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INSTABILITY OF NON-ABELIAN GAUGE THEORIES AND IMPOSSIBILITY OF CHOICE OF COULOMB GAUGE*

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ABSTRACT

In this lecture it is demonstrated that by virtue of the impossibility of introducing Coulomb gauge for large fields and of the growth of the invariant charge at large distances, non-Abelian gauge theories may not be formulated as a theory of interacting massless particles. This assertion appears as a strong argument in favor of the idea that the spectrum of states in non-Abelian theories is substantially different from the spectrum of states in perturbation theory.

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I. INTRODUCTION

In the formulation of free gauge theory, corresponding to the group SU(N), it is regarded by analogy with quantum electrodynamics like a theory describing N^2-1 interacting massless vector particles. Massless vector particles are described by three-dimensional transverse fields $B_i(x)$:

$$\frac{\partial B_i}{\partial x_i} = 0 .$$

In order to be convinced that this is really so, we are obligated to demonstrate that all variables, aside from $B_i(x)$, which formally enter the gauge Lagrangian, can be excluded, that is, the theory may be cast into only interacting massless fields. At first glance, it seems that such proof exists and appears trivial by virtue of the possibility of formulating the theory in Coulomb gauge. In the present work we demonstrate that this is not correct and the usual Lagrangian in coulomb gauge is not equivalent to the initial gauge invariant Lagrangian. The reason for this inequivalence is found in the fact that, in distinction to electrodynamics, in non-Abelian theories it is not possible to uniquely introduce three-dimensional transverse fields (in particular, transverse fields can be pure gauge). We demonstrate also that the infrared instability of non-Abelian theories (asymptotic freedom), demonstrated in perturbation theory, leads to the fact that this non-uniqueness is essential for the scales, where the invariant charge is of order unity. These assertions make most probable that the spectrum of states of non-Abelian theories doesn't contain massless particles. At last, by virtue of conservation of charge, it may result in the confinement of color.

II. COULOMB GAUGE

Since the Lagrangian of the Yang-Mills field

$$\mathscr{L} = \frac{1}{4g^2} \operatorname{tr} G_{\mu\nu} G_{\mu\nu} , \tag{1}$$

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \left[A_\mu , A_\nu\right]$$

is invariant with respect to gauge transformations

$$A_\mu = U^{-1} A'_\mu U + U^{-1} \frac{\partial}{\partial x_\mu} U , \tag{2}$$

where U is a unitary matrix; then it is always possible to choose U in such a manner that one of the components of the field goes to zero, for example, A_0. In this case $\mathscr{L}$ takes the form

$$\mathscr{L} = -\frac{\operatorname{tr}}{2g^2} \left\{\dot{A}_i \dot{A}_i - \frac{1}{2} H_{ij} H_{ij}\right\}^* , \tag{3}$$

$$H_{ij} = \frac{\partial}{\partial x_i} A_j - \frac{\partial}{\partial x_j} A_i + \left[A_i , A_j\right] ,$$

describing a mechanical system with potential energy $\frac{1}{4} H_{ij} H_{ij}$. However, by virtue of the fact that $H_{ij} H_{ij}/4$ is unchanged by the transformation

$$A_i = S^{-1} B_i S + S^{-1} \frac{\partial}{\partial x_i} S , \tag{4}$$

the potential energy does not depend on all components of A_i, that is, the mechanical problem contains a cyclical variable. If we fix the potentials B_i in (4) by some condition, then formula (4) may be understood as determining in the place of the variables A_i new variables: a cyclical coordinate S and non-cyclical

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*The minus sign in front of $\mathscr{L}$ is caused by A_i being anti-Hermitian matrices.

variables B_i. The passage to massless fields consists in that B_i is fixed by the condition

$$\frac{\partial}{\partial x_i} B_i = 0 \quad . \tag{5}$$

If B_i is determined by the condition (5), then the cyclic coordinate is determined by

$$\frac{\partial A_i}{\partial x_i} = \nabla_i (A)\, S^{-1} \frac{\partial S}{\partial x_i} \quad , \tag{6}$$

where

$$\nabla_i (A) \psi = \frac{\partial \psi}{\partial x_i} + \left[A_i, \psi\right] .$$

If (6) determines S uniquely, then (4) determines B_i uniquely. The kinetic energy takes the form in these variables:

$$-\frac{1}{2} \operatorname{tr} \left[\left(\dot{B}_i + \nabla_i (B) f\right)\left(\dot{B}_i + \nabla_i (B) f\right)\right] \equiv -\frac{1}{2} E_i E_i \ , \tag{7}$$

$$f = \dot{S} S^{-1} \ ,$$

and the momentum connected with the cyclical coordinate

$$\pi = \nabla^2 (B) f + \nabla_i \dot{B}_i = \nabla_i (B) E_i \ , \tag{8}$$

is conserved.

