Recent Higher-Order Corrections in the r/cMSSM Higgs Sector

Sven Heinemeyer, CERN

in collaboration with *W. Hollik, H. Rzehak and G. Weiglein*

Stanford, 03/2005

1. Motivation

2. Corrections of $\mathcal{O}(\alpha_b\alpha_s)$ in the rMSSM

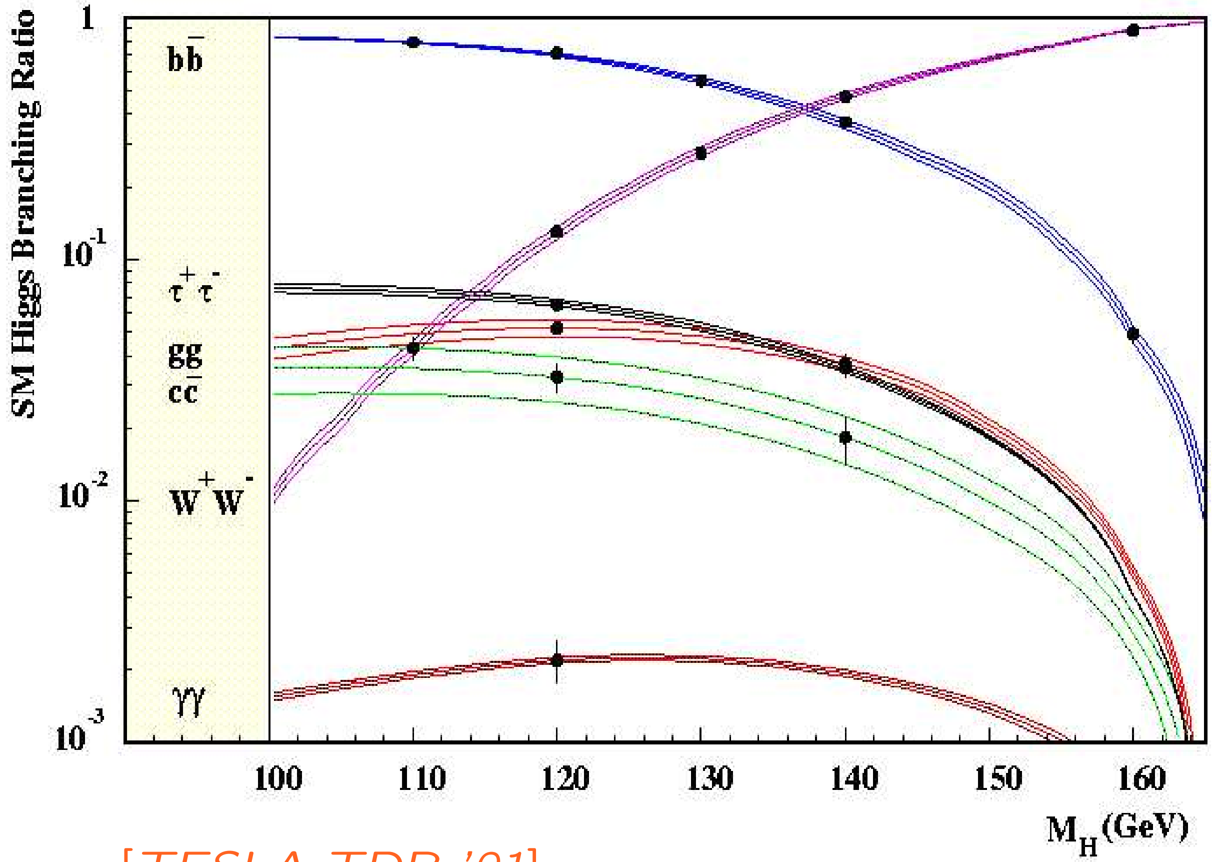
3. Corrections of $\mathcal{O}(\alpha_t\alpha_s)$ in the cMSSM

4. Conclusions

1. Motivation

SM Higgs @ LC:
Precise measurement of:

1. Higgs boson mass,
   $\delta M_H \approx 50$ MeV
2. Higgs boson width
   (direct/indirect)
3. Higgs boson couplings,
   $\mathcal{O}$ (few%) $\Rightarrow$
4. Higgs boson quantum
   numbers: spin, . . .



[*TESLA TDR '01*]

MSSM: similar precision expected (possible problems from loop corrections)

Q: Can this precision be utilized in the MSSM Higgs sector?

The Minimal Supersymmetric Standard Model (MSSM)

Superpartners for Standard Model particles

$$\Big[u, d, c, s, t, b\Big]_{L,R} \quad \Big[e, \mu, \tau\Big]_{L,R} \quad \Big[\nu_{e,\mu,\tau}\Big]_L \qquad \text{Spin } \tfrac{1}{2}$$

$$\Big[\tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}, \tilde{t}, \tilde{b}\Big]_{L,R} \quad \Big[\tilde{e}, \tilde{\mu}, \tilde{\tau}\Big]_{L,R} \quad \Big[\tilde{\nu}_{e,\mu,\tau}\Big]_L \qquad \text{Spin } 0$$

$$g \quad \underbrace{W^\pm, H^\pm} \quad \underbrace{\gamma, Z, H_1^0, H_2^0} \qquad \text{Spin 1 / Spin 0}$$

$$\tilde{g} \qquad \tilde{\chi}^\pm_{1,2} \qquad \tilde{\chi}^0_{1,2,3,4} \qquad \text{Spin } \frac{1}{2}$$

Enlarged Higgs sector: Two Higgs doublets

Problem in the MSSM: many scales

$\tilde{t}/\tilde{b}$ sector of the MSSM: (scalar partner of the top/bottom quark)

Stop, sbottom mass matrices ($X_t = A_t - \mu^*/\tan\beta$, $X_b = A_b - \mu^* \tan\beta$):

$$\mathcal{M}^2_{\tilde{t}} = \begin{pmatrix} M^2_{\tilde{t}_L} + m_t^2 + DT_{t_1} & m_t X_t^* \\ m_t X_t & M^2_{\tilde{t}_R} + m_t^2 + DT_{t_2} \end{pmatrix} \xrightarrow{\theta_{\tilde{t}}} \begin{pmatrix} m^2_{\tilde{t}_1} & 0 \\ 0 & m^2_{\tilde{t}_2} \end{pmatrix}$$

$$\mathcal{M}^2_{\tilde{b}} = \begin{pmatrix} M^2_{\tilde{b}_L} + m_b^2 + DT_{b_1} & m_b X_b^* \\ m_b X_b & M^2_{\tilde{b}_R} + m_b^2 + DT_{b_2} \end{pmatrix} \xrightarrow{\theta_{\tilde{b}}} \begin{pmatrix} m^2_{\tilde{b}_1} & 0 \\ 0 & m^2_{\tilde{b}_2} \end{pmatrix}$$

mixing important in stop sector (also in sbottom sector for large $\tan\beta$)

soft SUSY-breaking parameters A_t, A_b also appear in ϕ-$\tilde{t}/\tilde{b}$ couplings

$SU(2)$ relation $\Rightarrow M_{\tilde{t}_L} = M_{\tilde{b}_L}$

$\Rightarrow$ relation between $m_{\tilde{t}_1}, m_{\tilde{t}_2}, \theta_{\tilde{t}}, m_{\tilde{b}_1}, m_{\tilde{b}_2}, \theta_{\tilde{b}}$

Enlarged Higgs sector: Two Higgs doublets

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix}$$

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.})$$

$$+ \underbrace{\frac{g'^2 + g^2}{8}} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}} \, |H_1 \bar{H}_2|^2$$

gauge couplings, in contrast to SM

physical states: $h^0, H^0, A^0, H^\pm$

Goldstone bosons: $G^0, G^\pm$

Input parameters:

$$\tan\beta = \frac{v_2}{v_1}, \qquad M_A^2 = -m_{12}^2(\tan\beta + \cot\beta)$$

Contrary to the SM:

m_h is not a free parameter

MSSM tree-level bound: $m_h < M_Z$, excluded by LEP Higgs searches

Large radiative corrections:

Dominant one-loop corrections:

$$\Delta m_h^2 \sim G_\mu m_t^4 \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$$

> The MSSM Higgs sector is connected to all other sector via loop corrections (especially to the scalar top sector)

