MEASUREMENTS AND VISUALIZATION OF THE TRANSVERSE PHASE-SPACE TOPOLOGY AT LEP

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Abstract

The LEP Beam Orbit Measurement system (BOM) allows for the acquisition of the beam position at each Beam Position Monitor (BPM) for over 1000 consecutive turns. By synchronizing the acquisition with a kick given to the beam, we can investigate the behavior of the beam under different conditions. In particular, starting from the data of one BPM, we can apply a simple mathematical manipulation to build a ”virtual” BPM, with a phase advance of 90 degrees. Plotting the real BPM against the virtual one, we can observe the evolution of the beam in the phase-space. An appropriate coloring technique is used, to help the User finding his way through the data. Fixed points in the phase-space can be put in evidence, as well as the beam behavior in their neighbourhood. Quantities like the tune, the beam detuning as function of the position amplitude and the beam damping can be studied in this way. Significant examples from real life will be shown.

1 INTRODUCTION

The LEP Beam Orbit Measurement system (BOM) is made of 500 Beam Position Monitors (BPMs). Each BPM can record the position of each particle bunch in LEP for more than 1024 consecutive turns. Some of these BPMs also record a signal roughly proportional to the bunch currents. The data acquisition is synchronized all around LEP, and it can be triggered by timing events (for instance, by the injection kick). Via software, the information relative to a specific bunch can be extracted. Fig. 1 shows a typical data set, plotting the evolution of the horizontal position of the selected bunch at a given BPM. In this case, the non integer part of the horizontal tune of the beam before the kick was just below 1/3. The large oscillations induced by the kick, combined with the detuning with amplitude effect, initially brought the beam (or part of it) to the third order resonance. Eventually the oscillations were damped, and the beam moved back towards the closed orbit.

Our goal is to use this kind of data to get a better understanding of phenomena happening in the transverse phase space[1]. We will explain the method we developed, and we will show a variety of different applications using real data sets from LEP.

2 PHASE SPACE REPRESENTATION USING A VIRTUAL PICKUP

In order to visualize the behavior of the beam in the phase space in one plane, we have to plot the position of the beam against its derivative. The kind of phenomena we are interested in (slowly damped betatron oscillations) are well described in terms of sinus and cosinus functions, and therefore we can approximate the differentiation operator by a phase shift of 90 degrees. While traditionally this was done by selecting two BPMs separated by a phase advance of (approximatively) 90 degrees, we found that the most practical way of implementing this phase shift is the creation of a Virtual BPM. This is done by manipulating the position data coming from one single BPM, to reconstruct a representation of their derivative[1](see also [2]).

2.1 How to create a Virtual BPM

1. Our starting point is the data array containing the beam position at a given BPM for 1024 turns (Fig.1).

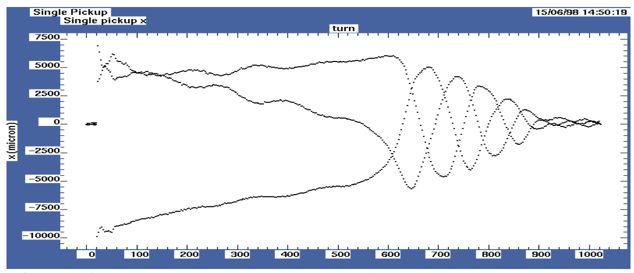

Figure 1: X position for 1024 turns. In this case the beam was kicked close to the third order resonance.

2. We perform an FFT on this position array, to get a frequency spectrum (preserving the sinus and cosinus components of each frequency).
3. We then rotate in the complex plane every spectrum component by 90 degrees. We also apply a linearly decreasing weight to the spectrum tails, to reduce errors (Fig.2).