Letting $\pi = 0$, we find an equation for the exclusion of the longitudinal components of E_i. Dividing E_i into longitudinal and transverse parts,

$$E_i = \pi_i + \frac{\partial \phi}{\partial x_i} , \quad \frac{\partial \pi_i}{\partial x_i} = 0 \tag{9}$$

and comparing (9) and (7), results in

$$\nabla^2 \phi = \partial_i \ \nabla_i (B) f \equiv - \Box \ (B) f \ . \tag{10}$$

From the condition $\pi = 0$, we find equations for ϕ and f in the form

$$\Box(B)\phi = \Box \frac{1}{\partial^2} \Box f = -\rho \quad , \tag{11}$$

$$\rho = \left[B_i, \pi_i\right] .$$

The Hamiltonian of the system after this may be written as

$$H = \frac{tr}{2g^2} \left\{ \pi_i \pi_i + f \Delta f + \frac{1}{2} H_{ij} H_{ij} - 2\rho f \right\} , \tag{12}$$

$$\Delta = \Box \frac{1}{\partial^2} \Box \quad , \tag{13}$$

or, using (11),

$$H = -\frac{tr}{2g^2} \left\{ \pi_i \pi_i - \rho \frac{1}{\Delta} \rho + \frac{1}{2} H_{ij} H_{ij} \right\} . \tag{14}$$

This completes the usual proof that the initial Lagrangian describes interacting massless gluons.

However, for the proof of (14) we made an important assumption that the procedure of introducing transverse fields is determined uniquely by Eq. (4) and (6). If this procedure is not unique, that is, one and the same A_i corresponds to several B_i, then using B_i as an independent variable, we will, by summing on the fields B_i, repeat the same A_i several times. If for different fields A_i, the number of repetitions will be different, then we obtain a senseless result.

III. IMPOSSIBILITY OF UNIQUE INTRODUCTION OF TRANSVERSE FIELDS

The question about the unique introduction of transverse fields amounts to the following: We imagine that we found such a B_i, which appears to obey Eq. (4), (6). Does there exist another field, B_i', which is transverse, connected with the field B_i by the gauge transformation

$$B'_i = U^+ B_i U + U^+ \frac{\partial U}{\partial x_i} \quad ? \tag{15}$$

Since B'_i and B_i are transverse, then U must satisfy the equation

$$\nabla_i (B) \frac{\partial U}{\partial x_i} U^+ = 0 \ . \tag{16}$$

This equation may be gotten from the condition of the extremum of the action

$$W = \int d^3 x \, \mathrm{tr} \left\{ \frac{\partial U^+}{\partial x_i} \frac{\partial U}{\partial x_i} - 2 \frac{\partial U}{\partial x_i} U^+ B_i \right\}^* \tag{17}$$

with the supplementary condition $U^+ U = 1$. From this expression for the action, it is almost evident that for sufficiently large fields B_i there exist solutions of Eq. (16) with $U \to \mathbb{1}$ as $r \to \infty$.

For this, in order to be convinced, we note that for sufficiently small B_i, $W \geq 0$ and achieves the absolute minimum for $U = \mathbb{1}$.

We examine the significance of W of "trajectories" near $\mathbb{1}$

$$U = \mathbb{1} + iv \ , \quad v^+ = v \ . \tag{18}$$

On these trajectories

$$W = \int d^3 x \, \mathrm{tr}\big(v \, \Box (B) \, v\big), \tag{19}$$

$$\Box (B) v = - \frac{\partial^2 v}{\partial x_i^2} - \left[B_i \, , \, \frac{\partial v}{\partial x_i} \right] \ . \tag{20}$$

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*The author is indebted to A. M. Polyakov for bringing attention to the possibility of writing the action in this form.

