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# Comparison of a New Calculation of Energy-Energy Correlations with $e^+e^- \to$ Hadrons Data at the $Z^0$ Resonance

The SLD Collaboration*

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## ABSTRACT

We have compared a new QCD calculation by Clay and Ellis of energy-energy correlations ($EEC$) and their asymmetry ($AEEC$) in $e^+e^-$ annihilation into hadrons with data collected by the SLD experiment at SLAC. From fits of the new calculation, complete at $\mathcal{O}(\alpha_s^2)$, we obtained $\alpha_s(M_Z^2) = 0.1184 \pm 0.0031(\text{exp.}) \pm 0.0129(\text{theory})$ ($EEC$) and $\alpha_s(M_Z^2) = 0.1120 \pm 0.0034(\text{exp.}) \pm 0.0036(\text{theory})$ ($AEEC$). The $EEC$ result is significantly lower than that obtained from comparable fits using the $\mathcal{O}(\alpha_s^2)$ calculation of Kunszt and Nason.

## I. INTRODUCTION

The theory of Quantum Chromodynamics (QCD) [1] contains in principle only one free parameter, the fundamental scale of strong interactions $\Lambda_{\overline{MS}}$,* which can be expressed in the form of the strong coupling $\alpha_s$. Tests of QCD in various hard processes and at different hard scales can therefore be reduced to comparison of the resulting values of $\alpha_s$ from fits of QCD to the data from these different reactions. For this purpose it has become standard to express such measurements in terms of $\alpha_s(M_Z^2)$ ($\overline{\text{MS}}$ scheme).

In $e^+e^-$ annihilation $\alpha_s$ may be determined from inclusive measures of the topology of hadronic events. We have previously determined $\alpha_s$ by applying such measures to hadronic decays of $Z^0$ bosons collected by the SLD experiment at SLAC [3,4]. A complementary technique is to measure $\alpha_s$ using energy-energy

* Throughout this paper we use the *modified minimal subtraction* ($\overline{\text{MS}}$) scheme [2] convention.

correlations ($EEC$) and their asymmetry ($AEEC$) [5]. These are inclusive two-particle correlations that can be used to probe the structure of hadronic events in more detail than the event topology variables and can be calculated perturbatively in QCD. Comparison of $\alpha_s$ determined in this way with that measured from event topology variables provides a significant consistency check of the validity of perturbative QCD.

The $EEC$ is defined [5] to be the normalized energy-weighted sum over all pairs of particles whose opening angles $\chi_{ij}$ lie between $\chi - \Delta\chi/2$ and $\chi + \Delta\chi/2$:

$$EEC(\chi) = \frac{1}{N_{event}} \sum_{1}^{N_{event}} \left( \frac{1}{\Delta\chi} \int_{\chi-\frac{\Delta\chi}{2}}^{\chi+\frac{\Delta\chi}{2}} \sum_{i,j=1}^{n_{particle}} \frac{E_i E_j}{E_{vis}^2} \delta(\chi' - \chi_{ij}) d\chi' \right), \tag{1}$$

where $\chi$ is the opening angle to be studied for the correlations; $\Delta\chi$ is the bin width; $E_i$ and $E_j$ are the energies of particles $i$ and $j$ and $E_{vis}$ is the sum of the energies of all particles in the event. In the central region, $\chi \sim 90^\circ$, the shape of the $EEC$ is determined by hard gluon emission; hadronization contributions are expected to be large in the collinear and back-to-back regions, $\chi \sim 0^\circ$ and $180^\circ$, respectively. The asymmetry of the $EEC$ is defined as

$$AEEC(\chi) = EEC(\pi - \chi) - EEC(\chi). \tag{2}$$

Several groups have performed perturbative QCD calculations, complete at $\mathcal{O}(\alpha_s^2)$, of the $EEC$ and $AEEC$: Richards, Stirling and Ellis (RSE) [6], Ali and Barreiro (AB) [7], Falck and Kramer (FK) [8], and Kunszt and Nason (KN) [9]. These calculations, valid in the central region, have the general form

$$EEC(\chi) = \frac{\alpha_s(\mu^2)}{2\pi} A(\chi) + \left( \frac{\alpha_s(\mu^2)}{2\pi} \right)^2 \left[ A(\chi) 2\pi b_0 \ln(\mu^2/s) + B(\chi) \right], \tag{3}$$

where, to the same order in perturbation theory, $\alpha_s(\mu^2)$ is related to the QCD scale $\Lambda_{\overline{MS}}$ by [10]

$$\alpha_s(\mu^2) = \frac{1}{b_0 \ln(\mu^2/\Lambda^2_{\overline{MS}})} \left[ 1 - \frac{b_1}{b_0^2} \frac{\ln\left[\ln(\mu^2/\Lambda^2_{\overline{MS}})\right]}{\ln(\mu^2/\Lambda^2_{\overline{MS}})} \right]; \tag{4}$$