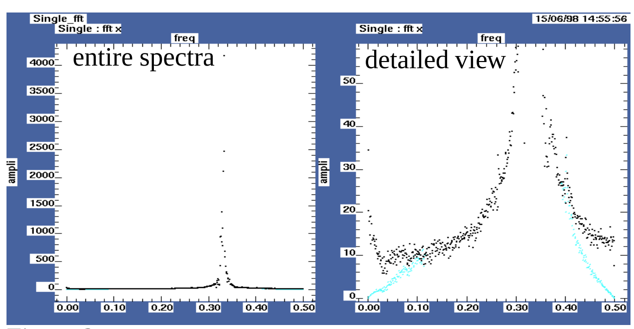

Figure 2: The FFTs of the Real (black) and Virtual (cyan) BPM. In parallel to the rotation operation (amplitude preserving) we apply a linearly decreasing weight to the spectrum tails.

4. Now we perform a Reverse FFT on the rotated spectrum, to get the position array of the Virtual BPM (Fig.3).
5. We plot the Real BPM against the Virtual one (Fig.4). We can also recenter the two arrays around the origin of the phase space, and renormalize on β.

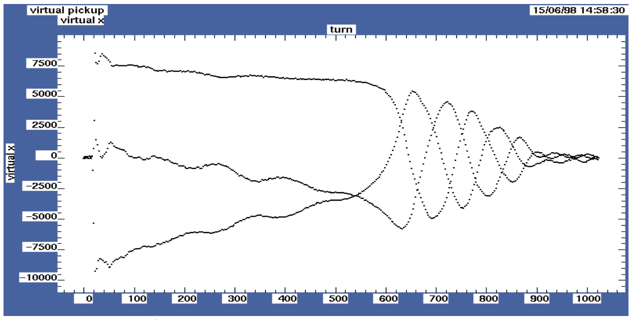

Figure 3: The position array for the Virtual BPM.

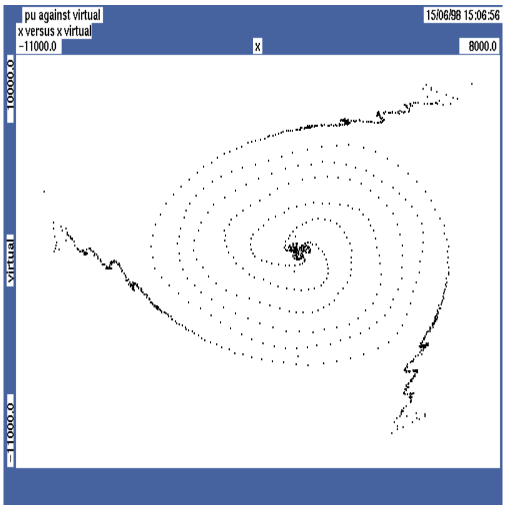

Figure 4: The phase space plot (not yet normalized).

6. We can then add fancy colours depending on the tune value, to emphasize phenomena to be visualized (Fig. 5). In our example, we will use the same colour for every third point, as the dominant phenomena is the oscillation close to the third order resonance. In some cases, connecting points with the same colour may help in understanding the dynamic behavior.

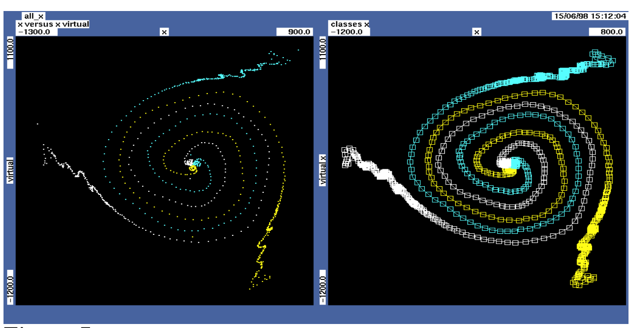

Figure 5: a) Every third point is given the same colour. b) Points with the same colour are connected.

2.2 *The Stroboscopic Effect*

The visualization of the tune evolution within our samples is also helped by what we call the "Stroboscopic Effect". It consists in the fact that every time the tune is close to a fraction with a small denominator, nicely identifiable arms appear in the phase space plot. In Fig. 6 we can identify an outer region, where the tune is close to 2/7 (0.286), and where we give every seventh point the same color, and an inner region, with tune close to 3/11 (0.273) where we give every 11th point the same colour.