Since the operator $\Box(B)$ is analogous to the Schrödinger operator for a particle in the field $[B, \partial/\partial x]$, dependent on the velocity, it is evident that for fields B_i of sufficient size, we will produce field amplitudes in a wide region where the field B_i is different from zero and the operator $\Box(B_i)$ will have negative eigenvalues. In this circumstance, the action W will be negative, evaluated with the eigenfunctions corresponding to these negative eigenvalues. Since the minimum value of the action (17) W_m is less than or equal to W_m^0 (W_m^0 is the minimum value on the "trajectories" near $\mathbb{1}$), then we arrive at the conclusion that for the fields B_i for which $\Box(B)$ achieves negative eigenvalues, the action (17) achieves a non-trivial extremum, and consequently (16) has a solution with $U \to \mathbb{1}$ as $x \to \infty$.

In such a manner we arrive at the conclusion that for such field values, the Coulomb gauge is not uniquely determined. Moreover, as is well known, the linear action (19) achieves "n" extrema, if $\Box(B)$ achieves "n" eigenvalues. If this situation persists for the action on all "trajectories," we will have had the next mapping of non-unique transverse potentials. We have broken the whole space of fields B_i into regions depending on the spectrum of the operator $\Box(B)$ (Fig. 1) such that each region C_n belongs to a field for which $\Box(B)$ achieves "n" eigenvalues. The boundaries between regions, L_i, correspond to fields for which $\Box(B_i)$ achieves a zero eigenvalue. Thus in the region C_0 the transverse potentials are determined uniquely. In all other regions, C_n, from each potential B_i may be found, at the most, n potentials in that or other region, except C_0, connected with the potential B_i via a gauge transformation. Below we demonstrate these effects in an example of a particular solution to Eq. (16), but immediately note that analogous problems of uniqueness arise and in a relativistically invariant formation of the theory. For example, in the gauge $\partial A_\mu/\partial x_\mu = 0$ the condition of uniqueness is determined by the same Eq. (16), but for four variables.

If, as it often happens, the theory is formulated in four-dimensional Euclidian space, the situation exactly coincides with that examined above. Consequently, in the Euclidian case, the gauge $\partial A_\mu/\partial x_\mu = 0$ also may not be introduced for large fields.

In the pseudo-Euclidian case, the situation doesn't stand clear and for clarifying questions, additional analysis is required.

IV. INCREASE IN THE INVARIANT CHARGE IN NON-ABELIAN THEORIES

The calculation of the invariant charge in Coulomb gauge was carried out by I. B. Khriplovich even before the discovery of asymptotic freedom in non-Abelian theories. We cite this calculation for this: In order to explain the reason of the growth of the invariant charge and to demonstrate that a significant part of the invariant charge originates from averaging over fields for which it is not possible to introduce the Coulomb gauge.

For the calculation of the invariant charge we use, following I. B. Khriplovich, one of its possible definitions, namely the quantity which is determined by the energy of two interacting heavy sources. For this end in the expression for the energy of the system, we substitute for ρ, $\rho + \rho_h$, and calculate the correction to the vacuum energy to second order in ρ_h. This correction takes the form

$$V_{coul} = \frac{-tr}{2g^2} \int d^3x\, \rho_h(x) \left\langle 0 \left| \frac{1}{\Delta} \right| 0 \right\rangle \rho_h(x) + \sum_n \frac{|V_{on}|^2}{E_0 - E_n} , \tag{21}$$

where

$$V_{on} = \frac{1}{g^2} \int d^3x\, tr\, \rho_h(x) \left\langle 0 \left| \frac{1}{\Delta} \rho(x) \right| n \right\rangle , \tag{22}$$

$$\Delta\psi = \square \frac{1}{\partial^2} \square \psi = \partial^2 \psi + 2\left[B_i, \partial_i \psi\right] + \left[B_i, \partial_i \frac{1}{\partial^2}\left[B_j, \partial_j \psi\right]\right] .$$

The first term describes the usual Coulomb interaction, connected with the propagation of the Coulomb field from one charge to another with only this distinction, that the Coulomb field does not propagate freely (the propagator is not equal to ∂^{-2}) but in the field of zero-point oscillations of the gluons B_i. The second term, which is negative and therefore diminishes the interaction, produces the usual polarization of the vacuum in an external field.