$\mu$ is the renormalization scale, often expressed in terms of the factor $f = \mu^2/s$; $\sqrt{s}$ is the center-of-mass energy of the experiment; $b_0 = (33 - 2n_f)/12\pi$; $b_1 = (153 - 19n_f)/29\pi^2$; and $n_f$ is the number of active quark flavors. Here we have assumed the definition of $\Lambda_{\overline{MS}}$ for five active flavors. The first order coefficients $A(\chi)$ can be calculated analytically, and the second order coefficients $B(\chi)$ are calculated numerically. The main difference among the four theoretical calculations mentioned above is in the method used to treat the soft and collinear singularities appearing in the second order coefficients.

In our recent comprehensive study [11] we compared all four calculations with our data and found large discrepancies between the values of $\alpha_s(M_Z^2)$ determined from the $EEC$ of up to 10% in magnitude (Table I). Given that *a priori* one has no reason to disregard any of these calculations, this situation represents a serious limitation to our ability to measure $\alpha_s(M_Z^2)$ using the $EEC$. Furthermore, for fixed values of $f$ the different calculations typically yielded different values of $\alpha_s(M_Z^2)$ from fits to the $EEC$ than from fits to the $AEEC$ [11]. However, it is interesting, and perhaps significant, that the three more recent calculations (AB, FK and KN) yielded consistent values of $\alpha_s(M_Z^2)$ from the $AEEC$.

In an attempt to resolve these discrepancies two groups have recently recalculated the $EEC$ and $AEEC$ complete at $\mathcal{O}(\alpha_s^2)$ in perturbative QCD. Glover and Sutton (GS) rederived the next-to-leading coefficients $B$ using three numerical techniques [12], and found essentially the same results as Kunszt and Nason, leading them to the conclusion that the KN calculation is correct and that the RSE, AB, and FK calculations are somehow deficient. Given that all of the KN calculations of observables at $\mathcal{O}(\alpha_s^2)$ [9] were based upon the same methods as the KN $EEC$ and $AEEC$ calculations, and are the benchmarks for measurements of $\alpha_s$ at SLC/LEP (see, *e.g.*, Ref. [4]), such confirmation is of extreme importance. However, in an independent calculation using a modification of the method of RSE, Clay and Ellis (CE) rederived the coefficients $B$ [13], but found that their results are not consistent with those of RSE or KN. In this paper we present the results of a comparison of the new CE calculation with our data, and compare these results with those from our previous comparisons of RSE, AB, FK, and KN.

## II. MEASUREMENT OF *EEC* AND *AEEC*

The data used in this analysis were recorded in 1992 and 1993 by the SLC Large Detector (SLD) from electron-positron annihilation events at the $Z^0$ resonance produced by the SLAC Linear Collider (SLC). The detector is described in Ref. [14]. This analysis is based on charged tracks. Details of the trigger, hadronic event, and charged track selection critera are given in Refs. [3,4].

The $EEC$ and $AEEC$ were calculated using all pairs of selected charged tracks, assigning each the charged pion mass. The data were corrected [11] for initial state radiation and detector effects using the JETSET [15] and HERWIG [16] Monte Carlo programs which simulate the hadronic decays of $Z^0$ bosons, combined with a simulation of the SLD. The bin width was chosen to be 3.6°, which is much larger than the two-particle angular resolution of the detector, so as to minimize bin-to-bin migration effects in the data correction procedure. The data were further corrected [11] for the effects of hadronization using both JETSET and HERWIG. The differences between the JETSET and HERWIG correction factors were taken into account in the systematic errors [11].