Measurement of m_h, Higgs couplings $\Rightarrow$ test of the theory

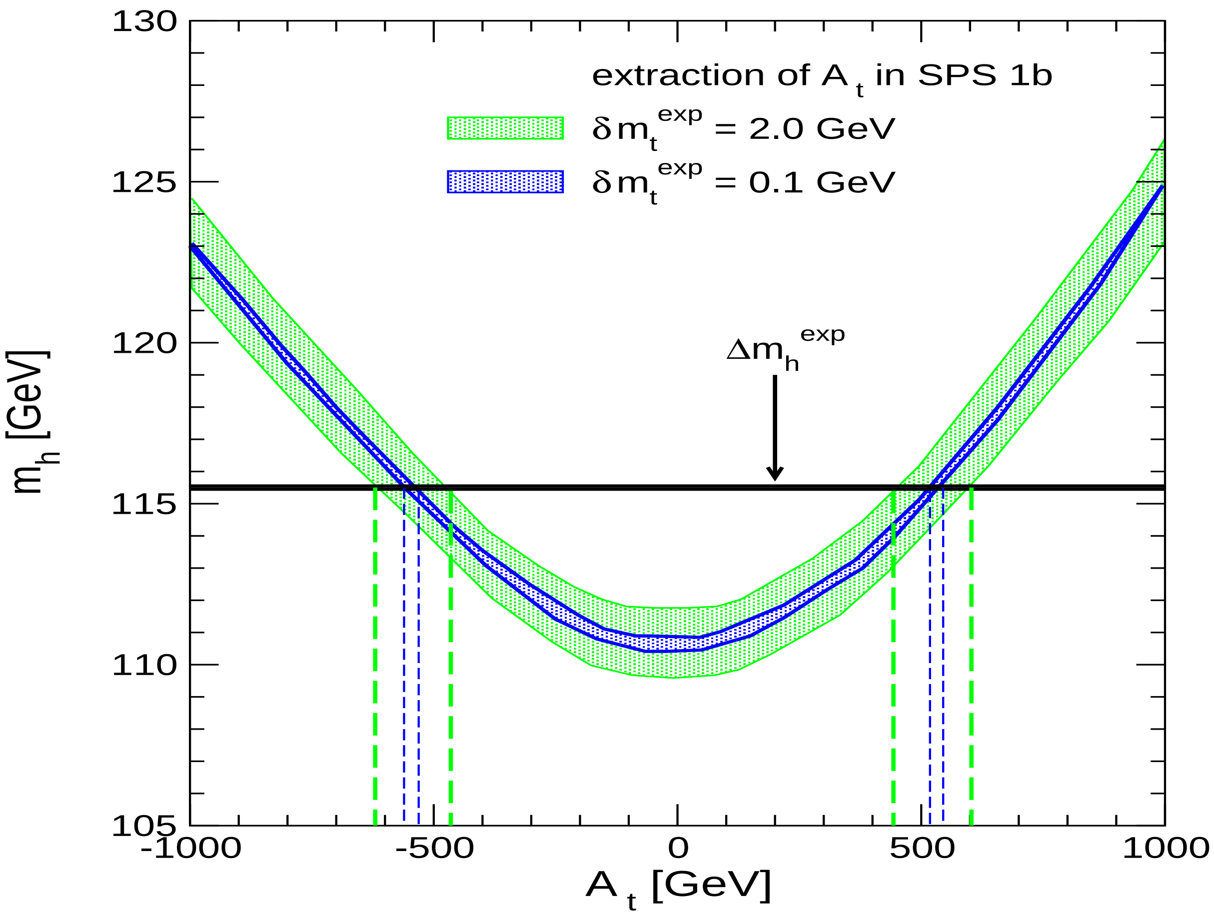
LC: $\Delta m_h \approx 0.05$ GeV

$\Rightarrow$ aim for theoretical precision!

($\Rightarrow m_h$ will be (the best?) electroweak precision observable)

Example of application: m_h prediction as a function of A_t

[S.H., S. Kraml, W. Porod, G. Weiglein '02]



SPS1b:

$m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{b}_1}, m_{\tilde{b}_2}$ known,

A_t unknown

$\tan\beta, M_A$ known,

realistic parametric errors assumed (from SUSY exp. errors)

$\Rightarrow$ extraction of A_t possible
Theory error neglected

$\Rightarrow$ m_h is crucial input for SUSY fit programs (Fittino, Sfitter)

Experimental situation:

LC will provide high accuracy measurements !

Theory situation:

measured observables have to be compared with theoretical predictions (in the MSSM)

Measured data is only meaningful if it is matched with theoretical calculations (masses, couplings) at the same level of accuracy

> Theoretical calculations should be viewed as an essential part of all future High Energy Physics programs

$\Rightarrow$ concentrate on Higgs masses and couplings here

2. Correction of $\mathcal{O}(\alpha_b \alpha_s)$ in the rMSSM

Evaluation of Higgs boson masses in the MSSM with real parameters:

Two-point vertex function:

$$\Gamma(q^2) = \begin{pmatrix} q^2 - m^2_{H,\text{tree}} + \hat{\Sigma}_{HH}(q^2) & \hat{\Sigma}_{hH}(q^2) \\ \hat{\Sigma}_{hH}(q^2) & q^2 - m^2_{h,\text{tree}} + \hat{\Sigma}_{hh}(q^2) \end{pmatrix}$$

determination of $\det(\Gamma(q^2)) = 0 \Rightarrow M_h, M_H, \alpha_{\text{eff}}, \ldots$

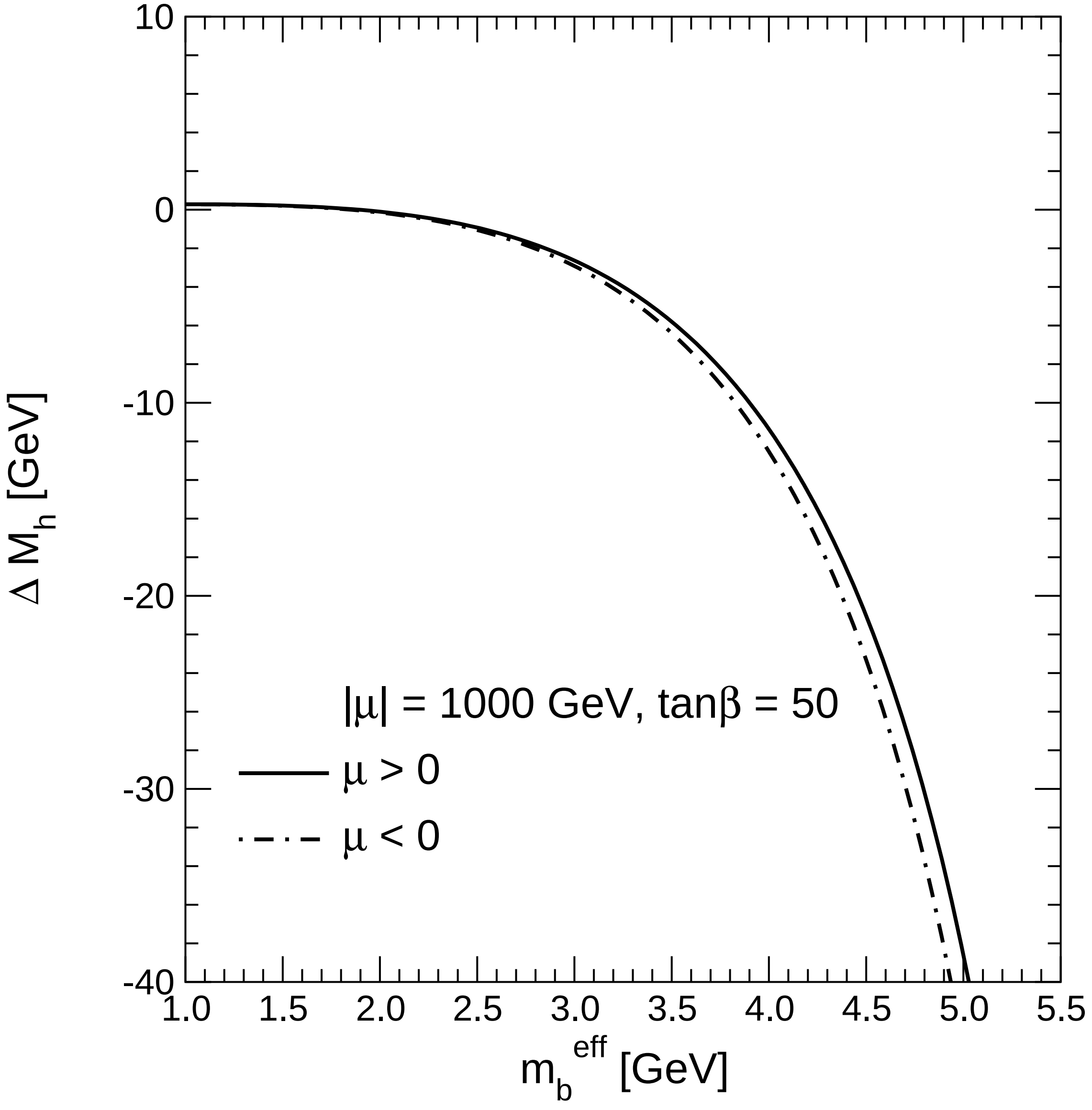
Main task: calculation of $\hat{\Sigma}(q^2)$, including renormalization

Here:

– evaluation of 2-loop corrections of $\mathcal{O}(\alpha_b \alpha_s)$

– comparison of 4 different renormalization schemes

Motivation: Why 2-loop corrections in the $b/\tilde{b}$ sector?



1-loop corrections $\mathcal{O}(\alpha_b)$ to M_h can be sizable

Precise M_h prediction
$\Rightarrow$ 2-loop corrections necessary

The Higgs self-energy at 2-loop:

$\to \alpha_s$ correction to the leading 1-loop term $\sim m_b^4$

Approximations:

– only m_b^2 terms

– gauge couplings are set to 0

– external momentum is set to 0

$$\Rightarrow \hat{\Sigma}_{22}^{(2)}(q^2) \approx \Sigma_{22}^{(2)}(0) + \cos^2\beta\,\delta M_A^{2\,(2)} - \frac{e}{2\,M_W s_W}\left(\sin^2\beta\,\cos\beta\,\delta t_1^{(2)} - \sin\beta\,(1+\cos^2\beta\,)\delta t_2^{(2)}\right)$$

in the $\phi_1\phi_2$ basis with

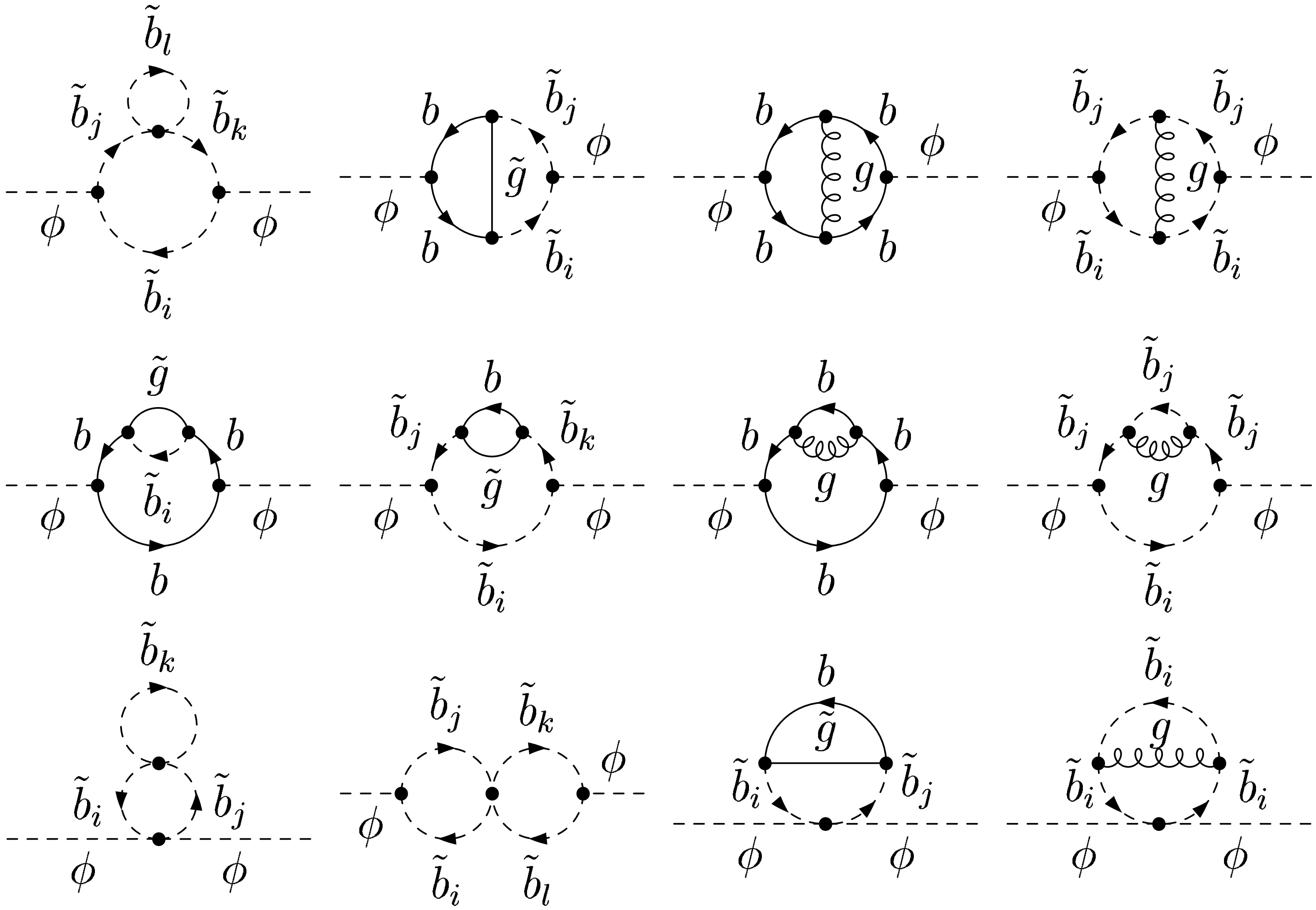
$\Sigma_{22}^{(2)}(0)$: unrenormalized 2-2 self-energy