In this manner, the increase of the interaction may originate only from the first term, that is from the change in the conditions of propagation of the Coulomb field. So this change of the conditions of propagation leads to the stronger interaction at large distances as is clear from the following. Consider the operator $-1/\Delta$ before averaging of the field B_i:

$$G(r,r') \equiv (r \mid -1/\Delta \mid r') = \int \frac{d^3k}{(2\pi)^3} \frac{\psi_k(r)\,\psi_k(r')}{\epsilon\,(k)} , \tag{23}$$

where ϵ (k), ψ_k are the eigenvalues and eigenfunctions of the operator Δ with momentum "k" at infinity. It is clear that if one calculates $G(r,r')$ for a field B_i lying near L_n (the boundary of the region C_n in Fig. 1), where the operator $\Box$ achieves a zero eigenvalue, then Δ also will achieve a null eigenvalue and $G(r,r')$ will go to ∞. If we calculate G for a potential within the region C_0, where the potential is either small or repulsive, then the change in the density of states would not be essential. Even if the average value of B_i is equal to zero, then it is all the same, inasmuch as attractive action is more essential than a repulsive one, for the presence of zero point oscillations leads to a strengthening of the interaction. It is also clear that a more intense action of the medium grows with growing distance between the charges, inasmuch as in this case, the probability grows that in the region between the charges is achieved a zero point vibration field lying near L_n, that is, guaranteeing a level of zero energy of the operator Δ.

The phenomenon being discussed is clearly visible in the calculation of the invariant charge in perturbation theory. For the calculation of the first term in (21) we must expand in a series of powers of B the operator $1/\Delta$ and, consequently, $1/\Box$:

$$\frac{-1}{\Delta} = \frac{-1}{\partial^2} + \frac{2}{\partial^2}\left[B_i, \partial_i\right]\frac{1}{\partial^2} - \frac{3}{\partial^2}\left[B_i, \partial_i\right]\frac{1}{\partial^2}\left[B_i, \partial_i\right]\frac{1}{\partial^2} \quad . \tag{24}$$

After averaging on the fields B_i, we receive the following expression for the contribution of this term to the Coulomb energy in the momentum representation (with $\rho_h = g^2\delta(\vec{x})$):

$$V^{(1)}_{coul} = \frac{g^2}{k^2}\left[1 + 3g^2\int \frac{d^3k'\, C_2^2}{(2\pi)^3 (k-k')^2\, 2k'}\left(1 - \frac{(kk')^2}{k^2k'^2}\right)\right] \approx$$

$$\approx g^2/k^2\left[1 + \frac{g^2 C_2^2}{4\pi^2}\,\ell n\, \Lambda^2/k^2\right] \quad , \tag{25}$$

corresponding to the diagram in Fig. 2, where the solid line corresponds to the Coulomb propagator, but the dotted line, to the average value of the products of fields B_i, that is, the transverse Green function of gluons at equal times. For the calculation of the second term in (21), it is possible to substitute for Δ, ∂^2. With this $V^{(2)}_{coul}$ leads to the usual expression for the vacuum polarization, corresponding to the diagram of Fig. 3:

$$V^{(2)}_{coul} = -\left(C_2^2 g^4 \ell n(\Lambda^2/k^2)\right)/k^2\, 48\pi^2 \ . \tag{26}$$

Combining (25) and (26), we receive

$$V_{coul} = g^2/k^2\left(1 + C_2^2 g^2 \frac{11}{48\pi^2}\,\ell n\,\frac{\Lambda^2}{k^2}\right) \quad , \tag{27}$$

which coincides with the usual expression for the invariant charge in first order of g^2. In the calculation of higher orders of g^2 in logarithmic approximation, the corrections to the dominant parts and Green function reduce to the same form as in QED, and

$$V_{coul} = g^2 \Big/ \left[k^2 \left(1 - C_2^2 \; \frac{11}{48\,\pi^2} \, g^2 \ell n \; \Lambda^2 / k^2 \right) \right] \quad . \tag{28}$$