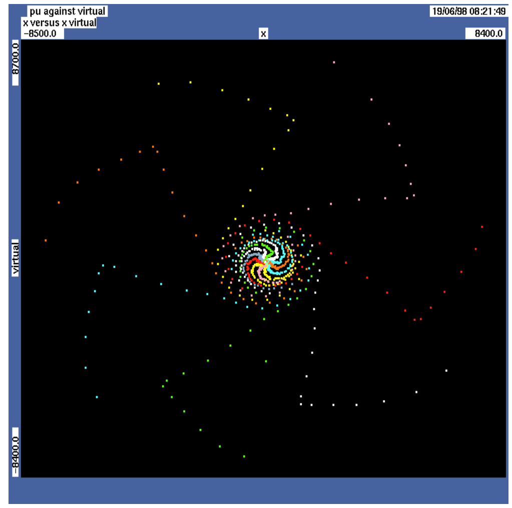

Figure 6: Stroboscopic effect. Tune = 2/7 (0.286) outside, 3/11 (0.273) inside.

2.3 *How accurate is the Virtual BPM ?*

Due to the discreteness and finiteness of the original sample, the process of generating the Virtual BPM cannot give an exact result. But from the result of some simulations one can see that, apart from a few bad points at the beginning and at the end of the data set, almost all the data points of the Virtual BPM array are correct within 1% (Fig. 7).

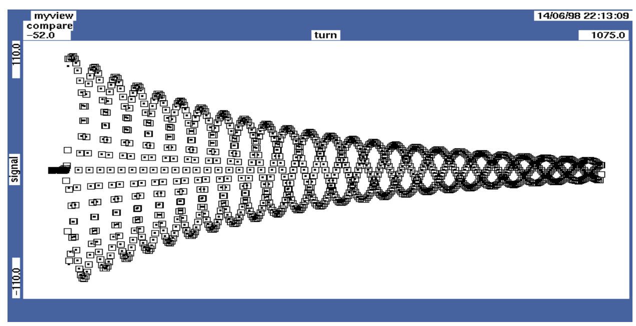

Figure 7: Exponential Decay after kick. Dots = expected BPM, Boxes = result of Virtual BPM Creation algorithm.

3 APPLICATIONS FROM LEP

3.1 *Third Order Resonances*

Our first example consists in the observation of the beam approach to the third order resonance fixed points. The beam, originally at a tune below 1/3, was strongly kicked in the horizontal plane, and, because of the tune growth with amplitude, it was trapped into the 3rd order resonance, and partially lost. In the following figures, every third point is given the same colour. In Fig. 8 the data arrays for the Real and the Virtual BPM are shown. In Fig. 9 the two arrays are plotted against each other, showing how the original oscillations around a tune of 1/3 are slowly damped and three fixed points are approached. Fig. 10 represents an enlargement on one of the three areas of the previous picture.

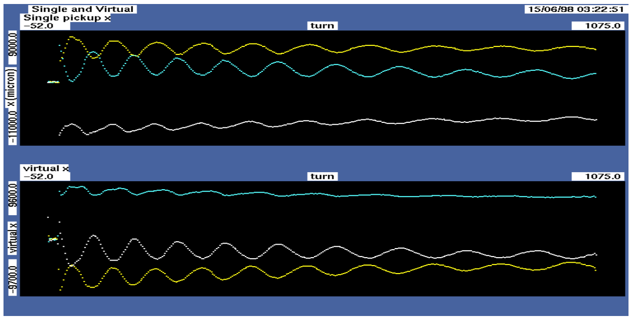

Figure 8: Real and Virtual BPM.

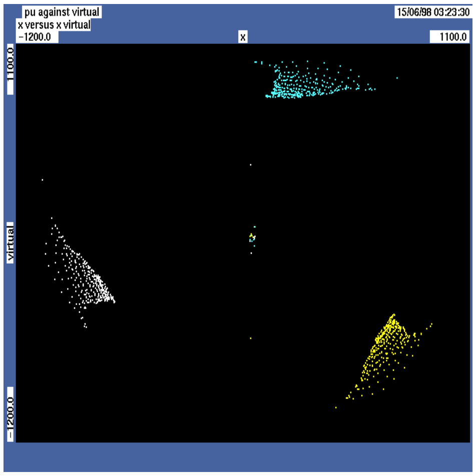

Figure 9: Phase space plot.