$\delta M_A^{2\,(2)} = \Sigma_A^{(2)}(0)$: A mass counter term

$\delta t_i^{(2)} = -T_i^{(2)}$: ϕ_i tad-pole

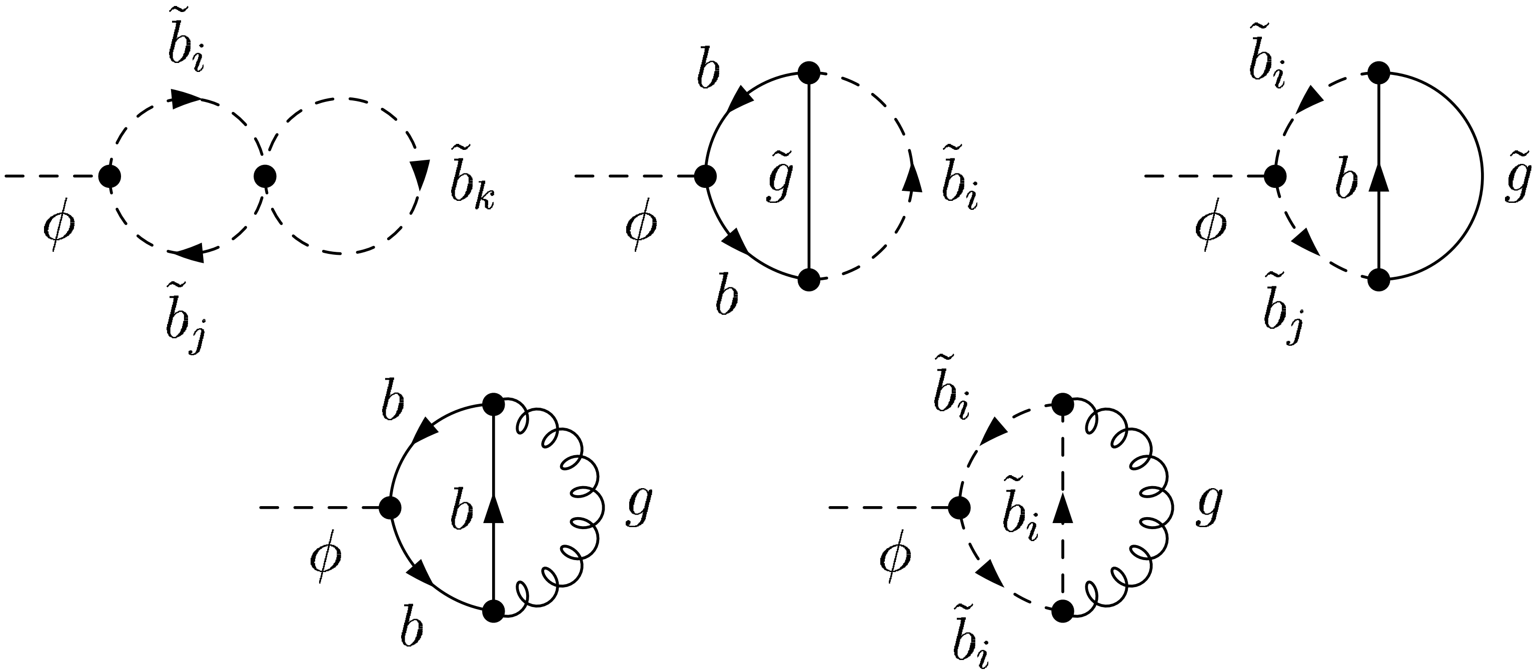
Contributions to the 2-loop self-energy:

2-loop self-energy diagrams:



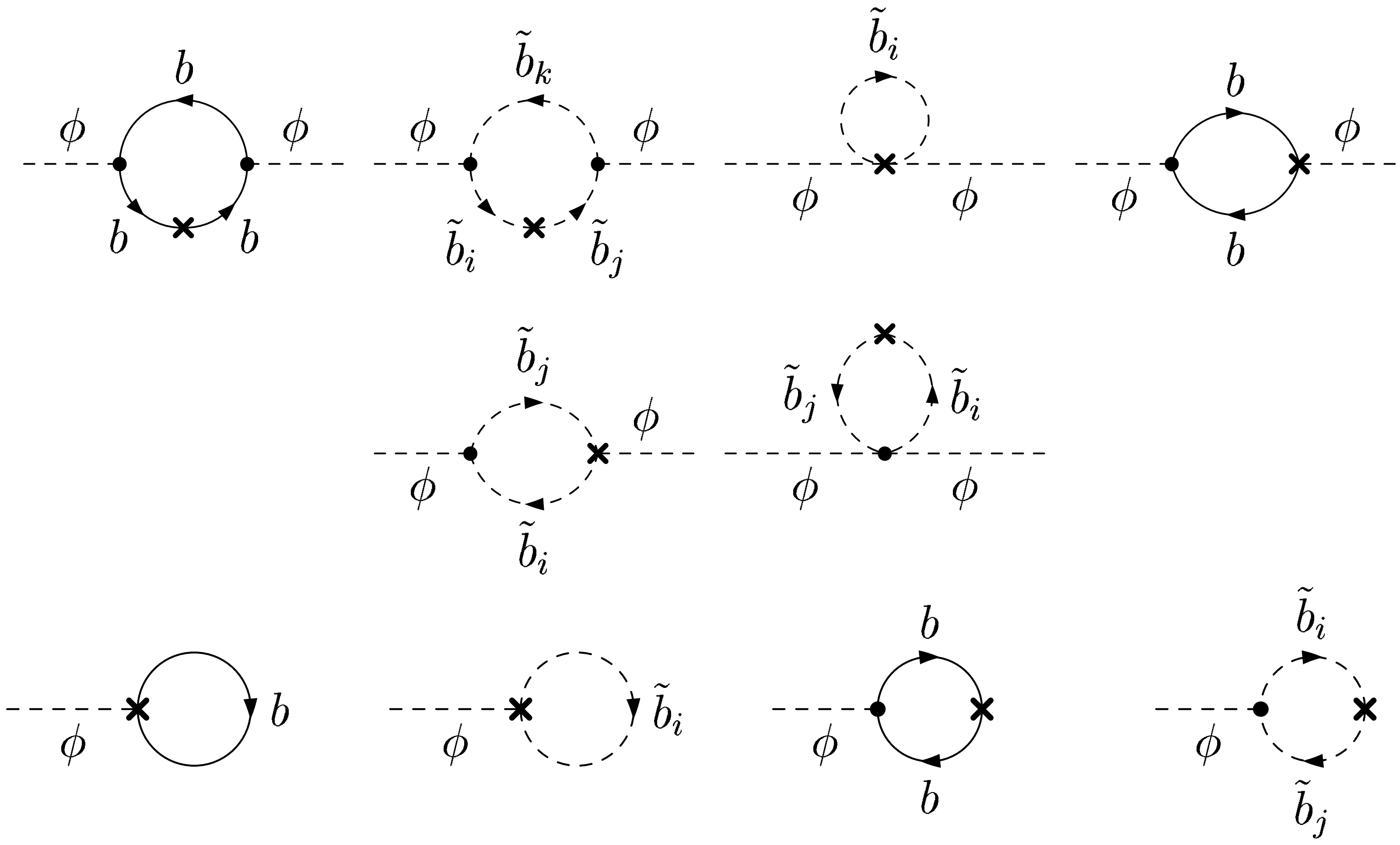
Contributions to the 2-loop self-energy:

2-loop tad-pole diagrams:



Contributions to the 2-loop self-energy:

diagrams with counter term insertion:



$\rightarrow$ different renormalization schemes enter

Evaluation of 2-loop diagrams:

1. Generation of diagrams and amplitudes with FeynArts
   [*Küblbeck, Böhm, Denner '90*] [*Hahn '00 - '03*]

2. Algebraic evaluation and tensor integral reduction to scalar integrals: TwoCalc

   (works for two-loop self-energies)

   [*G. Weiglein '92*] [*G. Weiglein, R. Scharf, M. Böhm '94*]

3. Further evaluation: insertion of integrals, expansion in $\delta = \frac{1}{2}(4 - D)$
   $\to$ algebraical check: cancellation of divergencies

4. Result:
   - algebraic Mathematica code
   - Fortran code (planned: implementation into *FeynHiggs*)

Renormalization:

Calculation of two-loop corrections of $\mathcal{O}(\alpha_t \alpha_s)$ and $\mathcal{O}(\alpha_b \alpha_s)$

$\Rightarrow$ parameters of the $t/\tilde{t}$ and $b/\tilde{b}$ are defined at the 1-loop level

$\Rightarrow$ different choices of renormalization possible

$t/\tilde{t}$ sector:

> one renormalization scheme:
> 4 independent parameters:
> $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$, $\theta_{\tilde{t}}$, m_t on-shell
> $\rightarrow A_t$ given in terms of the others

$b/\tilde{b}$ sector: four schemes analyzed

Investigation of scheme dependence:

$\Rightarrow$ information about size of missing higher order corrections

$\Rightarrow$ estimate of theory uncertainty

Renormalization schemes in the $b/\tilde{b}$ sector:

$$SU(2) \text{ relation} \Rightarrow M_{\tilde{t}_L} = M_{\tilde{b}_L}$$

$\Rightarrow$ out of $m_{\tilde{b}_1}$, $m_{\tilde{b}_2}$, $\theta_{\tilde{b}}$, A_b, m_b only 3 are independent

$\Rightarrow$ two parameters (incl. CTs) are given in terms of the others

In all four schemes: $m_{\tilde{b}_1}$ dep. ($SU(2)$ relation), $m_{\tilde{b}_2}$ OS

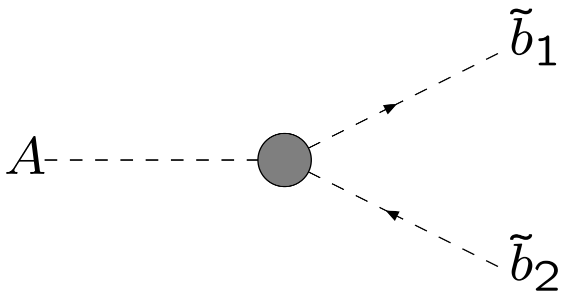
scheme	b-mass m_b	A_b	mixing angle $\theta_{\tilde{b}}$
m_b $\overline{\text{DR}}$	$\overline{\text{DR}}$	$\overline{\text{DR}}$	dep.
A_b, $\theta_{\tilde{b}}$ OS	dep.	OS	OS
A_b, $\theta_{\tilde{b}}$ $\overline{\text{DR}}$	$\overline{\text{DR}}$	dep.	$\overline{\text{DR}}$
m_b OS	OS	dep.	OS

Some more details:

– scheme m_b OS: analogous to the $t/\tilde{t}$ sector

→ obvious choice ?

– A_b OS: determined via



analogous to [*A. Brignole, G. Degrassi, P. Slavich and F. Zwirner '02*]

– $\theta_{\tilde{t}}$ OS: $\delta\theta_{\tilde{b}} = \dfrac{\text{Re}\,\Sigma_{\tilde{b}_{12}}(m^2_{\tilde{b}_1}) + \text{Re}\,\Sigma_{\tilde{b}_{12}}(m^2_{\tilde{b}_2})}{m^2_{\tilde{b}_1} + m^2_{\tilde{b}_2}}$

Resummed bottom quark mass:

→ absorb the leading corrections in a resummed form
in the bottom quark mass at the 1-loop level

$$m_b^{\overline{\text{DR}}} = \frac{\tilde{m}_b^{\text{pole}} + \Sigma_b^{\tan\beta \text{non-enh.}}{}_{|\text{fin}}}{1 + \Delta m_b}$$

with

$$\Delta m_b = \frac{2\,\alpha_s}{3\,\pi} \tan\beta\,\mu\,m_{\tilde{g}}\, I(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, m_{\tilde{g}}^2)$$

$$\Sigma_b^{\tan\beta \text{non-enh.}}{}_{|\text{fin}} = \tan\beta \text{ non-enhanced terms in } \Sigma_{b,s}$$

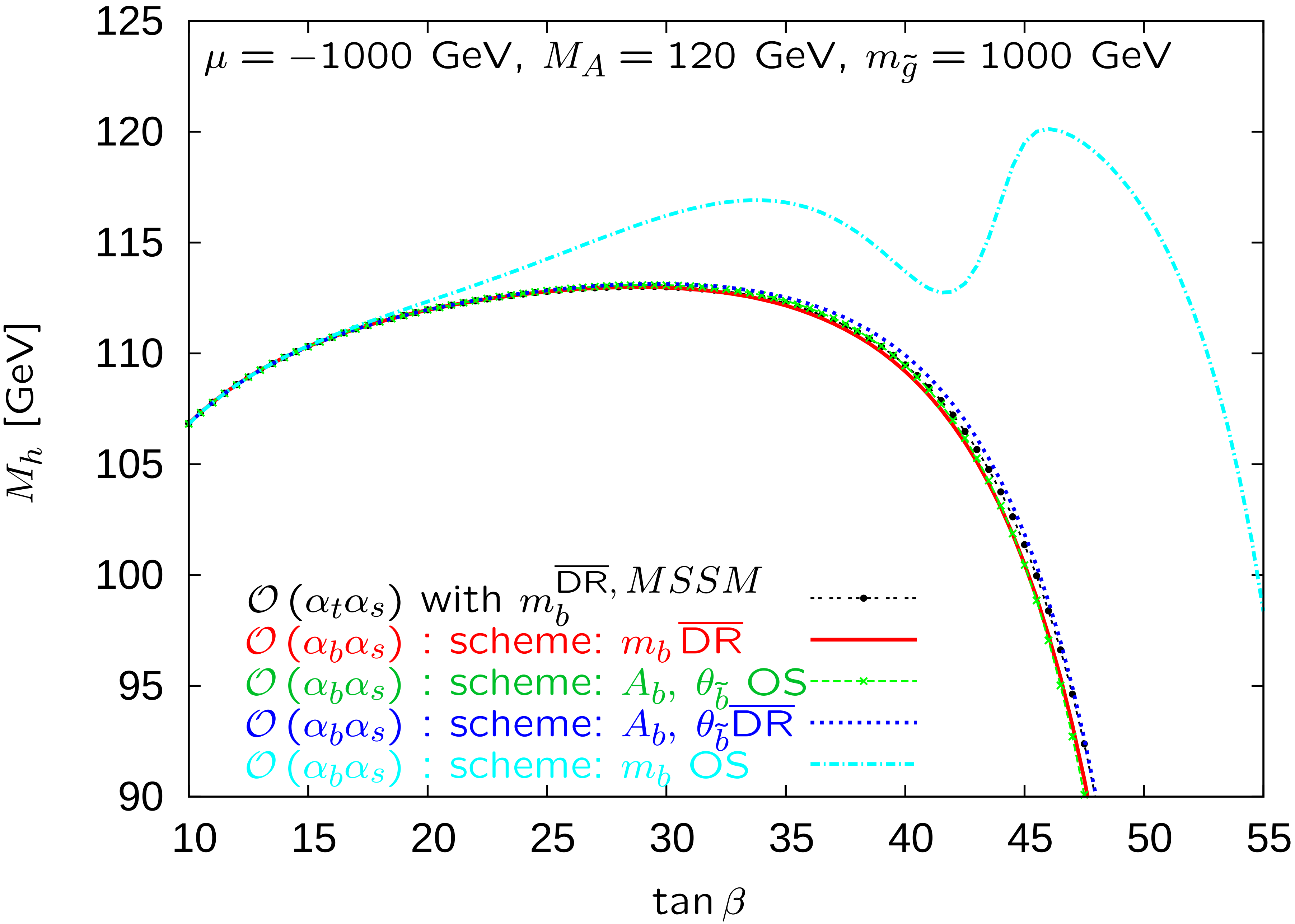
$$\tilde{m}_b^{\text{pole}} = m_b^{\overline{\text{MS}}}(M_Z) \times \left[1 + \frac{\alpha_s}{\pi}\left(\frac{4}{3} - \log\frac{(m_b^{\overline{\text{MS}}})^2}{M_Z^2}\right)\right]$$

"formal" pole mass obtained from the $\overline{\text{MS}}$ mass

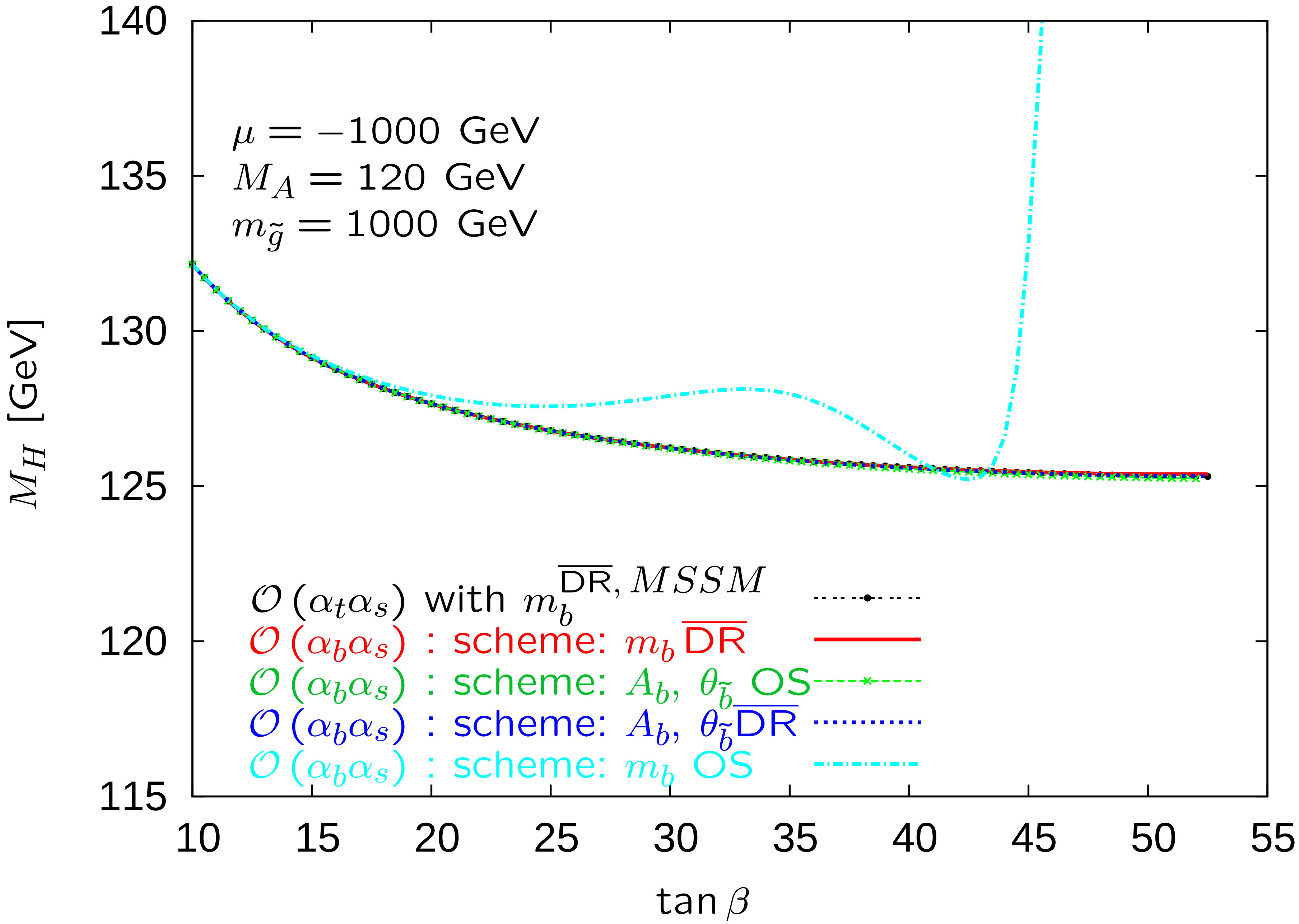
⇒ large higher-order corrections included at the 1-loop level