Comparing (21) and (28) and remembering that the second term in (21) is negative, we see that in the region

$$1 - C_2^2 \, g^2 \frac{11}{48\,\pi^2} \; \ell n \, \frac{\Lambda^2}{k^2} \sim g^2 \; ,$$

where the logarithmic approximation is still applicable, begins a huge (for $g^2 < 1$) strengthening of the interaction, so that $\langle 0 \mid 1/\Delta \mid 0 \rangle \sim 1/k^2 g^2$. Such an increase in $\langle 0 \mid 1/\Delta \mid 0 \rangle$ may originate only by virtue of the essential role played during the averaging by fields for which $\Box(B)$ achieves almost a null eigenvalue, that is, near to L_0 or even another of the L_i in Fig. 1. In such a manner the invariant charge of order unity originates from the average on fields for which, generally speaking, the usual theory is already incorrect by virtue of the nonuniqueness in introducing transverse fields.

The same may be said in another way. On the space scale at which the invariant charge is of order unity, the fluctuations of the transverse field are so large that they may not be described interacting massless transverse fields of gluons. As was already noted in the introduction, this may signify that on such scales, massless gluons do not exist. Finally, in the same line, it may signify that these scales determine the radius of color confinement.

V. PARTICULAR SOLUTIONS OF THE CHIRAL EQUATION

Below we shall confine ourselves to the SU(2) group and shall consider first the case $B_i = 0$, i.e., we shall ascertain whether there exist purely transverse gauge fields. We shall seek in this case a very simple spherically symmetric solution in the form

$$U = e^{i\alpha(r)\vec{n}\vec{\tau}} \tag{29}$$

where the τ_i are Pauli matrices, and

$$\vec{n}_i = \frac{\vec{x}_i}{r} , \qquad r^2 = x_i x_i .$$

Substituting (29) into (17), we get

$$W = \int r^2 dr \left[\left(\frac{\partial \alpha}{\partial r} \right)^2 + \frac{2 \sin^2 \alpha}{r^2} \right] \tag{30}$$

Hence the equation for $\alpha(r)$ is

$$r \frac{\partial^2}{\partial r^2} (\alpha r) - \sin 2\alpha = 0 \quad . \tag{31}$$

If we introduce the variable $t = \ell n\, r$, we shall arrive at the equation for a pendulum in a gravitational field with damping (Fig. 4):

$$\ddot{\alpha} + \dot{\alpha} - \sin 2\alpha = 0 . \tag{32}$$

The potential energy of the pendulum is $v(\alpha) = -2 \sin^2 \alpha$ (Fig. 5). It is obvious that the solution, nonsingular for $r \to 0$ ($t \to -\infty$), is for a pendulum which, when $t \to -\infty$, is in the position of unstable equilibrium $\alpha = 0$ ($\alpha = n\pi$).

When $t \to -\infty$, we have

$$\alpha(t) = \gamma r = \gamma e^t . \tag{33}$$

When $t \to +\infty$, we have $\alpha(t) \to \pm\frac{\pi}{2}$, and for any t—by virtue of the presence of the damping—we get $|\alpha(t)| < \pi$. The field B_i' corresponding to such an $\alpha(r)$ is not singular for finite values of r, and for $r \to \infty$ it has the form:

$$B_i'^{\alpha} = \epsilon_{\alpha\rho i} \frac{x_\rho}{r^2} \tag{34}$$

i.e., it decreases as $1/r$.

The solution found for U(r), and consequently for B'(r), is characterized by four parameters: three parameters $\vec{r}_0$, which define the reference point, and a parameter γ, which has the meaning of an inverse radius of the region beyond which B_i' decreases.

We have shown that there exists an indeterminacy which is characterized by at least four parameters, i.e., corresponding to the zeroth field A_i we have at least a four-parameter family of fields B_i.

To understand how significant this indeterminacy is, we deem it useful to ascertain where the B_i fields that have been found are situated from the standpoint of describing the space of the B_i fields with the aid of Fig. 1. The first thing that we note is that the equation $\Box(B)\psi = 0$ in the field B_i has a solution with ψ, which decreases as $r \to \infty$. This is clear from the fact that B_i depends on four arbitrary parameters. With an infinitely small variation of the parameters, we switch to a field that satisfies the same conditions, i.e., we perform a gauge transformation with U, which is close to unity, without altering the behavior of the field as $r \to \infty$, which does not depend on the parameters. This latter means that $\psi(r) \to 0$ as $r \to \infty$. We are then left with two possibilities: either $\psi(r)$ is square integrable, in which case B_i lies on one of the lines L_n, or $\psi(r)$ is not square integrable. In this latter case, in the field B_i there should be a bunching of the levels toward zero, i.e., an infinite number of the levels and $B_i(r)$ lies in the region C_∞.