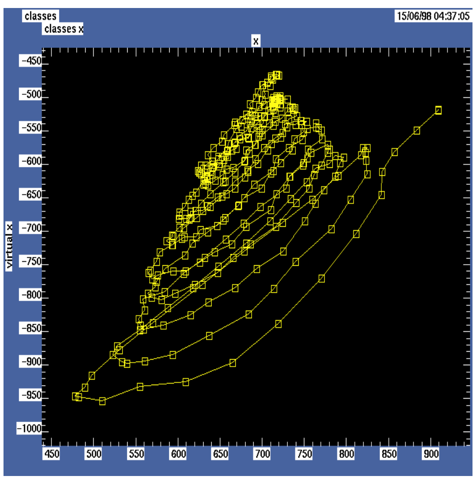

Figure 10: A detail on one of the three regions, as the beam moves toward the fixed point.

3.2 *Detuning With Amplitude*

The angle between consecutive turns in the phase space plot represents the tune at that turn. By plotting it against the Courant-Snyder invariant (kind of normalized amplitude) one can show how the tune changes as function of the

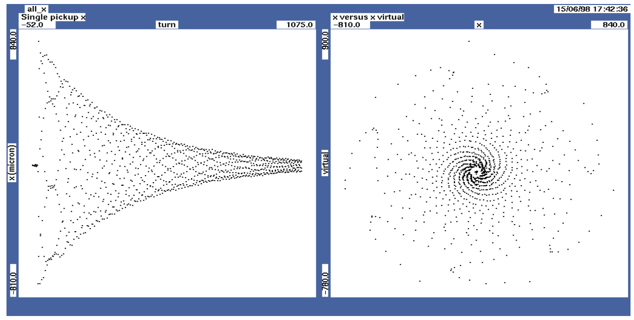

Figure 11: Position data and phase space plot for a good dataset.

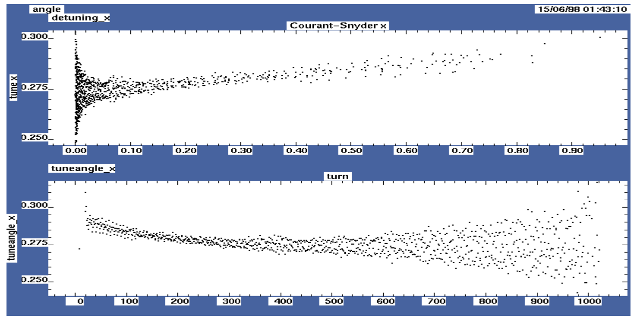

Figure 12: Detuning with amplitude (top).

beam oscillations amplitude. One must anyway be careful to stay away from resonances, which can modify the behavior of the beam and make the measurement meaningless. In Fig. 11 we show a data set suitable for this measurement, with smooth and regular damping of the oscillations after a large kick. Once the phase space plot has been centered and normalized, the Courant-Snyder invariant at every turn is just the square of the distance from the origin of the point corresponding to that turn. In Fig. 12 we plot the detuning with amplitude for this dataset, and we can see that it follows a linear behavior over the entire data range[1]. On the

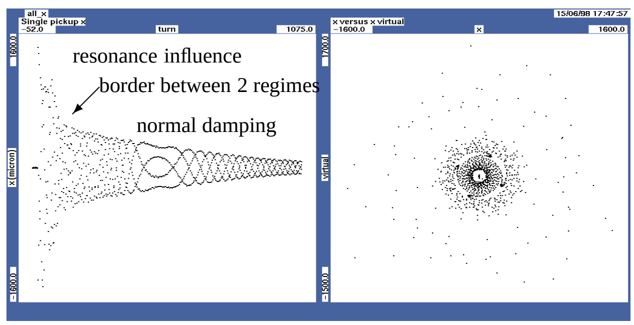

Figure 13: Position data and phase space plot for a bad dataset.

contrary, in Fig. 13 and 14 we can see what happens when we are too close to a resonance. By looking at the BPM data one can already notice that the behavior in the first 150 turns after the kick is quite different from the behavior later. This eventually reflects in the detuning with amplitude plot, where we can identify two different regions : the first, on the right, corresponds to the first turns after the kick, and it shows a slow detuning with amplitude, mainly

[1] Of course the errors become large as the amplitude gets too small.