→ other renormalization schemes by finite shift

M_h as a function of $\tan\beta$, $\mu < 0$:



M_H as a function of $\tan\beta$, $\mu < 0$:



Observations:

- Scheme m_b OS gives very large corrections

  Reason: A_b is a dependent quantity $\Rightarrow$ large corrections via δA_b

$$\delta A_b = \frac{1}{m_b}\left[-\frac{\delta m_b}{2\,m_b}(m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2)\sin 2\theta_{\tilde{b}} + \ldots\right]$$
$$= \frac{1}{m_b}\left[-\delta m_b(A_b - \mu \tan\beta) + \ldots\right]$$

$$\hat{\Sigma}_{HH} \sim (\cos\alpha\, A_b)^2, \quad \hat{\Sigma}_{hh} \sim (\sin\alpha\, A_b)^2$$

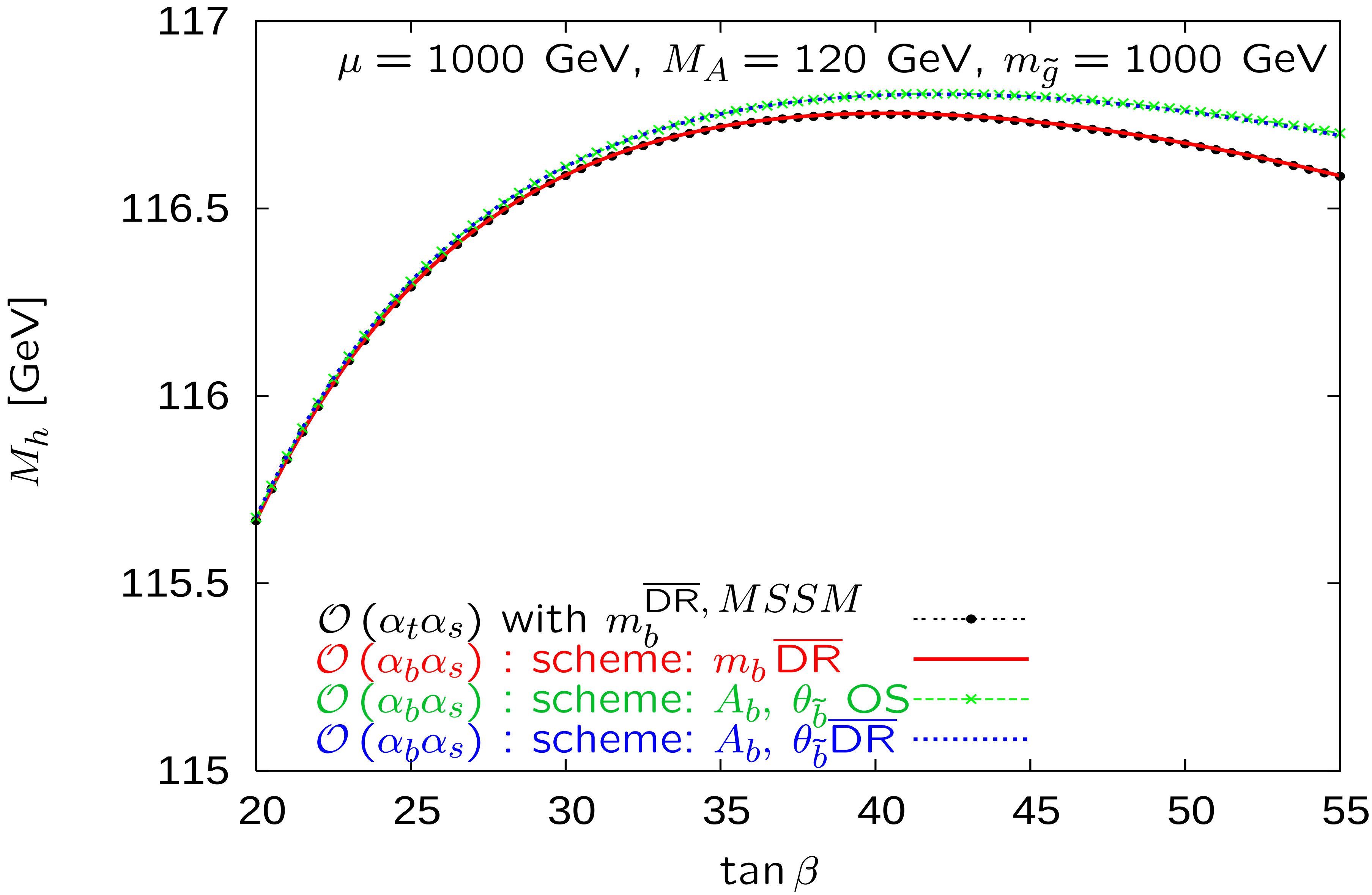
  $\Rightarrow$ effect more pronounced for M_H

  $\Rightarrow$ Scheme m_b OS is discarded as a useful renormalization scheme

- Other schemes: differences of $\mathcal{O}$ (1 GeV) for large $\tan\beta$

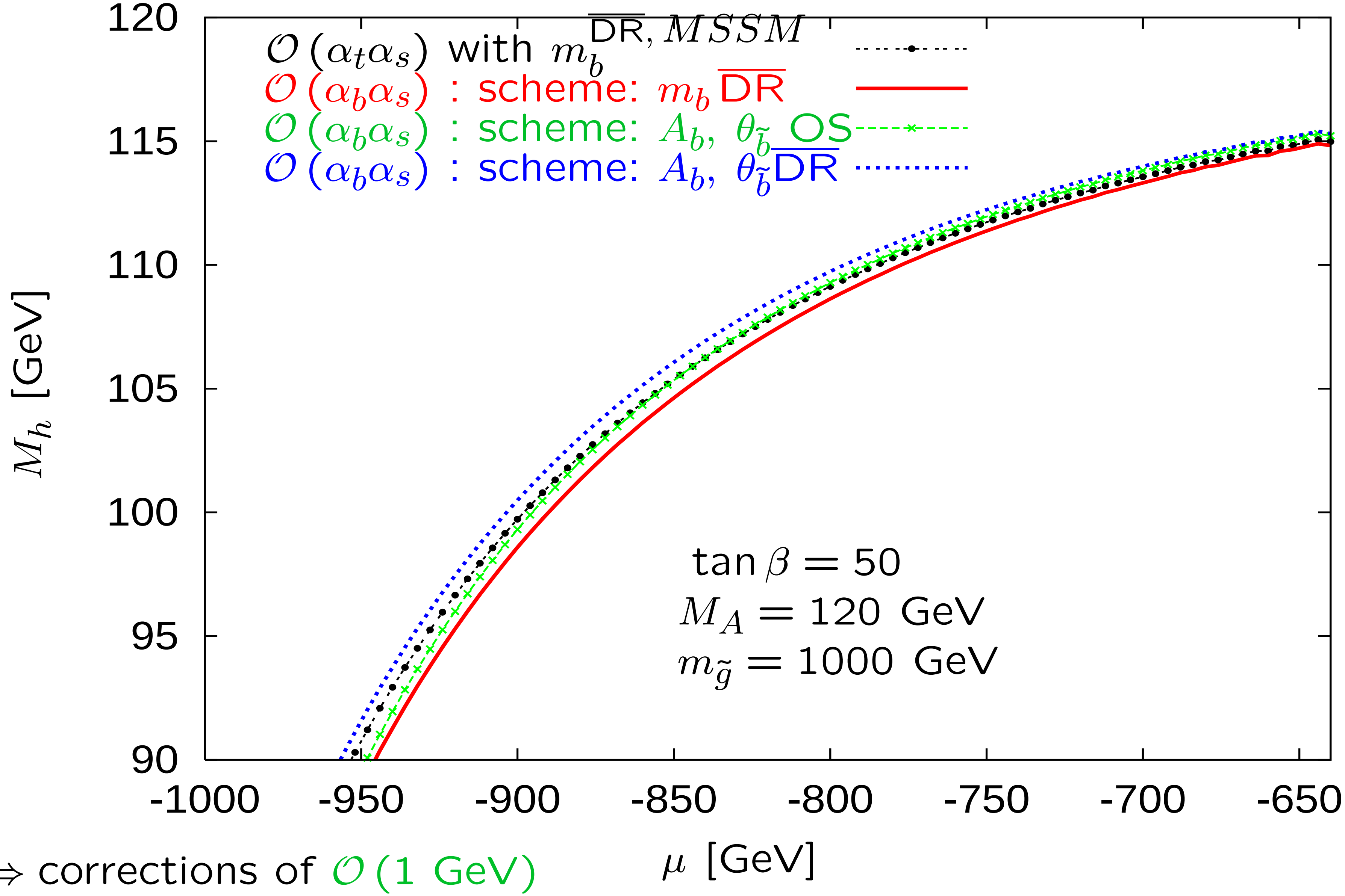
  $\Rightarrow$ non-negligible

M_h as a function of $\tan\beta$, $\mu > 0$:



$\Rightarrow$ small corrections, scheme m_b $\overline{\mathrm{DR}}$: "no" correction