A direct substitution of B_i into the equation $\Box\psi = 0$ leads to an equation which reduces to the Schrödinger equation with a potential that decreases as $1/r^2$ and has an infinite number of levels.

Hence the B_i' fields corresponding to the zeroth A_i from the standpoint of Fig. 1 are situated "infinitely far away" from $B_i = 0$. One can adduce arguments in support of the contention that all solutions of Eq. (16) for $B_i = 0$ possess this property.

We shall now discuss Eq. (16) for the presence of the field B_i, and for simplicity's sake we shall consider the field B_i as having a simple form convenient for solving the equation.

We let $B_i(x)$ have the form:

$$B_i^{\alpha}(x) = \frac{1}{2}\epsilon_{\alpha\rho i}\frac{x_\rho}{r^2}\, f(r)\ , \tag{35}$$

where $f(r)$ is an arbitrary function. This field is transverse. Selecting U in the same form (29) and substituting (35) and (29) into (17), we get an action in the form

$$W = \int r^2 dr \left\{ \left(\frac{\partial\alpha}{\partial r}\right)^2 + 2\,\frac{\sin^2\alpha}{r^2}\,[1 - f(r)] \right\} \tag{36}$$

and the equation

$$\ddot{\alpha} + \dot{\alpha} - \sin 2\alpha\,[1 - f(\tau)] = 0\ , \tag{37}$$

i.e., the equation for a pendulum in a time-dependent field. This equation has solutions of the same type as does the equation for $f = 0$, i.e., solutions in which $\alpha \to \frac{\pi}{2}$ as $\tau \to \infty$. However, if B_i is sufficiently great, Eq. (37) has other solutions too: $\alpha \to 0$ as $\tau \to \infty$ or $\alpha \to \pi$ as $\tau \to \infty$, which will lead to fields B_i' that rapidly decrease as $r \to \infty$. It is clear that one can select a field such that the pendulum, after starting to move from a position of unstable equilibrium with an energy equal to zero, will at a certain instant change its direction of velocity under the influence of the field and, after acquiring a positive energy, will start to damp out as it asymptotically approaches its starting position $[\alpha(\infty) = 0]$. One can also select a field f such that the pendulum, after increasing its velocity at a certain instant τ, will start to approach the starting position from the opposite direction $\alpha(\infty) = \pi$. If $f(\tau)$ has a more complex dependence on τ, then the pendulum can several times pass through a position of unstable equilibrium and only then start to approach it asymptotically.

We therefore arrive at the conclusion that, if the transverse field B_i is sufficiently great, one can have corresponding to it a whole set of transverse fields B_i' related to the field B_i through a gauge transformation and decreasing fairly rapidly as $r \to \infty$.

We shall now discuss what determines the magnitude of the field $f(\tau)$ in which the pendulum returns to the equilibrium position. Numerical analysis shows that a minimal restriction on $f(\tau)$ occurs if $f(\tau)$ acts at those instants τ at which the deviation $\alpha(\tau)$ is still much smaller than 1. In this case, instead of (37, we have

$$\ddot{\alpha} + \dot{\alpha} - 2\alpha(1 - f) = 0 \quad . \tag{38}$$

Equation (38) is somewhat different from the equation $\Box\psi = 0$ when $\psi = \alpha(r)\hat{n}$, in the particular form of B_i in expression (35). Hence the "minimal" field is the field on the curve L_0. However, for "minimal" $f = f_0$, $B_i' = B_i$, and in order to have the indeterminacy, f has to be greater than f_0. There will then exist a negative $-\epsilon$ eigenvalue of the operator $\Box$.