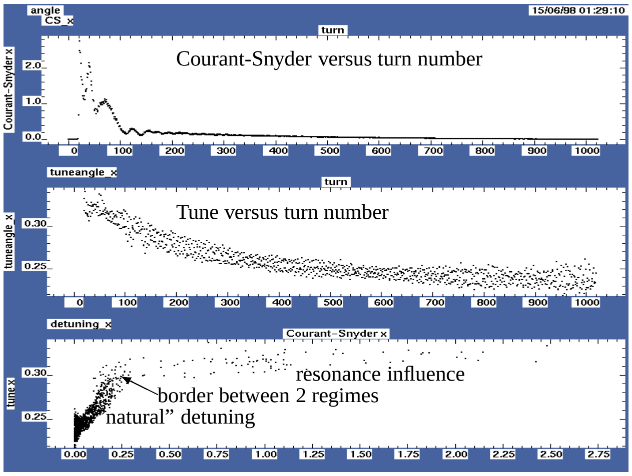

Figure 14: Detuning with amplitude (bottom). Notice two distinct regions.

because the tune is partially locked on the 3rd order resonance. Once the tune unlocks itself from the resonance, the behavior changes, and the detuning gets faster (left part of the picture).

3.3 *Chaotic Approach To Resonance Islands*

In this example we will follow the behavior of one very peculiar beam injection into LEP. For some unknown reasons the injection oscillation was not damped in the normal way (Fig. 15); rather, the beam moved toward some attraction point in the phase space. On its road to these points, a chaotic behavior was clearly observable. Almost all of the injected beam was lost within the first 1000 turns, and the losses were correlated with the period of the synchrotron oscillation, whose frequency is clearly visible in the FFT of the position data (Fig. 16). We can observe this behavior in a much clearer way by looking at the phase space, where we can also observe the same periodicity in the vertical plane (Fig. 17). The beam is captured by the attractors, and it is only released in the last turns of our sample (see Fig. 18). On its way to the attractor, the system oscillates between two states, as shown by Fig. 19 . In the left part of the picture, only every 7th point in the phase space is shown (and joined to the previous one). In the right part, only every 14th point is shown, and this eliminates completely the

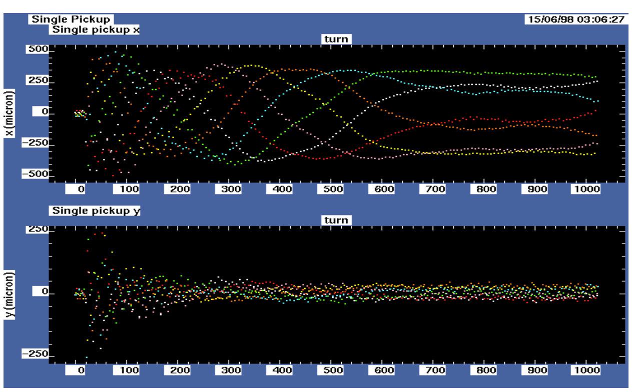

Figure 15: The beam moves towards a 7th order resonance.

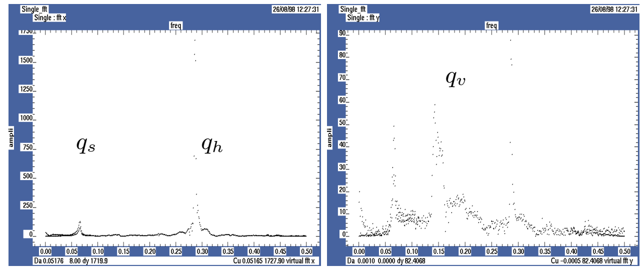

Figure 16: X and Y FFT of the position data.