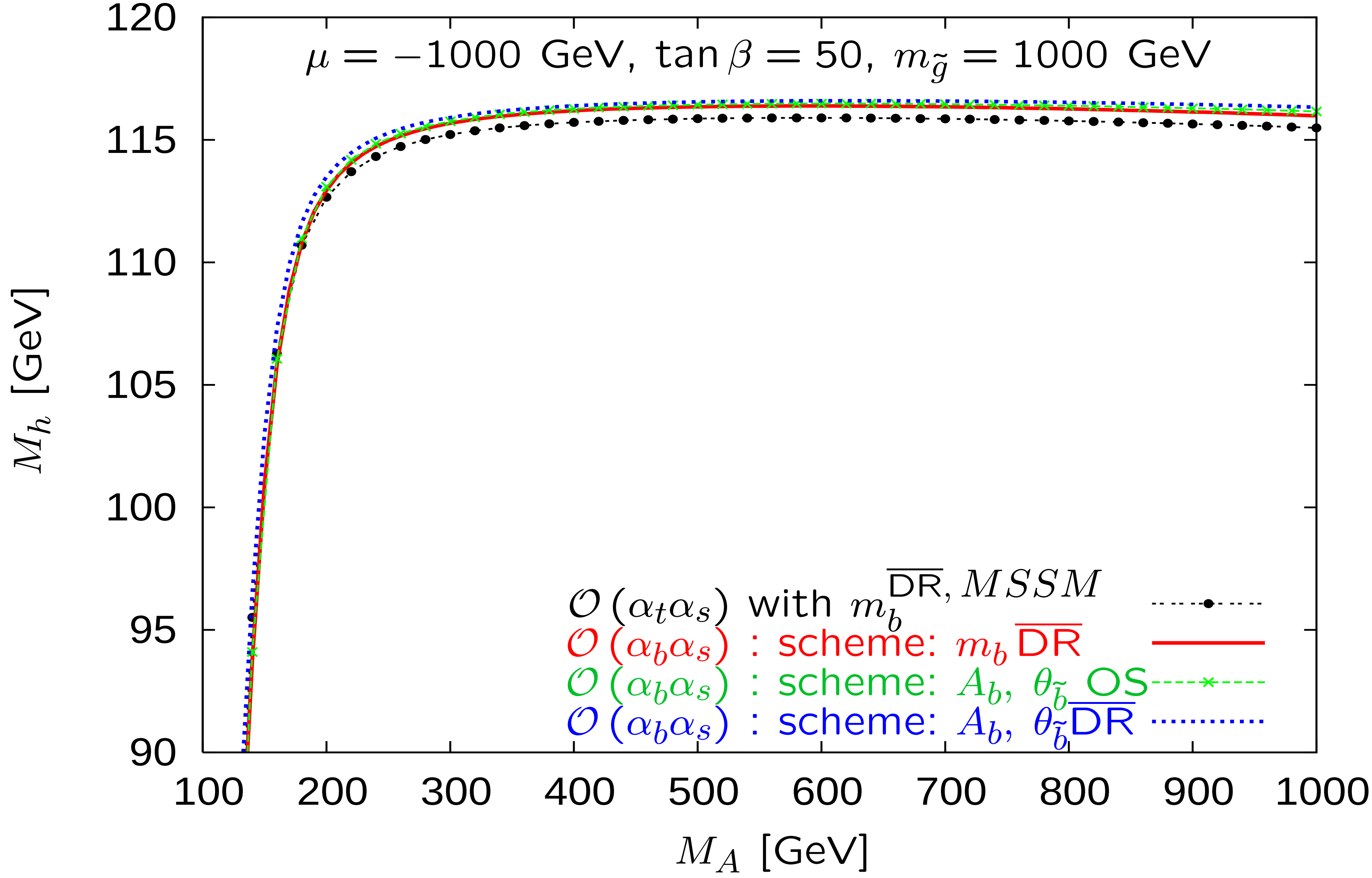
M_h as a function of μ, $\mu < 0$:



$\Rightarrow$ corrections of $\mathcal{O}(1$ GeV$)$

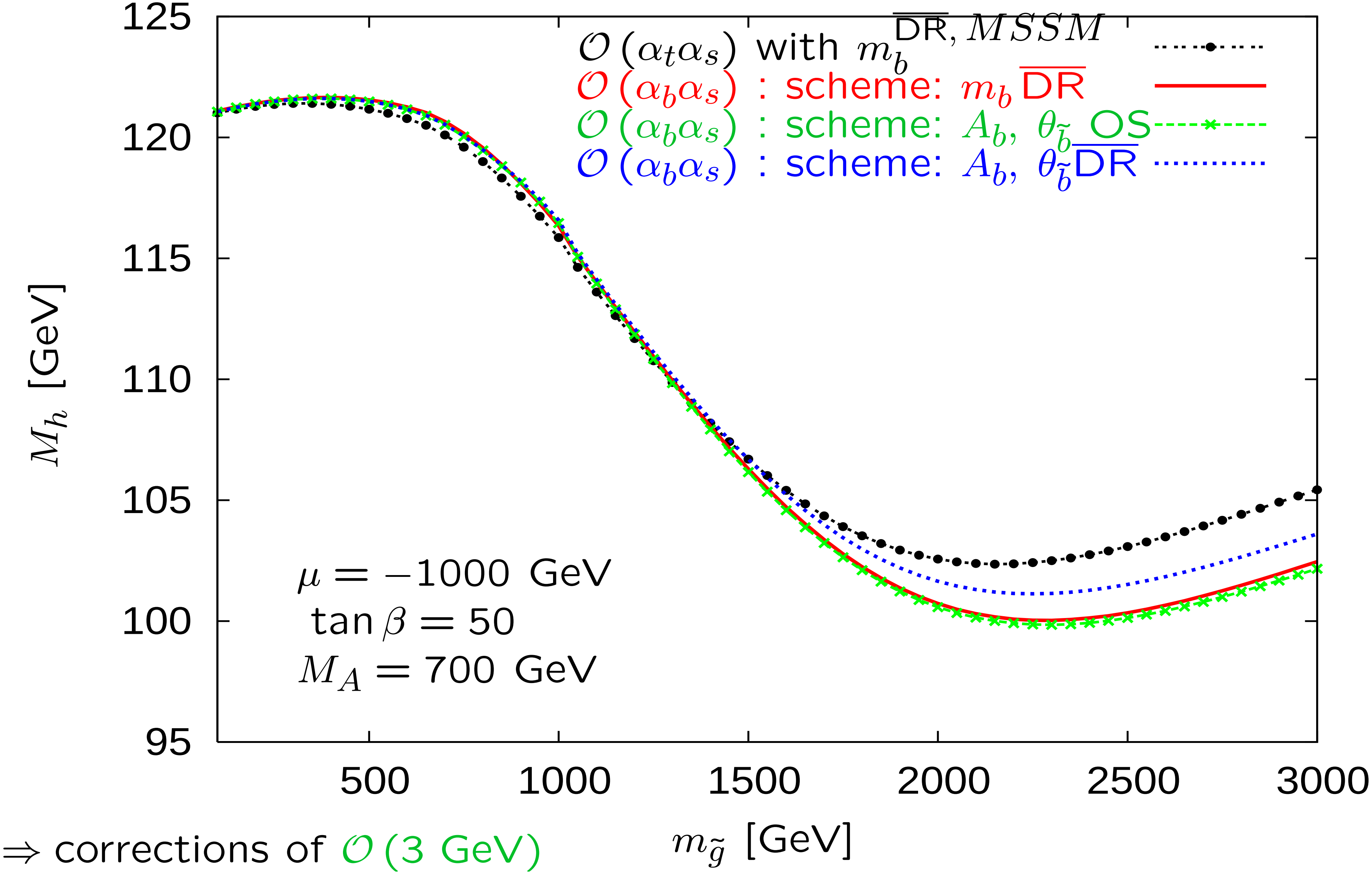
scheme difference similar

M_h as a function of M_A, $\mu < 0$:



$\Rightarrow$ subleading corrections of $\mathcal{O}$(1 GeV)

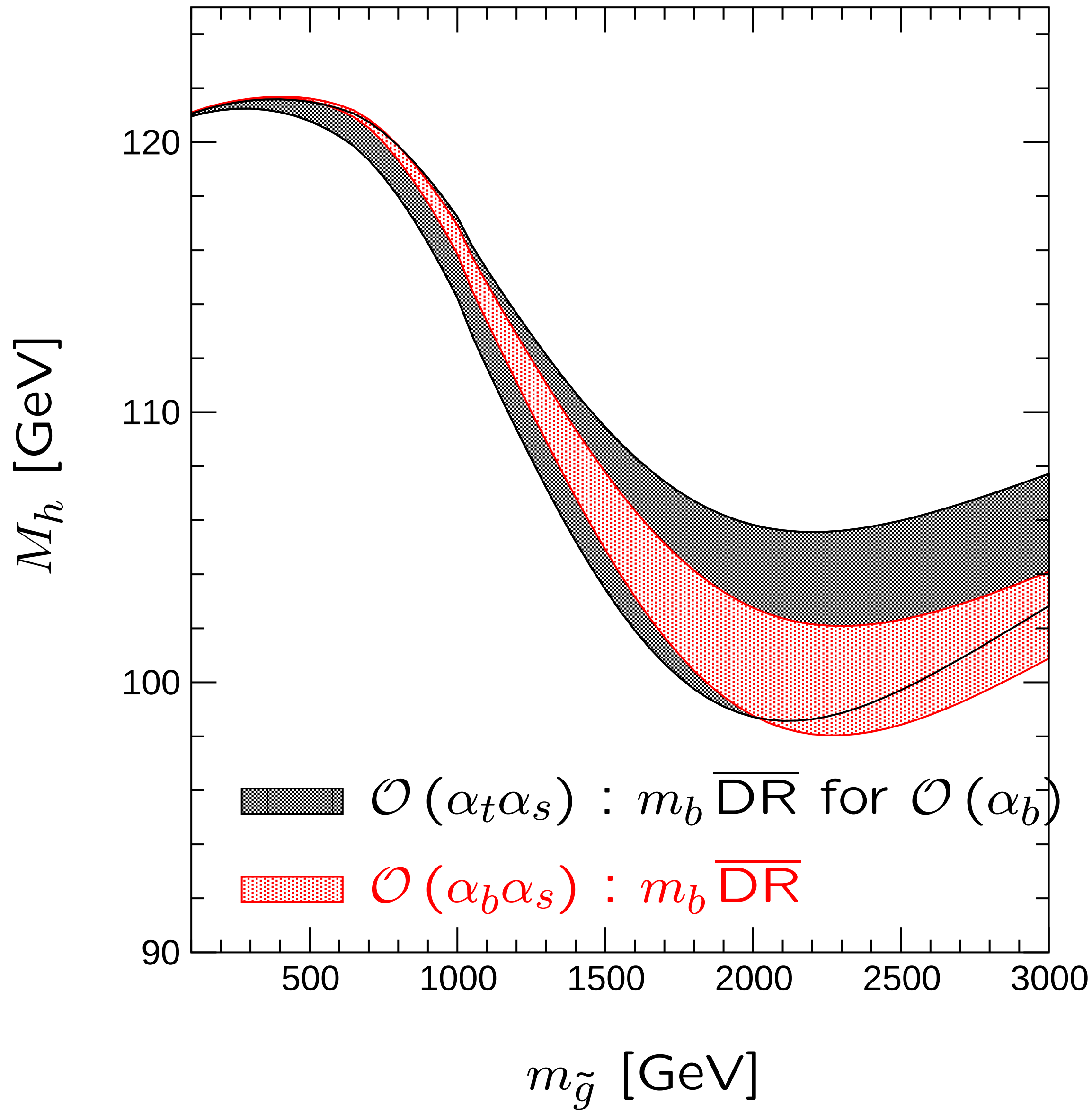
M_h as a function of $m_{\tilde{g}}$, $\mu < 0$:



$\Rightarrow$ corrections of $\mathcal{O}$(3 GeV)

scheme difference $\mathcal{O}$(2 GeV)

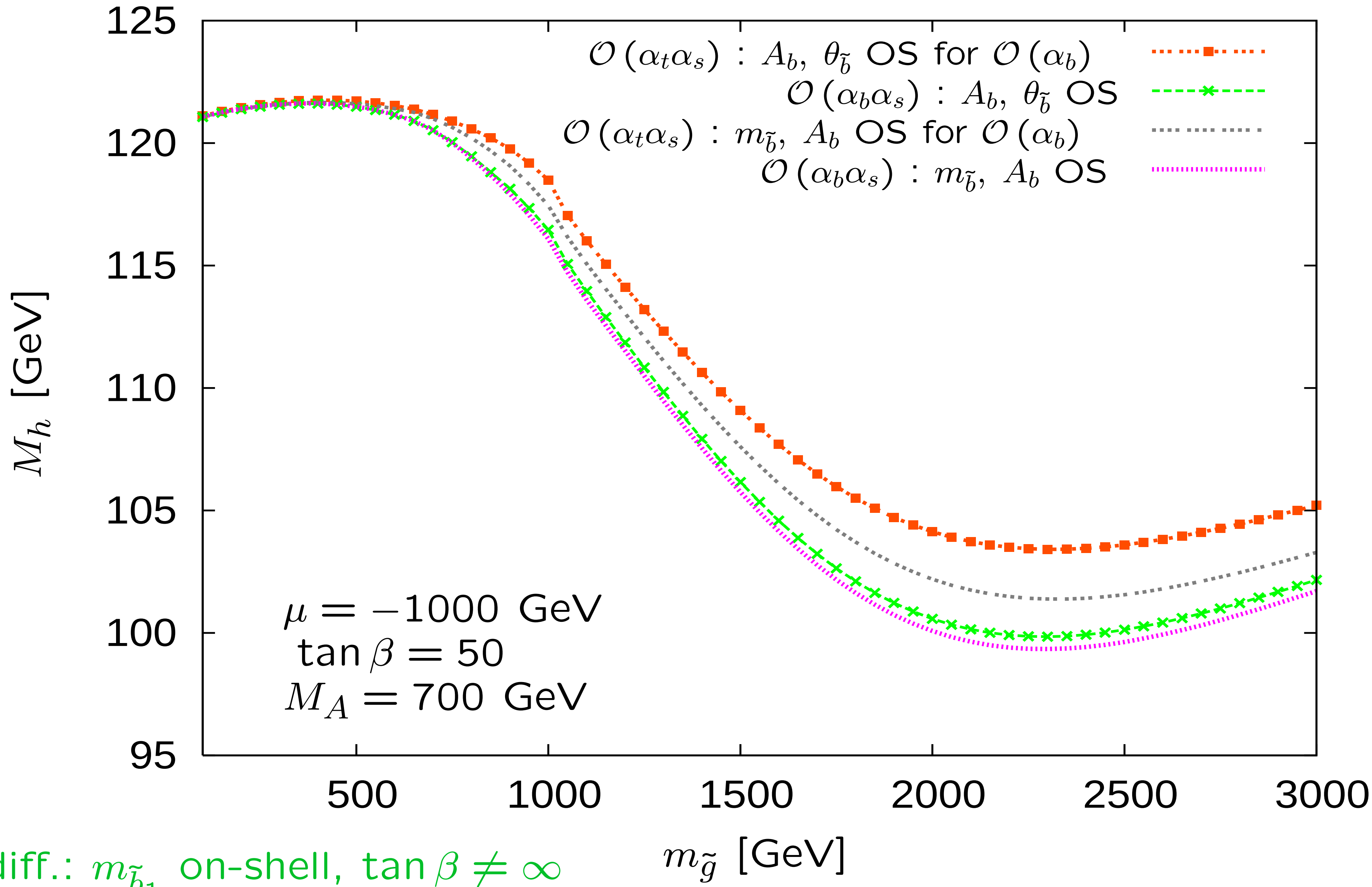
Dependence on renormalization scale $\mu^{\overline{\rm DR}}$:



$M_A = 700$ GeV
$\mu = -1000$ GeV
$\tan\beta = 50$
$m_t/2 \leq \mu^{\overline{\rm DR}} \leq 2\,m_t$

$\Rightarrow$ scale dependence
$\mathcal{O}(\pm 2$ GeV$)$ for large $m_{\tilde g}$

Comparison with existing calculation: [A. Brignole et al. '02]



diff.: $m_{\tilde{b}_1}$ on-shell, $\tan\beta \neq \infty$

$\Rightarrow$ 2-loop: differences $\mathcal{O}(0.5$ GeV$)$

3. Corrections of $\mathcal{O}(\alpha_t \alpha_s)$ in the cMSSM

Higgs potential of the cMSSM contains two Higgs doublets:

$$
\begin{aligned}
H_1 &= \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix} \\
H_2 &= \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = e^{i\xi} \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix}
\end{aligned}
$$

$$
\begin{aligned}
V &= m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.}) \\
&\quad + \underbrace{\frac{g'^2 + g^2}{8}}_{} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}}_{} |H_1 \bar{H}_2|^2
\end{aligned}
$$

gauge couplings, in contrast to SM

Five physical states: $h^0, H^0, A^0, H^\pm$ (no $\mathcal{CP}$V at tree-level)

2 $\mathcal{CP}$-violating phases: ξ, arg(m_{12}) $\Rightarrow$ can compensate each other

Input parameters: $\tan\beta = \frac{v_2}{v_1}$, M_A or $M_{H^\pm}$

Status of calculations in the cMSSM and uncertainties:

– fermion/sfermion corrections at 1-loop, $q^2 = 0$

– some leading logs from remaining sectors

– leading 2-loop corrections

[*A. Pilaftsis '98*] , [*A. Pilaftsis, C. Wagner '99*] , [*A. Demir '99*] , [*S. H. '01*]
[*S. Choi, M. Drees, J. Lee '00*] , [*M. Carena, J. Ellis, A. Pilaftsis, C. Wagner '00, '01*]
[*T. Ibrahim, P. Nath '01, '02*] , [*S. Ham, C. Kim, S. Oh, D. Son, E. Yoo '02*]
[*S.H., W. Hollik, H. Rzehak, G. Weiglein '05*]

– remaining sectors at 1-loop (rMSSM: 5 GeV)

– q^2 dependence at 1-loop (rMSSM: $\sim$ 2 GeV)

[*M. Frank, S. H., W. Hollik, G. Weiglein '02*]

Effects of complex parameters in the Higgs sector:

Complex parameters enter via loop corrections:

- μ : Higgsino mass parameter
- $A_{t,b,\tau}$: trilinear couplings $\Rightarrow X_{t,b,\tau} = A_{t,b} - \mu^* \{\cot\beta\,, \tan\beta\}$ complex
- $M_{1,2}$: gaugino mass parameter (one phase can be eliminated)
- $m_{\tilde{g}}$: gluino mass

$\Rightarrow$ can induce $\mathcal{CP}$-violating effects

Result:

$$(A, H, h) \to (h_3, h_2, h_1)$$

with

$$m_{h_3} > m_{h_2} > m_{h_1}$$

Inclusion of higher-order corrections:

($\to$ Feynman-diagrammatic approach)

Propagator / mass matrix with higher-order corrections:

$$\begin{pmatrix} q^2 - M_A^2 + \hat{\Sigma}_{AA}(q^2) & \hat{\Sigma}_{AH}(q^2) & \hat{\Sigma}_{Ah}(q^2) \\ \hat{\Sigma}_{HA}(q^2) & q^2 - m_H^2 + \hat{\Sigma}_{HH}(q^2) & \hat{\Sigma}_{Hh}(q^2) \\ \hat{\Sigma}_{hA}(q^2) & \hat{\Sigma}_{hH}(q^2) & q^2 - m_h^2 + \hat{\Sigma}_{hh}(q^2) \end{pmatrix}$$

$\hat{\Sigma}_{ij}(q^2)$ $(i, j = h, H, A)$: renormalized Higgs self-energies

$\hat{\Sigma}_{Ah}, \hat{\Sigma}_{AH} \neq 0 \;\Rightarrow\; \mathcal{CP}$V, $\mathcal{CP}$-even and $\mathcal{CP}$-odd fields can mix

Our result for $\hat{\Sigma}_{ij}$:

– full 1-loop evaluation: dependence on all possible phases included

– New: $\mathcal{O}(\alpha_t \alpha_s)$ corrections in the FD approach

rMSSM: difference between FD and RGiEP approach $\mathcal{O}$(few GeV)

Result: $(A, H, h) \to (h_3, h_2, h_1)$ with $m_{\mathrm{h}_3} > m_{\mathrm{h}_2} > m_{\mathrm{h}_1}$

Higgs boson couplings:

(in $q^2 = 0$ approximation)

$$\begin{pmatrix} h_3 \\ h_2 \\ h_1 \end{pmatrix} = \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ u_{21} & u_{22} & u_{23} \\ u_{31} & u_{32} & u_{33} \end{pmatrix} \cdot \begin{pmatrix} A \\ H \\ h \end{pmatrix}$$