To determine $\alpha(r)$ in this case, we set

$$\alpha(r) = \alpha_\epsilon + \tilde{\alpha} \ , \tag{39}$$

where

$$\ddot{\alpha}_\epsilon + \dot{\alpha}_\epsilon - 2\alpha_\epsilon (1 - f) = +\epsilon\, \alpha_\epsilon \quad , \tag{40}$$

in which α_ϵ decreases with large r. Substituting (39) into (37) and considering $\widetilde{\alpha}$ and α_ϵ to be small, and that $\widetilde{\alpha} \ll \alpha_\epsilon$, we get

$$\ddot{\widetilde{\alpha}} + \dot{\widetilde{\alpha}} - 2\,\widetilde{\alpha}\,(1 - f) = \epsilon\, \alpha_\epsilon - \frac{4}{3}\,\alpha_\epsilon^3\,(1 - f) \quad . \tag{41}$$

For (41) to have a non-singular solution for $\epsilon \rightarrow 0$, it is necessary that the right-hand side be orthogonal to α_ϵ:

$$\int d\tau \left[\epsilon\, \alpha_\epsilon^2 - \frac{4}{3}\,\alpha_\epsilon^4\,(1 - f)\right] = 0 \quad . \tag{42}$$

This relation determines the normalization of α_ϵ : $\alpha_\epsilon^2 \sim \epsilon$, i.e., $\alpha_\epsilon \sim \sqrt{\epsilon}$. Then $\widetilde{\alpha} \sim \epsilon^2$ also decreases with large r. As f further increases, ϵ (the binding energy) increases, and the expansion in powers of ϵ deteriorates. However, a solution still exists. When f increases to the point where a second level with a small binding energy ϵ' appears, then, by repeating the above operations, we shall show that a second solution appears. We therefore get a degree of indeterminacy equal to the number of eigenvalues of the operator $\Box$. Actually, owing to the fact that $\alpha(\infty)$ can equal not only zero but "$n\pi$" as well (the pendulum can sail through the equilibrium position several times from different directions), the degree of indeterminacy of the fields B_i is even greater.

VI. TRANSVERSE GAUGE FIELDS

In conclusion, we shall discuss whether a significant role in the interaction of gluons may not be played by the purely transverse gauge fields which slowly decrease as $r \to \infty$ (34), found in the preceding section. The existence of such transverse fields means that even in transverse gauge we have, along with the vacuum $B_i = 0$, vacua described by transverse fields with the asymptotic form (34). From the topological standpoint, these fields have a half-integer topologic charge. If between such vacua there existed tunneling transitions (analogous to the tunneling transitions between the vacua with integer topologic charges, achieved with the instanton solutions of Ref. 4), then, regardless of the magnitude of the coupling constant and of the instability of perturbation theory, large transverse fields would always be present in a physical vacuum. Then the entire theory would change completely. But if the probability amplitude for such transitions equals zero, we can confine ourselves to the transverse fields which decrease more slowly than $1/r$, and we shall have one vacuum in the transverse gauge.

If one attempts to ascertain the probability of such transitions, one gains the impression that this probability equals zero in the purely local theory. To understand the reason for this, we shall take as an example a field of the form

$$B_i^{\alpha}(\vec{r}, t) = \frac{1}{2}\, \epsilon_{\alpha \rho i} \frac{x_\rho}{r^2} \Big(1 - U(r,t)\Big) \quad , \tag{43}$$

where $U = 1$ corresponds to the zeroth field, and $U = -1$ to the transverse gauge field with a zero radius. The transition from the field with $U = 1$ to the field with $U = -1$ is the one that interests us.

The action for a Yang-Mills field of the form (43) has the form

$$W = \frac{1}{2g^2} \int dr\, dt \left\{ \dot{U}^2 - U'^2 - \frac{1}{4r^2} (1 - U^2)^2 \right\} . \qquad (44)$$

This action describes a string with the two equilibrium positions $U = 1$, $U = -1$. Between these equilibrium positions, and beyond them, there is a potential barrier whose height at the point $r = 0$ is infinite and tends toward zero as $r \to \infty$. The energy of the system is finite only under the condition $U(0,t) = \pm 1$. If $U(0,t)$ is fixed by one of these values, e.g., $U = 1$, then for any excitations of the string involving a jump across the barrier for any values of r, except $r = 0$, the string will slip into the region of large r and, after a finite length of time, will arrive at the equilibrium state $U = 1$ for any finite r. However, even a tunneling variation of $U(0,t)$ is impossible, for the barrier has a finite width and an infinite height. Hence any state of the string is characterized by the values $U(1,t) = \pm 1$, between which there are no transitions. Transitions would occur only in the event that we introduced a cutoff of the interaction at $r = 0$.