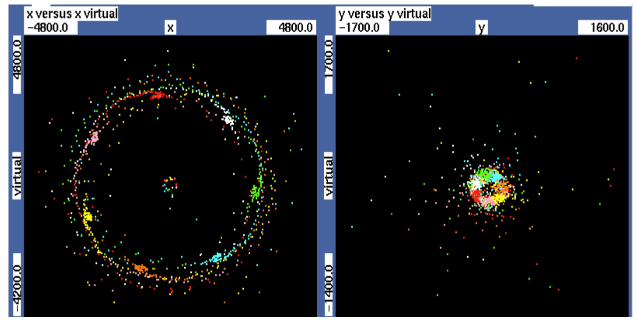

Figure 17: Phase space plots.

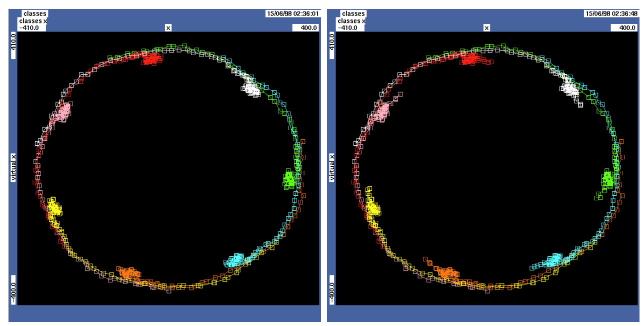

Figure 18: a) Turns 400-980. b) Turns 400-1020.

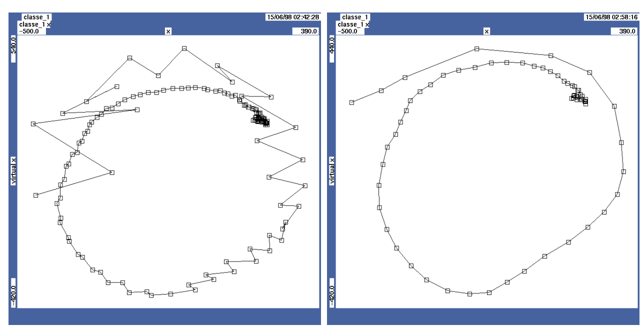

Figure 19: a) Every 7th point. b) Every 14th point.

oscillations.

3.4 *Filamentation*

One limitation of our BPM system is that we can only observe the behavior of the center of mass of the particle distribution. This is not always representative of the behavior of the individual particles, as it can be seen from the following example. After a strong kick close to a resonance, the oscillations of the center of mass of the particle distribution decay much earlier than the real oscillations of the individual particles. This effect is due to a rapid spreading of the particle distribution in the phase space ("Filamentation"). In our data set the oscillations seem to decay rapidly, but we are still loosing current at the end of the 1000 turns.

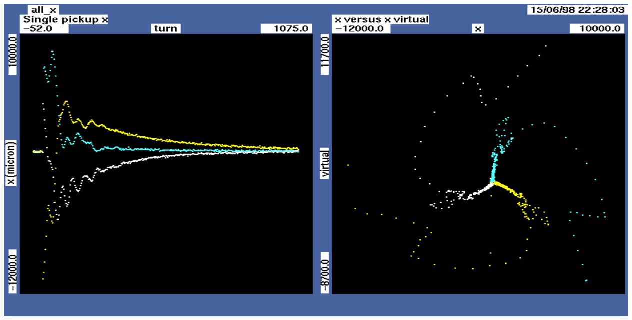

Figure 20: Position and phase space plots.

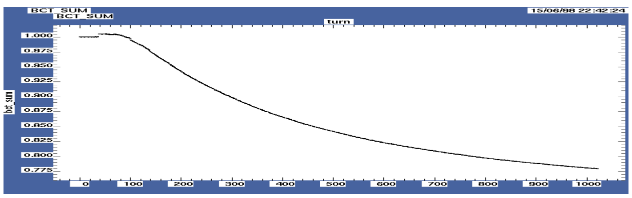

Figure 21: Evolution of bunch current during the 1000 turns.