Our previous study of energy-energy correlations [11] using the RSE, AB, FK, and KN calculations was based upon a comparison with the 1992 data sample. Our more recent compendium of $\alpha_s(M_Z^2)$ measurements [4] used only the KN calculations and was based upon comparison with our combined 1992 and 1993 data samples; it includes $\alpha_s(M_Z^2)$ values from fits to the $EEC$ and $AEEC$ that are consistent with our earlier KN results within experimental statistical errors. Our earlier KN results have already been quoted in the literature [12]. To avoid confusion over the slightly different central values and experimental errors given in Refs. [11] and [4] we list both sets of our KN results in Table I.

## III. DETERMINATION OF $\alpha_s$

The CE calculation was fitted to the fully corrected measured $EEC$ and $AEEC$ by minimizing $\chi^2$ under variation of $\Lambda_{\overline{MS}}$ for fixed renormalization scale factor $f$. The fits were restricted to the angular region $36° \leq \chi \leq 154.8°$ for the $EEC$ and $21.6° \leq \chi \leq 79.2°$ for the $AEEC$ [4]. For illustration the CE fit to our $EEC$ data for $f = 1$ is shown in Fig. 1, where the corresponding KN fit [4] is shown for comparison. The CE and KN fits are practically indistinguishable and both describe the data well. However, the fitted $\Lambda_{\overline{MS}}$ values are different. This is illustrated in Figs. 2(a) and 3(a), where $\alpha_s(M_Z^2)$ derived from $\Lambda_{\overline{MS}}$ is shown for different values of $f$, from fits using the $EEC$ and $AEEC$ respectively. The corresponding fit qualities $\chi^2_{dof}$ are shown in Figs. 2(b) and 3(b). While the CE and KN fits are of comparable quality (Figs. 2(b) and 3(b)), and the $\alpha_s(M_Z^2)$ values derived from the $AEEC$ are very similar (Fig. 3(a)), in the case of the $EEC$ the CE $\alpha_s(M_Z^2)$ values are systematically lower than the KN $\alpha_s(M_Z^2)$ values (Fig. 2(a)) by between 0.005 and 0.009 in the range $f > 10^{-3}$, where perturbation theory can be applied reliably [17].

Following the procedure defined in Ref. [4] to quote a single value of $\alpha_s$ for the CE calculation we obtained:

$$EEC : \alpha_s(M_Z^2) = 0.1184 \pm 0.0031(exp.) \pm 0.0021(had.) \pm 0.0127(scale)$$

$$AEEC : \alpha_s(M_Z^2) = 0.1120 \pm 0.0034(exp.) \pm 0.0017(had.) \pm 0.0032(scale),$$

where the total experimental error is the sum in quadrature of the statistical and experimental systematic errors [4], and the hadronization and scale uncertainties are defined in Ref. [4].*

## IV. SUMMARY AND DISCUSSION

We have compared our measurements of energy-energy correlations and their asymmetry in hadronic $Z^0$ decays with a new $\mathcal{O}(\alpha_s^2)$ perturbative QCD calculation by Clay and Ellis. This calculation describes our data well, and fits to the $AEEC$ yield similar values of $\alpha_s(M_Z^2)$ and its renormalisation scale uncertainty

* The scale ranges given in Ref. [4] and used here to quote average $\alpha_s(M_Z^2)$ values for the CE fits are slightly different than those used in Ref. [11]. In fact we obtain the same results for CE with either scale range.

as previous calculations by Ali and Barreiro, Falck and Kramer, and Kunszt and Nason presented in our previous studies [11,4]. However, in the case of fits to the $EEC$ the CE $\alpha_s(M_Z^2)$ value is consistent only with the FK value, and is about 0.006 lower than the KN value.

Our $\alpha_s(M_Z^2)$ results are summarised in Table I. With the exception of the RSE calculation, the remarkable degree of consistency between the $AEEC$ results, compared with differences at the level of 10% between the corresponding $EEC$ results, may provide some clue as to the theoretical origin of the discrepancies.

The Clay-Ellis $EEC$ result does not appear to be consistent with the claim of Ref. [12] that the Kunszt-Nason $EEC$ calculation has been demonstrated to be correct, and the data do not favour either calculation over the other. As the Kunszt-Nason calculations of the $EEC$, $AEEC$, and other event shapes have been used universally in $\alpha_s(M_Z^2)$ determinations at SLC/LEP, an application of the techniques used by Clay and Ellis to the other event shapes would seem to be highly desirable as a consistency check.