The situation changes if there are monopoles in the theory. Since the monopole field is defined by (43) for $U = 0$ and decreases at infinity only as $1/r$, it is already impossible for us to confine ourselves to fields which decrease more slowly than $1/r$. Then any solution of Eq. (16) is available, and it is consequently impossible to fix the transverse gauging in the presence of a monopole. It is interesting to note, though, that at great distances the monopole field is invariant with respect to a gauge transformation with a matrix having the form (29).

In conclusion, I wish to express my profound thanks to A. A. Belavin, Yu. Dokshitser, A. M. Polyakov, V. Korepin, and I. B. Khriplovich for their numerous and extremely helpful comments.

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Note added:

After this translation was completed, we learned of a very fine translation of Gribov's lecture by J. Bartels and W. Nahm from CERN. The translation and editing by Abarbanel attempt to retain as much as possible the flavor (and color!) of the original Russian article. We feel that this SLAC translation No. 176 should go forth and be widely circulated as it will benefit the physics community at large.

H.D.I. Abarbanel

H. C. Tze, SLAC Coordinator
of this translation project

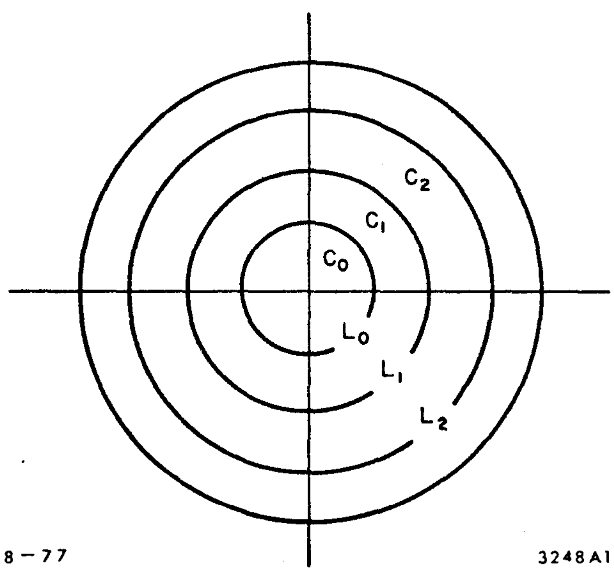

Figure 1

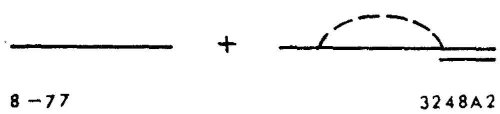

Figure 2

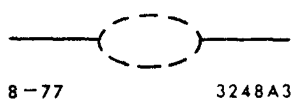

Figure 3

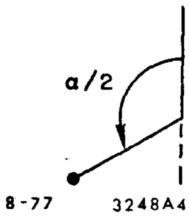

Figure 4

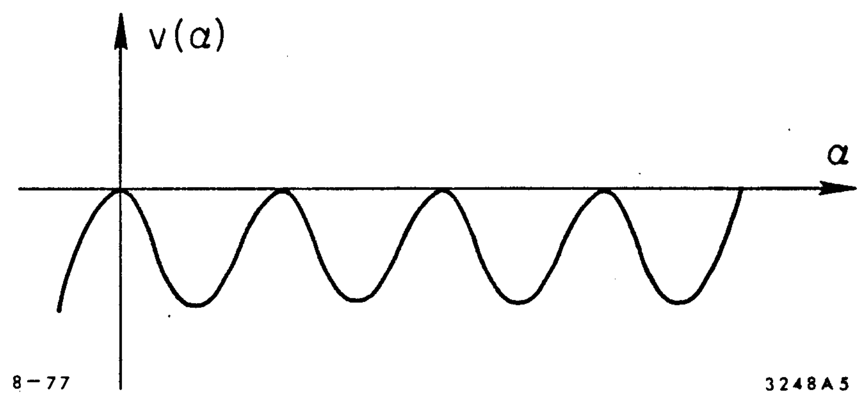

Figure 5