c(z_2-z_3)} \ldots e^{iq_z^d(z_{n-1}-z_n)} f_{ca} \cdot \chi(\rho, z_1 \ldots z_n), \tag{A-7}$$

where $\sum_{b,c,d}$ is the summation over all possible intermediate states, $f_{b,c}$ the amplitude of the transformation of a group of particles b into a group of particles c on a nucleon (Figure 7). Unlike the usually considered amplitudes of the interaction of individual particles (not groups of particles), these amplitudes are not matrix elements of an S-matrix. Infact, the absorption part, for example, in the variable S_3, is determined by the product $f_{ac} f_{ca}^x$. In other words, the contribution of the real intermediate states to f_{ac} is defined in the usual manner by changing $S_3 \to S_3 + i\varepsilon$, in f_{ca}^x by changing $S_3 \to S_3 - i\varepsilon$. If next we calculate the absorption part in S_2, then f_{ac} becomes $f_{ab} f_{bc}^x$, where f_{bc}^x is determined by changing S_2 to $S_2 - i\varepsilon$. Hence f_{bc}^x is defined as $f_{bc}(S_2 - i\varepsilon, S_3 + i\varepsilon)$. At first glance it might appear that introducing such quantities may lead to difficulties. Actually that is not so, and quantities of that kind are a natural generalization of the usual amplitudes for the case of the interaction of groups of particles. One may verify this by picturing the amplitude $F_{aa}^{(n)}$ as the integral of the time-ordered product of the nucleon operators: $\langle a | T A(x_1, x_1') A(x_2, x_2') \ldots A(x_n, x_n') | a \rangle$,

where $A(x_1 x_1') \sim \bar{\Psi}(x_1) \Psi(x_1')$, and the product can be decomposed with respect to the intermediate states

$$\langle a | A(x_1, x_1') | n \rangle \langle n | A(x_1, x_1') | m \rangle \langle \quad \rangle \ldots \langle \, .$$

The amplitudes $\langle n | A(x_2, x_2') | m \rangle$ are not matrix elements of an S-matrix unless one of the states $\langle n | , m \rangle$ is a single-particle state and coincides with the quantities discussed above.

Taking the final step to obtain formula (21), we adopt the assumption that the path length is large compared to the distance between nucleons. This mean that the points r_1, r_2, r_3 are, on the average, in the integral (A-4) and at a distance from one another that exceeds the distance between particles, hence under the condition (A-5) we can confine ourselves to only the correlations of the nearest nucleons, i.e., can write

$$\begin{gathered} \chi(\rho_1 z_1, z_1 - z_2, z_2 - z_3 \ldots z_{n-1} - z_n) = \varphi(\rho_1, z_1) \chi(z_1 - z_2) \times \\ \times \chi(z_2 - z_3) \ldots \chi(z_{n-1} - z_n) \\ \chi(z_i - z_{i-1}) \to 1 \quad \text{when} \quad z_i - z_{i-1} \to \infty \end{gathered} \tag{A-8}$$

where $\varphi(\rho_1 z_1)$- is constant inside the nucleus and equal to zero outside it. By virtue of the normalization condition (A-2')

$$\varphi(\rho_1 z_1) = \frac{N}{V^n} \, , \tag{A-9}$$

where V is the volume of the nucleus. Substituting (A-8) and (A-9) into (A-7), we get the formula (21) used in the text.