3.5 Coupling

In all the previous examples we were mainly interested in investigating the behavior of the beam in the plane to which the kick was applied, neglecting the other plane. In the data set we will examine now the effect of the coupling is dominating. The energy originally provided to the beam via the horizontal kick is transferred from one plane to the other and viceversa. Amplitude maxima in one plane correspond to minima in the other plane. If we look at the phase space plots of the individyual planes, we see that the X and Y distances from the center of the shase space beat strongly. Only by combining the two oscillations together we recover a smooth situation.

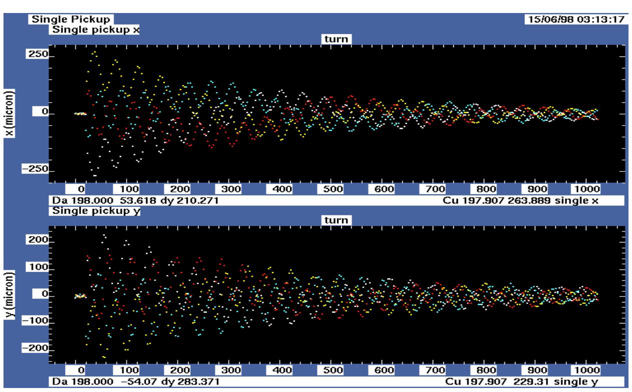

Figure 22: X and Y position.

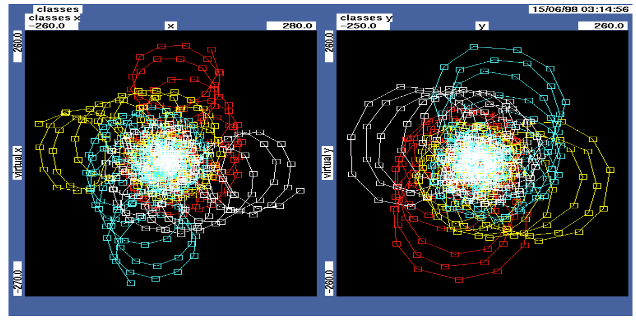

Figure 23: Phase space plots.

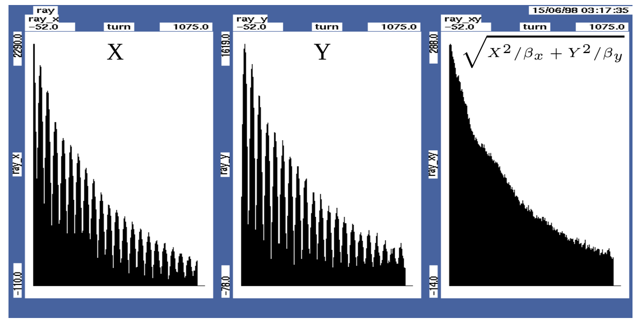

Figure 24: X,Y, and combined distance from the phase space center.

3.6 Non-Linearities Due To Beam-Beam Forces

The same method was applied to data obtained by submitting the beam to a continuous excitation of the betatron frequency. In the data set presented here, the two beams were colliding, and the beam-beam forces are probably responsible for the nonlinear behavior.

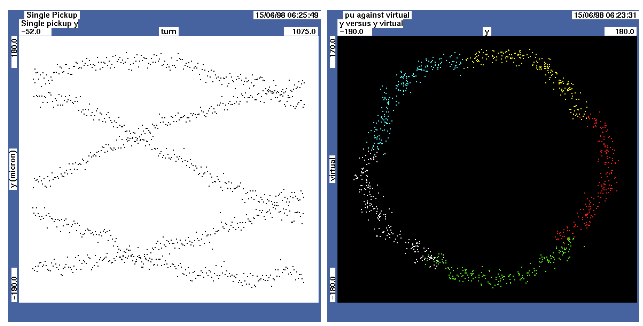

Figure 25: Phase space plot deformation induced by beam-beam forces.

4 REFERENCES

[1] The BOM 1000 Turn Display : A Tool To Visualize The Transverse Phase-Space Topology At LEP, G.Morpurgo; Proceedings of EPAC98, Stockholm, 1998.

[2] Applications Of Beam Diagnostic System At The VEPP-4, A.N.Dubrovin, A.S.Kalinin, D.N.Shatilov, E.A.Simonov, V.V.Smaluk; Proceedings of EPAC96, Sitges, 1996.