− h_1, h_2, h_3 : neutral Higgs boson with $\mathcal{CP}$V couplings

− $u_{12}, u_{13}, u_{21}, u_{31}$: $\mathcal{CP}$V mixings

− u_{ij} determine Higgs-fermion and Higgs-gauge boson couplings

The Higgs self-energy at 2-loop:

$\to \alpha_s$ correction to the leading 1-loop term $\sim m_t^4$

Approximations:

– only m_t^2 terms

– gauge couplings are set to 0

– external momentum is set to 0

$$\Rightarrow \hat{\Sigma}_{hh}^{(2)}(0) = \Sigma_{hh}^{(2)}(0) + (\cos\alpha \cos\beta + \sin\alpha \sin\beta)^2\, \delta M_{H^\pm}^{2\,(2)} - \frac{e}{2\, M_W s_W}\left(f_1(\alpha,\beta)\delta t_1^{(2)} + f_2(\alpha,\beta)\delta t_2^{(2)}\right)$$

in the hH basis with

$\Sigma_{hh}^{(2)}(0)$: unrenormalized hh self-energy

$\delta M_{H^\pm}^{2\,(2)} = \Sigma_{H^\pm}^{(2)}(0)$: $H^\pm$ mass counter term

$\delta t_i^{(2)} = -T_i^{(2)}$: ϕ_i tad-pole

$\hat{\Sigma}_{hA}$, $\hat{\Sigma}_{HA}$: $\delta t_A = -T_A$ enters

Differences to the real case:

- use $M_{H^\pm}$ as on-shell mass, since M_A receives loop corrections
  $\Rightarrow$ $\tilde{b}$ sector enters via $\Sigma_{H^\pm}$
  $\Rightarrow$ renormalization of the $\tilde{b}$ sector

- A_t complex $\Rightarrow$ renormalization of $|A_t|$ and ϕ_{A_t}

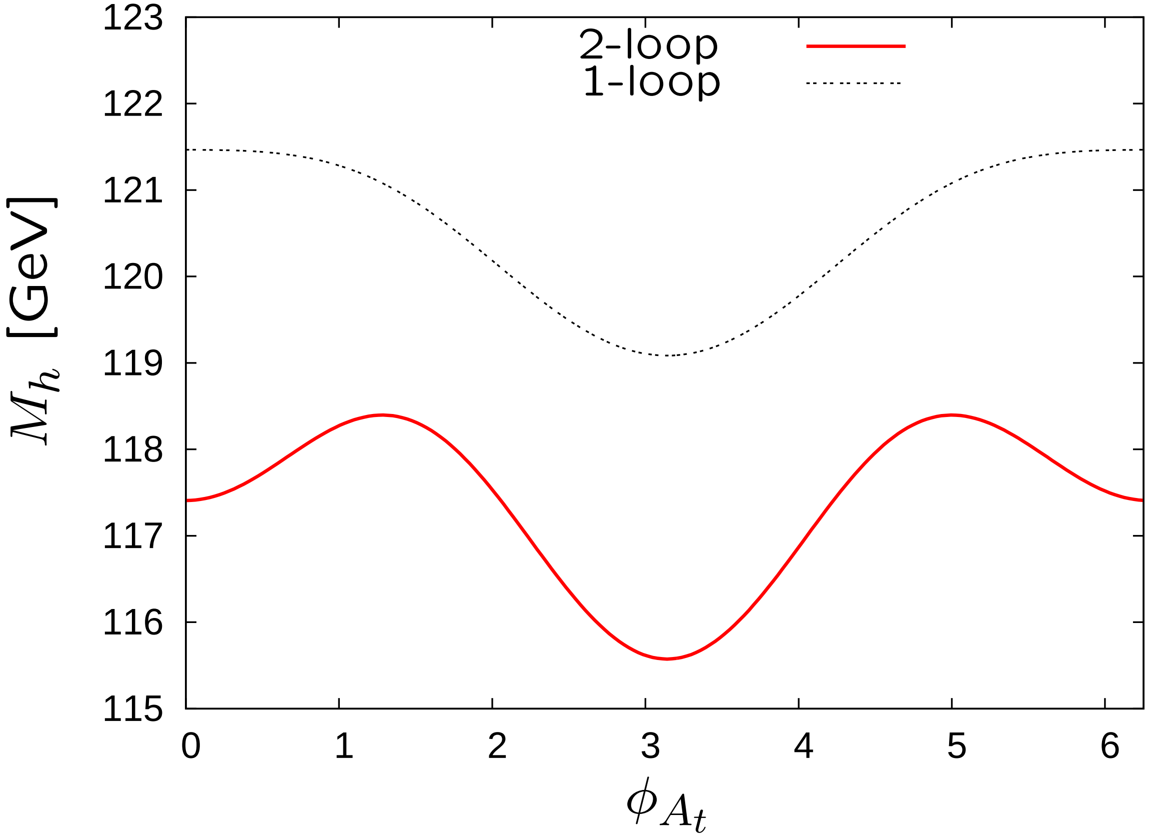
  (no renormalization of μ, no $\mathcal{O}(\alpha_s)$ corrections)

- M_3 complex, but $m_{\tilde{g}}$ is real (and positive)
  $\Rightarrow$ phase of M_3 enters gluino vertices

- $T_A \neq 0$ $\Rightarrow$ renormalized to zero
  $\Rightarrow$ δt_A enters renormalized self-energies

$\rightarrow$ so far all results preliminary!

M_h as a function of ϕ_{A_t}:



"normal SUSY parameters"

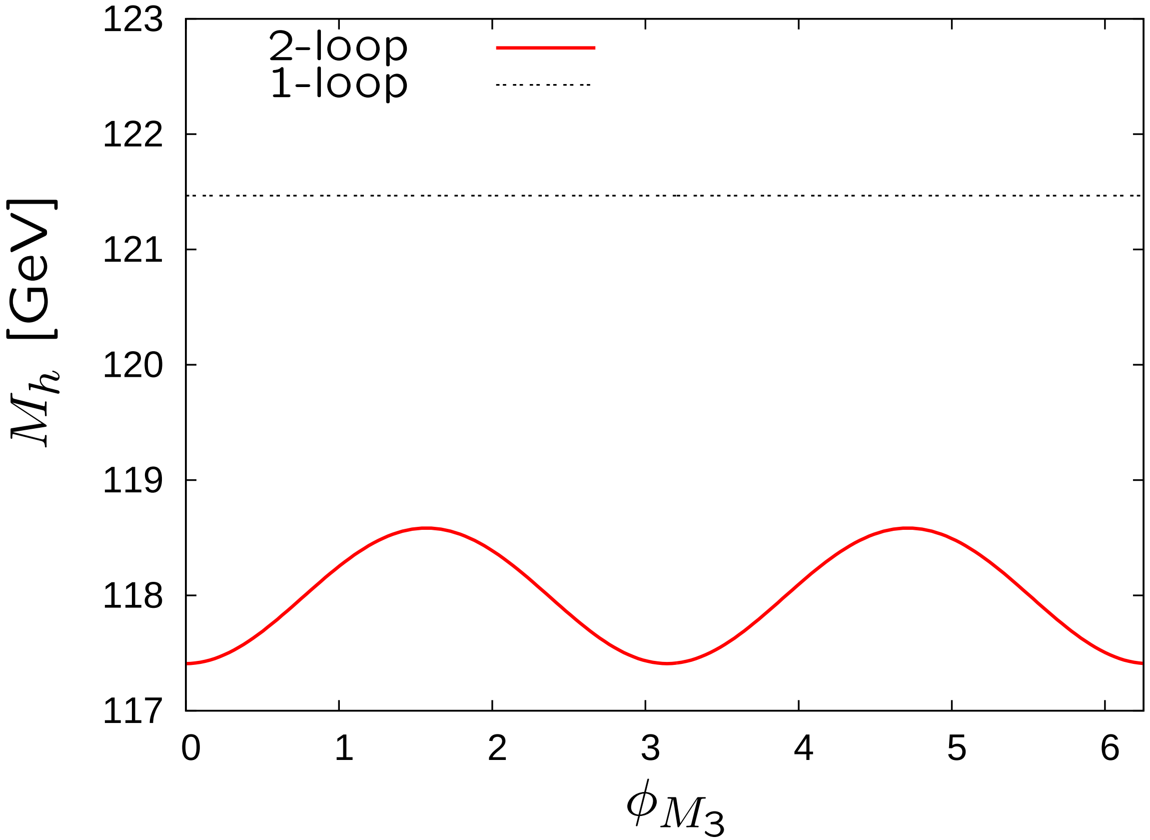
$M_{\text{SUSY}} = 500$ GeV

$|A_t| = 1000$ GeV

$\Rightarrow$ modified dependence
on ϕ_{A_t} at the 2-loop level

$\longrightarrow$ qualitative agreement
with existing calculations

M_h as a function of ϕ_{M_3}:



"normal SUSY parameters"

$M_{\text{SUSY}} = 500$ GeV

$|m_{\tilde{g}}| = 1000$ GeV

$\Rightarrow$ effect of phase $\mathcal{O}$ (1 GeV)

$\Rightarrow$ has to be taken into account

4. Conclusinos

- The LC will provide high precision results for a light r/cMSSM Higgs
- MSSM Higgs masses and couplings is connected via radiative corrections to all other sectors
- Evaluation of $\mathcal{O}(\alpha_s\alpha_s)$ corrections in the rMSSM:
  - new result for $\tan\beta \neq \infty$
  - investigation of different renormalization schemes
    $\Rightarrow$ error estimate from scheme and scale dependence
  - $\mu > 0$: corrections $\mathcal{O}(100\text{ MeV})$ $\Rightarrow$ under control
  - $\mu < 0$: corrections $\mathcal{O}(2-3\text{ GeV})$
    error estimate $\mathcal{O}(2\text{ GeV})$ $\Rightarrow$ not under control
- Evaluation of $\mathcal{O}(\alpha_s\alpha_t)$ corrections in the cMSSM:
  - new renormalization for complex parameters
  - $\tilde{b}$ sector enters
  - ϕ_{A_t} dependence modified
  - ϕ_{M_3} dependence $\mathcal{O}(1\text{ GeV})$
- Results will be implemented into *FeynHiggs* (www.feynhiggs.de)
  $\rightarrow$ talk by Thomas Hahn