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## Figure captions

Fig. 1: The measured parton-level (see text) $EEC$ (data points) compared with fits of the Clay-Ellis (solid line) and Kunszt-Nason (dashed line) $\mathcal{O}(\alpha_s^2)$ QCD calculations. The fit range is indicated by vertical lines.

Fig. 2: (a) $\alpha_s(M_Z^2)$ and (b) $\chi^2_{dof}$ from $\mathcal{O}(\alpha_s^2)$ QCD fits to the $EEC$ as a function of renormalization scale factor $f$. The statistical error at each $f$ value is typically $\pm 0.0008$ ($EEC$) or $\pm 0.0012$ ($AEEC$) and is not shown.

Fig. 3: As Fig. 2, but for the $AEEC$.

**Table I**: $\alpha_s(M_Z^2)$, experimental errors, and scale uncertainties from $\mathcal{O}(\alpha_s^2)$ QCD fits to the *EEC* and the *AEEC* using SLD data.

| QCD Calc. | *EEC* $\alpha_s(M_Z^2)$ | *EEC* Exp. error | *EEC* Scale uncertainty | *AEEC* $\alpha_s(M_Z^2)$ | *AEEC* Exp. error | *AEEC* Scale uncertainty | Reference |
|---|---|---|---|---|---|---|---|
| RSE | 0.133* | $^{+0.002}_{-0.003}$ | $\pm$ 0.011 | 0.124 | $\pm$0.005 | $\pm$ 0.008 | [11] |
| AB | 0.132 | $^{+0.002}_{-0.003}$ | $\pm$ 0.011 | 0.114 | $\pm$0.005 | $\pm$ 0.004 | [11] |
| FK | 0.119 | $^{+0.002}_{-0.003}$ | $\pm$ 0.013 | 0.113 | $\pm$0.005 | $\pm$ 0.003 | [11] |
| KN, GS | 0.125 | $^{+0.002}_{-0.003}$ | $\pm$ 0.012 | 0.114 | $\pm$0.005 | $\pm$ 0.004 | [11,12] |
| KN | 0.1240 | $\pm$0.0031 | $\pm$ 0.0121 | 0.1121 | $\pm$0.0034 | $\pm$ 0.0031 | [4] |
| CE | 0.1184 | $\pm$0.0031 | $\pm$ 0.0127 | 0.1120 | $\pm$0.0034 | $\pm$ 0.0032 | This analysis |

* Due to a typographical error in press this value is incorrectly given as 0.113 in Ref. [11].

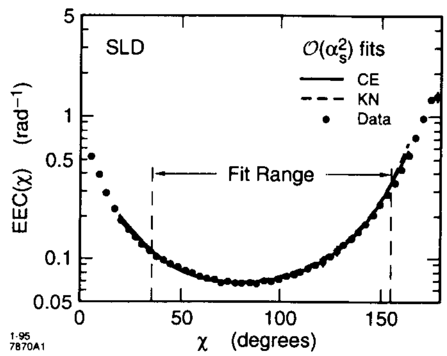


Figure 1:

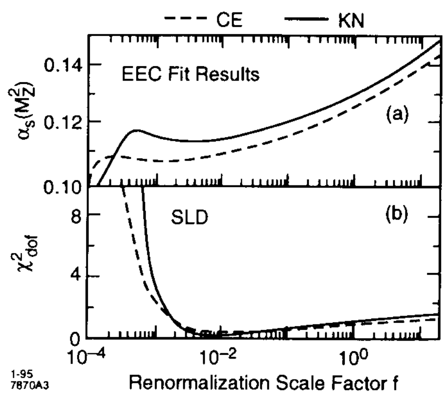


Figure 2:

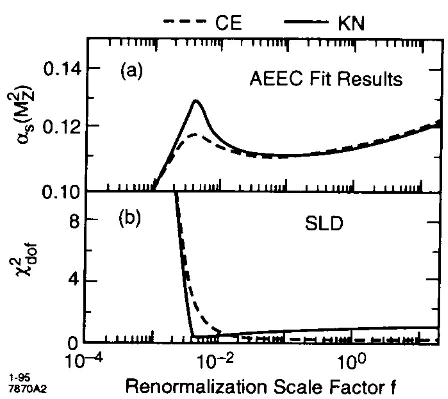


Figure 3: